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Posted Date: 24 October 2023

doi: 10.20944/preprints202310.1472.v1

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Article

A Flexible FPGA-Based Stochastic Decoder for 5G LDPC codes

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Abstract: Iterative Stochastic decoding is an alternative to standard fixed-point decoding of Low-density parity check codes (LDPC) to reduce inter-node routing. In this paper, we propose a Flexible Field-Programmable Gate Array (FPGA)-Based Stochastic decoding (SD) hardware architecture for LDPC codes in the Fifth-Generation (5G) New Radio (NR) Standard that supports decoding of set of various code rates. This decoder's runtime flexibility is desirable to switch a better performing code rate automatically based on the channel conditions without the extra time needed for reprogramming of FPGA. An offline design method is implemented to generate the hardware description language (HDL) code description of the decoder for the required code-rate set, which is further synthesized and integrated into the Xilinx Kintex-7 series FPGA board to determine the hardware resource utilisation and processing throughput. The Synopsys design tools were employed during both the simulation and synthesis stages, in combination with TSMC 65-nm CMOS standard cell technology, to facilitate comparative analysis. Compared with state-of-the-art designs, the proposed architecture reduces hardware utilization by up to 26% and achieved energy efficiency by 52%.

Keywords: 5G NR standard; error correcting codes; Low-density parity check codes (LDPC); Stochastic decoding (SD); Field-Programmable Gate Array (FPGA)

1. Introduction

Low-density parity check codes (LDPC) [1] have become one of the essential channel codes in many communication standards, such as DVB-S2 [2], IEEE 802.11 (WiFi) [3], and IEEE 802.16e (WiMax) [4], including Fifth-Generation (5G) wireless technology [5], because of their higher error correction capabilities close to Shannon's limit [6,7]. In 5G New Radio (NR) specifications, Quasi-Cyclic (QC) LDPC codes are selected as the channel coding scheme for data channels to achieve high throughput and low latency [8]. These QC LDPC codes have adopted two Base Graph Matrices (BGMs): H_{b1} and H_{b2} , and fifty one lifting sizes or expansion factors z_c to support various code rates [8]. The H_{b1} has the dimension of 46 block rows and 68 block columns, and it supports code rates ranging from 1/3 to 8/9. The H_{b2} has a dimension of 42 block rows and 52 block columns which supports code rates from 1/5 to 2/3.

The runtime flexibility of the decoder is desirable for the decoding received messages associated with different BGMs of the multiple code rates at the runtime [9]. Hence the decoder dynamically switches between a given set of LDPC code rates having diverse BGMs. This decoder has various commercial applications, such as switching to a better performing code rate automatically based on the channel conditions [10], without the extra time needed for reprogramming of FPGA. Another application is eliminating the re-synthesis of FPGA to test the performance of different code rates.

The specifications of 5G NR show that the BGM H_{b1} can support a vast range of code rates from 1/3 to 8/9. The irregular degrees and diverse connections within these BGMs create a more complex routing network using traditional FPGA-based fixed-point LDPC decoders based on the Sum product algorithm (SPA) [11] and Min sum algorithm [12]. This problem is further aggravated by using multi-bit, wide-channel Logarithmic-Likelihood Ratio (LLR) messages in the decoder, which require high FPGA resources. One alternative solution is converting the channel probability values corresponding to their respective channel LLRs into stochastic bit sequence representation in Stochastic

decoding (SD) [13]. This conversion helps each stochastic bit sequence need a single wire to exchange extrinsic values in a bit-wise manner between the nodes, rather than the W -wires needed for multi-bit wide LLR in conventional decoders. It significantly helps to reduce the inter-node routing complexity of the decoder [14]. The single bit-wise computations in Stochastic variable node (SVN) and Stochastic check node (SCN) units require simple logic units for the implementation. These Stochastic LDPC decoders provide error correction capability comparable to traditional fixed-point decoders [15]. The number of node-interconnects (I) of a decoder is calculated from (1) to determine the routing complexity [16].

$$I = 2 \times N \times e_l \times w_v \quad (1)$$

N is codeword length, e_l is extrinsic message length, and w_v is column weight. For instance, 1/3-rate codeword length $N = 3808$ LDPC code having average column weight $w_v = 4.56$ is decoded using SPA and SD. The number of node-interconnects required for SPA and SD are 138916 and 34729, respectively. The number of node-interconnects in SD is reduced by four times due to extrinsic message length $e_l = 1$ for SD, whereas at least $e_l = 4$ for fixed-point LDPC decoders using SPA.

Our Contributions:

- This work's main contribution is to propose a new partially parallel decoder architecture for Bit-wise Stochastic decoding for 5G NR standard LDPC codes. This architecture has been designed for code word length $N = 3808$, having code rates $R = 1/3, 2/5, 1/2, 2/3, 3/4, 5/6$, and $8/9$ for BGM1.
- Our proposed automated design flow procedure enables this flexibility in design. It creates an optimal FPGA-based Stochastic LDPC decoder design for any selected code rate set. This approach helps to reduce the time needed to design hand-coded interconnections in Hardware Description Language (HDL).

This paper is structured as follows. Section 2 introduces the basic concepts of Stochastic decoding and its Algorithm. Section 3 focuses on constructing BGM of 5G NR standard QC LDPC codes suitable for Stochastic decoders. Section 4 provides architecture details. Design flow is discussed in Section 5. Section 6 discusses Simulation results. Finally, the conclusion is noted in Section 7.

2. Preliminaries

2.1. Stochastic bit sequence generation

A Stochastic approach to LDPC decoding was introduced in [17]. This decoding process converts the received channel probability values into a Stochastic bit sequence equivalent. In general, the LLR of the s -th symbol x_s of a code word is found from the s -th received channel symbol y_s at SVN_s , which is represented as L_{ch,v_s} . By considering the Binary Phase-Shift Keying (BPSK) modulation over Additive White Gaussian noise (AWGN) channel with zero mean and variance σ^2 , the channel LLR computed as $L_{ch,v_s} = 2 \times y_s / \sigma^2$. The channel probability is also known as the intrinsic message, calculated as

$$P_{ch,v_s} = P(x_s = 1/y_s) = \left[\frac{\exp(L_{ch,v_s})}{\exp(L_{ch,v_s}) + 1} \right] \quad (2)$$

The P_{ch,v_s} value is represented with W binary symbols and compared with W -bit Pseudo-Random Number (PRN) $U \in [0, 1]$ of the Comparator[18], which varies with every clock cycle to generate the equivalent Stochastic bit sequence of length $f = 8$ as shown in Figure 1. For example, $P_{ch,v_s} = 0.3125$ is represented as $W = 4$ -bit fractional binary symbols $P = 0101$. At each clock cycle, a PRN generator based on Linear Feedback Shift Register (LFSR) [19] produces a new $W = 4$ -bit PRN U . Suppose $P > U$, the Comparator output at that clock cycle is 1 and otherwise 0. As shown in Table 1, after eight clock cycles, the Comparator output is 00100101, which is the Stochastic bit sequence belonging to $P_{ch,v_s} = 0.3125$. This sequence has a mean value of $3/8$, close to $P_{ch,v_s} = 0.3125$ [20].

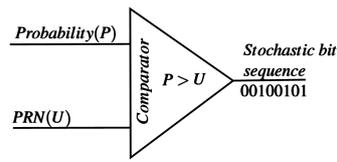


Figure 1. Schematic of Comparator.

Table 1. Example of Stochastic bit sequence generation.

Clock cycle(k)	U	$u(\text{realvalue})$	comparator output($P > U$)
0	1101	0.8125	0
1	0111	0.4375	0
2	0011	0.1875	1
3	0110	0.375	0
4	1001	0.5625	0
5	0010	0.125	1
6	1100	0.75	0
7	0100	0.25	1

2.2. Stochastic decoding algorithmic description

A binary (N, K) LDPC code in 5G is characterized by the null space of Parity Check Matrix (PCM) H with dimension $M \times N$ over $GF(2)$. The PCM is also visualized graphically as a bipartite Tanner graph [21,22]. A set of M Parity or Check nodes of the Tanner graph representing the rows of H , and a set of N Bit or Variable nodes of the Tanner graph representing the columns of H . In Stochastic decoding of LDPC code, the Stochastic bit sequence of the channel probability values corresponds to their respective channel LLRs exchanged iteratively bitwise between the SCNs and the SVNs until the desirable codeword is found or reaches the maximum iteration limit [23]. To illustrate the simplification of the decoding process, a degree-3 SCN and a degree-3 SVN are adopted. Both SCN and SVN elements perform the bitwise Modulo-2 arithmetic operations in SD. The steps are shown in the following:

1) Initialization: Once the channel probability values are reached at SVNs, compare the W -bit fractional binary equivalent sequence of P_{ch,vn_j} with W -bit PRN U and initialize generated stochastic bit sequence to its corresponding SVNs vn_j .

2) SCN update: As shown in Figure 2, the SCN_2 is connected to three SVNs SVN_2 , SVN_3 , and SVN_4 . The arriving SCN to SVN single-bit extrinsic message for node SVN_4 is computed as

$$F_{c_2 \rightarrow v_4} = a \cdot (1 - b) + b \cdot (1 - a) \quad (3)$$

Here a is the single-bit of stochastic bit sequence $\{a[k]\}$, $k = 0, \dots, f - 1$, belonging to its P_{ch,v_2} , which has been received message from SVN_2 to SCN_2 in previous clock cycle and b is the single-bit of stochastic sequence $\{b[k]\}$, belonging to its P_{ch,v_3} , which has been received message from SVN_3 to SCN_2 in a previous clock cycle.

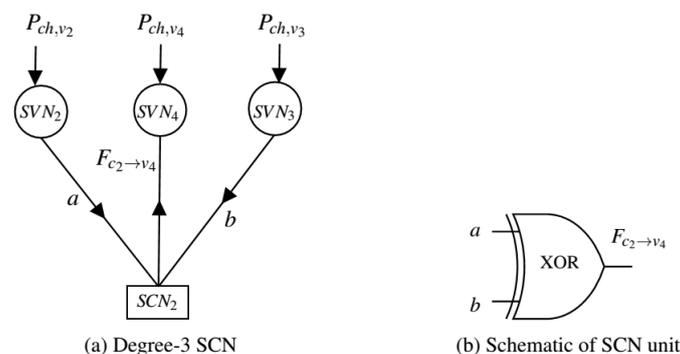


Figure 2. SCN and its implementation unit.

3) SVN update: In Figure 3, the SVN_4 is connected to three SCNs SCN_1 , SCN_2 , and SCN_4 . The arriving SVN to SCN single-bit extrinsic message for node SCN_2 is computed as

$$F_{v_4 \rightarrow c_2} = \frac{e.d}{e.d + (1-e).(1-d)} \quad (4)$$

Here e is the single-bit of stochastic bit sequence $\{e[k]\}$, belonging to the received message from SCN_1 to SVN_4 in the previous clock cycle and d is the single-bit of stochastic bit sequence $\{d[k]\}$, belonging to the received message from SCN_4 to SVN_4 in a previous clock cycle.

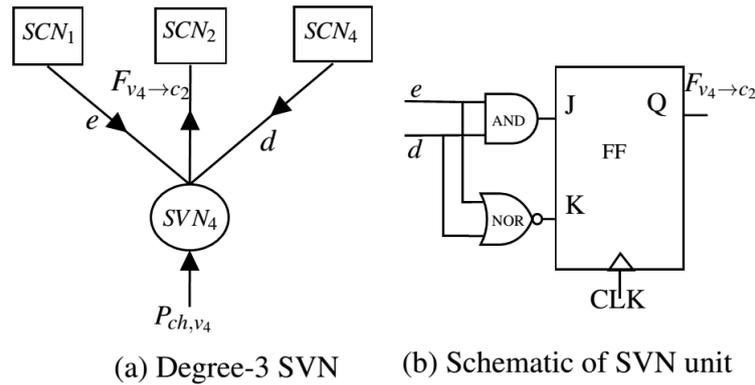


Figure 3. SVN and its implementation unit.

4) Termination: This iterative process will stop if it reaches the maximum Decoding cycles (DC) limit or all parity check equations are satisfied.

3. Construction of BGM in 5G NR standard

The PCM H is represented and constructed from its BGM H_b [24]. The H_b has dimension $m_b \times n_b$, where $M = m_b \times z_c$ and $N = n_b \times z_c$. The entries of H_b are expanded with $z_c \times z_c$ square sub-matrix in H , where z_c is known as the lifting size or expansion factor. The entry value '-1' of H_b is replaced by a zero matrix of dimension $z_c \times z_c$ in H , and the value '0' is replaced by the Identity matrix of dimension $z_c \times z_c$, and another non '-1' entry value $1 \leq S_{i,j} \leq z_c$, also known as shift value, is replaced by circulant permutation matrix $I(S_{i,j})$. The subscripts i, j represent the row index and column index of the entry. The sub-matrix $I(S_{i,j})$ is obtained by each row of the identity matrix having a right-shift value of $S_{i,j}$ positions. The value of $S_{i,j}$ is calculated from the function (5). The value of $V_{i,j}$ is obtained from Tables 5.3.2-2, and 5.3.2-3 of 5G-NR standard specification 3GPP TS 38.212 [8] according to the selected BGM and the set index i_{LS} .

In this paper, we constructed BGM H_{b1} of (N, K) LDPC code having a message or information block length $K = 1232$ and code rate $R = K/N = 1/3$. The five steps are listed below for constructing the H_{b1} having k_b as the length of message block columns. The terms k_b of H_{b1} and K of H are related as $K = k_b \times z_c$.

1. Selection from the two BGMs: As per the specification of 3GPP TS 38.212 [8], since the code rate $R \geq 1/4$, BGM1 is selected.
2. Calculate the value k_b after selecting BGM: From the specification of 3GPP TS 38.212 [8], the BGM1 has $k_b = 22$.
3. Find the expansion factor z_c : The selection of the minimum z_c from Table 5.3.2-1 [8], such that $k_b \times z_c \geq K$. For given $K = 1232$, $k_b = 22$, z_c is calculated as $1232/22 = 56$.
4. Selection of set index i_{LS} : After z_c is determined, the suitable shift coefficient matrix set from Table 5.3.2-1 [8] must be selected. Since $z_c = 56$, the set index $i_{LS} = 3$ is considered.

5. Compute the BGM entry values: Utilize the function (5) to determine the entry values $S_{i,j}$ by means of the modular z_c operation.

$$S_{i,j} = f(V_{i,j}, z_c) = \begin{cases} -1, & \text{if } V_{i,j} = -1; \\ \text{mod}(V_{i,j}, z_c), & \text{else} \end{cases} \quad (5)$$

6. Construction of the PCM H : Substitute each entry of the BGM by the corresponding circulant permutation matrix or zero matrix of size $z_c \times z_c$ in H .

From the Step 5, first entry $h_{0,0}$ of H_{b1} is calculated using $\text{mod}(V_{0,0}, z_c) = \text{mod}(223, 56) = 55$, where the selected $V_{0,0} = 223$ value obtained from Table 5.3.2-2 [8], which correspond to row index $i = 0$, column index $j = 0$ under set index $i_{LS} = 3$. Similarly, the remaining entry values are calculated by using a function (5) and $V_{i,j}$ values obtained from Table 5.3.2-2 [8] based entry's row and column indexes. The Table 2 shows calculations of the corresponding values of few entries of H_{b1} . The constructed H_{b1} has dimension of 46×68 . The BGM H_{b1} is shown in Table 4.

Table 2. Calculation of the entry values of H_{b1} .

Entry of H_{b1}	$V_{i,j}$	$\text{mod}(V_{i,j}, z_c)$	Corresponding values
$h_{0,0}$	$V_{0,0} = 223$	$\text{mod}(223, 56)$	55
$h_{0,1}$	$V_{0,1} = 16$	$\text{mod}(16, 56)$	16
$h_{0,2}$	$V_{0,2} = 94$	$\text{mod}(94, 56)$	38
$h_{0,3}$	$V_{0,3} = 91$	$\text{mod}(91, 56)$	35
$h_{0,4}$	$V_{0,4} = -1$	$\text{mod}(-1, 56)$	-1
$h_{0,5}$	$V_{0,5} = 74$	$\text{mod}(74, 56)$	18
$h_{0,6}$	$V_{0,6} = 10$	$\text{mod}(10, 56)$	10
$h_{0,7}$	$V_{0,7} = -1$	$\text{mod}(-1, 56)$	-1
$h_{0,8}$	$V_{0,8} = -1$	$\text{mod}(-1, 56)$	-1
$h_{0,9}$	$V_{0,9} = 0$	$\text{mod}(0, 56)$	0

In order to achieve the desired information lengths and rate adaptation in 5G NR QC-LDPC codes, a process of shortening and puncturing is carried out. The characteristics of the BGM1 are described in the Table 3.

Table 3. Parameters of BGM1.

Characteristics	BGM1 (H_{b1})
Number block columns (n_b)	68
Number block rows (m_b)	46
Number edges	316
Column weights (w_v)	1 to 30
Row weights (w_c)	3 to 19
Base code rate	1/3

Table 4. Base Graph Matrix

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	65	66	67
0	55	16	38	35	-1	18	10	-1	-1	0	37	48	21	47	-1	14	14	-1	29	30	48	25	1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	29	-1	45	39	46	7	-1	45	21	31	-1	38	37	-1	23	9	6	26	-1	31	-1	19	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1
2	39	35	31	-1	8	12	18	39	41	9	14	-1	-1	21	46	21	-1	30	5	55	34	-1	-1	-1	0	0	-1	-1	-1	-1	-1	-1	-1
3	33	18	-1	53	5	-1	45	30	16	-1	34	43	45	35	13	-1	40	18	43	-1	30	46	1	-1	-1	0	-1	-1	-1	-1	-1	-1	-1
4	2	10	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
5	52	3	-1	30	-1	-1	-1	-1	-1	-1	-1	-1	24	-1	-1	14	-1	-1	-1	-1	-1	18	41	-1	-1	-1	-1	0	-1	-1	-1	-1	-1
6	46	-1	-1	-1	-1	7	-1	-1	-1	-1	21	-1	7	-1	-1	-1	51	24	-1	4	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1
7	17	20	-1	-1	48	-1	-1	44	38	-1	-1	-1	-1	-1	46	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	
8	33	39	-1	4	-1	-1	-1	-1	-1	-1	49	-1	-1	36	-1	-1	39	-1	2	44	-1	33	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
9	9	37	-1	-1	-1	-1	-1	-1	45	49	-1	33	-1	-1	17	53	-1	50	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
10	-1	26	53	-1	6	-1	19	26	-1	-1	-1	47	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
11	52	11	-1	-1	-1	-1	-1	-1	-1	-1	-1	2	-1	-1	35	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
12	30	7	-1	-1	-1	-1	-1	-1	24	3	-1	28	-1	-1	-1	14	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
13	25	-1	-1	0	-1	-1	16	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	49	-1	-1	22	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
14	14	-1	-1	-1	-1	-1	-1	-1	-1	-1	7	-1	-1	43	23	51	-1	-1	43	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
15	34	8	-1	-1	-1	-1	-1	-1	19	-1	41	-1	-1	-1	-1	41	-1	-1	-1	-1	-1	25	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
16	-1	42	-1	52	-1	-1	-1	-1	43	-1	-1	-1	-1	-1	-1	-1	21	-1	45	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
17	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	54	-1	32	7	-1	-1	4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
18	-1	31	-1	-1	-1	-1	-1	-1	-1	-1	-1	54	32	-1	-1	-1	31	18	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
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22	53	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	3	-1	-1	36	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
23	-1	32	35	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	10	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
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27	-1	40	-1	-1	-1	29	-1	6	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
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29	-1	38	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	11	-1	-1	53	-1	2	-1	-1	12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
30	34	-1	-1	-1	-1	-1	-1	-1	18	-1	42	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
31	-1	7	-1	-1	-1	-1	49	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	9	-1	-1	16	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
32	24	-1	-1	-1	-1	-1	-1	-1	-1	-1	41	-1	2	-1	-1	-1	-1	-1	-1	-1	-1	30	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
33	-1	2	49	-1	-1	-1	-1	-1	-1	-1	49	-1	-1	-1	-1	-1	-1	25	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
34	26	-1	-1	-1	-1	-1	18	-1	-1	-1	-1	-1	12	-1	38	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
35	-1	24	-1	-1	-1	5	-1	-1	-1	-1	26	-1	-1	-1	-1	-1	-1	19	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
36	54	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	45	0	-1	-1	6	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
37	-1	25	-1	-1	-1	-1	-1	-1	-1	-1	27	-1	-1	-1	-1	-1	-1	-1	-1	-1	26	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
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39	-1	12	-1	21	-1	-1	49	-1	-1	-1	-1	-1	-1	-1	-1	-1	2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
40	11	-1	-1	-1	-1	-1	45	-1	-1	-1	-1	-1	-1	-1	40	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
41	-1	23	-1	47	-1	-1	-1	4	-1	-1	-1	-1	-1	-1	-1	55	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
42	2	-1	-1	-1	35	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	22	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
43	-1	38	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	22	-1	22	-1	-1	-1	-1	-1	-1	-1	-1	-1	49	-1	-1	-1	-1	-1	
44	28	-1	-1	-1	-1	-1	4	-1	9	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
45	-1	16	-1	-1	-1	-1	9	-1	-1	-1	29	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	

4. Proposed architecture

The suggested FPGA-based partially parallel Stochastic decoder's top-level design presents in this section. It has runtime flexibility and the capability of decoding the received messages corresponding to the set of seven code rates $R = 1/3, 2/5, 1/2, 2/3, 3/4, 5/6$, and $8/9$ of codeword length $N = 3808$ QC LDPC code compliant to 5G NR standard. Initially, the architecture determines the maximum matrix dimensions, such as the number of block columns $N_B = 68$, the number of block rows $M_B = 46$, lifting size $Z_c = 56$, column-weight $W_v = 30$, and row-weight $W_c = 19$, for this decoder from the supported code rate set. The architecture comprises various basic modules such as Stochastic Variable Node Decoder (SVND), Stochastic Check Node Decoder (SCND), BGM Read-Only Memory (ROM), Controller unit, Intrinsic Message Memory Unit (IMMU), and Routing network between modules SVND and SCND. These modules of the design are explained in the following subsections.

4.1. Layered decoding schedule

In partially-parallel architectures, only a few SVNs and SCNs are instantiated simultaneously. Figure 4 shows the proposed architecture that connects the SCND to the SVND. The SCND consists of M D Flip-Flops, used as memories to store updated SCN messages received from SVND. Here, M is the total number of SCNs of the Tanner graph of the decoded LDPC code. The architecture adopted Shuffled iterative decoding [25], in which the total number of block-columns of the BGM is subdivided into vertical layers, each layer consists of one block column of BGM or η_v columns of the PCM, where η_v is the SVN parallelism factor, these columns are mapped as SVNs of the Tanner graph of the decoded LDPC code. Before the next vertical layer is processed, the SVNs of each layer are processed in parallel, and the results are used to update the connected SCNs. Using this scheduling, the 3808 columns of the $1/3$ -rate codeword length $N = 3808$ LDPC code PCM are divided into $N_B = 68$ vertical layers, each layer consists of $\eta_v = 56$ columns. Accordingly, the proposed decoder has a SVND, which comprises $\eta_v = 56$ Stochastic variable node processing units (SVNPU), which function based on SVN update. These units process a layer of $\eta_v = 56$ columns of the PCM during each clock cycle,

with these updated values being written back to SCND immediately. Hence each Decoding cycle (DC) require $\tau_D = N_B = 68$ clock cycles. Each DC creates one new bit in each stochastic bit sequence.

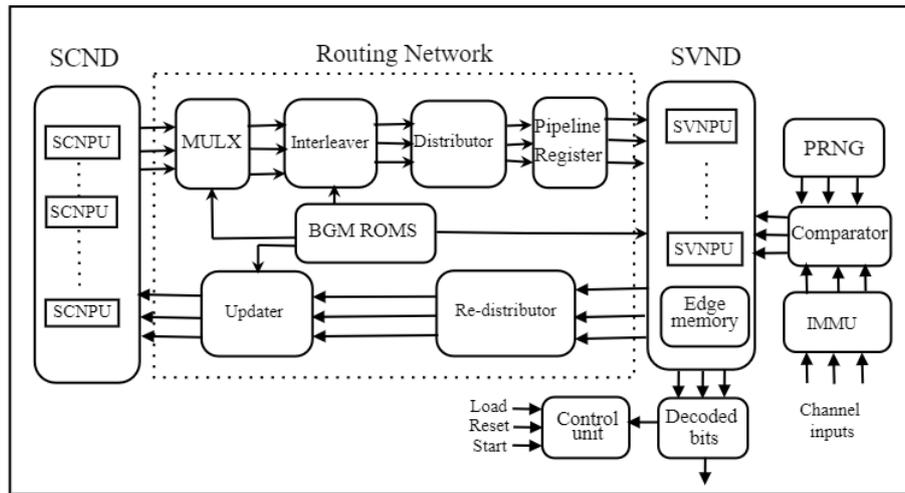


Figure 4. Block diagram of the proposed Stochastic decoder.

4.2. BGM ROMs

It is possible to regulate the routing and shifting of intrinsic and extrinsic messages between the routing network, SVNPU and SCNPUs by employing a set of ROM blocks that hold the location and shift values of non '-1' entries of each BGM in a hardware-optimized form: the three ROMs, namely ROM1-location, ROM2-shift, and ROM3-weight. Firstly, the row indices of all non '-1' entries within each block column of BGM are represented by the values in the ROM1-location. Second, ROM2-shift saves the shift value difference between each non '-1' entry in each block column and the previous non '-1' entry in the same block row. This value can be determined at design time and saved as described. Lastly, each block column of BGM's weight is stored in the ROM3-weight. For the proposed architecture, all three ROMs contain $N_b = 68$ locations, each with $W_v = 30$ values i.e., $N_b \times W_v = 2040$ locations.

4.3. Routing network

The Routing network carries the single-bit SCN to SVN messages from the SCND registers to the $\eta_v = 56$ SVNPU in the SVND, which are updated and sent back to the SCND. This routing network combines separate modules called Multiplexer (MULX), Interleaver, Distributor, Re-distributor, and Updater. The functions of these blocks are explained later in this section. The maximum number of SCN to SVN message inputs needed for any SVNPU at any time is $W_v = 30$, the maximum number of non '-1' entries in any block column within the allowed code rate set.

4.3.1. Multiplexer

Using the row-index values kept in the ROM1-location, the MULX unit chooses $W_v = 30$ blocks of $Z_c = 56$ bits from the SCND. However, instead of connecting each input to each output, this module requires fewer hardware resources by adopting the method of storing ROM1-location row index values in ascending order [27]. For instance, the block column of any BGM has weight w_v , such that $w_v \leq W_v$. Out of the total W_v values of ROM1-location arranged with top w_v block row indices of the non '-1' entries in that block column in ascending order, while remaining $W_v - w_v$ values are empty data. The multiplexer designed to support the seven LDPC BGMs of the allowed code rate set having maximum column weight $W_v = 30$ requires 210 connections. By using this method [27], the number of connections decreased to 39%.

4.3.2. Interleaver

The $W_v = 30$ MULX output blocks from the SCND are cyclically shifted by $W_v = 30$ parallel Barrel Shifters in the Interleaver module based on shift values stored in ROM2-shift. Before being read by the SVND, a group of $\eta_v = 56$ messages of sub matrix must be converted from a row-centric to a column-centric representation based on the corresponding shift value in BGM H_{b1} by a Barrel shifter. Each Barrel shifter has $Z_c = 56$ inputs and outputs because the shift value can be any integer between 0 and $Z_c = 56$. The hardware description language (HDL) used for the BSs integrated with the proposed flexible decoder architecture is customised for the supported BGMs during the design phase. The choice of multiplexer input for every BGM is influenced by the expansion factor $Z_c = 56$ value linked to the corresponding BGM, based on the cyclic shift decomposition algorithm mentioned in [26].

4.3.3. Distributor and Re-distributor

The distributor takes $W_v = 30$ blocks of $\eta_v = 56$ bits from SCND and performs a rearrangement process to form $\eta_v = 56$ blocks of $W_v = 30$ bits. The pipeline registers may optionally latch these blocks before the SVND processes them. After undergoing SVND processing, the resulting $\eta_v = 56$ blocks of $W_v = 30$ bits are again subjected to a similar rearrangement process by the Re-distributor to form $W_v = 30$ blocks of $\eta_v = 56$ bits.

4.3.4. Pipeline registers

Stochastic layered decoding involves binary operations, causing each layer's message updates to either alter or leave prior layer updates unchanged. The decoding schedule needs to store updates from each previous layer before moving on to the next. Adding a single pipeline stage to the suggested architecture can sometimes improve BER performance without negatively impacting the decoding process. In some BGMs, a column-wise permutation occurs, allowing for column-orthogonality in block columns. Adding a pipeline stage during the data path does not negatively impact decoding performance when processed in this order [27]. If a permutation is not found, introducing a few τ vacant stall layers among non-orthogonal columns can be developed. Finding the necessary minimum values of τ for each BGM and adding this supplementary pipeline stage is advantageous. Decreased block rows while keeping block columns constant can enhance coding rate. Simulations show that high-rate codes require more τ vacant stall layers among non-orthogonal columns compared to low-rate codes.

4.3.5. Updater

Using the values from the ROM1-location, this component transfers these $W_v = 30$ blocks of $\eta_v = 56$ bits back into the SCND at the proper location. Moreover, the ROM3-weight is utilized in this case to guarantee that just the most recent W_v values are written, removing any potential interference from non '-1' entries or "don't care" data. As D-Flip Flops are being used to implement the SCND, each layer can be written right away.

4.4. Stochastic variable node decoder (SVND)

The SVND module contains $\eta_v = 56$ parallel Stochastic variable node processing units (SVNPU). These SVNPU can process one block column of the BGM H_{b1} . Hence each SVNPU process one column of the PCM. Each SVNPU has multiple inputs and outputs equal to $W_v + 1$, where W_v is the highest column weight among the supported BGM code-rate set. The extra input and output are utilized, respectively, for channel-intrinsic messages and the estimation of the decoded bit. It is essential to consider that this largest number of inputs and outputs must be taken into account to guarantee that each SVNPU can decode any arbitrary column-weight column in the code-rate set. The SVND accepts the maximum SVN weight $W_v = 30$ input bits from the connected SCNs, and an intrinsic bit from

the Intrinsic Message Memory unit (IMMU). By considering such values, SVND implements the SVN update operation and the updated extrinsic values sent to the corresponding SCNPUs.

4.4.1. Stochastic variable node processing unit (SVNPU)

In this Stochastic architecture, the chance of SVNPU output being in the Hold state increases as inputs of SVNPU increase [20]. The SVNPU forces the current output to continuously repeat the previous output over a certain period when the two input bits of the SVNPU are not equal, known as the Hold state of SVNPU [23]. Due to this state, the decoder is unable to make correct decisions. It interrupts the decoder convergence and its Bit Error Rate (BER) performance degradation. Hence, the architecture of high column-weight SVNPU is built using low-weight sub-nodes with memory blocks; typically, $w_v \leq 4$ sub-nodes are employed.

As shown in Fig. 5, the column-weight $w_v = 6$ SVNPU is constructed using two column-weight $w_v = 3$ and one $w_v = 2$ sub-nodes. The memory blocks are Internal memory (IM) which has a length of $I = 2$ bits for each sub-node, and Edge memory (EM) has a length of $E = 64$ bits at the output edge. The critical role of EMs and IMs is to minimize the correlation produced by the Hold state in a stochastic sequence and disrupt the correlation of stochastic sequence by randomizing stochastic bits [28]. We used a dual-tree design [27] to build a high column-weight SVNPU that would be flexible enough to handle any active inputs and outputs up to the maximum $\lambda = W_v + 1$, which is a power of 2. For instance, column weight $W_v = 30$ of SVNPU has $\lambda = 32$ inputs and outputs. As a result, the adopted SVNPU dual-tree design requires a total of $S = 3 \times \lambda - 6 = 90$ sub-nodes. Two significant components, summing and combining, each with $t = \log_2(\lambda) - 1 = 4$ stages, make up the dual-tree structure.

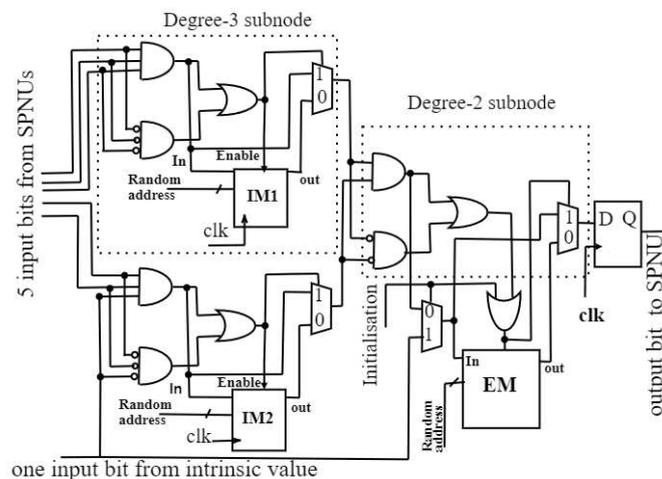


Figure 5. Block diagram of SVNPU with high $w_v = 6$ [20].

4.5. Control unit

The Control unit of the decoder manages internal and external control signals, including to stop or proceed with the iterative decoding. A new group of $\eta_v \times w$ -bit intrinsic probabilities may need to be loaded into the IMMU, and this can be indicated by the LOAD signal. The decoder must decode a new frame, and all modules must be reset to their default conditions using the RESET signal. The ability of the proposed architecture to swap the current BGM to decode the message contained in the IMMU during a single clock cycle is one of its essential characteristics. The supporting BGMs are parameterized in ROMs at compile-time to enable this flexibility level. The process of decoding is started and maintained by the START signal. As long as this signal is active, the decoder will keep doing iterative decoding; the decoding operation is stopped once it does.

5. Design flow

This section outlines the suggested design flow, which enables the automated flexibility in design and creates an optimal FPGA-based Stochastic LDPC decoder for a selected set of 5G QC LDPC codes. The flow chart in Fig. 6 depicts the design flow. In the first step, the Construction of the required code-rate BGM set is based on the user inputs like message length, code rate, and lifting sizes of the 5G LDPC code. The second step has two tasks: Determining the decoder's parameters, such as the maximum of the matrix dimensions and weights, namely N_b , M_b , Z_c , W_v , and W_c , respectively, which describe the chosen set of supported BGMs. Based on lifting size Z_c , it is possible to determine the parallelism factor η_v from these. For $N = 3808$ LDPC code-rate set, the maximum values are $N_b = 68$, $M_b = 46$, $Z_c = 56$, $W_v = 30$, and $W_c = 19$. Another task is extracting the positions and shift values of the non '-1' entries in each BGM and arranging them consistent with the ROMs. Considering the parameters derived in the earlier step, the design flow utilizes High-Level Synthesis (HLS) tool [29] to produce the Hardware Description Language (HDL) SystemVerilog code for the suggested architecture written in the High-level language C++. After the Register Transfer Level (RTL) modelling of the decoder, it is synthesized using Xilinx Synthesis Tool to measure the hardware requirements of the decoder. The Bit Error Rate (BER) simulations have been implemented on the FPGA test setup.

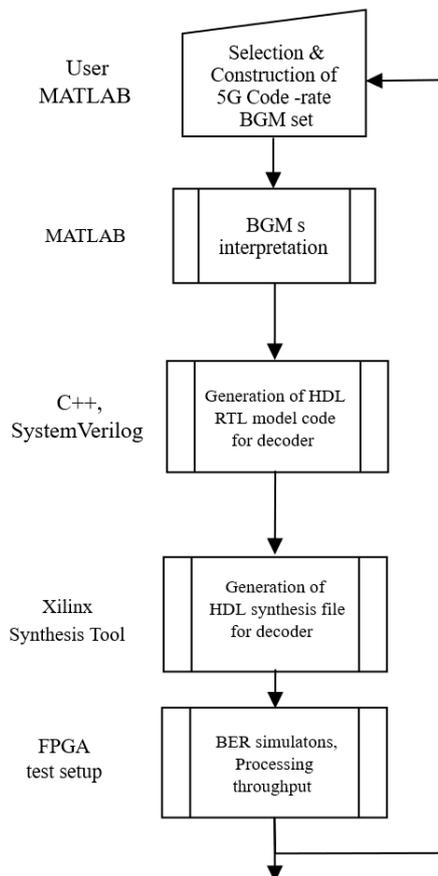


Figure 6. Flow chart for Offline design and implementation of the proposed decoder.

6. Implementation results and discussion

6.1. Approach

After the offline design method is completed, the HDL code description of the decoder for the required code-rate set is generated, which is further synthesized and integrated into the Xilinx Kintex-7 XC7K160T series FPGA board to determine the hardware resource utilisation and processing throughput. An additional parameter that is measured is the transmission energy efficiency of the

synthesized decoder in terms of the channel's signal-to-noise power ratio per bit E_b/N_o at a required BER of 10^{-6} for each target BGM. Simulations are carried out to evaluate the transmission energy efficiency of the synthesized decoder. The simulations entail a minimum of 100 frame errors per BER measurement and a maximum of 600 DCs per frame.

The intrinsic channel LLRs serve as input to the decoder module of the FPGA board, which is received via the RS232 port of the computer. In order to facilitate communication with the computer, an RS232 transceiver module has been designed. The MATLAB environment in the computer is utilised to send and receive the same set of decoded LLRs after completing the decoding process on FPGA. Subsequently, MATLAB compares the input and output LLRs of FPGA and estimates the BER performance.

6.2. Results

In order to analyse the performance of the proposed decoder, we consider three parameters: BER performance, Hardware utilisation, and Processing throughput.

6.2.1. BER performance

Figure 7 shows BER performance of the proposed decoder which delivers $BER = 10^{-6}$ at 2.65 dB of E_b/N_o for block length of 3808 with base code rate of 1/3. It has been observed that SD provides approximately nearer error correction performance compared with the conventional Sum-product (SPA) and Min-sum (MS) algorithms. Notably, decoding iterations of SD require more clock cycles than conventional designs. Additionally, it has been observed that applying noise-dependent scaling of 0.86 [28] to the received channel probabilities improves performance for lower code rates such as 1/3 and 2/5. Conversely, this performance declines for higher code rates such as 5/6 and 8/9.

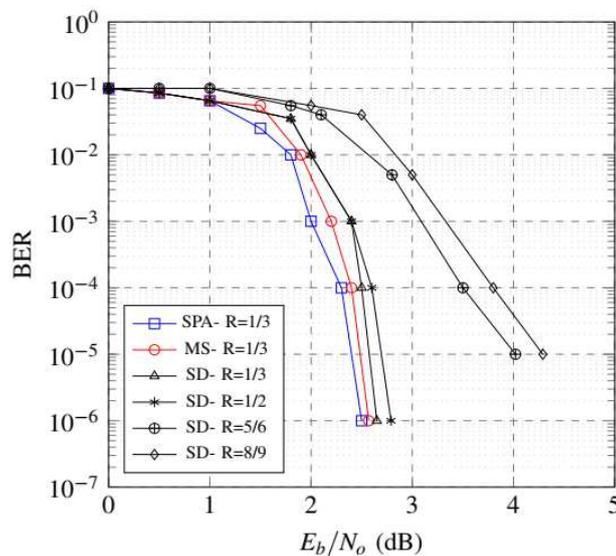


Figure 7. BER plot of various algorithms and code rates for $N = 3808$.

6.2.2. Hardware utilisation

The proposed design's advantage lies in the significant reduction of decoder complexity. Table 5 compares the suggested design and the min-sum-based decoder architectures, indicating that the former requires less hardware by approximately 37%. A crucial parameter that affects the decoder's area and routing complexity is the number of interconnecting wires between its nodes. In this regard, the suggested design outperforms the Min-sum-based decoder by requiring 34729 fewer interconnect wires. Other parameters of the suggested design are comparable to those of the Min-sum-based decoder. The results in Table 6 indicate that the increase in coding rate significantly impacts the SD

decoder's decoded processing throughput. However, at the same time, it negatively affects the error correction performance.

Table 5. FPGA Implementation results.

Standard	5G	5G
Code length	3808	3808
Base Code rate	1/3	1/3
Sub-matrix size	56	56
Implementation	Kintex-7 FPGA	Kintex-7 FPGA
Decoding algorithm	Stochastic Decoding	Min-Sum
Scheduling	Column-layered	Row-layered
No.of interconnects	34729	138916
Intrinsic message width	8-bit	4-bit
Extrinsic message width	1-bit serial	4-bit
LUTs	8,278	12,962
Slice Registers	1,767	2,041
DCs or Itrs	≈ 620 DCs	15
Avg. throughput	≈ 953 Mbps	1.5 Gbps
E_b/N_o at BER= 10^{-6}	2.65dB	2.57dB

Table 6. SD results of various code rates of $N = 3808$.

Active Code-rate	No.of clock cycles per DC	LUTs(k)	Slice registers(k)	Throughput(Mbps)	E_b/N_o at BER= 10^{-6}	No.of DC per frame
1/3	68	8.2	1.7	953.4	2.65dB	620
2/5	68	8.2	1.7	964.3	2.69dB	530
1/2	68	8.2	1.9	1100.9	2.79dB	450
2/3	68	8.2	1.9	1189.3	3.28dB	430
3/4	68	8.2	1.9	1240.5	3.87dB	400
5/6	68	8.2	1.9	1267.6	4.02dB	360
8/9	68	8.2	1.9	1298.2	4.29dB	330

6.2.3. Processing throughput

The proposed architecture requires a reduction in hardware resources, although at a considerably lower processing throughput expense. This lower throughput of Stochastic decoders is due to the high number of Decoding cycles (DC) needed to reach a correct code word. Due to the high number of DCs and partially parallel decoder architecture, the problem is made worse by the fact that each DC requires many clock cycles, which reduces decoding throughput. The SD architecture requires maximum 620 DCs per frame, and each decoding cycle necessitates 68 clock cycles for code length $N = 3808$. Table 5 compares both designs and concludes that the processing throughput of the SD design is about 38% lower than that of the Min-sum design. Table 6 demonstrates that an increase in code rate results in a concurrent increase in processing throughput.

6.3. Comparative analysis

To facilitate a comparative analysis, the Verilog hardware description language (HDL) was employed to model the architecture. It was subsequently subjected to simulation to verify its functionality using a test pattern generated from a C++ simulator. The design functions were verified successfully, following which the architecture was synthesized while adhering to suitable time and area

constraints. The Synopsys design tools were employed during both the simulation and synthesis stages, in combination with TSMC 65-nm CMOS standard cell technology. The results obtained post-synthesis are presented in Table 7. Specifically, the proposed SD architecture occupies an area of 1.10 mm^2 and achieves a throughput of 1.12 Gbps, while the power consumption is 410 mW.

Table 7. Comparative results.

Design	Proposed	[31]	[32]
Standard	5G-NR	5G-NR	802.16e
Code length	3808	3808	2304
Base code rate	1/3	1/3	1/2
Decoding algorithm	SD	CMS	NMS
Scheduling	Column-layered	Row-layered	Row-layered
Extrinsic message width	1-bit	4-bit	4-bit
Sub-matrix size	56	56	96
DCs or Itrs	620	10	10
Area (mm^2)	1.10	1.49	2.9
Throughput (Gbps)	1.12	3.04	2.20
Power (mW)	410	259	870

The comparison of the proposed SD with other LDPC decoders is provided in Table 7, considering implementation and performance. The proposed design has been shown to reduce the decoder architecture's complexity significantly. These designs exhibit varying implementation parameters, including code length and decoding algorithms such as Combined Min-Sum (CMS) and Normalised Min-Sum (NMS). Compared to the reported decoders, the proposed architecture exhibits area efficiency by 26% compared to [31] while delivering energy efficiency by 52% compared to [32].

7. Conclusions

In this paper, we constructed BGM of QC LDPC code of 5G NR standard. We presented Stochastic decoding to LDPC codes as a hardware-efficient alternative to SPA and MS-based LDPC decoders. From simulations, it has been observed that with the inclusion of features like Scaling and Edge memory, Stochastic decoding performs almost nearer to SPA-based LDPC decoder in terms of BER performance.

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