Exact Formulas of the Gravitational Redshift

Askar Abdukadyrov *

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Article

Exact Formulas of the Gravitational Redshift

Askar Abdukadyrov

Kyrgyz State Technical University, Bishkek, Kyrgyzstan; E-mail: abdukadyrov64@mail.ru

Abstract: In 1911, A. Einstein suggested that gravity affects at the electromagnetic radiation originating from the distant sources, namely, it shifts the spectral lines in the red side of the spectrum and derived the gravitational redshift formula. A modern science claims that Einstein's formula is mathematically immaculate. We show that this is not true: in fact, this formula is approximate. Using the law of conservation of mechanical energy, we derived two exact formulas of the gravitational redshift.

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Scientific reasoning and treatises should be short and clear, in no case verbose. It must be remembered that science is for those who do not know it yet.

N. Gogol, writer

1. Introduction

In astrophysics, gravitational redshift (GRS) is the process by which electromagnetic radiation originating from a source that is in gravitational field is reduced in frequency, or redshifted, when observed in a region of a weaker gravitational field.

The gravitational redshift can be interpreted in two ways.

One is that GRS is a direct result of *gravitational time dilation* – as one moves away from a source of gravitational field, the rate at which time passess is increased.

The other approach is that the photon reddens because *it loses the energy* when overcoming the attraction of the gravitational field.

Supporters of the first explanation often try to discredit the second point of view, for example, by arguing that a moving photon has no mass.

In fact, a moving photon has the mass m, energy $E = mc^2$ (c is the speed of light in vacuum), and momentum $p = E/c = mc$ and exerts pressure on the obstacle.

Phenomenon GRS was predicted by A. Einstein. In 1911, in article «Over the influence of gravity on the propagation of light» [1] he wrote: «The theory of relativity led to the conclusion that the inertial mass of a body increases with the increase of energy contained in it; if the increment of the energy is E, the increment of the inertial mass is equal to E/c^2… But does this increment of the inertial mass also correspond to the increment of the gravitating mass?».

Exploring the process of emission of light in a gravitational field, Einstein came to the conclusion that «the increment of the gravitating mass equal to the increment of the inertial mass» (the equivalence principle). He suggested that gravity affects at the electromagnetic radiation originating from the distant sources, namely, it shifts the spectral lines in the red side of the spectrum on the relative value

$$(\nu_1 - \nu_2)/\nu_1 = (\Phi_1 - \Phi_2)/c^2 = GM/r_1c^2 - GM/r_2c^2, \quad (1)$$

where ν_1 is the frequency of the electromagnetic radiation emitted by cosmic source, ν_2 is the frequency of the electromagnetic radiation measured by the observer (at the Earth's surface), $\nu_1 - \nu_2 = \Delta\nu$ is a difference of frequencies, $\Phi_1 = GM/r_1$ and $\Phi_2 = GM/r_2$ are the values of the gravitational field potential at the points of emission and observation, G is the gravitational constant, M is the mass of

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the gravitating body, r_1 is the distance from the the body's center of mass to the point of radiation, r_2 is the distance from the the body's center of mass to the point of observation.

Formula (1), like any other, must be analyzed, but in this case there is the following difficulty.

In the 20th century, the cult of Einstein was created (like the cult of Aristotle in medieval Europe): he was proclaimed an infallible genius and no one should doubt this. Therefore, there is an unwritten rule in the editorial offices of scientific journals: not to allow publication of articles that criticize Einstein's works, even if it is obvious that there are weighty grounds for criticism. However, there are exceptions; let's give an example.

At the beginning of the 20th century, it was discovered that the mass of a charged elementary particle (electron) depends on its speed. In 1904, H. Lorentz [2] and in 1905 Einstein [3] proposed that this dependence is described by the formula

$$m = m_0/(1 - v^2/c^2)^{1/2}, \quad (2)$$

where m_0 is the mass of a rest particle, m is its relativistic mass, v is speed.

Hence, the assertion arose that when the speed of a particle approaches the speed of light ($v \to c$), then its mass (energy) increases indefinitely ($m \to \infty$). However, this statement is only a historical misconception.

The article [4] was recently published, which shows that the mass (energy) of a moving particle cannot increase infinitely, and it has reached the maximum value $M = \lim m$ on the maximum velocity v_M, which is very close to the speed of light ($v_M \approx c$), moreover, the relativistic mass limit cannot exceed the Stoney mass $m_s = (ke^2/G)^{1/2} = 1.859 \times 10^{-9}$ kg (k is the proportionality coefficient, e is the elementary charge, and G is the gravitational constant):

$$m = m_0/(1 - v^2/c^2)^{1/2}, \quad v \le v_M, \; m \le M, \; M \le m_s. \quad (3)$$

For example, the proton energy limit $M_p c^2 = 9.19 \times 10^{21}$ eV; this is the upper limit of the cosmic ray spectrum.

Thus, new ideas, although with difficulty, but still make their way. Therefore, we will continue the consideration of GRS and obtain two exact formulas.

2. Method

We use the known relation for the kinetic energy E of a light quantum (photon):

$$E = mc^2 = hv, \quad (4)$$

where m is the mass of a moving photon, v is its frequency, h is Planck constant.

Let at the starting point, for example on the surface of star or the Earth of radius r_1, is emitted a quantum with initial mass m_1 and frequency v_1. When photon is moving in the gravitational field of the body up to a height $\Delta r = r_2 - r_1$ (r_2 is the distance from the body's mass center to the endpoint of the photon move), is performed negative work and as a result *a kinetic energy of the photon is decreased* (by reducing its mass and frequency) by magnitude

$$\Delta E = (m_1 - m_2)c^2 = h(v_1 - v_2), \quad (5)$$

where m_2 and v_2 are final mass and final frequency of the photon.

The equality of the masses, characterizing the inertial and gravitational properties of ordinary objects, tested in experiments to high relative accuracy (10^{-13}), so it is natural to assume the validity of the equivalence principle for photon. Let's see if that's the case.

According to Equation (1):

$$(v_1 - v_2)c^2 = v_1(\Phi_1 - \Phi_2) \quad (6)$$

or (considering that $v = mc^2/h$)

$$(m_1 - m_2)c^2 = m_1(\Phi_1 - \Phi_2). \quad (7)$$

Note that the left side of this equation includes the values m_1 and m_2 of the photon's inertial mass, while the right side contains the value of its gravitational mass m_1.

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Thus, from formula (1) it follows that when a photon moves upward, its inertial mass (m_i) *decreases* from m_1 to m_2, and its gravitational mass (m_g) *is not changed*, remaining equal to the initial value m_1 (i.e. there is no increment of the gravitating mass).

Is obtained from Eq. (1) that in all the way the inertial mass of the photon *does not coincide* with its gravitational mass, $m_i \neq m_g$, which violates the equivalence principle.

In addition, the quantity $m_1\Phi_1 = |U_1|$, i.e. is equal to the absolute value of the potential energy of a photon in the initial position, but the quantity $m_1\Phi_2$ *has no a physical sense*. Thus, Einstein's formula needs to be corrected.

To get the exact equation it is necessary to apply *the law of conservation of mechanical energy*, according to which the sum (W) of the kinetic ($E = mc^2$) and potential ($U = -m\Phi$) energy of the bodies, interacting with each other by the forces of gravitation, is not changed:

$$W = E + U = [mc^2 - m\Phi] = \text{const.} \quad (8)$$

Thus, the total energy of the photon

$$W = m_1c^2 - m_1\Phi_1 = m_2c^2 - m_2\Phi_2. \quad (9)$$

Therefore, an decrease in the kinetic energy ΔE of the photon is compensated by an increase in its potential energy $\Delta U = U_2 - U_1$:

$$\Delta E = m_1c^2 - m_2c^2 = m_1\Phi_1 - m_2\Phi_2 = (\nu_1\Phi_1 - \nu_2\Phi_2)h/c^2. \quad (10)$$

This process corresponds to the reduction of frequency:

$$\Delta\nu = \Delta U/h = (\nu_1\Phi_1 - \nu_2\Phi_2)/c^2. \quad (11)$$

From here we find *the first exact formula* of the relative change in the radiation frequency:

$$\Delta\nu/\nu_1 = [\Phi_1 - \Phi_2(\nu_2/\nu_1)]/c^2. \quad (12)$$

If the light goes way Δr small in comparison with the radius r_1 of the Earth or the star, $\Delta r << r_1$, then the frequency ratio $\nu_2/\nu_1 \approx 1$ and the exact formula (12) becomes an approximate which coincides with the Einstein's formula (1):

$$\Delta\nu/\nu_1 \approx (\Phi_1 - \Phi_2)/c^2 = g_{av}\Delta r/c^2, \text{ if } \Delta r << r_1,\ \nu_2/\nu_1 \approx 1, \quad (13)$$

where $g_{av} = GM/r_1r_2$ is the average acceleration of free fall, created by the gravity of a star or the Earth in the way Δr.

In 1960, American scientists Paund and Rebka [5] first confirmed the existence of the gravitational redshift. In their experiment, the gamma-quanta directed vertically upwards to a height $\Delta r \approx 22.5$ m near the Earth's surface ($r_1 \approx 6370$ km, $g_{av} \approx 9.8$ m/s) and is measured decrease in the frequency: the measured value $\Delta\nu/\nu_1 \approx 2.56\times10^{-15}$ was close to the theoretical value 2.46×10^{-15} calculated by the formula (1).

It is clear that if the researchers had known exact formula (12), then they would have the theoretical value closer to the experimental (i.e. more than 2.46×10^{-15}), as

$$\nu_2/\nu_1 < 1,\ \Phi_2(\nu_2/\nu_1) < \Phi_2,\ (\Phi_1 - \Phi_2)/c^2 < [\Phi_1 - \Phi_2(\nu_2/\nu_1)]/c^2. \quad (14)$$

In addition, using equation (11) we can derive *the second exact formula* of the relative change in the radiation frequency [6]:

$$\Delta\nu/\nu_2 = [(\nu_1/\nu_2)\Phi_1 - \Phi_2]/c^2. \quad (15)$$

Likewise, if the light goes way Δr small in comparison with the radius r_1 of the Earth or the star, $\Delta r << r_1$, then the frequency ratio $\nu_1/\nu_2 \approx 1$ and the exact formula (15) becomes an approximate which coincides with the Einstein's formula (1):

$$\Delta\nu/\nu_2 \approx (\Phi_1 - \Phi_2)/c^2 = g_{av}\Delta r/c^2, \text{ if } \Delta r << r_1,\ \nu_1/\nu_2 \approx 1. \quad (16)$$

3. Conclusion

The time of boundless admiration for Einstein is long gone and his works should be analyzed in the same way as the works of other scientists. The gravitational redshift is a very important

phenomenon, which requires a simple and clear explanation. We have shown that Einstein's formula (1) is approximate and applicable when the light travels a way Δr negligible compared to the radius r_1 of the star or the Earth, $\Delta r << r_1$, i.e. when the frequency ratio $v_1/v_2 \approx 1$.

In the case when the frequencies v_1 and v_2 are significantly different, formula (1) gives an incorrect result. This is explained by the fact that in the derivation of the formula Einstein did not use the law of conservation of mechanical energy. Cannot continue to ignore this fact, because this formula is actively used in astrophysical calculations.

Note that the derivation of this formula takes 6 pages in Einstein's article. Thus, verbosity is not a guarantee of obtaining the correct scientific result.

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