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Is Bell's Experiment Really Telling Us Something?

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Is Bell's Experiment Really Telling Us Something?

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Abstract: Bell's correlation formula is based on a classical Kolmogorov probability theory. It is tested in experiment with an estimate correlation. The latter is the raw product moment (rpm) correlation that is classical Kolmogorov based as well. We note that the famous Bell inequalities are derived from Bell's correlation formula. The problem is that the hypothesis: "is it possible that in experiment Bell's correlation is equal to quantum", can only found to be false. This is not because the data generating process does not produce the required data. Or that the inequalities sufficiently show that the data cannot violate the inequalities. It is because the statistics of the Bell experiment is flawed. The rpm correlation in experiment cannot be equal to the quantum value in principle. This is because it requires the classical Kolmogorov rpm correlation to be not Kolmogorov.

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1. Introduction

In 1935 Einstein formulated his doubts about the completeness of quantum mechanics. His most clear formuation and the one he agreed with most [1] and [2], can be found in a German text [3]. Considerations of probabilty were important to Einstein in the early period [1].

In 1964, J.S. Bell formulated a correlation [4] between distant measurements of the spin of two entangled particles. Einstein separation can be found in the distance between the locations of measurement. Einstein's "brief interaction" arises from the entangled particles from a source. The idealised Bell experiment is well known and a proper description of it can be found in [5].

In the present letter it is the aim to show that the statistics of the *experiment* is flawed. One can find a proper description of the necessary classical probability in textbooks [6], [7] and [8].

1.1. Correlation

Let us start with acknowledging that Bell's correlation formula is representing Einstein hidden extra parameters in the form of values λ of classical probability random variables. From the perspective of probability theory Bell's starting point is a probability measure space (Λ, Σ, P_B) with Λ the universe set, Σ the sigma-algebra based on Λ and P_B the probability measure [10]. Bell's general idea is to have a probability density, $\rho(\lambda) \geq 0$ for the values of those hidden extra parameters leading to $\lambda \in \Lambda$. The Λ is the general universal set. And so for a set $A \subset \Lambda$, the theoretical probability to find the values in set A is

$$0 \leq P_B(A) = \int_{\lambda \in A} \rho(\lambda) d\lambda \leq 1 \tag{1}$$

Here $P_B(\Lambda) = 1$. This probability measure follows the laws of classical probability [6]. A short presentation of the axioms [9, page 10] that stand a the foundation of the probability laws, is perhaps instructive.

- Positivity: A probability, P, is *never* negative
- Certain event: The probability P of the universe set U is unity
- Additivity: If events Q and R are disjoint then $P(Q \cup R) = P(Q) + P(R)$.

Obviously, $Q \subset U$, $R \subset U$ and $Q \cup R \subset U$.

In the experiment we postulate two observers, Alice and Bob. They employ instruments to measure spin. Alice sets the paramenter vector of her spin measuring instrument with unit lengthe

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vector $\boldsymbol{a}$. Bob sets unit length vector $\boldsymbol{b}$. Bell's correlation function is then an expectation value for two measurement functions $A(\boldsymbol{a},\lambda) \in \{-1,1\}$ and $B(\boldsymbol{b},\lambda) \in \{-1,1\}$. Here -1 represents spin-down and 1 represents spin up. The result of measurement, according to Bell, not only is a function of the setting parameter vector. It also varies with the hidden extra parameters random variables with values λ. The formula is

$$E(\boldsymbol{a},\boldsymbol{b}) = \int_{\lambda\in\Lambda} \rho(\lambda)A(\boldsymbol{a},\lambda)B(\boldsymbol{b},\lambda)d\lambda \tag{2}$$

This formula is obviously also embedded in classical Kolmogorovian probability theory [6], [7].

2. Experiment

Bell's formulation (2) is not directly observable for individual pairs of entangled particles. The expression (2) of the role of Einstein's hidden extra parameters, anticipated a possible "counting registered result" experiment. To gain some insight into the physical set-up of a typical experiment a very schematic representation is provided below

$$[A(\boldsymbol{a})] \leftarrow\sim \cdots \sim\leftarrow\sim [S] \sim\rightarrow\sim \cdots \sim\rightarrow [B(\boldsymbol{b})]$$

Here, $[A(\boldsymbol{a})]$ represents Alice's measuring instrument with unit length setting vector $\boldsymbol{a}$. The $[B(\boldsymbol{b})]$ represents Bob's measuring instrument with unit length setting vector $\boldsymbol{b}$. The $[S]$ is the source of the entangled particle pairs. The $\sim$ represent the respective particles from a given pair in flight towards the respective distant measurement instruments. The axis of propagation of the two $\sim$ is in the plane of the paper.

In the experiment, e.g. Weiss et al [5], one employs the counting of spin measurements to estimate Bell's correlation (2). This estimate is what is called a raw product moment (rpm) correlation coefficient. In the present context of our discussion, we employ the angle $x = \angle(\boldsymbol{a},\boldsymbol{b})$ between the setting vectors of Alice and Bob. For photons it suffices to look at the angle in the plane orthogonal to the plane of the axis of propagation of the two photons.

The angle x is in the interval $[0,2\pi) = \{x \in \mathbf{R}; 0 \leq x < 2\pi\}$. Bob doesn't know the setting $\boldsymbol{a}$ of Alice and Alice doesn't know the setting $\boldsymbol{b}$ of Bob. The angle $x = \angle(\boldsymbol{a},\boldsymbol{b})$ is a continuous variable.

In the experiment, spins for angle setting x are registered by Alice and Bob. Then, given angle $x \in [0,2\pi)$, the correlation formula for equal spin countings $N(x,=)$ and unequal spin countings $N(x,\neq)$ is

$$R(x) = \frac{N(x,\neq) - N(x,=)}{N(x,\neq) + N(x,=)} \tag{3}$$

In a formal way, in this formula we can discern two events. They are "observation of unequal spin under an angle $x \in [0,2\pi)$". These are the (idealised) registered pairs: Alice registered ± 1, Bob registered ∓ 1. And "observation of equal spin under angle $x \in [0,2\pi)$". Alice registered ± 1 and Bob registered ± 1.

Let us furthermore suppose ideal measurement such that $N = N(x,\neq) + N(x,=)$ and we may write

$$R(x) = 1 - 2\left(\frac{N(x,=)}{N}\right) \tag{4}$$

$N = N(x)$. A similar sort of situation is survival length in a time scale, which is a continuous variable, together with the excluding smoker or non-smoker. One can employ compound random variables to describe the situation. Further, let us establish that in (4) the $\frac{N(x,=)}{N}$ complies with the definition of a

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classical probability; i.e. frequency of the event "observation of equal spin under angle $x \in [0, 2\pi)$" divided by total number of observations under $x \in [0, 2\pi)$. Therefore, we may write

$$R(x) = 1 - 2P(x, =) \tag{5}$$

And $P(x, =)$ the probability to find equal spin under an angle $x \in [0, 2\pi)$ which is estimated in experiment by $\frac{N(x,=)}{N}$. This is a classical probability that complies to the Kolmogorov axioms [7, page 2]. Bell's correlation (2) is a classical probability representation of additional Einstein hidden parameters. Therefore its estimation via an rpm correlation (5) in experiment must likewise be a part of classical probability theory as well. We also may note that the quantum correlation, $Q(x)$ for photons depends on x as a cosine.

$$Q(x) = \cos(x) \tag{6}$$

The quantum correlation is based on the quantum wave function ψ.

2.1. Hypothesis

In the experiment the question then is asked if it is possible that the $Q(x) = \cos(x)$ from (6) can be approximated (epsilon close) with (5). It is easy trigonometry to acknowledge that

$$\cos(x) = 1 - 2\sin^2(x/2) \tag{7}$$

And so, combining (5) with (6) and (7), the hypothesis can be reformulated as

$$H_0 : P(x, =) = \sin^2(x/2) \tag{8}$$

The question then subsequently is whether or not hypothesis H_0 from (8) can be true at all. It requires a Kolmogorovian probability $P(x, =)$ to be equal to (epsilon close in experiment) $\sin^2(x/2)$.

Because $x \in [0, 2\pi)$ is a continuous variable, we can only conclude that the event "observation of equal spin under angle $x \in [0, 2\pi)$" is associated to a continuous random variable X. It is a well known fact from probability theory that the probability of a point in a continuous random variable distribution of values, is zero [8]. The probability in a continuous case is the area under the probability density f curve. E.g.

$$P(0 \leq X < x) = \int_0^x f(y)dy \tag{9}$$

Its principle complies with (1). The event $G_x \equiv$ "$N(x, =)$ number of equal spins registered for angle x in a total of N observations", is represented by a set of numerical values of a continuous random variable X such that $X \in [0, x) \subset [0, 2\pi)$. The probability of an interval for X is e.g. $P(u \leq X < v)$. Note that in Riemannian integration open endpoints are the limit of integration. There is, hence, no difference between $P(u \leq X < v)$ and $P(u \leq X \leq v)$.

The above implies that in fact the hypothesis in (8) contains $P(x, =) = P(0 \leq X < x) = \sin^2(x/2)$ for $x \in [0, 2\pi)$. As an illustration of [8], we have $P(x - \Delta x \leq X < x + \Delta x)$ and $\{x; x - \Delta x \leq X < x + \Delta x\} \subset [0, 2\pi)$. This is for required $\sin^2(x/2)$ equal to $\sin^2[\frac{1}{2}(x + \Delta x)] - \sin^2[\frac{1}{2}(x - \Delta x)]$. We therefore find that $P(x - \Delta x \leq X < x + \Delta x) \to 0$ for $\Delta x \to 0$.

This in turn also implies that [6], the probability density function $f(y)$ in (9) is

$$f(y) = \frac{1}{2}\sin(y) \tag{10}$$

for $y \in [0, 2\pi)$. Note that $\frac{d}{dx}\sin^2(x/2) = \frac{1}{2}\sin(x)$ and $\sin^2(x/2)$ is what is required to see that H_0 is true for arbitrary $x \in [0, 2\pi)$.

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However, this density function in (10) is impossible for a Kolmogorov classical probability because $f(y)$ is not positive definite for $y \in [0, 2\pi)$. Looking at the area under density curve, and the Additivity above. We then find, for $S_1 = [0, \pi)$, $S_2 = [\pi, 3\pi/2)$ we have $S_1 \cup S_2 = [0, 3\pi/2)$ and note, $S_1 \cap S_2 = \emptyset$. Under $\sin^2(x/2)$ we then find $P(S_1 \cup S_2) = P(S_1) + P(S_2) = \sin^2(3\pi/4) = \frac{1}{2}$. But $P(S_1) = \sin^2(\pi/2) = 1$ and so $1 + P(S_2) = \frac{1}{2}$, hence, $P(S_2) = -\frac{1}{2}$ and this concurs with $u = \pi$ and $v = 3\pi/2$

$$P(u \leq X < v) = \sin^2(v/2) - \sin^2(u/2) = \sin^2(3\pi/4) - \sin^2(\pi/2) = \\ = \left(-\frac{1}{\sqrt{2}}\right)^2 - 1 = \frac{1}{2} - 1 = -\frac{1}{2} < 0 \tag{11}$$

The function $\sin^2(x/2)$ is not monotone on $[0, 2\pi)$. One cannot require that a classical Kolmogorov probability [7, page 2, axiom V] must produce in experiment, a series of data where negative probabilities are necessary in order to obtain H_0 is true in (8).

It is also interesting to note that the universe interval $[0, 2\pi)$ has a probability $P(0 \leq X < 2\pi)$. When a $\sin^2(x/2)$ rule is required, it is found that $P(0 \leq X < 2\pi) = \sin^2(2\pi/2) - \sin^2(0/2) = 0 - 0 = 0$ instead of the required unity. This requirement is also contrary to the Kolmogorov axioms determining classical probability [7].

In addition, Alice and Bob are unaware of each other's setting vectors. The whole range for x is therefore possible. Hence, for all $x \in [0, 2\pi)$ we have $P(0 \leq X < x)$. Moreover, quantum probability density $|\psi(\boldsymbol{x})|^2 < 0$ does not exist and the quantum wave function $\psi(\boldsymbol{x})$ is not a direct observable probability.

Let's now turn to a possible discreteness or coarse graining. The claim that the angle is a discrete variable is also not leading to a genuine probability. Apart from the fact that for each possible $x \in [0, 2\pi)$ one is interested in Einstein extra variables, the following is true. Because $\sin^2(\pi/2) = 1$, it follows that sums of hypothetical discrete probabilities will possibly be > 1. I.e. there are discrete countable $\mathcal{X} = \{x_1, x_2, ...x_N\} \subset [0, 2\pi)$ possible such that

$$\sum_{x \in \mathcal{X} \cup \{\pi\}} \sin^2(x/2) > 1 \tag{12}$$

That is also obviously contrary to classical probability definitions; i.e. Certain event.

The conclusion is therefore that the angle most definitely reflects a continuous random variable with ditto statistics. Further, it turns out that this statistical approach is flawed. A probability below one single point in continuous density is zero [8]. There are no negative probabilities in experiment. Bell's starting point (2) complies to classical Kolmogorov probability theory, viz. (1). Modeling (2) with an implicit negative probability requirement, is a defective statistical design of the experiment.

3. Discussion & conclusion

In the paper it was demonstrated that the experiment of Bell *always* will find that the hypothesis "there are Einstein hidden extra parameters modeled as classical probability random variables", is false. This derives from a defective statistical design. The random variable X contains both the continuous angle x information as well as equal vs unequal spin. Note we are looking here at a continuous random variable associated to the angle $x = \angle(\boldsymbol{a}, \boldsymbol{b})$ and $x \in [0, 2\pi)$ with the equal spin characteristic "=" to describe the statistical event G_x. And also note that a view of a discrete random variable in the experiment, violates the principles of classical statistics as well.

Furthermore, we may observe that Bell's formulation starts with proper classical probability modeling of Einstein variables. If one requires in experiment violations of classical probability, then little wonder that one does not find Einstein in the data. This has got nothing to do with inequalities.

The absence of Einstein hidden variables concluded from Bell's experiment is not because those variables are not possible in nature. We don't find them because the experiment asks the impossible

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from classical probability. Negative probabilities do not occur in experiments. In terms of [10], when $U = [0, 2\pi)$, then, with proper sigma algebra Σ', the space $(U, \Sigma', \sin^2(x/2))$ is *not* a probability space.

Moreover, it is not possible to rightfully claim that the experiment demonstrates a quantum correlation. This is actually nothing more than an attempt to draw a conclusion from a flawed statistical procedure. It is crooked statistics when the starting point of classical probability (Λ, Σ, P_B), like in Bell's correlation formula, is required in experiment to behave as a non-probability space like $(U, \Sigma', \sin^2(x/2))$.

One may wonder if there is really a form of *probability* theory possible to justify the Bell experiment. We point out that a quantum wave function $\psi(\boldsymbol{x})$, even obtained from an ensemble of ψ measurements (if that's possible) at a certain location, is definitely not a direct observable *probability*.

Hence, there are valid reasons to claim that a Bell experiment is by necessity embedded in classical probability theory. And so our inevitable result is that the generally accepted conclusion "no go Einstein" from Bell's experiment, is based on flawed statistics.

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