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Article

Alena Tensor as an Example of the Dualistic Approach and Its Possible Applications in Quantum Field Theory and Unification Theories

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Abstract: The Alena Tensor is a recently discovered class of stress-energy tensors that, as previous publications have shown, has extremely useful properties. In curved spacetime, this tensor reproduces Einstein Field Equations and in flat Minkowski spacetime, it describes a physical system with fields that can be widely configured. The use of the discussed tensor significantly simplifies Quantum Field Theory equations and provides canonical, generalized four-momentum with vanishing four-divergence that satisfies Klein-Gordon equation. This article discusses the possible applications of Alena Tensor in Quantum Field Theory and unification theories against the background of existing research directions. The advantages, disadvantages and potential limitations of the approach discussed are also considered.

Keywords: quantum field theory; quantum mechanics; general relativity; unification of interactions

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1. Introduction

The history of physics is also the history of unification. "In all the attempts at unification we encounter two distinct methodological approaches: a deductive-hypothetical and an empirical-inductive method". [1] where a good example of the first approach is Supersymmetry [2] and the second one, Grand Unification [3] and, in a sense, the Standard Model itself. The past teaches us that after the stage of research on individual phenomena and obtaining a satisfactory description of them, comes the phase of unification, in which the scattered puzzles of descriptions are put together into one whole picture, which soon turns out to also be just a part of bigger picture.

The unification was carried out by J.C. Maxwell, combining scattered descriptions of electricity and magnetism [4], which H. Minkowski unified into the electromagnetic field tensor, combining Maxwell's achievements with the special theory of relativity of Einstein [5], who also unified the previous works of Lorentz and Poincaré, adding his own, important added value to them.

Today, modern physicists are faced with many puzzles, most of which are huge pictures, entire sections of physics, composed of hundreds of smaller parts, the existence of which we owe to thousands of outstanding scientists. The largest and most famous descriptions requiring unification are, of course, General Relativity (GR) and Quantum Field Theory (QFT), but we cannot forget about others (so fundamental that it is easy to forget about them), such as Continuum Mechanics, Electrodynamics or Thermodynamics.

Part of the entire unification effort are dualistic theories [6], mainly adopting a deductive-hypothetical approach. They do not attempt to falsify existing methods and descriptions of physical phenomena to the benefit of others, but instead seek a theoretical model in which existing descriptions can be reconciled. They assume that contradictions between existing descriptions may be apparent and in fact they are only different, equally valid ways of describing the same phenomena.

Dualistic descriptions are so widely used that we sometimes forget how controversial they once were. Examples of such dualistic descriptions are wave–particle duality, Lagrangian and Hamiltonian mechanics or Noether's first theorem.

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In the context of the unification of GR and QFT, a dualistic solution to the puzzle may appear from a completely unexpected direction, as in the work of D. Grimmer describing topological redescription [7] and giving the possibility of changing the topology of space in a way similar to changing coordinate systems. It can also come from a rather obvious direction [8], because it can be expected that there is a mathematical transformation between accelerated motion and geodesic motion in curved space-time for all accelerations due to known fields.

The main benefit of using dualistic theories, apart from the cognitive value, seems to be the possibility of further, independent development and use of existing descriptions of reality, as well as, in many cases, the possibility of transforming the results between different descriptions. For this reason, it is worth taking a look at an example of using this approach.

This article will discuss one of the dualistic approaches called Alena Tensor against the background of known problems regarding field unification and QFT and GR unification. Possible areas of application of this dualistic approach, its strengths and weaknesses, and potential limitations will also be analyzed.

2. Alena Tensor and Its Properties

This chapter summarizes the state of knowledge regarding Alena Tensor based on previous publications and describes its properties in the context of further applications. The author uses the Einstein summation convention, metric signature $(+,-,-,-)$ and commonly used notations.

Alena Tensor is the central equation of the method described in [9,10]. It is a stress-energy tensor describing a physical system with fields, which can be interpreted in flat and curved space-time. The Alena Tensor $T^{\alpha\beta}$ has the following form

$$T^{\alpha\beta} = \varrho\, U^{\alpha} U^{\beta} - \left(c^2 \varrho + \Lambda_\rho\right)\left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) \tag{1}$$

Designations used:

- $g^{\alpha\beta}$ is the metric tensor of space-time in which the physical system is considered,
- $h^{\alpha\beta}$ is the metric tensor of curved space-time in which all motion takes place along geodesics and it is related to the field tensor,
- $1/\xi \equiv \frac{1}{4}\, g_{\mu\nu}\, h^{\mu\nu}$
- $\varrho \equiv \varrho_o \gamma$ where ϱ_o is rest mass density and γ is Lorentz gamma factor,
- ϱU^{α} is four-momentum density in the system as perceived by some stationary observer, in accordance with the postulate raised in the publication [9],
- Λ_ρ is related to the invariant of the field tensor that describes the field in the system.

The field present in the system is described by some field tensor, e.g. $\mathbb{F}^{\beta\gamma}$, which may be widely configured. To simplify the reasoning and further description, it will be assumed that field is described by $\mathbb{F}^{\beta\gamma}$ representing electromagnetic field, but the properties described here are general and apply to the field in a broader sense.

For $\mathbb{F}^{\beta\gamma}$ understood as electromagnetic field tensor one gets the following relationships

$$h^{\alpha\beta} \equiv 2\, \frac{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}}{\sqrt{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}\, g_{\mu\beta}\, \mathbb{F}_{\alpha\eta}\, g^{\eta\xi}\, \mathbb{F}^{\mu}{}_{\xi}}} \tag{2}$$

which provides the property $h^{\alpha\beta}\, g_{\mu\beta}\, h_{\alpha}{}^{\mu} = 4$, and

$$\Lambda_\rho \equiv \frac{1}{4\mu_o} \mathbb{F}^{\alpha\mu}\, g_{\mu\gamma}\, \mathbb{F}^{\beta\gamma}\, g_{\alpha\beta} \tag{3}$$

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where μ_o is vacuum magnetic permeability. The stress–energy tensor for electromagnetic filed, denoted as $\Upsilon^{\alpha\beta}$ may be presented as follows

$$\Upsilon^{\alpha\beta} \equiv \Lambda_\rho \left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) = \Lambda_\rho g^{\alpha\beta} - \frac{1}{\mu_o}\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma} \tag{4}$$

The pressure p in the system is equal to

$$p \equiv c^2\varrho + \Lambda_\rho \tag{5}$$

which allows (1) to be written as

$$T^{\alpha\beta} = \varrho\, U^\alpha U^\beta - \frac{p}{\Lambda_\rho}\Upsilon^{\alpha\beta} \tag{6}$$

The remaining tensors that describe the system are defined as follows

$$R^{\alpha\beta} \equiv 2\varrho\, U^\alpha U^\beta - p\, g^{\alpha\beta} \tag{7}$$

its trace R

$$R \equiv R^{\alpha\beta}\, g_{\alpha\beta} = -2p - 2\Lambda_\rho \tag{8}$$

and tensor $G^{\alpha\beta}$ as

$$G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2}R\,\xi\, h^{\alpha\beta} \tag{9}$$

which allows to rewrite (1) as

$$G^{\alpha\beta} - \Lambda_\rho\, g^{\alpha\beta} = 2\, T^{\alpha\beta} + \varrho c^2 \left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) \tag{10}$$

The above description allows to consider flat space-time, curved space-time, and all intermediate states, in which space-time is partially curved and part of the motion results from the existence of residual fields. One may at first look at boundary solutions: flat space-time with fields and curved space-time without fields.

2.1. Behavior of the System in Curved Space-Time

Considering $g^{\alpha\beta}$ as equal to $h^{\alpha\beta}$ one obtains that it yields $\xi = 1$, therefore the whole part of Alena Tensor related to fields vavish. It yields

$$T_{\alpha\beta} = \varrho\, U_\alpha U_\beta \tag{11}$$

The value of tensor $G_{\alpha\beta}$ becomes

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}R\, h_{\alpha\beta} \tag{12}$$

and (10) reduces to

$$G_{\alpha\beta} - \Lambda_\rho\, h_{\alpha\beta} = 2\, T_{\alpha\beta} \tag{13}$$

Therefore, in curved space-time, $R_{\alpha\beta}$ acts as Ricci tensor and $G_{\alpha\beta}$ acts as Einstein curvature tensor, both with an accuracy of $\frac{4\pi G}{c^4}$ constant, where cosmological constant Λ is related to the invariant of the field tensor

$$\Lambda = -\frac{4\pi G}{c^4}\Lambda_\rho \tag{14}$$

where Λ_ρ has a negative value due to the adopted metric signature.

The above result gives a chance to solve the puzzle of the "smile of the Chesire cat" [11] explaining the reason for the appearance of the cosmological constant in Einstein Field Equations.

Since covariant four-divergences of $T_{\alpha\beta}$ and $G_{\alpha\beta}$ vanish, therefore they represent curvature tensors, related to corresponding four-force densities present in flat Minkowski space-time. It is therefore worth taking a look at the four-force densities associated with these tensors in flat space-time.

2.2. Behavior of the System in Flat Minkowski Space-Time

Considering $g^{\alpha\beta}$ as equal to $\eta^{\alpha\beta}$ Minkowski metric tensor, thanks to the amendment to the continuum mechanics described in [9]

$$\partial_\alpha U^\alpha = -\frac{d\gamma}{dt} \quad \rightarrow \quad \partial_\alpha\, \varrho U^\alpha = 0 \tag{15}$$

total four-force density f^α acting in the system is equal to

$$f^\alpha \equiv \partial_\beta\, \varrho U^\alpha U^\beta \tag{16}$$

and it is the sum of electromagnetic (f^α_{EM}), gravitational (f^α_{gr}) and the sum of remaining (f^α_{oth}) four-force densities, where

$$f^\alpha = \begin{cases} f^\alpha_{EM} \equiv \partial_\beta\, \Upsilon^{\alpha\beta} \quad (electromagnetic) \\ + \\ f^\alpha_{gr} \equiv (\eta^{\alpha\beta} - \xi\, h^{\alpha\beta})\, \partial_\beta\, p \quad (gravitational) \\ + \\ f^\alpha_{oth} \equiv \frac{\varrho c^2}{\Lambda_\rho} f^\alpha_{EM} \quad (sum\, of\, remaining\, forces) \end{cases} \tag{17}$$

wha yields

$$\partial_\beta\, T^{\alpha\beta} = 0 \tag{18}$$

$$f^\alpha_{gr} + f^\alpha_{oth} = \partial_\beta\, G^{\alpha\beta} \tag{19}$$

The above result shows, that when using the Alen Tensor, it should be assumed that the Einstein tensor does not describe the curvature associated with gravity alone.

Neglecting other forces (as we currently do in known solutions for GR), one actually approximately obtains metric tensors responsible for gravity alone. However, the inclusion of other interactions in the Alen Tensor causes the Einstein tensor to correspond to the curvature associated with the four-force density from Equation (19). This means that the above approach can be used to search for the causes of disturbances between observations and the expected motion resulting from gravitational equations, which is currently attributed entirely to Dark Matter [12].

By introducing an additional tensor $\Pi^{\alpha\beta}$ defined as

$$\Pi^{\alpha\beta} \equiv -c^2 \varrho\, \xi\, h^{\alpha\beta} \tag{20}$$

one may rewrite Alena Tensor in flat Minkowski space-time as

$$T^{\alpha\beta} = \varrho\, U^\alpha U^\beta - p\, \eta^{\alpha\beta} - \Pi^{\alpha\beta} + \Lambda_\rho \xi\, h^{\alpha\beta} \tag{21}$$

In such picture, vanishing four-divergence of the above

$$f^\alpha = \partial^\alpha\, p + \partial_\beta\, \Pi^{\alpha\beta} + f^\alpha_{EM} \tag{22}$$

express relativistic equivalence of Cauchy momentum equation (convective form) [13], where $\Pi^{\alpha\beta}$ plays a role of deviatoric stress tensor [14]. The above representation therefore allows for the analysis of the system using the tools of continuum mechanics. From this perspective, f_{EM} appears as a body force, while the remaining forces are the effect of fluid dynamics [15] and could be modeled with help of Navier-Stokes Equations [16,17].

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By imposing following condition on normalized Alena Tensor as described in [10]

$$0 = \partial_\beta \left(\frac{T^{\alpha\beta}}{\eta_{\mu\gamma} T^{\mu\gamma}} \right) + \partial^\alpha \ln \left(\eta_{\mu\gamma} T^{\mu\gamma} \right) \tag{23}$$

one obtains farther simplification. Some gauge of electromagnetic four-potential denoted as $\mathbb{A}^\mu$ may be expressed as

$$\mathbb{A}^\mu \equiv -\frac{\Lambda_\rho}{p} \frac{\varrho_o}{\rho_o} U^\mu \tag{24}$$

where ρ_o denotes rest charge density in the system. It also simplifies Alena Tensor to

$$T^{\alpha\beta} = \frac{1}{\mu_o} \mathbb{F}^{\alpha\gamma} \partial^\beta \mathbb{A}_\gamma - \Lambda_\rho \eta^{\alpha\beta} \tag{25}$$

and leads to the explicit form of gravitational four-force density

$$f^\alpha_{gr} \equiv \varrho \left(\frac{d \ln(p)}{d\tau} U^\mu - c^2 \partial^\mu \ln(p) \right) \tag{26}$$

Both Lagrangian and Hamiltonian density for the systems appear to be related to invariant of the field tensor

$$\mathcal{L} = \mathcal{H} = \Lambda_\rho \tag{27}$$

where it was shown that

$$\frac{\partial \Lambda_\rho}{\partial \mathbb{A}_\alpha} = \partial_\nu \left(\frac{\partial \Lambda_\rho}{\partial(\partial_\nu \mathbb{A}_\alpha)} \right) = -J^\alpha \tag{28}$$

where J^α if electric four-current. It shows that in this solution there is no potential in the classical sense and dynamics of the system depends on itself. This is a clear analogy to main GR equation and something that should be expected from a GR-equivalent description of the system in flat space-time.

It was also shown, that

$$H^\beta \equiv -\frac{1}{c} \int T^{0\beta} \, d^3x \tag{29}$$

acts as canonical four-momentum for the point-like particle, and its four-divergence vanishes due to the Poynting theorem. The action S (Hamilton's principal function) for the point-like particle is

$$-S = H^\beta X_\beta \tag{30}$$

and this action vanishes for the inertial system. It clearly shows that inertial systems in this approach does not exist and should be considered as some abstract idealizations. Considered system without a field vanishes, what shows that space-time in this approach should be indeed understood as some method to perceive the field.

Mentioned canonical four-momentum is equal to

$$H^\mu = P^\mu + V^\mu = -\frac{\gamma L}{c^2} U^\mu + \mathbb{S}^\mu \tag{31}$$

where P^μ is four-momentum, L is for Lagrangian for point-like particle, $\mathbb{S}^\mu$ due to its properties, seems to be some description of the spin, and where V^μ describes the transport of energy due to the field

$$V^\mu = q\mathbb{A}^\mu + \frac{\varrho c^2 \gamma^2}{p} P^\beta + \frac{\varrho c^2}{p} \mathbb{S}^\mu + Y^\mu \tag{32}$$

where Y^μ is the volume integral of the Poyinting four-vector, and

$$\mathbb{S}^\beta \equiv \int \frac{\epsilon_o \Lambda_\rho}{\gamma c \rho_o} \, \mathbb{F}^{0\mu} \partial_\mu U^\beta \, d^3x \tag{33}$$

where ϵ_o is electric vacuum permittivity.

2.3. Quantum Picture in Flat Minkowski Space-Time and Generalizations

To ensure compatibility with the equations of QM one may consider properties of $\mathbb{S}^\mu \mathbb{S}_\mu$ and e.g. by setting

$$\mathbb{S}^\mu \mathbb{S}_\mu = m^2 c^2 - \left(\frac{\gamma L}{c} \right)^2 \tag{34}$$

and introducing quantum wave function Ψ in form of

$$\Psi \equiv e^{\pm i K^\mu X_\mu} \tag{35}$$

where K^μ is wave four-vector related to canonical four-momentum

$$\hbar K^\mu \equiv H^\mu \tag{36}$$

one obtains Klein-Gordon equation

$$\left(\Box + \frac{m^2 c^2}{\hbar^2} \right) \Psi = 0 \tag{37}$$

However, the second quantization seems much more interesting. If one considers solely the electromagnetic field within the system and replaces (27) with the current Lagrangian density employed in QED, one should derive equations that characterize the entire system involving the electromagnetic field. Remarkably, these equations would inherently encompass the system's gravitational behavior. This is because, in the method under consideration, gravity naturally emerges within the system as an outcome of the presence of energy-momentum tensors associated with the relevant fields, and the resultant Lagrangian density duly incorporates this aspect.

It is possible, that this might clarify the challenging quest for identifying quantum gravity as a distinct interaction within Quantum Field Theory. Additionally, it could potentially account for the remarkable precision of QED's predictions, provided it indeed characterizes the complete system involving an electromagnetic field.

One may also consider generalizing the Alena Tensor to other fields. At this point, however, it seems necessary to introduce a certain classification of fields that will explain the differences in the approach to their analysis in flat, curved space-time and in quantum perspective.

For example, remaining with the previous notation, one may describe the field (e.g. electroweak field) in the system by some generalized field tensor $\mathbb{W}^{\alpha\beta\gamma}$ providing more degrees of freedom, and express Alena Tensor in flat space-time as follows

$$T^{\alpha\beta} = \varrho \, U^\alpha U^\beta - \left(\frac{c^2 \varrho}{\Lambda_\rho} + 1 \right) \left(\Lambda_\rho \, \eta^{\alpha\beta} - \mathbb{W}^{\alpha\delta\gamma} \, \mathbb{W}^\beta{}_{\delta\gamma} \right) \tag{38}$$

where

$$\Lambda_\rho \equiv \frac{1}{4} \mathbb{W}^{\alpha\beta\gamma} \mathbb{W}_{\alpha\beta\gamma} \tag{39}$$

$$\xi \, h^{\alpha\beta} \equiv \frac{\mathbb{W}^{\alpha\delta\gamma} \, \mathbb{W}^\beta{}_{\delta\gamma}}{\Lambda_\rho} \tag{40}$$

$$\xi \equiv \frac{4}{\eta_{\alpha\beta} \, h^{\alpha\beta}} \tag{41}$$

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The Alena Tensor defined in this way retains most of properties described in the previous chapters, however, it now describes other four-force densities in the system. Total four-force density f^α can be now presented as

$$f^\alpha = \begin{cases} f^\alpha_{fun} \equiv -\partial_\beta \, \mathbb{W}^{\alpha\delta\gamma} \, \mathbb{W}^\beta_{\ \delta\gamma} \quad (fundamental\, forces) \\ + \\ f^\alpha_{gr} \equiv \left(\eta^{\alpha\beta} - \xi \, h^{\alpha\beta}\right) \partial_\beta \, \varrho c^2 \quad (related\, to\, gravity) \\ + \\ f^\alpha_{sec} \equiv \frac{\varrho c^2}{\Lambda_\rho} f^\alpha_{fun} \quad (secondary\, forces) \end{cases} \tag{42}$$

Therefore, interactions can be classified based on their properties as:

- fundamental interactions related to body forces f^α_{fun}
- gravitational or gravity with an additional field, related to f^α_{gr}
- secondary interactions related to four-force density f^α_{sec}

where each of above f^α_i four-force density should satisfy the condition

$$0 = U_\alpha f^\alpha_i \tag{43}$$

Interactions defined in this way can be analyzed both from the classical perspective and in the regime of QFT description.

3. Potential Applications against the Background of Existing Research

This chapter analyzes possible applications and generalizations of the Alena Tensor against the background of existing works. Their benefits, weaknesses and limitations resulting from the use of the considered method will also be presented.

Quantum Mechanics and Quantum Field Theory describe phenomena in a radically different way from the description used in the General Relativity, Electrodynamics or Continuum Mechanics. For this reason, it is worth first analyzing the possibility of remapping the way of describing reality resulting from the use of the Alena Tensor in relation to particular issues important from the point of view of the quantum conceptual apparatus.

3.1. Dark Sector and Perspectives of Unification

The first topic discussed will be the issue of the dark sector, which seems to be the easiest to analyze. Although Dark Energy and Dark Matter are concepts closely related to the General Relativity, their analysis is also carried out from the perspective of quantum theories and quantum cosmology [18–20].

The use of Alena Tensor indicates that the invariant of the field tensor is responsible for the vacuum energy and the associated cosmological constant [21]. This allows to replace "the worst theoretical prediction in the history of physics" [22] with an attempt to estimate the value of this field tensor invariant. This also means, that it becomes possible to search for the expected form of the field tensor based on the experimentally measured value of its invariant, and allows to look for an answer to the question of what fields, apart from the electromagnetic field, should constitute stress energy-tensor.

An example of such an approach seems to be an attempt to estimate the values of magnetic and electric fields based on available background radiation data [23] and an attempt to determine the value of the invariant of the electromagnetic field tensor. Importantly, it also seems that this invariant does not have to be the constant [24,25], which would be particularly important for solving the Hubble tension problem [26].

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The above research may also prove important from the perspective of Maxwell's equations with axion modifications [27] and attempts to explain Dark Matter based on these particles [28], especially in the context of the recent results regarding Sigma-8 tension [29].

Analyzing the possible directions of unification of interactions, it can also be noted that the Alena Tensor allows for testing hypotheses regarding the interconnections of fields and the connections of fields with gravity. Fields defined in the way presented in Section 2.3 allow for quite a lot of freedom in adapting them to the existing division of interactions that emerged in quantum mechanics: electroweak, strong and gravitational interactions. Perhaps this will shed new light on current work on the unification of these interactions [30–32].

Due to the fundamental importance of electroweak interaction (fermions are the building blocks of matter), it seems that the field strength tensor present in the system should be somehow related to this interaction, where the rest (related to gravity and secondary interactions) could be linked to the strong interaction and potentially to other fields [33]. It would be also supported by conclusions from research on Double Copy Theory [34–36], since it can be assumed that solutions should include perturbative duality between gauge theory and gravity.

Finally, when discussing the unification of interactions, it is impossible to ignore the importance of the Higgs field [37]. The adoption of an analysis model based on the Alena Tensor creates new possibilities for relating the geometry of space-time with the field [38]. Even based on the simple model presented in Section 2.1, it is possible to analyze relationships between the Higgs field and the electromagnetic field [39,40]. Additionally, due to the possibility of analyzing the system based on the proposed Lagrangian and generalized canonical four-momentum, it becomes possible to study individual classes of fields in terms of their impact on the phenomenon of symmetry breaking [41,42].

When building theoretical models, however, one should remember about the limitations related to the adopted analysis method. In curved space-time, the curvature described by the Einstein tensor will always be related to the four-force densities $f^{\alpha}_{gr} + f^{\alpha}_{sec}$. In flat space-time, conditions (23), (27) and (29) still seem reasonable.

3.2. Quantum Gravity

There is no universal agreement on the approach to developing quantum gravity [43] and so far research is being carried out using different methods in different directions. One of the research directions is canonical quantum gravity [44] with its attempt to quantify the canonical formulation of general relativity, the most promising example of which is Loop Quantum Gravity [45].

Work is also ongoing in the field of string theory, where M-theory [46] seems to be the leading area of research. There are also many other e.g. [47–49] less frequently cited studies that explore different, sometimes unusual [50] research areas.

Against the background of the above research directions, the dualistic approach represented by Alena Tensor seems very promising because it changes the research paradigm in two ways.

The first paradigm shift is that, according to the conclusions presented earlier, in the description provided by Alena Tensor, the Einstein tensor is not exclusively related to gravity. The introduction of additional interactions into the system causes an additional space-time curvature term related to secondary interactions to appear in the curved space-time in the Einstein tensor. This means a change in assumptions and a completely new way of perceiving the prospect of unifying the remaining interactions with gravity.

The second paradigm shift results from the very nature of the dualistic approach and concerns the lack of need to search for quantization methods in curved space-time. According to the reasoning presented earlier, if one describes the field in flat space-time by some field tensor and enters it into the Alena Tensor in the appropriate way, the equations in curved space-time will naturally turn into the Einstein Field Equations.

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The second paradigm shift in particular seems to be extremely important from the point of view of research on quantum gravity phenomena, because it opens new possibilities for studying quantum phenomena in a strong gravitational fields.

Current research approaches to quantum problems in a strong gravitational field each time require the construction of an appropriate model in which the obtained results can be interpreted, either through careful selection of the observer [51], or making direct use of the principle of equivalence [52], or own, specific approach [53]. It also needed consideration of the specific quantum phenomena occurring in the vicinity of very massive objects, such as the Unruh effect [54] or Hawking radiation [55]. Thanks to the dualistic approach, such research can now be conducted in flat space-time with fields and then the results can be easily analyzed in curved space-time.

One of the natural directions of research seems to be the development of a field tensor that, in curved space-time, provides the known metrics [56] used to describe gravity, extended by the term related to secondary interactions. The development of such a field tensor seems to be the first step towards building quantum gravity, this time - contrary to the direction described in the previous chapter - from the side of the General Relativity.

Interestingly, because the use of the Alena Tensor indicates the possibility of shaping the metric tensor of space-time using a field, it also sheds new light on research on new drives [57], including the quantum effects [58] needed to analyze them. Although many QM and QFT problems seem unsolvable [59,60] using current paradigms, such as the Plack scale problems [61], previously mentioned paradigm shift can change this situation.

It also seems interesting to search for solutions to the problem of quantization of interactions related to the tensor (9) in various space-times, thus the problem of quantization should be addressed.

3.3. Quantization

To get a full picture of the applicability of the approach based on Alena Tensor, one may consider an example of its application to gravity quantization.

One may start with a choice of proper representation of a metric $g^{\alpha\beta}$ so that the interpretation of time in first quantization will be "natural". By "natural interpretation" of time, it is understood the approach in which, after the first quantization of Hamiltonian, one gets a proper definition of the time evolution operator in the ”Schrödinger representation”, in such a way that

$$U(t, t_0) = e^{-iH\cdot(t-t_0)/\hbar} \tag{44}$$

fulfill classical conditions [62]

$$\begin{aligned} U^\dagger(t, t_0)U(t, t_0) &= I \\ |\psi(t_0)\rangle &= U(t_0, t_0)|\psi(t_0)\rangle \\ U(t, t_0) &= U(t, t_1)U(t_1, t_0) \end{aligned} \tag{45}$$

This means that, in general, it should be possible to incorporate the Lagrangian formalism for the Gauge fields. Therefore, for the field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{46}$$

one needs to define proper commutator

$$[t_a, t_b] = i f^{abc} t_c \tag{47}$$

As it was show in [63] this can be done by rewriting $g^{\alpha\beta}$ in the $(3+1)$-split in Geroch decomposition manner. This approach solves the proper initial value problem, since now space-time can be interpreted as the evolution of space in time, with interpretation of time that is consistent with Quantum Mechanics:

time as a distinguished, absolute, external, global parameter. A summary of full formalism has been presented many times, last and the modern one can be found in Viktor Bergstedt [64], where computation rules look as follows

$$\left\{\gamma_{ij}, \pi^{kl}\right\} = \frac{1}{2}\left(\delta_i^k \delta_j^l + \delta_j^k \delta_i^l\right) \delta^{(3)}(x - y) \tag{48}$$

The above approach makes it possible to introduce gravity into Quantum Mechanics in form of canonical quantization and couple this field with other interaction in regular manner. In such picture gravity acts as just another quantum field that could be incorporated into the Standard Model Lagrangian and interact with other fields on the same principles. The only difference is that we are bound to only one representation of the metric $g^{\alpha\beta}$ with $(3+1)$-split Geroch decomposition. However, it may be transformed to other, more convenient coordinate systems when quantum phenomena can be negligible.

Presented approach opens a natural way to implement that representation of tensor $g^{\alpha\beta}$ into the Alena Tensor (1) for better understanding overall interpretation of GR in the big scale. From the other point of view, it opens the possibility to look for a quantum gravity phenomena in the small scale, where perturbation approach as quantum and gravity interaction are in the same level of magnitude. The most promising application of this approach could be implementing this calculations to Hawking radiation phenomena on the Planck scales, as the original calculation are questioned by other authors [65,66].

New observation methods allow to look for a quantum gravity phenomenon in the present or near future data that could test the boundaries of GR in the classical approach. One of the most promising directions in the present observation is the rise of gravitational wave (GW) astronomy. It might be worth investigating the post-merge echoes that occur because of the stimulated emission of Hawking radiation after compact binary merger events involving stellar black holes. This could be a promising way to search for deviations from General Relativity and could serve as evidence for the quantum structure of black hole horizons. Present methods used to model this phenomenon in modified theories of gravity are extremely challenging in Numerical Relativity and could provide inconclusive observation interpretation [67]. The approach presented in this paper may also help obtain results without using effective model echoes within the framework of linear perturbation theory.

4. Conclusions and Discussion

As presented above, the possibility of using a new tool, Alena Tensor, seems to open up new research possibilities both in terms of searching for the relationship between QFT and GR, as well as in terms of connections between many phenomena previously analyzed separately: in quantum or classical description, curvilinear or in flat space-time, or, for example, the possibility of combining the interpretation of fluid dynamics with field theory.

By appropriately selecting field tensors and testing hypotheses regarding their relationship with the Einstein tensor in curved space-time, it is possible to search for new interpretations for Dark Matter, as well as to analyze the relationships of the invariants of these field tensors with the cosmological constant. By adopting a new interpretation of the cosmological constant as an invariant of the field tensor, possibilities also open up to explain contradictory experimental data for cosmological phenomena, because the field tensor invariant does not have to be constant in time.

Due to the high flexibility of the Alena Tensor in the selection of fields, it also seems to be a very good tool for testing hypotheses regarding the unification of interactions. Such research can be conducted in the regime of the QFT mathematical apparatus and, importantly, thanks to a clear interpretation of the four-divergence of the field stress-energy tensor (four-force density), obtained results would also lead to obtaining an interpretation of quantum interactions in the classical description, which would be a major milestone in combining known QFT results with the classical description of interactions.

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Finally, one can also seek a quantum description of gravity in new ways, taking advantage of the paradigm shift that Alena Tensor brings with it. This does not mean that the problems associated with quantizing fields in curved space-time disappear and the behavior of quantum fields when changing the metric tensor will still require careful analysis. However, it seems that thanks to the dualistic description provided by Alena Tensor, this analysis may be much easier.

Further research using the Alena Tensor may also lead to its further transformations, generalizations and interpretations, as well as to the design of experiments in terms of the sought properties that match the experimental data. And all this has a chance to bring us one step closer to the next image that will connect the previously scattered puzzles of knowledge.

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