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Marcos Cardoso Rodriguez and Ion Vasile Vancea *

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Article

Neutrino Masses in Supersymmetric Models with *R*-Symmetry

Marcos Cardoso Rodriguez [1,†] and Ion Vasile Vancea [1,*]

[1] Grupo de Física Teórica e Matemática Física, Departamento de Física, Universidade Federal Rural do Rio de Janeiro, BR 465 Km 7, 23890-000, Seropédica, RJ, Brazil; marcoscrodriguez@ufrrj.br

* Correspondence: ionvancea@ufrrj.br

† These authors contributed equally to this work.

Abstract: In this article, we give a brief review of the origin of the neutrino mass in some interesting non-linear supersymmetric models with R-symmetry. These models are able to address and solve the most important problems of the particle physics and provide mechanisms for neutrino mass generation and their mixing parameters in agreement with the current experimental data. Their prediction could be experimentally tested in the near future by collider experiments.

Keywords: neutrino mass; supersymmetric models; R-symmetry

1. Introduction

The internal symmetries of the Standard Model (SM) are described by the gauge group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{1}$$

where the subscripts C, L and Y refer to color, left chirality and weak hypercharge, respectively. At the weak scale, the electroweak symmetry subgroup

$$SU(2)_L \otimes U(1)_Y \tag{2}$$

is spontaneously broken to

$$SU(3)_C \otimes U(1)_{em}. \tag{3}$$

This spontaneous symmetry breakdown is driven by an $SU(2)_L$ doublet of the scalar Higgs field defined as

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, 1), \tag{4}$$

with the following vacuum expectation value (vev)

$$< H > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{5}$$

The Yukawa coupling

$$Y_f H \bar{f}_R f_L + hc, \tag{6}$$

induces Dirac masses for all charged leptons except for neutrinos which is massless at all perturbative level due to the lack of a right-handed neutrino component.

The SM successfully describes the particle phenomenology across the energy scales probed by Large Hadron Collider (LHC). However, SM also has some relevant problems such as

1. The coupling constants do not meet at a single definite value.
2. The mass hierarchy problem ($M_H \ll M_{Pl}$).

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3. The naturalness or fine tuning problem.
4. The CP-violation and matter anti-matter asymmetry.

Within the framework of particle physics, one promising class of theories that could solve the problems of the SM is formed by the supersymmetric (SUSY) extensions of the SM. SUSY arose in theoretical papers more than 30 years ago with the Minimal Supersymmetric Standard Model (MSSM) proposed by Pierre Fayet in [1]. (see, for more details, e. g. [1,2]).

The arguments in favour of SUSY are based on the belief that more fundamental a theory is, a higher internal symmetry it should have [3–5]. This construct is amply used in string theory where SUSY is promoted to a local symmetry and the supergravity obtained in this way represents a formal unification of gravity with the field theory. However, the SUSY is useful to particle physics without involving the string theory, as it provides a simple and natural mechanism for the cancelling of the quadratic divergences. Indeed, since SUSY is a symmetry between the bosonic states $|B\rangle$ and the fermionic states $|F\rangle$, there is a negative sign fermionic contribution at one-loop to the radiative correction of the scalar mass

$$\delta m_s^2 = O\left(\frac{\alpha}{2\pi}\right)\left(\Lambda^2 + m_B^2\right) - O\left(\frac{\alpha}{2\pi}\right)\left(\Lambda^2 + m_F^2\right) = O\left(\frac{\alpha}{2\pi}\right)\left(m_B^2 - m_F^2\right). \quad (7)$$

The cutoff $\Lambda \sim M_P$ no longer needs fine tuning across a 10^{32} energy scale. In order to stabilize the mass hierarchy, one has to set $|m_B^2 - m_F^2| \sim 1\text{TeV}^2$ (electroweak scale) which is the main reason to search for masses of superparticles in this range. This has an additional benefit, since it is a predicted mass scale by some Dark Matter models, where the best candidates are the lighest neutralinos or sneutrinos. Among other useful features of SUSY, one should mention its helpfulness in solving the coupling constant problem in the framework of the Grand Unification Theories (GUT). As is well known, GUT models based on $SU(5)$ and $SO(10)$ gauge groups do not have an intersection point of the three gauge couplings, display a proton instability and have a GUT hierarchy problem that involves the electroweak (light) and the GUT (heavy) Higgs bosons. In the supersymmetric GUT models, the first two problems above are improved by placing the predictions within the experimental bounds while the GUT hierarchy problem is completely solved.

Also, concerning the CP-violation, it is known that in the SM, the CP-violation originates in the quark sector. On the other hand, in the SUSY models, there are several new CP-violation phases, coming from gluino [6] and neutralino [4] sectors as well as from the Higgs sector [7,8].

Another reason to search for theories and models beyond the SM are some experimental and observational results which have not found a satisfactory explanation within the SM framework, e. g.

1. Neutrinos are massive and they oscillate.
2. The muon anomalous magnetic moment.
3. Dark Matter.
4. Dark Energy.

The shortcomings of the SM invite to build extensions of it that could solve its problems. From a different angle, a fundamental theory should predict the unification of the gravitational interaction with the other three fundamental interactions and should address all the above mentioned puzzles as well as questions usually raised in General Relativity such as the Dark Energy which can be addressed and possibly solved in SUSY models [9,10].

In this paper, we review the relation between the neutrinos and the R-symmetry in several non-minumal SUSY models: MSSM with R-Parity violation, NMSSM, MSSM3RHN, $\mu\nu$SM, SUSYB-L, SUSYLR and two SUSYGUTS models. (For a detailed discussion of one of these models see [12]). With the recent experimental bounds in the minimal SUSY models from the LHC, there has been a revival of interest in non-minimal models with R-symmetry [13,14]. Beside addressing the neutrino problems, the R-symmetric models also provide a fruitful framework to model Dark Matter with gaugino acquiring mass through R-symmetry breaking process [15].

2. Neutrinos Mass

Historically, the main phenomenological reason to introduce the neutrinos mass was the solar neutrino problem which basically states that the ration between the flux of electron neutrinos detected by the electron neutrinos predicted is roughly around one half, but can be as low as one third [16] (for a recent review, see e. g. [17]). Several crucial experiments have supported the idea that there are three neutrino flavours of low masses at eV scale ($< 0,12$ eV) and they participate to weak and gravitational interactions. Since the neutrino is a spin-half electrically neuter particle, it can be described by a Majorana field with the anti-neutrino characterized by the opposed helicity. The unknown nature of the neutrino that could be a Dirac or a Majorana spinor is under current experimental investigation at CUORE [18], GERDA [19], MAJORANA [20], SNO+ [21] and EXO [22]. The experimental results suggest that the neutrinos have non-zero masses and oscillations. The best-fit values at 1σ error level for the neutrino oscillation parameters in the three-flavour framework are [23]

$$\begin{aligned}
\sin^2\theta_{12} &= \sin^2\theta_{solar} = 0.304^{+0.022}_{-0.016}\,, \Delta m^2_{21} = \Delta m^2_{solar} = 7.65^{+0.23}_{-0.20}\times 10^{-5}\ \mathrm{e}V^2,\\
\sin^2\theta_{23} &= \sin^2\theta_{atm} = 0.50^{+0.07}_{-0.06}\,, |\Delta m^2_{23}| = \Delta|m^2_{atm}| = 2.40^{+0.12}_{-0.11}\times 10^{-3}\ \mathrm{e}V^2,\\
\sin^2\theta_{13} &= \sin^2\theta_{CHOOZ} = 0.01^{+0.016}_{-0.011}\,. \qquad (8)
\end{aligned}$$

3. R-Symmetry

We start off by recalling the important remark that the SM has two accidental symmetries $U(1)_L$ and $U(1)_B$, namely the lepton and baryon numbers, besides its gauge symmetries. The $U(1)_L$ and $U(1)_B$ are responsible for the stability of the nucleons under decay into light leptons and the existence of Dirac neutrinos. In general, the conservation of $U(1)_L$ implies that the neutrinos are Dirac, while if the $U(1)_L$ is broken, the breaking pattern determines the nature of the neutrinos [24]. In the SM model, all the interactions conserve both discrete symmetries. However, models beyond SM do not need to conserve $U(1)_L$ or $U(1)_B$ as is the case with several SUSY models that contain interactions that violate both $U(1)_L$ or $U(1)_B$, or both. Therefore, one has to impose a discrete symmetry to get a model where all the interactions do not violate L and B conservation. This discrete symmetry is known today as R-symmetry and it was introduced independently by A. Salam and J. Strathdee [13] and P. Fayet [14].

Let us recall the R-symmetry. Consider the commutation relations involving the fermionic generators of the super-Poincaré algebra

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} = 0\,,\\
[Q_\alpha, P_m] &= [\bar{Q}_{\dot\alpha}, P_m] = 0\,,\\
\{Q_\alpha, \bar{Q}_{\dot\alpha}\} &= 2\sigma^m_{\alpha\dot\alpha} P_m\,,\\
[M_{mn}, Q_\alpha] &= i\,(\sigma_{mn})_\alpha{}^\beta Q_\beta\,,\\
\left[M_{mn}, \bar{Q}^{\dot\alpha}\right] &= i\,(\bar\sigma_{mn})^{\dot\alpha}{}_{\dot\beta}\bar{Q}^{\dot\beta}\,. \qquad (9)
\end{aligned}$$

The R-symmetry $U(1)_R$ is a symmetry of the super-algebra (9) defined by the following commutation relations

$$[P_m, R] = [M_{mn}, R] = 0\,, \qquad (10)$$
$$[Q_\alpha, R] = Q_\alpha\,, \qquad (11)$$
$$\left[\bar{Q}^{\dot\alpha}, R\right] = -\bar{Q}^{\dot\alpha}\,. \qquad (12)$$

From the above commutators we conclude that

$$Q_\alpha \to Q'_\alpha = e^{-i\delta} Q_\alpha\,, \qquad (13)$$
$$\bar{Q}_{\dot\alpha} \to \bar{Q}'_{\dot\alpha} = e^{i\delta}\bar{Q}_{\dot\alpha}\,, \qquad (14)$$

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which means that the R-charges of Q and $\bar{Q}$ are -1 and $+1$, respectively.

In order to construct models with R-symmetry, it is necessary to define the action of the $U(1)_R$ operator on the superspace and superfields. Let us denote the generator of R-symmetry by $\mathbf{R}$ and the superspace coordinates by $\{x, \theta, \bar{\theta}\}$ [25]. Then the action of $\mathbf{R}$ on the fermionic coordinates θ and $\bar{\theta}$ is given by the following relations

$$\mathbf{R}: \quad \theta \to \theta' = e^{-i\delta}\theta\,, \qquad \bar{\theta} \to \bar{\theta}' = e^{i\delta}\bar{\theta}. \tag{15}$$

Hence, the R-charges are $\mathrm{R}(\theta) = -1$ and $\mathrm{R}(\bar{\theta}) = 1$. These charges are summarized in the following table

mathematical objects	Q	$\bar{Q}$	θ	$\bar{\theta}$	$d^2\theta$	$d^2\bar{\theta}$
R-charges	-1	$+1$	$+1$	-1	-2	$+2$.

(16)

The superfields on the superspace are the basic objects of the SUSY models. The operator $\mathbf{R}$ acts on the chiral $\Phi(x, \theta, \bar{\theta})$, anti-chiral $\bar{\Phi}(x, \theta, \bar{\theta})$ and vector superfields $V(x, \theta, \bar{\theta})$ as follows

$$\begin{aligned}
\mathbf{R}\Phi(x,\theta,\bar{\theta}) &= e^{2in_\Phi\delta}\Phi(x, e^{-i\delta}\theta, e^{i\delta}\bar{\theta}), \\
\mathbf{R}\bar{\Phi}(x,\theta,\bar{\theta}) &= e^{-2in_\Phi\delta}\bar{\Phi}(x, e^{-i\delta}\theta, e^{i\delta}\bar{\theta}), \\
\mathbf{R}V(x,\theta,\bar{\theta}) &= V(x, e^{-i\delta}\theta, e^{i\delta}\bar{\theta}).
\end{aligned} \tag{17}$$

where $2n_\Phi$ is the R-charge of the above chiral superfield while the vector superfield is invariant under this symmetry. The above relations show that in general the terms of the superpotential have charges 2, which is two times the charge of $d\theta$. Here and in what follows we are using the conventions and basic results from the standard reference [25].

The relations (17) show that proposals of discrete R-symmetries based on $\mathbf{Z}^N$ groups are constrained by the allowed values of the R-charges of superfields and their components. Consider, for example, the chiral superfield defined by the equation

$$\bar{D}_{\dot{\alpha}}\Phi = 0\,, \tag{18}$$

which has the following decomposition

$$\Phi = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)\,. \tag{19}$$

The relation between the charges $R(\Phi)$ and $R(A)$, $R(\psi)$ and $R(F)$ is the following

$$R(A) = R(\Phi)\,, \qquad R(\psi) = R\Psi - R(\theta)\,, \qquad R(F) = R(\psi) - 2R(\theta)\,. \tag{20}$$

Thus, the superpotential that corresponds to the $\theta\theta$ factor has charge either $0 \mod 2$ or $2R(\theta) \mod 2$ which prohibits the use of $\mathbb{Z}^2$ as R-symmetry group.

The Master Lagrangian of a supersymmetric model has the following general form

$$\int d^4x \left\{ \int d^4\theta K(\bar{\Phi}_i e^{2gV}, \Phi_j) + \left[\int d^2\theta \left(W(\Phi) + \frac{1}{4} W^\alpha W_\alpha \right) + hc \right] \right\}. \tag{21}$$

where the first term is known as the *Kähler potential* and has the general form $K(\bar{\Phi}, \Phi)$, $W(\Phi)$ is the superpotential and the last term describes the supersymmetric Yang-Mills action with

$$W_\alpha = -\frac{1}{8g}\bar{D}\bar{D}e^{-2gV}D_\alpha e^{2gV}, \tag{22}$$

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{8g}DD\bar{D}_{\dot{\alpha}}e^{-2gV}e^{2gV}. \tag{23}$$

The Master Lagrangian is invariant under R-Symmetry. By expanding the action from the Equation (21), we obtain

$$\int d^4x \left[\int d^4\theta \bar{\Phi}_i \Phi_i + d^2\theta \left(\lambda_i \Phi_i + \frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{g_{ijk}}{3} \Phi_i \Phi_j \Phi_k \right) + hc + \dots \right] , \tag{24}$$

where m_{ij} and g_{ijk} are parameters symmetric in all their indices and $\dots$ stands for terms from the Yang-Mills lagrangian. The Equation (24) displays the invariance of the Käller potential under R-symmetry. On the other hand, the R-symmetry can be broken by the superpotential term.

One important SUSY model is the supersymmetric quantum electrodynamics that contains two chiral superfields that have the following transformation properties under the R-symmetry

$$\begin{aligned} \Phi'_+ &= e^{-2ie\Lambda} \Phi_+, \ \mathbf{R}\Phi_+ = e^{2in} \Phi_+, \\ \Phi'_- &= e^{2ie\Lambda} \Phi_-, \ \mathbf{R}\Phi_- = e^{-2in} \Phi_- . \end{aligned} \tag{25}$$

Here, Λ is another chiral superfield and e is the $U(1)$ charge. The lagrangian of the supersymmetric quantum electrodynamics is given by the following equation [25]

$$\begin{aligned} \mathcal{L}_{SQED} &= \frac{1}{4} \left(\int d^2\theta W^\alpha W_\alpha + \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \right) + \int d^4\theta \bar{\Phi}_+ e^{2eV} \Phi_+ + \int d^4\theta \bar{\Phi}_- e^{2eV} \Phi_- \\ &+ M \left(\int d^2\theta \Phi_+ \Phi_- + \int d^2\bar{\theta} \bar{\Phi}_+ \bar{\Phi}_- \right) . \end{aligned} \tag{26}$$

One can easily verify that $\mathcal{L}_{SQED}$ is invariant under R-Symmetry defined by the Equations (17) and (25) above.

3.1. Continuous R-Symmetry in MSSM

The simplest SUSY model is the MSSM that extends the field content of the SM by a minimal set of fields and has been used as a main model in the investigations of the supersymmetry. The superpotential of MSSM has two terms

$$\begin{aligned} W_2 &= \mu\epsilon \hat{H}_1 \hat{H}_2 + \mu_{0a} \epsilon \hat{L}_{aL} \hat{H}_2, \\ W_3 &= f^l_{ab} \epsilon \hat{L}_{aL} \hat{H}_1 \hat{E}_{bR} + f^u_{ij} \epsilon \hat{Q}_{iL} \hat{H}_2 \hat{U}_{jR} + f^d_{ij} \epsilon \hat{Q}_{iL} \hat{H}_1 \hat{D}_{jR} \\ &+ \lambda_{abc} \epsilon \hat{L}_{aL} \hat{L}_{bL} \hat{E}_{cR} + \lambda'_{iaj} \epsilon \hat{Q}_{iL} \hat{L}_{aL} \hat{D}_{jR} + \lambda''_{ijk} \hat{D}_{iR} \hat{U}_{jR} \hat{D}_{kR}. \end{aligned} \tag{27}$$

In order to analyse the R-symmetry of MSSM, one makes the assumption that

$$\begin{array}{ll|l} \text{for}: & H_1,\ H_2 & n_{H_1} = n_{H_2} = 0 \\ \hline \text{for}: & Q,\ U,\ D\ L,\ E & n_Q = n_u = n_d = n_L = n_l = \frac{1}{2} . \end{array} \tag{28}$$

Then the R-symmetry acts on the components of the fields as follows

$$\begin{aligned} H_{1,2}(x) &\overset{\mathbf{R}}{\longmapsto} H_{1,2}(x) , \quad \tilde{H}_{1,2}(x) \overset{\mathbf{R}}{\longmapsto} e^{-i\alpha} \tilde{H}_{1,2}(x), \\ \tilde{f}_L(x) &\overset{\mathbf{R}}{\longmapsto} e^{i\alpha} \tilde{f}_L(x) , \quad \tilde{f}_R(x) \overset{\mathbf{R}}{\longmapsto} e^{-i\alpha} \tilde{f}_R(x), \\ \Psi(x) &\overset{\mathbf{R}}{\longmapsto} \Psi(x). \end{aligned} \tag{29}$$

This shows that the scalars $H_{1,2}$ and all the standard fermions are invariant under R-symmetry. One can easily see that the conserving R-symmetry terms from the superpotential are

$$W = \mu\epsilon \hat{H}_1 \hat{H}_2 + f^l_{ab} \epsilon \hat{L}_{aL} \hat{H}_1 \hat{E}_{bR} + f^u_{ij} \epsilon \hat{Q}_{iL} \hat{H}_2 \hat{U}_{jR} + f^d_{ij} \epsilon \hat{Q}_{iL} \hat{H}_1 \hat{D}_{jR} . \tag{30}$$

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It is important to note here that, in this case, all charged fermions get masses at tree-level. However, the neutrinos are massless as in the SM.

The neutrinos can be given masses in the presence of the unbroken R-symmetry in a Minimal R-Supersymmetric Standard Model (MRSSM) that generalizes MSSM. In MRSSM, both $H_{d,u}$ have $n_H = 0$ and the particle content of the MSSM is enlarged in the following way

$$\begin{aligned} \hat{R}_d &\sim \left(\mathbf{1}, \mathbf{2}, +\frac{1}{2}\right), \; n_{R_d} = 2, \\ \hat{R}_u &\sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right), \; n_{R_u} = 2, \end{aligned} \tag{31}$$

The adjoint chiral superfields $\hat{O}, \hat{T}$ and $\hat{S}$ and their R-charge are zero. The superpotential of this model is

$$\begin{aligned} W &= \mu_d \left(\hat{R}_d \cdot \hat{H}_1\right) + \mu_u \left(\hat{R}_u \cdot \hat{H}_2\right) + \Lambda_d \left(\hat{R}_d \cdot \hat{T}\right) \hat{H}_1 + \Lambda_u \left(\hat{R}_u \cdot \hat{T}\right) \hat{H}_2 + \lambda_d \hat{S} \left(\hat{R}_d \cdot \hat{H}_1\right) \\ &+ \lambda_u \hat{S} \left(\hat{R}_u \cdot \hat{H}_2\right) - f^l_{ab} \epsilon \hat{L}_{aL} \hat{H}_1 \hat{E}_{bR} - f^u_{ij} \epsilon \hat{Q}_{iL} \hat{H}_2 \hat{U}_{jR} - f^d_{ij} \epsilon \hat{Q}_{iL} \hat{H}_1 \hat{D}_{jR} , \end{aligned} \tag{32}$$

where the complex triplet is defined as follows

$$\hat{T} = \frac{1}{-} \begin{pmatrix} \frac{\hat{T}_0}{\sqrt{2}} & \hat{T}_+ \\ \hat{T}_- & -\frac{\hat{T}_0}{\sqrt{2}} \end{pmatrix} . \tag{33}$$

In MRSSM, the neutrinos acquire Majorana masses [26–29]. The Majorana neutrino mass term is given by

$$H_u H_u L_i L j. \tag{34}$$

3.2. A Problem of Continuous R-Symmetry. Discrete R-Parity

We already mentioned that the vector superfield V is invariant under R-Symmetry. However, the field components of the vector superfield transform as

$$\begin{aligned} A_m(x) &\xmapsto{\mathbf{R}} A_m(x) \\ \lambda(x) &\xmapsto{\mathbf{R}} e^{i\alpha}\lambda(x) \\ \bar{\lambda}(x) &\xmapsto{\mathbf{R}} e^{-i\alpha}\bar{\lambda}(x) \\ D(x) &\xmapsto{\mathbf{R}} D(x) \end{aligned} . \tag{35}$$

In order to break the supersymmetry, Girardello introduced a mass term for gauginos of the following form [30]

$$m_\lambda \left(\lambda\lambda + \bar{\lambda}\bar{\lambda}\right) , \tag{36}$$

that transforms under the R-symmetry (35) as follows

$$m_\lambda \left(e^{2i\alpha}\lambda\lambda + e^{-2i\alpha}\bar{\lambda}\bar{\lambda}\right) . \tag{37}$$

From the relations (35) one can see that the mass term given by the Equation (36) is not invariant under the R-symmetry.

The soft breaking of supersymmetry suggests to replace the continuous R-symmetry, by a discrete R-symmetry called R-parity denoted by $\mathbf{R}_d$ which solves the above problem by setting $\alpha = \pi$. In the supersymmetry models with R-parity, the gluinos and other gauginos become massive.

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For the following values of R-charges

$$
\begin{aligned}
n_{H_1} &= 0,\ n_{H_2} = 0,\ n_L = \frac{1}{2},\ n_Q = \frac{1}{2}, \\
n_E &= -\frac{1}{2}, n_U = -\frac{1}{2},\ n_D = -\frac{1}{2},
\end{aligned} \tag{38}
$$

the allowed terms in the superpotential are

$$
W = \mu\epsilon\hat{H}_1\hat{H}_2 + f^l_{ab}\epsilon\hat{L}_a\hat{H}_1\hat{l}^c_b + f^u_{ij}\epsilon\hat{Q}_i\hat{H}_2\hat{u}^c_j + f^d_{ij}\epsilon\hat{Q}_i\hat{H}_1\hat{d}^c_j. \tag{39}
$$

The above superpotential defines the known MSSM, and as in Equation (30) all the neutrinos are massless at all orders of perturbations. However, by choosing the following charges

$$
\begin{aligned}
n_{H_1} &= n_{H_2} = n_L = n_l = 0, \\
n_Q &= -n_u = -n_d = \frac{1}{2},
\end{aligned} \tag{40}
$$

the terms in the superpotential allowed by this new R-parity are the following ones

$$
\begin{aligned}
W_H &= \mu\epsilon\hat{H}_1\hat{H}_2 + \mu_{0a}\epsilon\hat{L}_a\hat{H}_2 + f^l_{ab}\epsilon\hat{L}_a\hat{H}_1\hat{l}^c_b + f^u_{ij}\epsilon\hat{Q}_i\hat{H}_2\hat{u}^c_j + f^d_{ij}\epsilon\hat{Q}_i\hat{H}_1\hat{d}^c_j \\
&+\lambda_{abc}\epsilon\hat{L}_a\hat{L}_b\hat{l}^c_c + \lambda'_{iaj}\epsilon\hat{Q}_i\hat{L}_a\hat{d}^c_j\,.
\end{aligned} \tag{41}
$$

As shown in [31–33], the superpotential (41) generates neutrino masses as showed numerically at [34]. We can, also, reproduce the mixing parameters [35].

The R-parity plays an important role in the baryon B and lepton L numbers violation in the MSSM. The most general superpotential of the MSSM contains interacting terms that violate the baryon and lepton numbers. In order to remove these terms that are not allowed by the SM, the R-parity is introduces. The general form of the R-parity in terms of B and L numbers is

$$
R = (-1)^{3(B-L)+2S} = (-1)^{2S}M,
$$

where M is the matter parity and takes the following values

$$
\begin{array}{cc|c}
M = +1 & \text{for} & \text{Higgs and Gauge superfields} \\
\hline
M = -1 & \text{for} & \text{Matter superfields}\,.
\end{array} \tag{42}
$$

This remark is important for the construction of the SUSYB-L models.

3.3. Nelson-Seiberg Theorem

Since the SUSY is not directly observed in nature, one has to consider that it is a broken symmetry. One important class of models that display the dynamical SUSY breaking of strongly-coupled gauge theories at low-energies are the generalized O'Raifeartaigh models which are weakly-coupled Wess-Zumino models in which the SUSY is broken by vacuum expectation values of tree-level F-term [36]. The generalized O'Raifeartaigh models are a particular case of generic calculable models that obey the Nelson-Seiberg theorem [37]

Theorem *(Nelson-Seiberg) The necessary and sufficient conditions for SUSY breaking at the true vacuum in a Wess-Zumino model with a generic superpotential are:*

- *Necessary: The model must have an R-symmetry.*
- *Sufficient: The R-symmetry should be spontaneously broken.*

Here, the term *generic* refers to the property that the superpotential must contain all renormalizable terms with complex coefficients. The Nelson-Seiberg theorem has been revisited lately in [38,39] and counterexamples have been found in [40,41].

The theorem relates the global $U(1)_R$ to the SUSY- breaking by the following argument. Since the superpotential R-charge is 2, then it has the following form

$$W = \phi_n^{\frac{2}{R(\phi_n)}} g\left(\frac{\phi_i}{\langle\phi_i\rangle^{\frac{R(\phi_i)}{R(\phi_n)}}}\right), \tag{43}$$

where $i = 1, 2, \ldots, n$ denotes the fields. However, the SUSY vacuum is a stationary point of the system

$$\frac{\partial W(\phi_i)}{\partial \phi_j} = 0, \tag{44}$$

for all i, j. Then it is easy to see that the effective equations (44) reduce to a system of $n-1$ equations with n unknowns which does not admit a solution, in general. Therefore, one cannot determine a supersymmetric vacuum. That leads to the conclusion that in the presence of the R-symmetry, the SUSY must be spontaneously broken.

4. Next to Minimal Standard Model

The NMSSM is characterized by the following new singlet superfielfield

$$\hat{S} \sim (\mathbf{1}, \mathbf{1}, 0), \tag{45}$$

given as a chiral superfield [12]

$$\hat{S}(y,\theta) = S(y) + \sqrt{2}\theta\tilde{S}(y) + \theta\theta F_S(y). \tag{46}$$

Here, the vev of the scalar field S is taken to be

$$\langle S\rangle \equiv \frac{x}{\sqrt{2}}. \tag{47}$$

The fermionic field $\tilde{S}$, defined by the Equation (46), is known as *singlino*.

The most general superpotential that contains the singlet extension of MSSM is given by the following relation [42]

$$\begin{aligned} W_{GSEMSSM} &= W_H + \left(\xi_F M_n^2\right)\hat{S} + \lambda\left(\hat{H}_1\hat{H}_2\right)\hat{S} + \lambda_i\left(\hat{\tilde{L}}_i\hat{H}_2\right)\hat{S} \\ &+ \frac{\mu_2}{2}\left(\hat{S}\right)^2 + \frac{\kappa}{3}\left(\hat{S}\right)^3, \end{aligned} \tag{48}$$

where W_H is defined by the Equation (41). The parameters λ, κ and ξ_F are dimensionless while the parameters μ_2 and M_n have mass dimension.

The the Next-to-the-Minimal Supersymmetric Standard-Model (NMSSM) can be obtained either from super-GUT models or from super-string $E(6)$ models [12,42]. To get NMSSM, one needs to set

$$\mu = \mu_{0i} = \xi_F = \mu_2 = \lambda''_{ijk} = 0. \tag{49}$$

Then the superpotential takes the following form

$$
\begin{aligned}
W_{NMSSM} &= \sum_{i,j=1}^{3}\left[f^{l}_{ij}\left(\hat{H}_1\hat{L}_i\right)\hat{l}^c_j + f^{d}_{ij}\left(\hat{H}_1\hat{Q}_i\right)\hat{d}^c_j + f^{u}_{ij}\left(\hat{H}_2\hat{Q}_i\right)\hat{u}^c_j \right] \\
&+ \lambda\left(\hat{H}_1\hat{H}_2\right)\hat{S} + \lambda_i\left(\hat{\hat{L}}_i\hat{H}_2\right)\hat{S} + \frac{\kappa}{3}\left(\hat{S}\right)^3 + \sum_{i,j,k=1}^{3}\left[\lambda_{ijk}\left(\hat{L}_i\hat{L}_j\right)\hat{l}^c_k + \lambda'_{ijk}\left(\hat{L}_i\hat{Q}_j\right)\hat{d}^c_k\right].
\end{aligned} \tag{50}
$$

The superpotential that generates neutrino masses in NMSSM was given in [43] where the following function was proposed

$$
\begin{aligned}
W_{NMSSM} &= \sum_{i,j=1}^{3}\left[f^{l}_{ij}\left(\hat{H}_1\hat{L}_i\right)\hat{l}^c_j + f^{d}_{ij}\left(\hat{H}_1\hat{Q}_i\right)\hat{d}^c_j + f^{u}_{ij}\left(\hat{H}_2\hat{Q}_i\right)\hat{u}^c_j \right] \\
&+ \lambda\left(\hat{H}_1\hat{H}_2\right)\hat{S} + \mu_i\left(\hat{\hat{L}}_i\hat{H}_2\right) + \frac{\kappa}{3}\left(\hat{S}\right)^3 \\
&+ \sum_{i,j,k=1}^{3}\left[\lambda_{ijk}\left(\hat{L}_i\hat{L}_j\right)\hat{l}^c_k + \lambda'_{ijk}\left(\hat{L}_i\hat{Q}_j\right)\hat{d}^c_k\right].
\end{aligned} \tag{51}
$$

From W_{NMSSM}, just one massive neutrino is generated as in MSSM with R-Parity violation terms. All conclusions presented after Equation (41) still hold in this case.

An important conclusion is that if we want to generate masses for all neutrinos at tree level, we need to introduce three right-handed neutrinos. We discuss this case in some detail below.

5. Minimal Supersymmetric Standard Model with three right-handed Neutrinos

The simplest supersymmetric model to explain the masses of the left-handed neutrinos is known as the Minimal Supersymmetric Standard Model with three generation Right-Handed neutrinos (MSSMRH). The particle content of this model is the same as MSSM with the addition of the right-handed neutrinos $\hat{N}_i \sim (\mathbf{1}, 0)$ (see, e. g. [4,44–46]). The superpotential of the MSSMRH is given by

$$
W_{MSSMRH} = W_H + \frac{1}{2}M^n_{ij}\hat{N}_i\hat{N}_j + \frac{1}{3}f^n_{ij}\left(\epsilon\hat{H}_2\hat{L}_i\right)\hat{N}_j + hc, \tag{52}
$$

where W_H is defined by the Equation (41). For this model, the R-charges described by the Equation (40) together with the condition $n_N = 0$.

In this MSSMRH, the masses of neutrinos are obtained from the terms

$$
-\left\{\frac{1}{2}\left[\mu_{0i}\left(\epsilon H_2 L_i\right) + M^n_{ij}N_iN_j\right] + \frac{1}{3}f^n_{ij}\left(\epsilon H_2 L_i\right)N_j\right\} + \text{h.c.}. \tag{53}
$$

Here, the symmetric matrices M^n_{ij} are the Majorana masses and f^n_{ij} are the source of the Dirac masses. This model is interesting because it can generate mass to all neutrinos at tree level and also to explain all mixing data about neutrinos, as shown in [47].

If we consider the $\mathcal{Z}_3$ symmetry that acts on the chiral superfields by a real phase

$$
\Phi \to \exp\left(\frac{2\pi\omega}{3}\right)\Phi, \tag{54}
$$

where ω is an entire number, then the Majorana masses can be avoided and only the Dirac mass term survives

$$
\frac{1}{3}f^n_{ij}\left(\epsilon H_2 L_i\right)n^c_j. \tag{55}
$$

In order to explain the lightness of the neutrino masses, one must have $f \leq 10^{-12}$ [48].

6. μ from ν Supersymmetric Standard Model ($\mu\nu$SSM)

The MSSM and also the MSSMRH suffer from the μ-problem [49] that can be formulated as the generation of a μ coupling in the $\mu \hat{H}_1 \hat{H}_2$ term of the order of the electro-weak scale. The μ-problem is solved by the Next-to-the-Minimal Supersymmetric Standard-Model (NMSSM). From the point of view of neutrino oscillations, NMSSM is not interesting because its neutrinos are massless [4]. Nevertheless, one can construct a non-minimal SUSY model that solves the μ-problem and at same time gives masses to all neutrinos. This is called the "μ from ν Supersymmetric Standard Model" ($\mu\nu$SSM) and it was proposed in [50].

The superpotential of $\mu\nu$SSM can be obtained by requiring that it be invariant under $\mathcal{Z}_3$-symmetry, see Equation (54). This leads to the following formula

$$\begin{aligned} W_{\mu\nu suppot} &= f^l_{ab}\epsilon\hat{L}_a\hat{H}_1\hat{E}_b + f^u_{ij}\epsilon\hat{Q}_i\hat{H}_2\hat{U}_j + f^d_{ij}\epsilon\hat{Q}_i\hat{H}_1\hat{D}_j \\ &+ \frac{1}{3}\sum_{i,j=1}^{3}\left[f^n_{ij}\left(\epsilon\hat{H}_2\hat{L}_i\right)\hat{N}_j + h^n_i\left(\epsilon\hat{H}_2\hat{H}_1\right)\hat{N}_i + \kappa^n_{ijk}\hat{N}_i\hat{N}_j\hat{N}_k\right]. \end{aligned} \quad (56)$$

This superpotential is consistent with the phenomenological models derived from the superstring theory. From it, one can see that the neutralino mass matrices have the general form

$$\begin{bmatrix} \mathrm{M}_{7\times7} & \mathrm{m}_{3\times7} \\ \mathrm{m}^T_{3\times7} & 0_{3\times3} \end{bmatrix} \quad (57)$$

The neutrinos are Majorana in this model. Therefore, several processes as the double beta decay, can take place without neutrinos. More details can be found in [12].

7. Supersymmetric $B-L$ Model

The TeV scale right-handed neutrino is naturally obtained in supersymmetric $B-L$ (SUSYB-L) extension of the SM which is one of the simplest non-linear class of models beyond the SM that provide a viable and testable solution to the neutrino mass. Also, it can account for the experimental results of the light neutrino masses and their large mixing angles.

The B-L number defined as the baryon minus lepton numbers can be obtained from a gauge symmetry that is broken at TeV scale [51,52]. Recall that this is the SUSY breaking scale necessary to explain the hierarchy problem in MSSM. In the SUSYB-L, as happen in the MSSM, the Higgs potential receives large radiative corrections that induce spontaneous $B-L$ symmetry breaking at TeV scale, in analogy to the electroweak symmetry breaking in MSSM [51].

One interesting properties of this kind of model is the new complex phases in the leptonic sector that can generate lepton asymmetry that is converted to baryon asymmetry [52]. The relevant terms from the Lagrangian that are responsible for that are the soft SUSY-breaking terms [53,54]

$$\mathcal{L}_{\mathrm{soft}} = \frac{\tilde{m}_n^2}{2}\left(\tilde{N}\right)^\dagger\tilde{N} + \frac{B_n^2}{2}\tilde{N}\tilde{N} + A_n Y_n\left(\epsilon\tilde{L}H_2\right)\tilde{N} + h.c. \quad (58)$$

The above Lagrangian describes the mixing between the sneutrino $\tilde{N}$ and the anti-sneutrino $\tilde{N}^\dagger$. The CP violation phase in this mixing generates lepton asymmetry in the final states of the $\tilde{N}$-decay and it is converted to baryon asymmetry through the sphaleron process [55].

7.1. The Minimal $B-L$ Supersymmetric Model

The light neutrino masses can be obtained from the minimal gauged $U(1)_{B-L}$ that is invariant under the following gauge group

$$SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}. \tag{59}$$

In this model, we have R-parity violation [56]. The matter chiral supermultiplets for leptons and their $SU(2)_L$, $U(1)_Y$ and $U(1)_{B-L}$ quantum numbers are given by

$$\hat{L}_i \sim (\mathbf{2},-1,-1)\,,\;\; \hat{E}_i \sim (\mathbf{1},2,1)\,,\;\; \hat{N}_i \;\sim\; (\mathbf{1},0,1)\,. \tag{60}$$

With this matter content, the superpotential can be defined as follows

$$W_{BL} = f^l_{ab}\epsilon\hat{L}_a\hat{H}_1\hat{E}_b + f^u_{ij}\epsilon\hat{Q}_i\hat{H}_2\hat{U}_j + f^d_{ij}\epsilon\hat{Q}_i\hat{H}_1\hat{D}_j + \frac{1}{3}\sum_{i,j=1}^{3} f^n_{ij}\left(\epsilon\hat{H}_2\hat{L}_i\right)\hat{N}_j. \tag{61}$$

Once the R-parity is broken, the neutralinos and neutrinos can mix as in MSSM. Due to this mixing, the masses to all the neutrinos is generated via the see-saw mechanism. Let us briefly review the basic arguments for the generation of light neutrino masses given in [56]. By breaking the R-parity, the mixing of the neutralinos and neutrinos fields takes place. These fields are

$$\left(\nu, \nu^c, \tilde{B}, \tilde{B}', \tilde{W}^0_L, \tilde{H}^0_d, \tilde{H}^0_u\right).$$

One particular simple case is given by the vev $v_L \to 0$ and $Y^D_\nu << 1$. Then

$$M_\nu = M^I_\nu + M^R_\nu,$$

where the vev's of the Higgs doublets are

$$\left\langle H^0_u \right\rangle = \frac{v_u}{\sqrt{2}}, \qquad \left\langle H^0_d \right\rangle = \frac{v_d}{\sqrt{2}},$$

and M^I_ν is the see-saw contribution and M^R_ν is from the R-parity violation. These two contributions have the following form

$$M^I_\nu = \frac{1}{2} Y^D_\nu M^{-1}_{\nu C} \left(Y^D_\nu\right)^T v_u^2\,, \qquad M^R_\nu = m M^{-1}_{\tilde{\chi}^0} m^T$$

where

$$M_{\nu C} \approx \left(M_{BL} + \sqrt{4M^2_{Z'} + M^2_{BL}}\right)/2\,, \qquad m = \mathrm{diag}\left(0,0,0,0,Y^D_\nu v_R/\sqrt{2}\right).$$

Here, $M_{\tilde{\chi}^0}$ is the neutralino mass matrix as calculated in the MSSM. This shows that the light neutrino masses is obtained from the R-symmetry breaking in the minimal $SUSYB-L$ model. For more details, we refer to the original paper [56].

8. Supersymmetric Left-Right Model

The minimal SUSYB-L model discussed in the above section is not the only one that can explain the light neutrinos. The SUSYLR is a different proposal which can also solve the strong CP problem [57]. In the SUSYLR model, the gauge symmetry group is

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}. \tag{62}$$

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The lepton content of SUSYLR is different from the lepton content of MSSM where the left-handed fermions belong to the doublet representation while the right-handed fermions are singlets of $SU(2)_L$. Here, both left-handed and right-handed leptons are doublets in the corresponding $SU(2)$ gauge groups

$$\hat{L}_i \sim (\mathbf{2}, \mathbf{1}, -1), \;\; \hat{L}_i^c \sim (\mathbf{1}, \mathbf{2}, 1). \tag{63}$$

In the literature, two different SUSYLR models have been discussed: the first one uses $SU(2)_R$ triplets (SUSYLRT) [58] and the second one has $SU(2)_R$ doublets (SUSYLRD) [59]. Since the neutrinos in SUSYLRD are massless, we will not discuss the details of this model here.

8.1. Triplet Model (SUSYLRT)

The scalars fields of this model are

$$\begin{aligned} \hat{\Delta}_L &\sim (\mathbf{3}, \mathbf{1}, 2)\,, \;\; \hat{\Delta}'_L \sim (\mathbf{3}, \mathbf{1}, -2)\,, \\ \hat{\delta}^c_L &\sim (\mathbf{1}, \mathbf{3}, -2)\,, \;\; \hat{\delta}'^c_L \sim (\mathbf{1}, \mathbf{3}, 2)\,, \\ \hat{\Phi} &\sim (\mathbf{2}, \mathbf{2}, 0)\,, \;\; \hat{\Phi}' \sim (\mathbf{2}, \mathbf{2}, 0)\,. \end{aligned} \tag{64}$$

The most general superpotential W is given by [58]

$$\begin{aligned} W &= M_\Delta Tr(\hat{\Delta}_L \hat{\Delta}'_L) + M_{\delta^c} Tr(\hat{\delta}^c_L \hat{\delta}'^c_L) + \mu_1 Tr\left[(i\sigma_2)\,\hat{\Phi}\,(i\sigma_2)\,\hat{\Phi}\right] + \mu_2 Tr\left[(i\sigma_2)\,\hat{\Phi}'\,(i\sigma_2)\,\hat{\Phi}'\right] \\ &+ \mu_3 Tr\left[(i\sigma_2)\,\hat{\Phi}\,(i\sigma_2)\,\hat{\Phi}'\right] + f_{ab} Tr\left[\hat{L}_a\,(i\sigma_2)\,\hat{\Delta}_L \hat{L}_b\right] + f^c_{ab} Tr\left[\hat{L}^c_a\,(i\sigma_2)\,\hat{\delta}^c_L \hat{L}^c_b\right] \\ &+ h^l_{ab} Tr\left[\hat{L}_a \hat{\Phi}\,(i\sigma_2)\,\hat{L}^c_b\right] + \tilde{h}^l_{ab} Tr\left[\hat{L}_a \hat{\Phi}'\,(i\sigma_2)\,\hat{L}^c_b\right], \end{aligned} \tag{65}$$

where $h^l, \tilde{h}^l$ are the Yukawa couplings for the leptons. This model can be embedded in a supersymmetric grand unified theory (SUSYGUT) with $SO(10)$, some of which will be briefly discussed in the next section.

The masses of neutrinos in the SUSYLRT are given by the following matrix [61]:

$$\begin{aligned} M^\nu_{ab} &= \frac{1}{\sqrt{2}}\left[k_1 h^l_{ab} + k'_2 \tilde{h}^l_{ab}\right](\nu_a \nu^c_b + hc) + \frac{v_R}{\sqrt{2}} f^c_{ab}(\nu^c_a \nu^c_b + hc) \\ &- \frac{v_L}{\sqrt{2}} f_{ab}(\nu_a \nu_b + h.c.). \end{aligned} \tag{66}$$

This result is in agreement with the one given in [60] for $v_L = 0$.

9. Supersymmetric Grand Unification Theory

In the standard minimal SUSYGUT scenario, the models possess both the supersymmetry and the unified gauge symmetry at the unification scale. The main properties of a finite SUSYGUT are [62]

1. The number of generations is fixed by the requirement of finiteness.
2. There are various realistic possibilities given by the gauge groups $SU(5)$, $SU(6)$, $SO(10)$ and $E(6)$.

The last point above leads to a huge class of models compatible with the SUSYGUT. Below, we are going to give a very brief highlight of two supersymmetric grand unification models that give masses to neutrinos, namely $SU(5)_{RN}$ and $SO(10)_M$.

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9.1. $SU(5)$ grand unified model with right-handed neutrinos $SU(5)_{RN}$

There are very nice reviews of the $SU(5)$ SUSY model are given at [63,64]. The SUSYGUT model with $SU(5)$ gauge group meets observational difficulties that come from the proton decay channel [65]

$$p \longrightarrow K^+ \nu^c .$$

Also, the neutrinos are massless and the lepton sector must be changed to provide a mechanism that can generate masses for neutrinos. For a discussion of the challenges posed by the $SU(5)$ models, see [66].

Let us focus now on the $SU(5)$ grand unified model with right-handed neutrinos $SU(5)_{RN}$. Similarly to the minimal $SU(5)$ model, we can introduce the charged lepton[1] as $\Phi_i \sim \bar{5}$. Also, we have to introduce the right-handed neutrinos which are as $\eta_i \sim 1$. The scalars of the model are given by $H \sim 5$ and $\overline{H} \sim \bar{5}$. Then the superpotential is given by [67,68]:

$$W_{SU(5)_{RN}} = f^n_{ij} \eta_i \Phi_{jA} H^A + \frac{1}{2} M_{ij} \eta_i \eta_j \,, \tag{67}$$

where only the terms that give neutrinos masses are written explicitly. Here, M_{ij} is the Majorana mass matrix for right-handed neutrinos and f^n is the Dirac mass. Both masses are generated via the see-saw mechanism.

9.2. Minimal SO(10) Supersymmetric Model $SO(10)_M$

The $SO(10)$ supersymmetric models are extensively reviewed in [69,70]. Since our focus is the generation of the neutrinos masses, we can exemplify the $SO(10)$ group with the model presented in [71,72]. Here, the lepton is introduced as $\psi_i \sim 16$ representation of $SO(10)$ and the Higgs fields in the representations $H_{10} \sim 10$ and $\Delta \sim 126^*$ of $SO(10)$. The terms of the superpotential that are responsible for the lepton masses, are given by the following relation

$$W_{SO(10)_M} \;=\; h_{ij} \psi_i \psi_j H_{10} + f_{ij} \psi_i \psi_j \Delta \,, \tag{68}$$

where h and f are symmetric matrices in (i,j) that denote generation indices. The neutrino masses can be calculated in this model from

$$M_{\nu D} \;=\; \left(h^* \cos\alpha_u - 3 f^* e^{i\gamma_u} \sin\alpha_u \right) \sin\beta \,, \tag{69}$$

where α_u is a new parameter of the model responsible for non-null CKM mixing angles [71].

The $SO(10)$ Supersymmetric Model has a second mass generating mechanism which produces the neutrino masses from the scalar of the 126 representations of $SO(10)$. A large mass mixing in the $\nu_\mu - \nu_\tau$ channel imposes to the low energy Majorana neutrino mass matrix the following form [73]

$$M_{\nu_{LL}} \sim \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1 \\ t & 1 & 1 \end{pmatrix} \Lambda \,, \tag{70}$$

which has the following eigenvalues in Λ units

$$|m_1| \simeq \frac{t}{\sqrt{2}} - \frac{t^2}{8} - \frac{3t^3}{64\sqrt{2}} \,, \quad |m_2| \simeq \frac{t}{\sqrt{2}} + \frac{t^2}{8} - \frac{3t^3}{64\sqrt{2}} \,, \quad |m_3| \simeq 2 + \frac{t^2}{4} \,. \tag{71}$$

[1] The 5^* representation of $SU(5)$ and the indices i, j, as usual, denote generations, and A is $SU(5)$ index that runs from 1 to 5.

By making assumptions about the texture of the Dirac mass matrix (that is, its form to be of the up-quark mass matrix), one has the other Majorana mass matrices

$$M_{\nu_{LR}} = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & \beta & \gamma \\ \alpha & \gamma & 1 \end{pmatrix} \eta$$

with $\alpha \simeq \beta \ll \gamma \ll 1$, and

$$M_{\nu_{RR}} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R$$

with

$$\delta_1, \delta_2, \delta_3 \ll 1\,.$$

Here, one has the following expressions

$$\delta_1 = \frac{\alpha^2}{2\alpha - 2\alpha\gamma + \gamma^2 t}\,, \quad \delta_2 = \frac{\beta^2 t}{2\alpha - 2\alpha\gamma + \gamma^2 t}\,, \quad \delta_3 \simeq \frac{\alpha(\gamma - \beta) + \beta\gamma t}{2\alpha - 2\alpha\gamma + \gamma^2 t}\,.$$

The see-saw mechanism imposes the following equation

$$M_{\nu_{LL}} = -M^T_{\nu_{LR}} M^{-1}_{\nu_{RR}} M_{\nu_{LR}}\,.$$

It is important to note that this model provides a large mixing and a large mass splitting by postulating the general form of the Dirac mass matrix. Also, it generates a hierarchical mass matrix for the right-handed neutrinos with a small mixing. For more details on this interesting SUSYGUT model, see [73].

10. Conclusions

We have reviewed the connection between the neutrino masses and the R-symmetry in some interesting non-linear SUSY models. The interest in this ideas, some older and some new, has been revived by the results obtained at recent collider experiments and by the atmospheric, astronomical and cosmological observations. More important, all the models presented here can be tested in high energy experiments.

The R symmetry can be used to solve the μ-problem since the later can be connected with the supersymmetry breaking process and allows constructing models with viable neutrino masses. The fermion mass hierarchies can be explained by constructing models with pseudo-anomalous $U(1)_R$ symmetry. Discrete non-abelian symmetries based on $\mathbb{Z}^N$ can be used to obtain the neutrino mixing. These models solve the fermion mass hierarchies and the hierarchy problem without fine-tuning while modelling a neutrino mixing by a non-abelian flavour symmetry. For details on these models, see [74].

To conclude, non-linear extensions of the SUSY with R-symmetry are to solve the main problems of the particle physics beyond SM, and represent viable theories for addressing other fundamental problems of quantum field theory and gravity, such as the Dark Matter and Dark Energy. These models can be tested experimentally, which makes them inviting for both theoretical and experimental explorations.

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