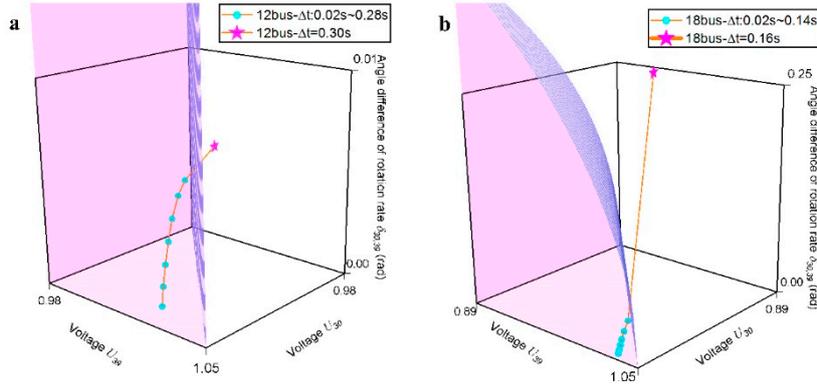
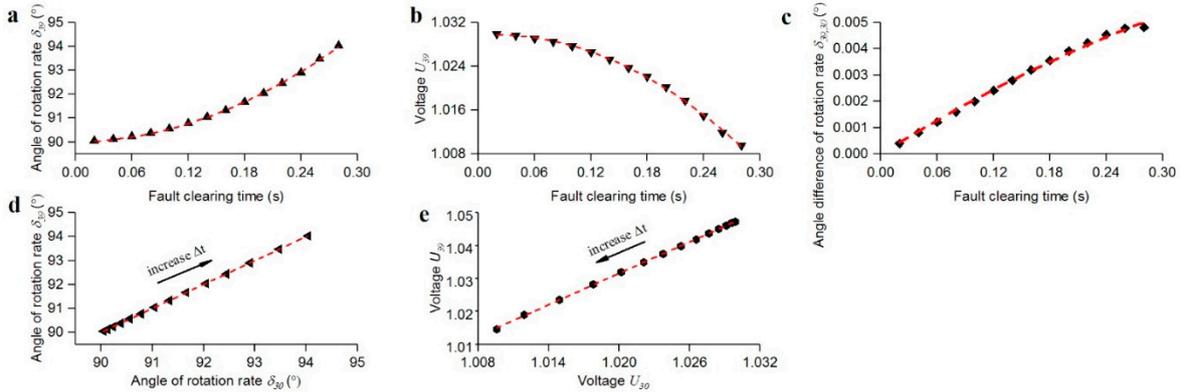


Extended Figure 1. Stability boundary for generators



a and b correspond to IEEE 39-BUS system, generator disturbed instability process Schematic diagram of the disturbed trajectory of the operating point crossing the boundary after a three-phase short-circuit ground fault at 12bus and 18bus in node IEEE39 with gradually increasing Δt , respectively. (Δt increases from 0.04s, until destabilization)

Extended Figure 2. Fitting results for generator perturbed trajectories of 12bus after a three-phase short circuit to ground fault



The expressions for the disturbed operating points of the meta-generators and generators have the same form before and after permutation.

a. The projection of the disturbed trajectory in the $\delta_{39} - \Delta t$ plane. The result of the fit is $\frac{a_w(1)}{2} = 48.96109 \pm 0.79143, b_w(1) = 0.52556 \pm 0.17753, c_w(1) = 90$, and the adjusted R-squared value is 1.

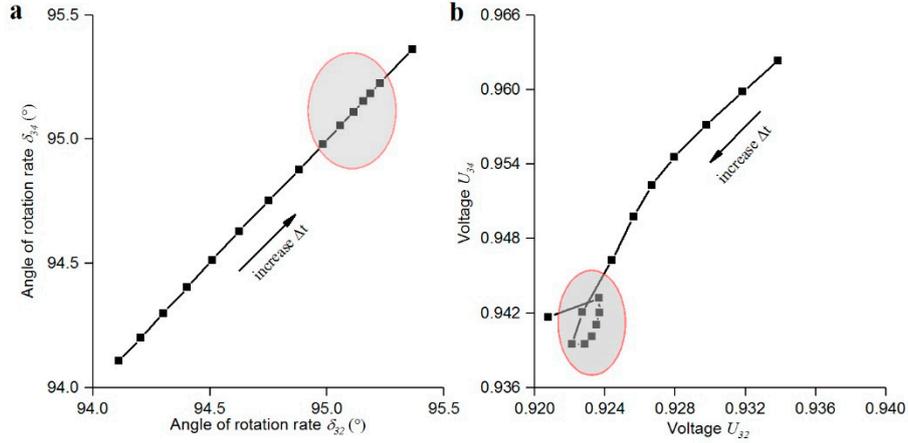
b. The projection of the disturbed trajectory in the $u_{39} - \Delta t$ plane. The result of the fit is $\frac{a_u(1)}{2} = -0.2968 \pm 0.00968, b_u(1) = 0.01069 \pm 0.00299$ and the adjusted R-squared value is 0.99905.

c. The projection of the disturbed trajectory in the $\delta_{39,30} - \Delta t$ plane. The result of the fit is $a_{12} = -0.01545 \pm 0.00238, b_{12} = 0.02222 \pm 5.34533E-4$ and the adjusted R-squared value is 0.9993.

d. The projection of the disturbed trajectory in the $\delta_{30} - \delta_{39}$ plane. The result of the fit is $a_\delta = 1.00019 \pm 3.59019E-4, b_\delta = -0.01749 \pm 0.03285$ and the adjusted R-squared value is 1.

e. The projection of the disturbed trajectory in the $u_{30} - u_{39}$ plane. The result of the fit is $a_u = 1.59015 \pm 0.00611, b_u = -0.59037 \pm 0.00625$ and the adjusted R-squared value is 0.99981.

Extended Figure 3. IEEE 39-BUS system, self-organizing behavior of generators near the boundary

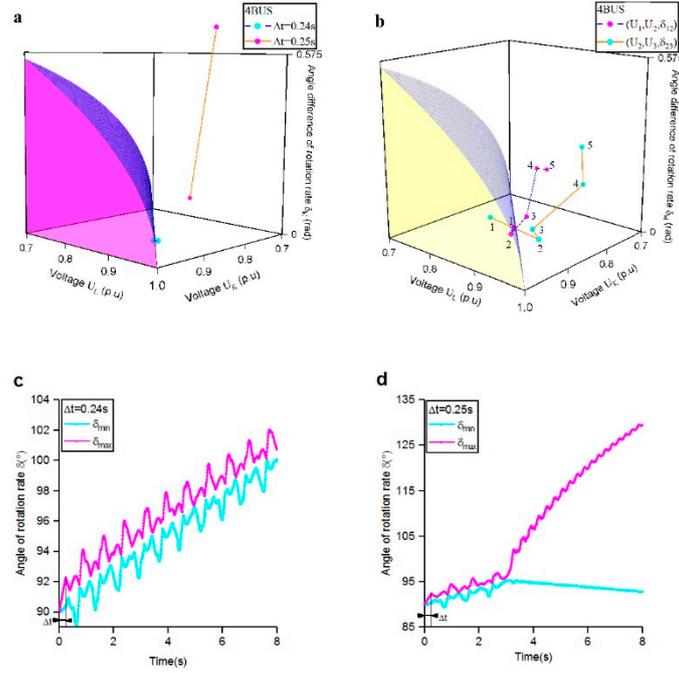


The 18bus three-phase short-circuit ground fault, increase in fault time Δt from 0.140s to 0.154s, and the trajectory of disturbed operating point near the boundary. The arrow shows the direction of increase of Δt . The generator has the same barrier and attractor as the meta-generator.

a. In the $\delta_{32} - \delta_{34}$ plane, running points appear to cross the barrier before they reach the boundary, and the elliptical area marks the position of the barrier. From 0.147s onwards the interval between running points decreases in the direction of increasing Δt

b. In the $u_{32} - u_{34}$ plane, the graph is presented as a critical state local attractor, which appears simultaneously with the synchronous barrier. The ellipse indicates the position of the attractor. The graph of the trajectory of the run point from 0.147s onwards is shown as an attractor (in the shaded area).

Extended Figure 4. Stability boundary for meta-generators for IEEE 9-BUS system

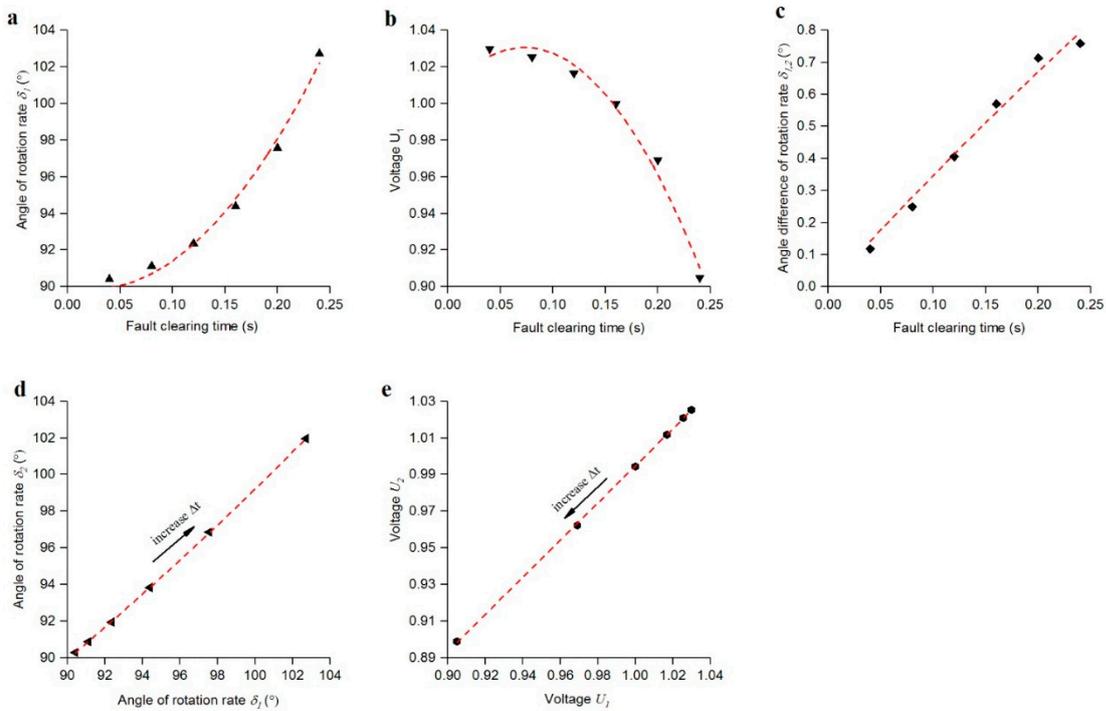


a. The same boundary equation apply to the 3-machine 9-node system. 4bus sets the location of the operating point after a three-phase short-circuit ground fault.

b. Results of the multiswing instability for $\Delta t=0.25s$.

c and d are the results of the 4BUS three-phase ground fault simulation, respectively. It is stable at $\Delta t = 0.24s$, unstable at $\Delta t = 0.25s$, and unstable in $(3s, 4s)$.

Extended Figure 5. Fitting results for meta-generator perturbed trajectories of 4bus after a three-phase short circuit to ground fault



Fitting results for perturbed trajectories for the 4bus trajectory of disturbed operating points after a three-phase short-circuit ground fault..

a. The projection of the disturbed trajectory in the $\delta_1 - \Delta t$ plane. The result of the fit is

$\frac{a_\omega(1)}{2} = 263.16677 \pm 27.27487, b_\omega(1) = -12.16449 \pm 5.45497, c_\omega(1) = 90$. The adjusted R-squared value is 0.99997.

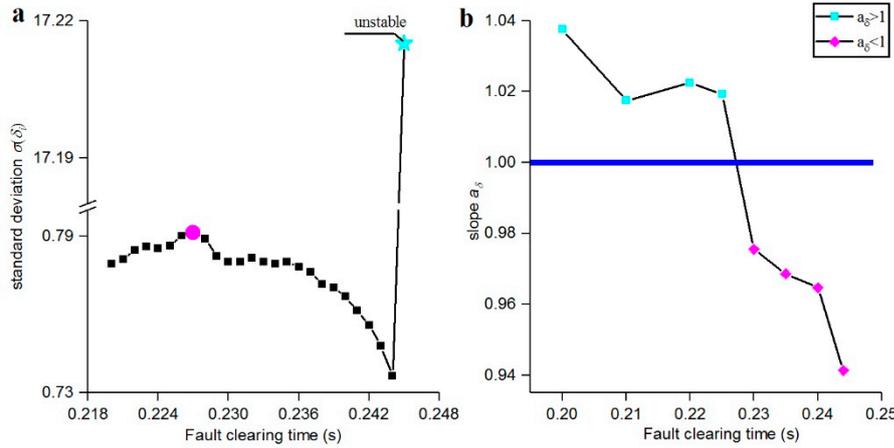
b. The projection of the disturbed trajectory in the $u_1 - \Delta t$ plane. The result of the fit is $\frac{a_u(1)}{2} = -4.32887 \pm 0.74225, b_u(1) = 0.63292 \pm 0.21231, c_u(1) = 1.00743 \pm 0.01298$, and the adjusted R-squared value is 0.97679.

c. The projection of the disturbed trajectory in the $\delta_{12} - \Delta t$ plane. The result of the fit is $a_{12} = -0.01956 \pm 0.03364, b_{12} = 0.06242 \pm 0.00673$, and the adjusted R-squared value is 0.99519.

d. The projection of the disturbed trajectory in the $\delta_1 - \delta_2$ plane. The result of the fit is $a_\delta = 1.05168 \pm 0.01287, b_\delta = -4.40363 \pm 1.21501$, and the adjusted R-squared value is 0.99925.

e. The projection of the disturbed trajectory in the $u_1 - u_2$ plane. The result of the fit is $a_v = 0.98731 \pm 0.00722, b_v = 0.01791 \pm 0.00713$, and the adjusted R-squared value is 0.99973.

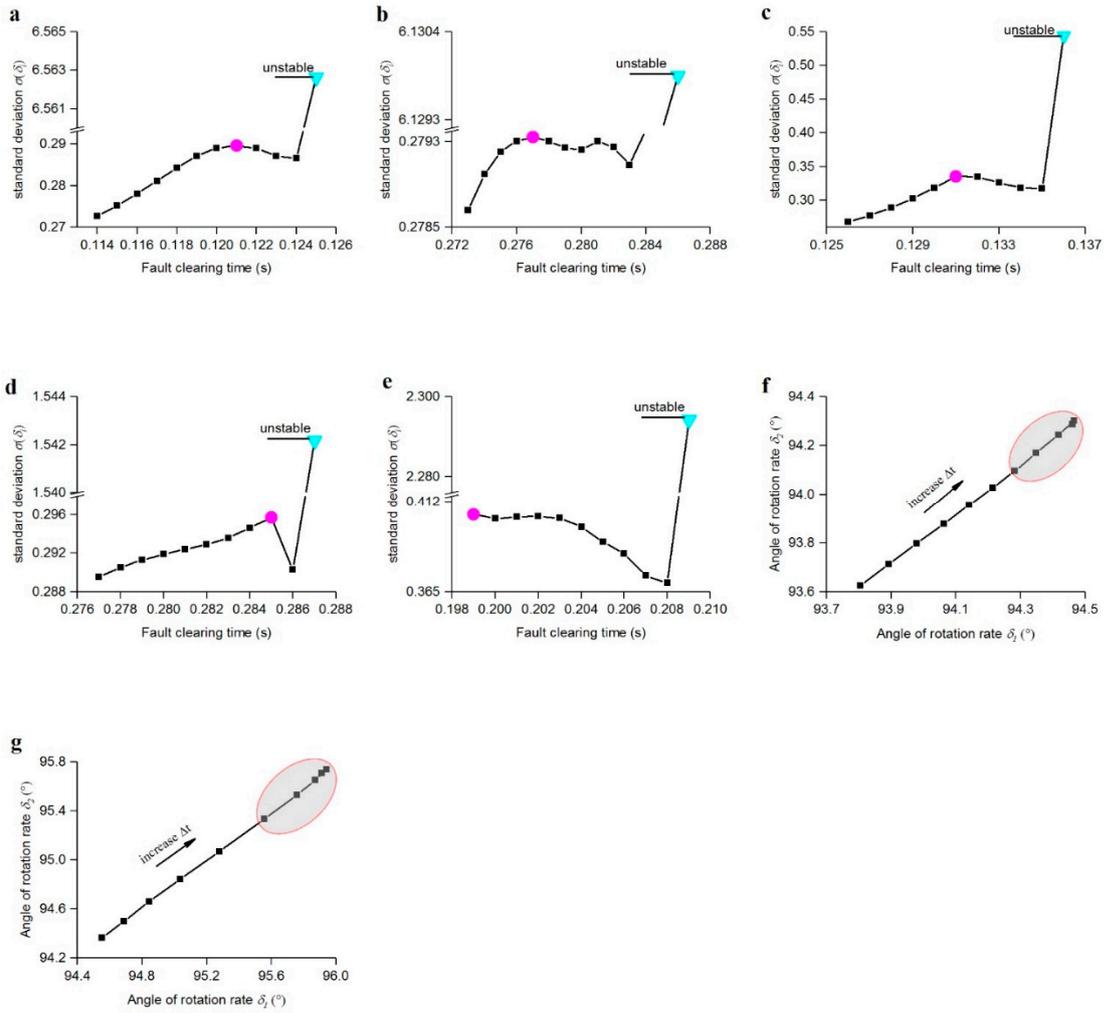
Extended Figure 6. Spontaneous synchronization behaviour of running points near the boundary



a. The standard deviation of δ decreases from $\Delta t = 0.227$ s. It rises by 2200% at $\Delta t = 0.245$ s, where there is an instability.

b. Derive $\frac{d\delta_{kl}}{d(\Delta t)} = \frac{d((a_\delta - 1)\delta_l + b_\delta)}{d(\Delta t)} = (a_\delta - 1) * \frac{d\delta_l}{d(\Delta t)}$ from the fitted equation (e). Due to the monotonicity of δ_i with respect to Δt , i.e. $\frac{d\delta_l}{d(\Delta t)} > 0, \Delta t \in (0, CCT)$, $a_\delta - 1$ changes from positive to negative and a_δ changes from greater than 1 to less than 1.

Extended Figure 7. IEEE 39bus system, Spontaneous synchronisation always occurs near the synchronisation stability boundary



A three-phase short-circuit ground fault is set at the corresponding node. a, b, c, d, e correspond to the critical behaviour of the operating points after the failure of 6bus, 12bus, 24bus, 30bus & 36bus respectively. All the results show that near the boundary, there is always a significant decrease in $\sigma(\delta_i)$. These results demonstrate a strong correlation between synchronisation stability boundary and spontaneous synchronisation.

f and g show that the behaviour of the operating points also show the phenomenon of synchronous barrier when faults are set at 6bus and 24bus (shaded area).