

Supplementary material 1. List of 21 white matter tracts

A. Projection Fibers

- Anterior_limb_of_internal_capsule
- Posterior_limb_of_internal_capsule
- Posterior_thalamic_radiation
- Anterior_corona_radiata
- Superior_corona_radiata
- Posterior_corona_radiata
- External_capsule
- Retrolenticular_part_of_internal_capsule
- Pontine_crossing_tract

B. Limbic Fibers

- Cingulum_(cingulate_gyrus)
- Cingulum_(hippocampus)
- Fornix(cres)_Stria_terminalis

C. Association Fibers

- Superior_longitudinal_fasciculus
- Superior_fronto-occipital_fasciculus
- Inferior_fronto-occipital_fasciculus
- Sagittal_Stratum
- Uncinate_fasciculus

D. Callosal Fibers

- Genu_of_corpus_callosum
- Body_of_corpus_callosum
- Splenium_of_corpus_callosum
- Tapatum

Supplementary material 2: Nonlinear least square optimization

The nonlinear sigmoidal growth function in Model (1) can be fitted by iterative nonlinear least square optimization algorithms such as Gauss-Newton and Levenberg-Marquardt algorithm (10) (8-10). We used the latter algorithm since it provides better convergence due to its flexible step size at each iteration. The iteration is based on a linear approximation with the first order Taylor series expansion of a likelihood function of Model (1) at $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)' = \boldsymbol{\beta}^0$, that is,

$$f(\mathbf{x}_i; \boldsymbol{\beta}) = f(\mathbf{x}_i; \boldsymbol{\beta}_0) + \tilde{\mathbf{X}}_i(\boldsymbol{\beta}_0)(\boldsymbol{\beta} - \boldsymbol{\beta}_0), \quad (\text{S1})$$

where $\tilde{\mathbf{X}}_i = [\partial f(\mathbf{x}_i; \boldsymbol{\beta}) / \partial \boldsymbol{\beta}]$ and $\boldsymbol{\beta}^0$ is the initial values of the three β coefficients. Specifically, the iteration starts with the initial values $\boldsymbol{\beta} = \boldsymbol{\beta}^0$ and updates $\boldsymbol{\beta}$ after each iteration until convergences of all of the parameters are met in Equation (S1). Each row of $\tilde{\mathbf{X}}$ represents the gradient of the nonlinear curve at the i -th observation and thus $\tilde{\mathbf{X}}$ is a Jacobian matrix. The least square estimate that minimizes the sum of squares of residuals is the same as the maximum likelihood estimate (MLE) under Gaussian assumption. Since an MLE estimated from an iterative search algorithm can be a local maximum of the likelihood function, we used 100 different sets of starting values of parameters and then selected the parameter set that maximizes the log-likelihood function value. However, it is possible that optimization of nonlinear equation fails to converge for some datasets with all 100 sets of starting values. Such failures were suspected due to the over-parameterization which makes some parameters in Model (1) redundant and thus makes Jacobian matrix singular.

Supplementary material 3: Likelihood ratio test

We use a likelihood ratio test (LRT) to test significance of parameters since LRT is robust to potentially non-symmetric sampling distribution of the parameter estimates compared with other tests such as a Wald test as demonstrated and small samples in (8). The LRT statistics for testing $H_0: \beta_j = \beta_j^0$ is expressed as below.

$$T_1 = l(\hat{\boldsymbol{\beta}}) - l(\hat{\boldsymbol{\beta}}_{-j}; \beta_j = \beta_j^0) \sim \frac{1}{2} \chi^2(1), (j=1, 2, 3) \quad (\text{S.2})$$

where $\hat{\boldsymbol{\beta}}$ is the MLE for the entire p -dimensional column vector of $\boldsymbol{\beta}$, and $\hat{\boldsymbol{\beta}}_{-j}$ is the MLE of $\boldsymbol{\beta}$ with fixed $\hat{\beta}_j = \beta_j^0$. The sampling distribution of $2 \times T_1$ in Equation (S.2) is the chi-square distribution with degrees of freedom equal to one, denoted by $\chi^2(1)$. A $(1-\alpha) \times 100\%$ profile likelihood confidence interval (C.I.) for β_j can then be constructed from the sampling distribution as follows.

$$\{\beta_j^0 \mid l(\hat{\boldsymbol{\beta}}) - l(\hat{\boldsymbol{\beta}}_{-j}; \beta_j = \beta_j^0) \leq \frac{1}{2} \chi_{1-\alpha}^2(1)\}. (j=1, 2, 3)$$

Supplementary material 4: Model Comparison

A. Model Comparison based on MSE

WM Track	MSE			
	Linear	Power	Sigmoid	Diff
Posterior_thalamic_radiation	692.3	686.7	640	52.3
Cingulum_cingulate_gyrus	957	954.1	920.5	36.5
Retrolecticular_part_of_internal_capsule	775.1	768.4	748.2	26.9
Body_of_corpus_callosum	1465.9	1465.8	1447.8	18.1
Posterior_corona_radiata	706	702.9	688.8	17.2
Uncinate_fasciculus	1547.8	1543.8	1533.2	14.6
Cingulum_hippocampus	966.4	966.4	958.4	8
Posterior_limb_of_internal_capsule	832.5	832.1	828.6	3.9
Splenium_of_corpus_callosum	925	924.4	921.5	3.5
Inferior_fronto-occipital_fasciculus	693.7	692	690.7	3
Sagittal_Stratum	628.9	625	629.5	-0.6
Fornixres _Stria_terminalis	696.3	695.3	698.1	-1.8
Superior_longitudinal_fasciculus	739.2	735.6	741.1	-1.9
Anterior_corona_radiata	655.8	653.3	659.3	-3.5
Anterior_limb_of_internal_capsule	661.9	661.6	669.3	-7.4
Genu_of_corpus_callosum	1191.3	1191.2	1199.8	-8.5
Superior_corona_radiata	531.8	531.8	541.3	-9.5
External_capsule	509.2	508.4	521.3	-12.1
Superior_fronto-occipital_fasciculus	1870.9	1870.4	1889.5	-18.6
Pontine_crossing_tract	1152.1	1147.5	1175.5	-23.4
Tapatum	2542.4	NA	2580.4	-38

1. Diff= linear model – threshold model
2. NA: Nonlinear fit failed to converge.
3. White matter tracts were ordered by decreasing order of difference.

B. Model Comparison based on AIC

WM Track	AIC			
	Linear	Power	Threshold	Diff
Posterior_thalamic_radiation	997	998.1	991.7	5.3
Cingulum_cingulate_gyrus	1031.3	1033	1030.2	1.1
Retrolenticular_part_of_internal_capsule	1009	1010.1	1008.2	0.8
Posterior_corona_radiata	999.1	1000.6	999.4	-0.3
Body_of_corpus_callosum	1076.5	1078.5	1078.2	-1.7
Uncinate_fasciculus	1082.3	1084	1084.3	-2
Cingulum_hippocampus	1032.4	1036.4	1034.5	-2.1
Posterior_limb_of_internal_capsule	1016.6	1018.5	1019	-2.4
Inferior_fronto-occipital_fasciculus	997.2	999	999.7	-2.5
Splenium_of_corpus_callosum	1027.7	1029.7	1030.3	-2.6
Sagittal_Stratum	986.8	988.2	989.9	-3.1
Superior_longitudinal_fasciculus	1004	1005.4	1007.2	-3.2
Fornixres_Stria_terminalis	997.6	999.5	1000.9	-3.3
Anterior_corona_radiata	991.3	992.9	994.8	-3.5
Genu_of_corpus_callosum	1054.6	1056.5	1058.3	-3.7
Superior_fronto-occipital_fasciculus	1102.4	1104.4	1106.4	-4
Anterior_limb_of_internal_capsule	992.3	994.21	996.4	-4.1
Tapatum	1134.9	NA	1139.4	-4.5
Superior_corona_radiata	969.1	971.1	973.9	-4.8
Pontine_crossing_tract	1051	1052.6	1056.1	-5.1
External_capsule	964.5	966.3	969.9	-5.4

1. Diff= linear model – threshold model
2. NA: Nonlinear fit failed to converge.
3. White matter tracts were ordered by decreasing order of difference.

C. Model Comparison based on BIC

WM Track	BIC			
	Linear	Power	Threshold	Diff
Posterior_thalamic_radiation	1005	1008.8	1005	0
Cingulum_cingulate_gyrus	1039.3	1043.7	1043.5	-4.2
Retrolenticular_part_of_internal_capsule	1017	1020.7	1021.5	-4.5
Posterior_corona_radiata	1007.1	1011.3	1012.8	-5.7
Body_of_corpus_callosum	1084.5	1089.2	1091.5	-7
Uncinate_fasciculus	1090.3	1094.7	1097.6	-7.3
Cingulum_hippocampus	1040.4	1049.7	1047.8	-7.4
Posterior_limb_of_internal_capsule	1024.5	1029.2	1032.3	-7.8
Inferior_fronto-occipital_fasciculus	1005.2	1009.6	1013	-7.8
Splenium_of_corpus_callosum	1035.7	1040.3	1043.6	-7.9
Sagittal_Stratum	994.8	998.8	1003.2	-8.4
Superior_longitudinal_fasciculus	1012	1016.1	1020.5	-8.5
Fornixres_Stria_terminalis	1005.6	1010.1	1014.2	-8.6
Anterior_corona_radiata	999.3	1003.5	1008.1	-8.8
Genu_of_corpus_callosum	1062.5	1067.2	1071.6	-9.1
Superior_fronto-occipital_fasciculus	1110.4	1115	1119.7	-9.3
Anterior_limb_of_internal_capsule	1000.2	1004.9	1009.7	-9.5
Tapatum	1142.9	NA	1152.8	-9.9
Superior_corona_radiata	977.1	981.7	987.2	-10.1
Pontine_crossing_tract	1059	1063.2	1069.4	-10.4
External_capsule	972.4	976.9	983.2	-10.8

1. Diff= linear model – threshold model
2. NA: Nonlinear fit failed to converge.
3. White matter tracts were ordered by decreasing order of difference.