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Article

Optimized FIR Filter Using Genetic Algorithms: A Case Study of ECG Signals Filter Optimization

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Abstract: The advancement of technology and the availability of specialized digital signal processing chips have made digital filter design and implementation more feasible in a variety of fields, including biomedical engineering. This paper makes two key contributions. First, it uses a genetic algorithm to optimize the coefficients of Finite Impulse Response (FIR) filters. Second, it conducts a case study on using genetic algorithms to optimize FIR filters for electrocardiogram (ECG) biomedical signal noise removal. The goal of the proposed filter design approach is to achieve the desired signal bandwidth while minimizing the side lobe level and eliminating unwanted signals using a genetic algorithm. The results of the comprehensive analysis of the impact of different genetic operators show that the genetic algorithm-based filter outperforms conventional filter designs.

Keywords: genetic algorithms; digital filters; metaheuristic optimization; Finite Impulse Response (FIR); side lobe level (SLL)

1. Introduction

Digital filters are used in many different fields to remove noise, split multiple signals [1], missing data imputation [2], and enhance the quality of received signals [3,4]. In recent years, there has been a lot of research into the use of Genetic Algorithms to optimize the design of digital filters. A Genetic algorithm [5] is a metaheuristic optimization algorithm that can be used to find optimal, or near optimal, solutions to problems that are too computationally expensive and are difficult to solve using traditional methods. In the case of digital filter optimization, the goal is to find a set of coefficients that optimizes the performance of the filter using some performance metric.

Electrocardiography (ECG) is a well-known and reliable tool that is widely used in the medical field to analyze and diagnose the condition of the heart. This analysis is very crucial for the early detection of heart diseases. ECG signals are very weak signals and hence susceptible to disturbances in the measurement environment. Using digital filters to remove unwanted parts of these signals is very critical for the performance and accuracy of the algorithms and software tools that are used to detect abnormal heart conditions [6].

The side lobe level (SLL) [7] is a measure of the noise that is produced by the filter outside of its passband. A low side lobe level indicates that the filter is good at removing noise. The genetic algorithm works by iteratively generating new populations of filters, each of which is a slight variation of the previous population. The filters in each population are evaluated based on their side lobe level, and the best filters are selected to create the next generation. This process continues until a filter with a sufficiently low side lobe level is found.

The genetic algorithm approach to digital filter optimization has several advantages over traditional methods [8,9]. First, it is not limited to finding local minima, which can be a problem with traditional methods. Second, it can be used to find solutions to problems that are too complex for

traditional methods. Third, it is relatively easy to implement. This approach is a promising technique that has the potential to significantly enhance the design of digital filters used in different fields, including biomedical engineering.

In this paper, a GA-based digital filter is proposed to optimize the coefficients of FIR filters for ECG signals. The search space for the filter is defined by 20 coefficients. Finding the optimal values of these coefficients is practically extremely large due to the high non-linearity of the frequency response of the filters. The objective, or fitness, function to be optimized is the minimization of the side lobe level (*SLL*). As discussed earlier in this section, the *SLL* is a measure of the noise that is produced by the filter outside of its passband. A low *SLL* indicates that the filter is good at removing noise. The actual coefficients of the filter are mapped to the genes of a chromosome. Each coefficient represents a gene, while the whole coefficients represent a chromosome. The genetic algorithm works by iteratively generating new populations of filters, each of which is a slight variation of the previous population. The filters in each population are evaluated based on their *SLL*, and the best filters are selected to create the next generation. This process continues until a filter with a sufficiently low *SLL* is found.

The remainder of the paper is organized as follows. Section 2 presents the related work. Section 3 explain the research methodology. Section 4 highlight and compare different options of the proposed methodology. At the same time, Section 5 discuss the results for the case study of using GA to filter ECG signal. Eventually, Section 6 concludes the paper.

2. RELATED WORK

Meta-heuristic optimization algorithms such as Genetic Algorithms (GA) [10], Ant Colony Optimization (ACO) [11], and Chemical Reaction Optimization (CRO) [12] are used to solve different optimization problems. These algorithms have been extensively used in recent years to implement digital filters in a wide range of applications [13].

The authors in [14] proposed a comprehensive fitness function for an infinite impulse response (*IIR*) digital filter design with six terms, including a low delay parameter. Low delay filters are preferred since they achieve quasi-real-time processing. Other parameters can be included in the fitness function to meet linear phase, minimum phase and stability constraints.

The authors in [15] proposed a method to detect breast malignancies. The filter bank that has been used to obtain the filter response is the Maximum Response Eight Filter Bank. Genetic Algorithm and Linear Discriminant Analysis have been used for feature selection and feature reduction, respectively.

An optimal fourth-order Butterworth active low-pass filter design by the hybrid intensified current search (*HICuS*) algorithm was proposed in [16]. With AI-based modern optimization, the *HICuS* algorithm is claimed to be one of the most efficient metaheuristic optimizers.

The authors in [17] presented a differential evolution algorithm that uses a combination of rectangular and polar coordinates. The algorithm was developed using thirteen commonly used numerical optimization test functions. The algorithm was then applied to design an infinite impulse response (*IIR*) digital filter.

A digital filter design problem that involves multiple, often conflicting filter parameters, has been presented in [18]. Hence an optimization algorithm is needed. A low pass finite impulse response (LPFIR) filter optimal design has been achieved using Particle Swarm Optimization (PSO) and Dynamic Adjustable PSO (DAPSO). Other recent works in [19–23] presented design of FIR with variable multiple stop band, coefficients optimization in adaptive equalization and FIR design using arithmetic optimization and genetic algorithms.

3. METHODOLOGY

The research methodology is described in four stages as depicted in Figure 1. Stage one, FIR filter design and coefficient generation. Stage two uses a genetic algorithm to map filter coefficient. Stage three, predict the best solution of the filter coefficient using a genetic algorithm. Stage four evaluates the genetic algorithm prediction using impulse response.

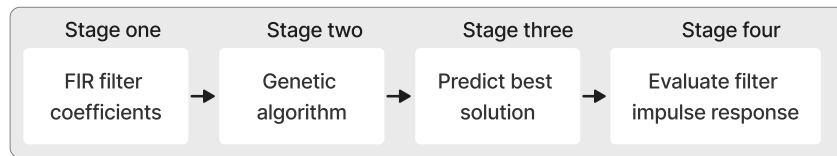


Figure 1. Research Methodology Flow Chart.

Stage one (FIR filter design) : The filtering process uses a direct form discrete-time FIR filter of order M . The top part is an M -stage delay line with $M + 1$ taps shown in Figure 2. Each unit delay is a z^{-1} operator in Z-transform notation. For a causal discrete-time FIR filter of order M , each value of the output sequence is a weighted sum of the most recent input values as in Equation 1.

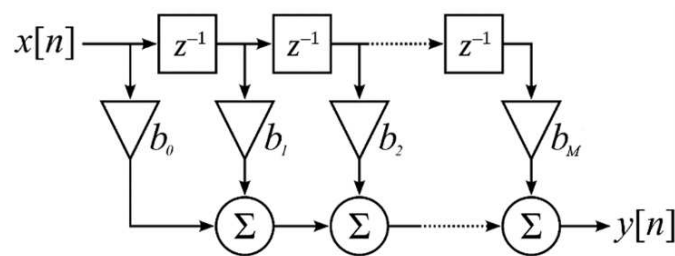


Figure 2. Structure of a direct form discrete-time FIR of order M .

$$y[n] = \sum_{i=0}^M b_i x[n-i] \quad (1)$$

where $x[n]$ and $y[n]$ are the discrete input and output signals respectively. M is the filter order, and b_i is the value of the impulse response at the i^{th} instant for M of an M -order FIR filter.

The frequency response $H(z)$ is expressed in terms of the Z-transform transfer function as shown in Equation 2:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} \quad (2)$$

Ideally, we would like to have M negligible for small computation with a target frequency response given by the ideal low-pass filter as follow in Equation 3:

$$H(\omega) = \begin{cases} 1, & \text{if } |\omega| \leq |\omega_c| \\ 0, & \text{if } \omega_c \leq |\omega| \leq \pi \end{cases} \quad (3)$$

Stage two (Map and process FIR coefficients using GA) : The general structure of GA is depicted in Figure 3. An initial population of 20 chromosomes, each one containing 20 genes, was created randomly in the range $[0,1]$. The genes represent the filter coefficients. The initial set was passed to a fitness function for frequency response evaluation and side lobe level (*SLL*) estimation. The selection is done for the minimum *SLL*. If the condition of the minimum *SLL* level does not meet the desired threshold, the GA selects new genes or filter coefficient randomly using crossover and re-evaluates the fitness function *SLL*.

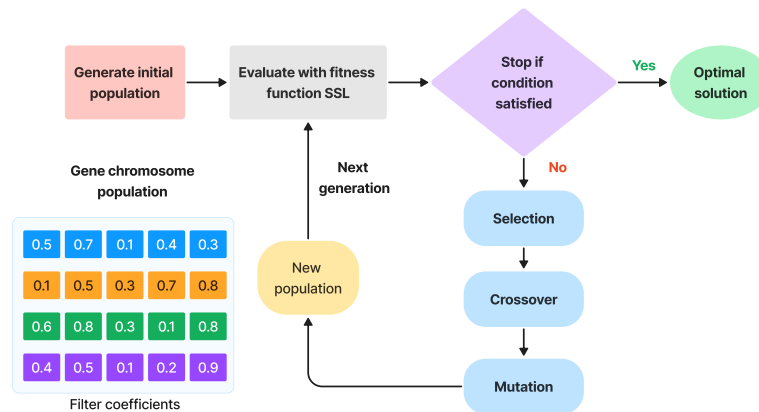


Figure 3. General structure of a genetic algorithm for digital filter applications.

Stage three (Predict the best solution) : The mapped filter coefficients are filtered out by using GA to select the best solution among the coefficients. Figure 4 depicts 20 frequency responses for the population and emphasizes the best solution shown in red with a thick solid line. Figure 5 illustrates the frequency response in magnitude for the best solution extracted from the initial population. The coefficients of the best solution are shown below. The *SLL* is shown in Figure 5 with -16.1660 dB.

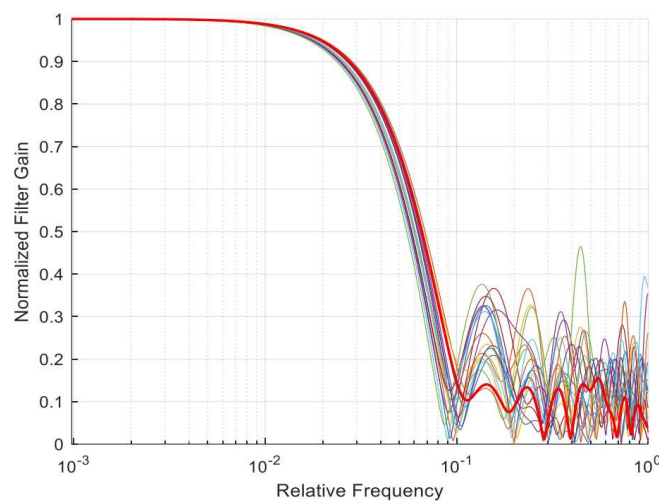


Figure 4. A population of 20 filters (Chromosomes). The best filter is shown in red with minimum side lobe level (*SLL*). Relative or normalized frequency is in logarithmic scale. The filter order is $N-1=19$.

Stage four (Evaluate the best solution) : Finally, the best solution is evaluated using the impulse response to select the minimum *SSL*. This step is important to make sure the best solution is optimized in the desired frequency bandwidth and is at the extreme minimum level for other frequencies. The GA has some randomness in the process, and this step is essential as a final quality control measure for choosing the best solution. Figure 5 depicts the impulse response coefficient of the best solution extracted from the initial population given by $b[n]$ coefficients. In this example, Filter Coeffs ($b[n]$) = [0.5009 0.4457 0.6175 0.5180 0.4395 0.8174 0.7331 0.9891 0.8428 0.7717 0.6087 0.9935 0.4332 0.3967 0.4869 0.6107 0.3800 0.0383 0.8333 0.7669]. The best *SLL* from the initial population is -16.1660 dB.

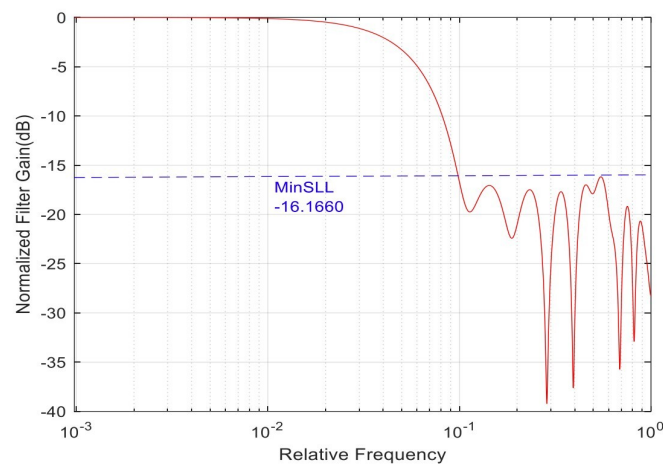


Figure 5. The best filter based on the minimum *SLL* selected from the initial population. The *SLL* is at -16.166 dB. Relative or normalized frequency is in logarithmic scale. Filter order is $(N-1) = 19$.

4. Comparative analysis of the GA algorithm options for FIR filter optimization

The analysis of the GA algorithm is formulated as an optimization problem. The core of an optimization problem is based on the minimization of a fitness function. This latter has to be set meticulously to achieve our objective of reaching a minimum *SLL*. The fitness function has input variables given by the filter coefficients and the resolution of the frequency response of the filter. The objective is to minimize the maximum of the *SLL* as an output. This output is supplied to the genetic algorithm (GA) for minimization.

Detailed results and analysis for the study that includes 16 cases) are listed and discussed in Sections 4.1–4.4, with addition to detailed information in Appendix A and Appendix B. These Sections summarize the comprehensive study options. Each section show results on using optimization solver functions with sub-functions for each GA algorithm option. Results are given by the *SLL* and the filter coefficients, along with graphs on convergence and frequency response. The GA is set with different options in order to study the effect of each parameter on the result of the minimization. The GA uses other options given by the solver. Options are algorithm settings, Section 4.1, population settings in Section 4.2, run time limits in Section 4.3, and tolerance in Section 4.4.

The fitness function based on estimating the maximum of the *SLL* of the normalized discrete frequency response is shown in the following sections. For each solver option from the four alternatives, a graph compares the four *SLL* cases, and another graph compares the convergence. The GA results compared with the conventional FIR results generated by Matlab. Where the seed of the random number in the Matlab generator is fixed.

4.1. Algorithm Settings Results and Analysis

The study of algorithm setting has two categories. Cross-over function and mutation function. The cross-over function has two options heuristic and cross-over two points. At the same time, the mutation options are adapt-feasible and Gaussian.

Figure 6 shows the gain with respect to normalized frequency for cases (1-4); see Appendix A. The lowest performance in Figure 6 is given by the heuristic function. The crossover 2-points, Gaussian mutation and adapt-feasible mutation, have similar performance up to 1 dB margin.

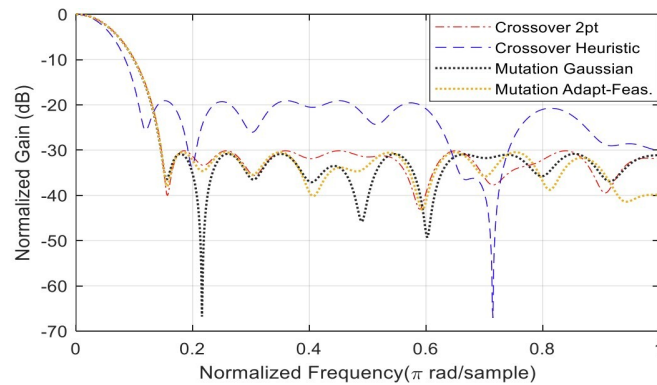


Figure 6. Minimization of the *SLL*. The magnitude spectrum shows two algorithm settings, each with two functions, as described in the legend.

At the same time, for the above-mentioned four cases, the convergence speeds are different, as shown in Figure 7. Considering the *SLL* and the filter coefficients are listed for each case, see Appendix. A. The crossover heuristic has the best performance since it converges in 71 generations, followed by the crossover 2-pts with 193 generations, and then comes the mutation with adapt-feasible, followed by the Gaussian mutation with 235 generations and 326 generations, respectively.

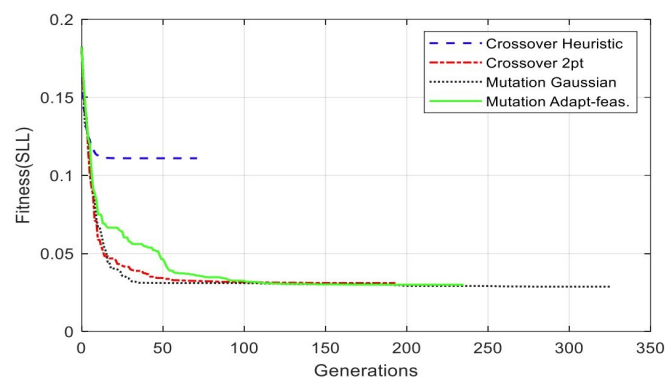


Figure 7. Convergence speed of the *SLL* minimization.

The results of various options of the Algorithm setting are listed in Table 1. The population size is 200 for all cases. By analyzing Figure 6 and Figure 7, we can observe the optimal solution is given by the Crossover with a two-point function since it achieves an *SLL* close to the best solution with moderate convergence.

Table 1. Algorithm setting evaluation results.

Alg. setting	No.	Sub-function solver setting	<i>SLL</i> (dB)	Gen.No. Convergence
Cross Over	1	Heuristic	-19.0941	71
Cross Over	2	Two points	-30.1599	193
Mutation	1	Adapt Feas.	-30.4694	235
Mutation	2	Gaussian	-30.8327	236

Further analysis and experiment results for mutation and cross-over results are presented in the Appendix. B

4.2. Population Settings Results and Analysis

The population setting has two solver options. Max gens and Stall time limit. Max gens were evaluated using two sub-functions, settings 100 and 200. While the Stall time sub-functions are set

as 2 and 4 seconds. Table. 2 summarizes the result for various options on the configurations of the GA- population setting category. Each solver option is studied, with two sub-functions each. The *SSL* values are presented in *dB*.

Table 2. Population setting evaluation results.

<i>Solver Option</i>	<i>No.</i>	<i>Sub-function solver setting</i>	<i>SSL (dB)</i>	<i>Gen.No. Convergence</i>
Max gens	1	100	-30.0219	100
Max gens	2	200	-30.1599	193
Stall time limit	1	2 Sec	-30.1599	193
Stall time limit	2	4 Sec	-30.1599	193

In Figure 8, when max generation is set to 100, the *SLL* stops at -30.0219, while the *SLL* stops at *SLL* of -30.1599 *dB* at generation 193 before the max generation of 200. For a stall time limit of 2 and 4 seconds, the *SLL* converges at the same value with the same filter coefficients.

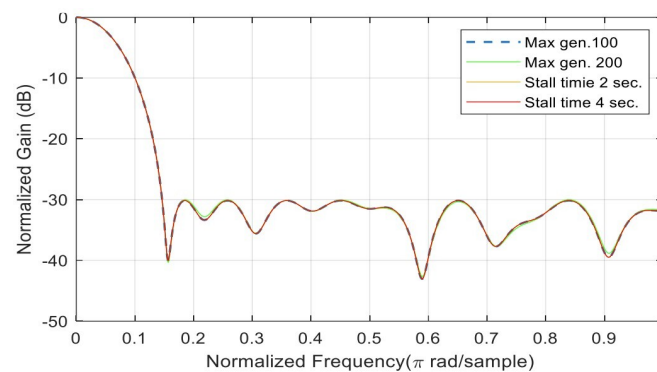


Figure 8. Minimization of the *SLL* (max gen. 100 and 200).

Figure 9 shows the convergence speed of the *SLL*. By analyzing Figure 8 and Figure 9, we can observe that increasing the number of generations or stall time limit will not improve the *SLL* if the fitness converges before reaching the set values.

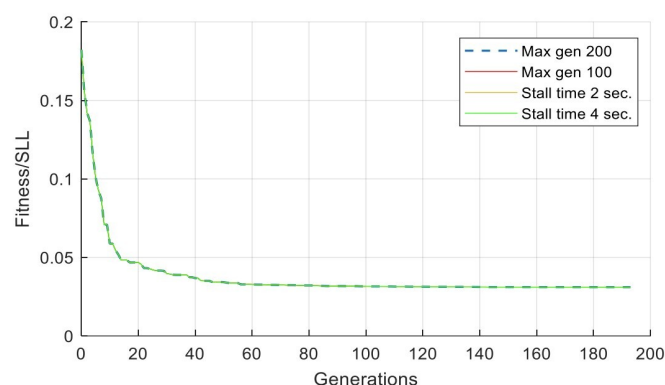


Figure 9. Convergence speed of the *SLL* minimization. For max generations of 100, the fitness stops exactly at 100 generations, while for the three other cases, the generations stop at 193 generations.

4.3. Run Time Limits Results and Analysis

The run-time limit has two categories. The population size and initial population. Population size has two sub-functions solver settings the 100 and 300. In contrast, the initial population has two solvers setting x_0 and x_1 ; see Appendix A for more details. The *SSL* convergence of the number of population settings category is depicted in Figure 11 with respect to the population sample of different

generations number 202, 235, 253 and 274, as listed in Table. 3. The initial value x_1 , population size 300, population size 100 and initial x_0 , respectively. Changing the population size or initial condition will shift the convergence speed. Increasing population size will decrease the convergence iterations.

Table 3. Run Time Limits.

Pop. setting	No.	Sub-function solver setting	SLL (dB)	Gen.No. Convergence
Pop. Size	1	100	-30.7976	253
Pop. Size	2	300	-31.5810	235
Initial. Pop	1	Initial Pop. (x_0)	-31.3811	274
Initial. Pop	2	Initial Pop. (x_1)	-31.3141	202

Figure 10 shows the default value of the GA is 200 chromosomes. When the population is set to 300, the SLL is almost constant when the initial condition is varied. It changes from -31.3811 dB to -31.3141 dB with a difference of 0.067 dB.

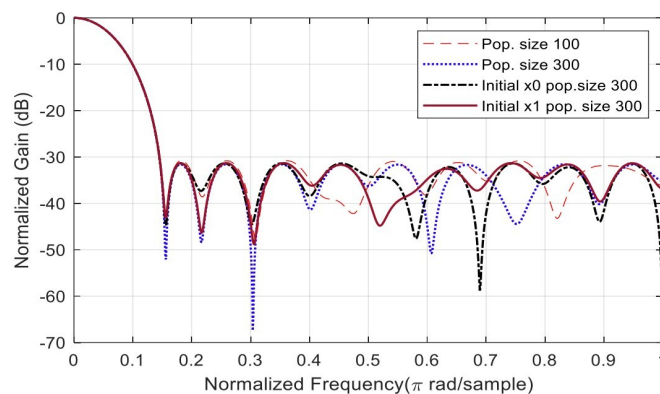


Figure 10. Minimization of the SLL. For population size increasing from 100 to 300, the SLL decreases from -30.7976 dB to -31.5810 dB with a slight difference of less than 1 dB.

By analyzing Figure 10 and Figure 11, we can observe the effect of changing the population size and the change of the initial condition on the algorithm convergence for different setting categories.

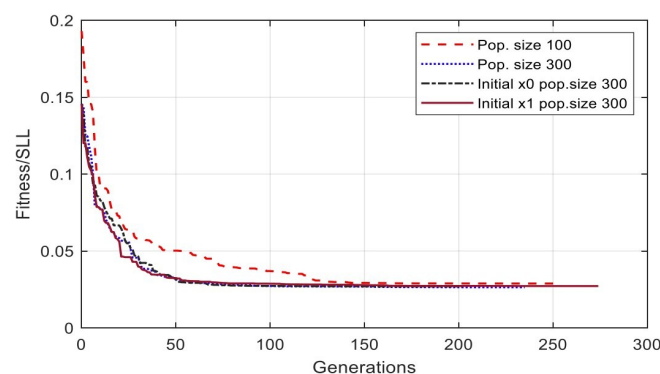


Figure 11. Convergence speed of the SLL minimization.

4.4. Tolerances Results and Analysis

The tolerance option has two categories, function tolerance and fitness limit. The function tolerance solver setting is set to the e^x , and the fitness limit study selects 0.01 and 0.001 as two different decimal sub-function solvers. Table. 4 summarizes the tolerance option setting of GA algorithm. The table also lists different solver options and different generation numbers. Altering the fitness limit from 0.01 to 0.05 change the SLL from -30.1599 dB to -26.3296 dB; see Figure 12 for more details.

For instance, e^x in figures corresponds to 10^x (base 10, not base $e=2.71$). For the convergence speed Figure 13, When the function tolerance decreases from $1.e^{-5}$ to $1.e^{-4}$, the convergence decreases from 148 gen. to 92 gen. When the fitness limit varies from 0.01 to 0.05, the convergence gains high speed from 193 generations to 14 generations.

Table 4. Tolerance evaluation results.

<i>Solver. options</i>	<i>No.</i>	<i>Sub-function solver setting</i>	<i>SSL (dB)</i>	<i>Gen.No. Convergence</i>
Fun. tolerance	1	$1.e^{-4}$	-29.9617	92
Fun. tolerance	2	$1.e^{-5}$	-30.1490	148
Fitness limit	1	0.01	-26.3296	14
Fitness limit	2	0.001	-30.1599	193

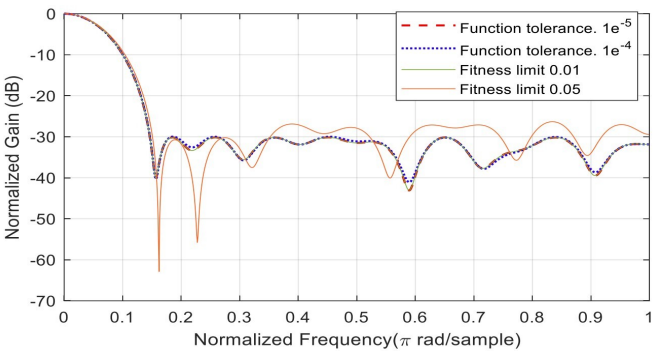


Figure 12. Minimization of the *SLL* changing the function tolerance from $1.e^{-5}$ to $1.e^{-4}$ shift the *SLL* from -30.1490 dB to -29.9617 dB with faster convergence.

By analyzing Figure 12 and Figure 13, we can observe the effect of changing the function tolerance and the fitness limit on both the *SLL* and the convergence speed (number of generations required for the stop criteria) as depicted in the figures captions.

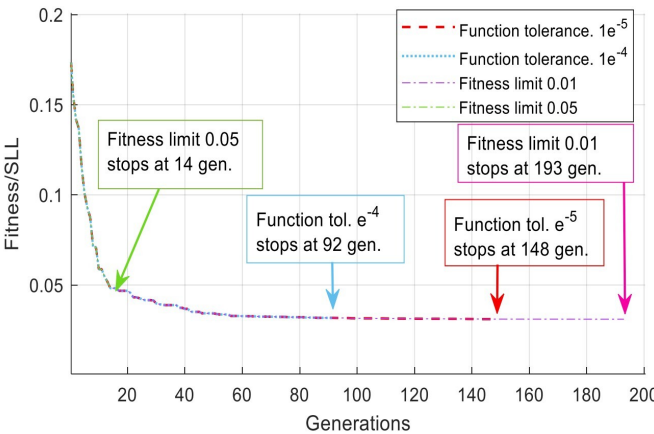


Figure 13. Convergence speed of the *SLL* minimization of different tolerance options.

5. ECG Filtering Results Discussion

This section presents a case study for the optimized FIR filter coefficients using GA. The validation of the proposed algorithm is performed on an ECG signal where an existing noise is to be removed, and the comparison of both time and frequency signals is carried out. The signal was then analyzed to assess the quality of the filtered signal. Comprehensive results are shown below in different figures.

Figure 14 depicts the ideal ECG signal for a normal person. The figure shows the main components of the ECG, which are the P-wave, QRS complex and T-wave. The ideal signal act as a reference while removing the noise to make sure all components are preserved.

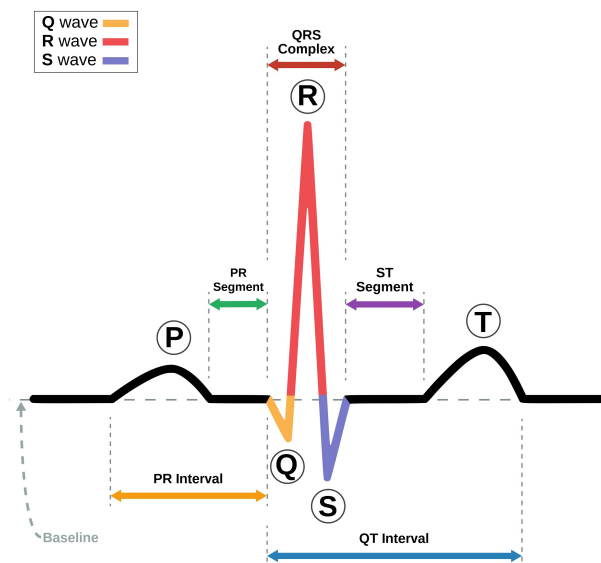


Figure 14. Ideal ECG signal for a normal person.

The case study discusses a noisy ECG signal contaminated with 60 Hz noise and other artifacts. The study removes the artifact from a noisy ECG signal using GA. The 60 Hz noise from the power line is the common source of interference that affect the ECG signal. It has a considerable effect as its amplitude is significant to interfere with the low amplitude ECG signal, which is in the range of Millivolts (mV) in most cases. Furthermore, there are different type of artifacts and interference that affects ECG signal like body movement and external electromagnetic waves that are generated from other devices, mainly from medical imaging devices at hospitals.

As illustrated in Figure 15, the noise effect is significant on P-wave and T-wave as they have lower amplitude than the QRS complex of the ECG. Furthermore, there is an artifact at the beginning of the signal; this artifact is typically generated from motion or improper electrode attachments. The noise in the signal misleads the medical doctor while identifying the patient's case.

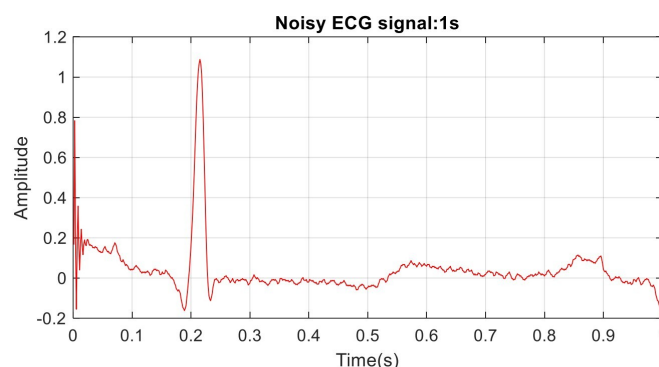


Figure 15. Example of one second of the noisy ECG signal.

To analyze the source of the noise on the complex structure of the ECG signal, we need to analyze the magnitude and frequency response of the spectrum. This is important to identify the signal that has a dominant spectrum over the typical range of ECG.

Figure 16 is the magnitude spectrum displaying the level of noise in dB with respect to the frequency in Hz . Visualizing the noisy signal in the frequency domain help to identify the dominant noisy signal on the original data.

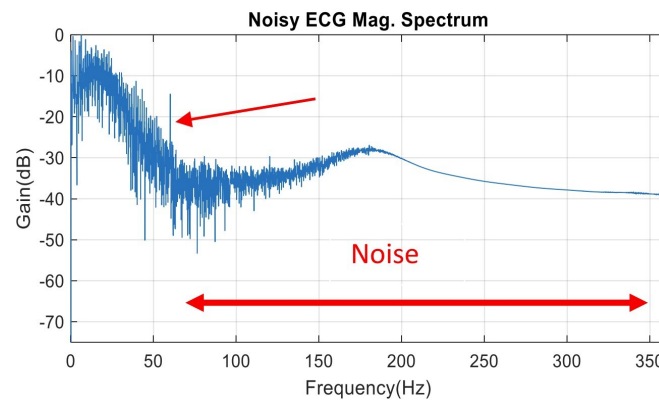


Figure 16. The magnitude spectrum of the noisy ECG signal. The ECG signal is contaminated with a 60 Hz signal from the power line. The sampling rate is 720 samples/sec. Half sampling is at 360 Hz.

Figure 17 is the spectrum of the filtered signal. The filtered ECG using GA with Gaussian mutation category which shows an attenuation of noise with more than 30 dB . At half sampling rate, the signal is at -70 dB . The interference signal of 60 Hz is attenuated with more than 30 dB . The analysis is for case 3; see Appendix. A for details.

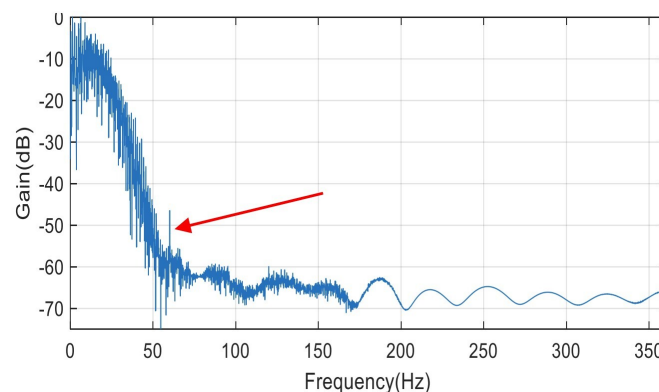


Figure 17. Filtered ECG spectrum magnitude using GA Gaussian mutation.

The final filtered ECG is shown in Figure 18 with respect to the time domain. Comparing Figure 15 and Figure 18 shows the excellent filtering behaviour of the Gaussian mutation GA filter. The ECG signal in the Figure 18 may be different from the ideal ECG in Figure 14 as the real recording has some variations for normal cases between different patients.

Figure 19 shows the difference between GA and standard FIR filter frequency response. The figure shows standard FIR attenuation gain as -40 dB compared to -31 dB for the GA. The GA FIR presents a sharp passband relative to the standard FIR magnitude for the same order, even though the relative cut-off frequency is set to 0.01 (1%).

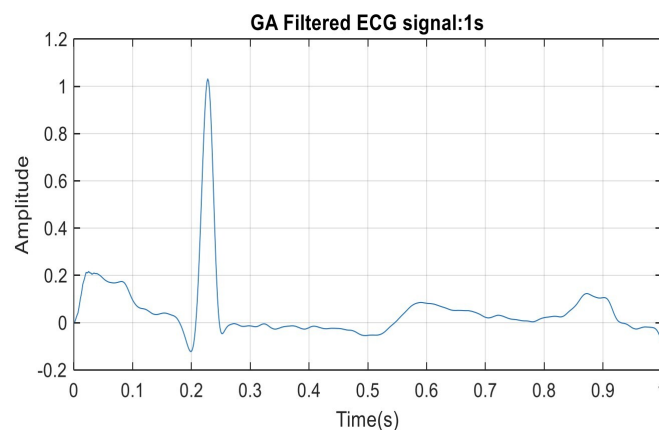


Figure 18. One second at the beginning of the filtered signal showing complete removal of the artifact, 60 Hz power signal interference with smoothed ECG signal using Gaussian mutation genetic algorithm.

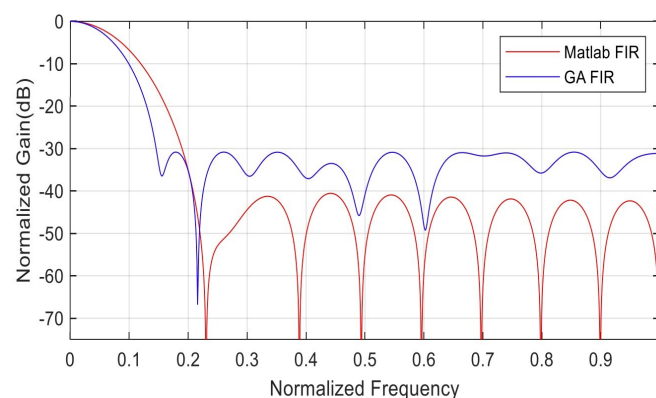


Figure 19. Comparison between GA FIR filter and conventional FIR for low pass filters that generated by Matlab.

6. Conclusions

In this study, a genetic algorithm is used to comprehensively analyze the synthesis of digital low-pass filters. Sixteen cases are analyzed, and results are shown to demonstrate the effectiveness of the method. The fitness function is derived from the frequency response of the finite impulse response low-pass filter, and the inputs of the fitness are the coefficients of the filter, which start with a random sequence, and the frequency resolution of the frequency response. The output is the side lobe level that has to be minimized for the best filter synthesis. The different options were analyzed, and their effects were shown in both graphs and summarized. A Gaussian filter was used to perform digital filtering of a noisy ECG real recording. The filter shows enhanced results in suppressing different kinds of noise.

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Abbreviations

Abbreviations
The following abbreviations are used in this manuscript:

ACO	Ant Colony Optimization
Alg.	Algorithm
CRO	Chemical Reaction Optimization
DAPSO	Dynamic Adjustable Particle Swarm Optimization
DSP	Digital signal Processing
dB	Decibel unit
ECG	Electrocardiogram
EEG	Electroencephalogram
FIR	Finite impulse response
GA	Genetic algorithm
Gen.	Generation
IIR	Infinite impulse response
PSO	Particle Swarm Optimization
SLL	Side lobe level

Appendix A. Comparative analysis of different GA categories and options

This section presents the results of 16 cases, as listed in Table A1. The results were analyzed using different GA algorithms and different categories. For each option, there are selected sub-function solvers. The filter coefficient and the SSL are also listed to show the best solution among different options.

Table A1. Comparison between different methodologies.

Case No.	GA algorithm option	Category	Sub-function solver setting	SSL (dB)	Filter Coefficients
1	Algorithm Setting	Cross-over function	Heuristic	-19.0941	[0.1728 0.5336 0.5986 0.2363 0.4960 0.7534 0.6947 0.5135 0.4703 0.6803 0.6439 0.6650 0.7603 0.2939 0.4812 0.3335 0.4335 0.3923 0.3140 0.3530]
2	Algorithm Setting	Cross-over function	Crossover two points	-30.1599	[0.1119 0.1180 0.1937 0.3267 0.3476 0.4794 0.5589 0.6952 0.7050 0.7677 0.8614 0.7511 0.7524 0.7646 0.6211 0.5923 0.4037 0.4056 0.3399 0.2514]
3	Algorithm Setting	Mutation Function	Gaussian mutation	-30.8327	[0.1359 0.2439 0.2813 0.3658 0.4837 0.6359 0.6791 0.8025 0.8434 0.8723 0.9373 0.9739 0.8859 0.8013 0.6949 0.5921 0.5364 0.4193 0.2180 0.3143]
4	Algorithm Setting	Mutation Function	Adapt-feasible mutation	-30.4694	[0.2063 0.2968 0.3638 0.5322 0.6796 0.7954 0.8321 0.9273 1.0000 1.0000 0.9863 0.9579 0.9579 0.8361 0.6728 0.4515 0.5123 0.2934 0.2521 0.2428]
5	Population Setting	Population Size	Size=100	-30.7976	[0.1575 0.2661 0.2467 0.4623 0.5886 0.5737 0.7992 0.9097 0.9834 0.9999 0.9954 1.0000 0.9933 0.8660 0.7521 0.6720 0.5120 0.4924 0.2738 0.2611]
6	Population Setting	Population Size	Size=300	-31.5810	[0.1916 0.2769 0.3671 0.3364 0.6227 0.6609 0.7728 0.8591 0.9155 0.9637 0.9567 0.9225 0.8709 0.7921 0.6803 0.5446 0.5021 0.3453 0.2449 0.2105]
7	Population Setting	Initial Population	value x_0 =[0.5425 0.1422 0.3733 0.6741 0.4418 0.4340 0.6178 0.5131 0.6504 0.6010 0.8052 0.5216 0.9086 0.3192 0.0905 0.3007 0.1140 0.8287 0.0469 0.6263]	-31.3811	[0.3034 0.2449 0.3330 0.4338 0.5682 0.7429 0.8100 0.8999 0.9730 0.9947 0.9979 0.9346 0.8853 0.8487 0.6816 0.5995 0.5000 0.3849 0.2243 0.1835]
8	Population Setting	Initial Population	value x_1 =[0.5476 0.8193 0.1989 0.8569 0.3517 0.7546 0.2960 0.8839 0.3255 0.1650 0.3925 0.0935 0.8211 0.1512 0.3841 0.9443 0.9876 0.4563 0.8261 0.2514]	-31.3141	[0.2438 0.2903 0.4232 0.5260 0.5379 0.7947 0.8925 0.8748 0.9824 1.0000 0.9999 0.9441 0.8710 0.7934 0.6194 0.5883 0.4087 0.3561 0.2031 0.1954]
9	Run Time Limits	Max Generations	value = 100	-30.0219	[0.1087 0.1176 0.1907 0.3214 0.3442 0.4764 0.5565 0.6933 0.7032 0.7667 0.8613 0.7482 0.7511 0.7637 0.6208 0.5923 0.4035 0.4112 0.3396 0.2504]

Table A1. Cont.

Case No.	GA algorithm option	Category	Sub-function solver setting	SSL (dB)	Filter Coefficients
10	Run Time Limits	Max Generations	value = 200	-30.1599	[0.1119 0.1180 0.1937 0.3267 0.3476 0.4794 0.5589 0.6952 0.7050 0.7677 0.8614 0.7511 0.7524 0.7646 0.6211 0.5923 0.4037 0.4056 0.3399 0.2514]
11	Run Time Limits	Stall Time Limit	value (T) =2 sec	-30.1599	[0.1119 0.1180 0.1937 0.3267 0.3476 0.4794 0.5589 0.6952 0.7050 0.7677 0.8614 0.7511 0.7524 0.7646 0.6211 0.5923 0.4037 0.4056 0.3399 0.2514]
12	Run Time Limits	Stall Time Limit	value (T) =4 sec	-30.1599	[0.1119 0.1180 0.1937 0.3267 0.3476 0.4794 0.5589 0.6952 0.7050 0.7677 0.8614 0.7511 0.7524 0.7646 0.6211 0.5923 0.4037 0.4056 0.3399 0.2514]
13	Tolerances	Function Tolerances	value =1.e-5	-30.1490	[0.1114 0.1179 0.1935 0.3265 0.3472 0.4791 0.5588 0.6947 0.7047 0.7677 0.8613 0.7507 0.7521 0.7644 0.6208 0.5923 0.4037 0.4054 0.3399 0.2510]
14	Tolerances	Function Tolerances	value =1.e-4	-29.9617	[0.1085 0.1163 0.1896 0.3215 0.3422 0.4723 0.5575 0.6922 0.7006 0.7659 0.8626 0.7454 0.7475 0.7630 0.6218 0.5917 0.4032 0.4120 0.3399 0.2487]
15	Tolerances	Fitness Limit	Fitness Limit Value=0.01	-30.1599	[0.1119 0.1180 0.1937 0.3267 0.3476 0.4794 0.5589 0.6952 0.7050 0.7677 0.8614 0.7511 0.7524 0.7646 0.6211 0.5923 0.4037 0.4056 0.3399 0.2514]
16	Tolerances	Fitness Limit	Fitness Limit Value=0.05	-26.3296	[0.0376 0.0092 0.1892 0.2362 0.3246 0.4347 0.4748 0.6437 0.6288 0.7338 0.8300 0.7065 0.7334 0.7504 0.6110 0.5910 0.3996 0.3229 0.2517 0.3651]

Appendix B. ANALYSIS OF RANDOM SOLUTIONS FOR CROSSOVER AND MUTATION

This section compares random seed solutions (No fixed seed) using Crossover and mutation functions. For each function, the sub-functions are analyzed, and figures are shown. Four figures show the SLL and how the SLL varies. In each caption, the SLL worst case and best case are specified with mean and median values followed by the SLL solutions. Figures A1–A4 show the filter's magnitude frequency response with ten solutions for the Crossover/ Heuristic, Crossover/ Two-points, Mutation adapt-feasible, and Mutation/ Gaussian, respectively. By analyzing the mean and median values, we observe that Mutation/Adapt-Feasible is the best, followed by Mutation/ Gaussian, then Crossover/ two-points, then the worst case is crossover/Heuristic.

Figure A1 depicts ten solutions for the Crossover/ Heuristic sub-settings. The worst case is SLL: -19.0941 dB, and the best case is SLL: -25.5974 dB, Mean=-21.7458 dB, Median=-21.8616 dB. Ten SLL solutions = [-19.0941, -22.1809, -25.5974, -22.0978 -20.8282, -21.9670, -21.7562 -19.2285, -21.4284, -23.2790].

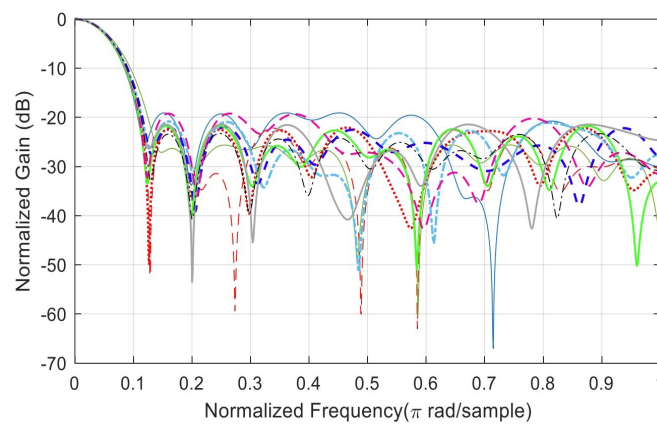


Figure A1. Minimization of the SLL. Ten (10) solutions for the Crossover/ Heuristic.

Figure A2 shows the minimization of the SLL. Ten (10) solutions for the Crossover/ Two-points. Worst case SLL: -28.8705 dB, Best case SLL: -31.7864 dB, Mean= -30.8800 dB, Median= -30.9479 dB. Ten SLL solutions = [-31.1692, -31.7864, -31.5800, -30.6253, -31.3180, -30.6638, -28.8705 -30.7265, -31.6496, -30.4102].

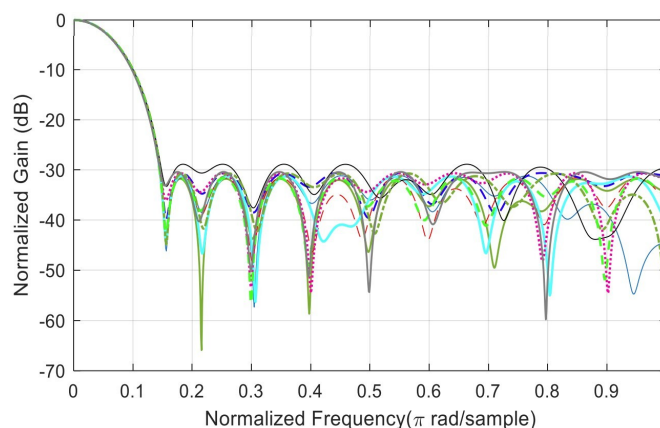


Figure A2. Minimization of the SLL. Ten (10) solutions for the Crossover/ Two-points.

Figure A3 depicts the minimization of the SLL. Ten (10) solutions for the Mutation/ Adapt-Feasible. Worst case SLL: -30.7847 dB, Best case SLL: -31.7460 dB, Mean= -31.3768 dB, Median= -31.3983 dB, Ten SLL solutions = [-31.2148 -31.1336, -30.7847, -31.5693, -31.6100, -31.6331, -31.4939 -31.3027, -31.2802, -31.7460].

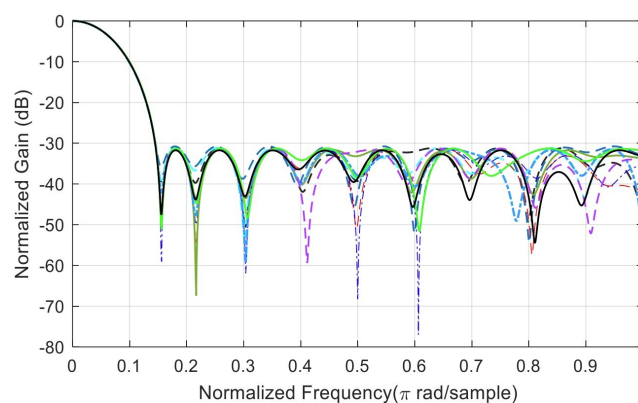


Figure A3. Minimization of the SLL. Ten (10) solutions for the Mutation/ Adapt-Feasible.

Figure A4 displays the minimization of the SLL. Ten (10) solutions for the Mutation/ Gaussian. Worst case SLL: -29.1817 dB, Best case SLL: -32.0514 dB, Mean= -30.9744 dB, Median=-31.2508 dB, Ten SLL solutions = [-29.8972, -31.7044, -31.2590, -31.4947, -29.1817, -31.2425, -31.0011, -32.0514, -31.8377, -30.0746]

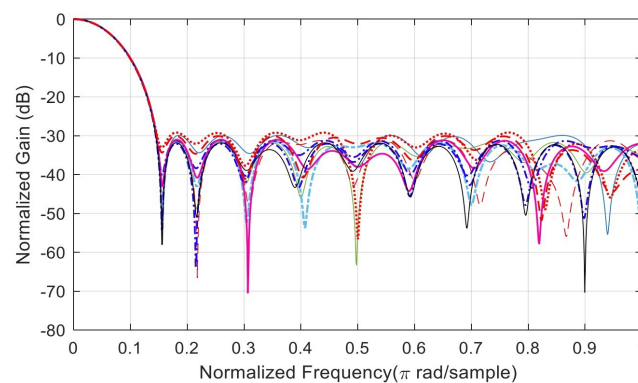


Figure A4. Minimization of the SLL. Ten (10) solutions for the Mutation/ Gaussian.

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