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Extension of Fuzzy ELECTRE I for Evaluating Demand Forecasting Methods in Sustainable Manufacturing

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Abstract: The selectin of a demand forecasting method is critical for companies aiming to avoid manufacturing overproduction or shortages in pursuit of sustainable development. Various qualitative and quantitative criteria with different weights must be considered during the evaluation of a forecasting method. The qualitative criteria and criteria weights are usually assessed in linguistic terms. Aggregating these various criteria and linguistic weights for evaluating and selecting demand forecasting methods in sustainable manufacturing is a major challenge. This paper proposes an extension of fuzzy Elimination and Choice Translating Reality (ELECTRE) I to resolve this problem. In the proposed method, fuzzy weighted ratings are defuzzified with the signed distance to develop a crisp ELECTRE I model. Moreover, an extension to ELECTRE I is developed by suggesting an extended modified discordance matrix and a closeness coefficient for ranking alternatives. The proposed extension can overcome the problem of information loss which can lead to incorrect ranking result when using the Hadamard product to combine concordance and modified discordance matrices. A comparison is conducted to show the advantage of the proposed extension. Finally, a numerical example is used to demonstrate the feasibility of the proposed method; furthermore, a numerical comparison is made to display the advantage of the proposed method.

Keywords: demand forecasting methods; fuzzy ELECTRE I; extended modified discordance matrix; Hadamard product; closeness coefficient

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1. Introduction

Manufacturing is a key driver of growth and global development and is a major contributor to the creation of prosperity and employment, especially in growing economies; however, industrial activities have a substantial environmental burden (Haapala et al., 2013). The key considerations in sustainable manufacturing (green manufacturing) are the efficient use of resources through the enhancement of resource productivity (Dornfeld, 2014) and minimization of waste (Abdul Rashid et al., 2008). Therefore, to avoid overproduction or shortages during production, accurately forecasting demand is necessary to ensure the production of an adequate number of intermediate parts and final products. Manufacturing systems based on advanced forecasting subsystems play a key role in supply chains and can facilitate environmental protection and long-term sustainable development (Abdul Rashid et al., 2008).

A forecasting method is a method of predicting the solution to a task to enable users to accurately predict outcomes (Pilinkienė, 2008). A poor manufacturing forecast could cause a buildup of product stock, leading to increasing part ordering and holding costs (Choudhury, 2018). Therefore, demand forecasting is a crucial element of the planning process for companies with the goal of sustainable development. More than 200 forecasting methods are described in the economic literature; these

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methods can be classified on the basis of the following criteria: type of information, forecast time span, forecast object, and forecast goal (Pilinkienė, 2008). Dweiri et al. (2015) used the analytic hierarchy process (AHP) to select a production planning forecasting method in a supply chain. Dahooie et al. (2019) provided a hybrid method of fuzzy MULTIMOORA approach for multi-criteria decision making and objective weighting method (CCSD) to select a forecasting method for technology. Various qualitative criteria, such as ease of use and data validity, and quantitative criteria, including implementation cost and forecast accuracy, must be considered when evaluating demand forecasting methods; these criteria may differ in importance. Evaluating demand forecasting methods is thus a multiple criteria decision-making (MCDM) problem. Therefore, how to aggregate various criteria and their weights to select the most suitable demand forecasting method is a key challenge in forecasting research. To overcome this challenge, this paper proposes an extension of the fuzzy ELECTRE method for selecting the best demand forecasting method.

The ELECTRE method (Roy, 1968) is an MCDM method based on outranking relations. An advantage of the ELECTRE method that it achieves a more realistic decision-making process by including both the criteria weights and the preferences of the decision-maker in the selection process (Singh and Kaushik, 2019). The ELECTRE method is one of the most effective decision-making techniques; the method outputs a reduced set of suitable alternatives by using outranking relations to remove options outranked by other options (Akram et al., 2019). Several versions of the ELECTRE method have been proposed, namely ELECTRE I, II, III, IV, IS, and TRI. Among these versions, ELECTRE I involves a choice problem and attempts to select a small group of favorable alternatives to facilitate the ultimate selection of a single alternative (Zandi and Roghanian, 2013). However, because some decision makers provide their opinions using linguistic terms, the performance ratings and criteria weights in the ELECTRE method cannot be measured precisely (Hatami-Marbini and Tavana, 2011). Scholars have investigated combining ELECTRE I with fuzzy set theory for addressing the imprecise or vague nature of linguistic assessments. Shojaie et al. (2018) used fuzzy ELECTRE to evaluate green health suppliers, and they conducted a case study of Tehran Chemie Pharmaceutical Company. Akram et al. (2020) introduced bipolar fuzzy TOPSIS and bipolar fuzzy ELECTTRE I to determine a disease that explains a patient's symptoms. Moreover, Massami and Manyasi (2021) used fuzzy ELECTRE to determine the importance of various criteria and sub-criteria for evaluating the performance of sailors.

To the best of our knowledge, fuzzy ELECTRE I has yet to be applied for selecting demand forecasting methods selection has never been studied before. To fill this gap, this paper proposes an extension of fuzzy ELECTRE I for selecting the most suitable demand forecasting method. In the proposed method, the membership function of the fuzzy weighted rating of each alternative for each qualitative criterion is defuzzified by applying the signed distance (Yao and Wu, 2000) to form a crisp ELECTRE I model. Defuzzification formulas can be precisely derived to improve the model for assisting in decision-making. In addition, the proposed ELECTRE I model uses a closeness coefficient, derived on the basis of an expanded modified discordance matrix, to rank alternatives. The proposed closeness coefficient can resolve the problem of information loss, which can otherwise lead to incorrect ranking result when the Hadamard product is used to combine the concordance matrix and modified discordance matrix. A comparison with some other methods will be used to present the advantage of the proposed extension based on closeness coefficient. Moreover, a numerical example will be provided to display the feasibility of the proposed method. In addition, a numerical comparison with other methods will be conducted to display the advantage of the proposed method. The rest of this paper is organized as follows. Section 2 presents a review of literature. Section 3 introduces the basic concepts of fuzzy set theory. Section 4 describes the model establishment process, with Section 4.1 presenting a comparison of the developed extension with other approaches to demonstrate the advantages of the proposed closeness coefficient using expanded modified discordance matrix. Section 5 provides a numerical example to demonstrate the feasibility of the proposed method, with Section 5.1 presenting a numerical comparison and analysis to display the advantage of the proposed method. Finally, Section 6 addresses the conclusion.

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2. Literature review

2.1. Sustainable manufacturing

Since the 1980s, the core goal of sustainable manufacturing has been waste reduction, and the aim of cleaner manufacturing is to increase available resources and reduce energy usage in manufacturing (Seliger et al., 2008). Recently, sustainable manufacturing has been defined as "the ability to smartly use natural resources for manufacturing, by creating products and solutions that, thanks to new technology, regulatory measures and coherent social behaviours, are able to satisfy economical, environmental, and social objectives, thus preserving the environment while continuing to improve the quality of human life" (Garetti and Taisch, 2012). The pursuit of sustainability affects operations and manufacturing activities in which input materials and energy are converted into commercial products (Haapala et al., 2013). Moreover, material and equipment that are adaptable to various situations are required for flexible manufacturing, which is responsive to variations in material flows, and flexible manufacturing can enhance sustainability while maintaining competitiveness (Rosen and Kishawy, 2012). Owing to the increased complexity and performance expectations in supply chains for high-tech products, forecasting product demands is now key for efficiently managing operations (Dweiri et al., 2015). Therefore, an accurate demand forecasting method that can avoid overproduction or shortages and facilitate sustainable manufacturing should be the cornerstone of a sustainable supply chain. Forecasting methods in sustainable manufacturing have drawn the attention of numerous scholars in various fields. Hart et al. (2017) introduced effective manufacturing systems for supply chains based on demand forecasting. Rivera-Castro et al. (2019) presented diagonal feeding, a useful technique for forecasting build-to-order lean manufacturing supply chains. A review of the literature regarding sustainable manufacturing strategies can be found in the article by Garetti and Taisch (2012).

2.2. Demand forecasting method selection

Demand forecasting is critical for industrial firms because many decision-making processes require accurate forecasts for the selection of appropriate strategies for sales budgeting, production planning, new product launches, and other business activities (Choudhury, 2018). Moreover, accurate demand forecasts are necessary to create an effective master plan that can facilitate all managerial processes involved in internal and external material flows, enabling comprehensive supply chain management (Hart et al., 2017). Forecasting methods can be classified as quantitative methods, including the simple moving average method and the exponential smoothing method, and qualitative methods, including the Delphi method and the nominal group technique. Various categories of forecasting methods are available to businesses; therefore, selecting an appropriate method is key. Qualitative forecasting methods are based on experts' opinions and may thus marred by several biases; by contrast, quantitative methods analyze previously acquired data, which may not be applicable if the business environment changes substantially (Dweiri et al., 2015). Combining qualitative and quantitative methods could achieve substantially improved results compared with those produced with a single approach (Dweiri et al., 2015). Therefore, various quantitative criteria, such as accuracy and maintenance cost, and qualitative criteria, including method adaptability and data validity, are necessary for selecting the optimal demand forecasting method; the weights of these criteria are also necessary for this selection process. Accordingly, evaluating demand forecasting methods is an MCDM problem. Selecting the best forecasting method is a critical task for many manufacturers, and the inappropriate selection of a forecasting method can result in reduced sales and market share. To overcome this challenge, this paper proposes an extension of fuzzy ELECTRE for selecting the most suitable method. The selection of forecasting methods has been investigated by researchers in various fields. For example, Acar and Gardner (2012) used tradeoff curves, considering total costs and customer service, to select the optimal forecasting method. Intepe et al. (2013) used TOPSIS, an intuitionistic fuzzy environment method, to select the optimal forecasting method. Dahooie et al. (2019) developed a hybrid method of fuzzy MULTIMOORA approach for MCDM and objective weighting method (CCSD) for selecting a forecasting method for technology. Furthermore,

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Taghiyeh et al. (2020) proposed a new forecasting method selection scheme by considering intermediate classifications. Meira et al. (2021) used prediction interval to select and enhance the predictive power of forecasting models. Hanifi et al. (2020) studied the literature regarding wind power forecasting with physical, statistical, and hybrid methods. introduced Kuznietsova et al. (2021) introduced a data technique for evaluating and forecasting roaming cell services in Ukrain.

2.3. Fuzzy ELECTRE method

The ELECTRE method was developed in 1965 and is suitable for selecting the best action from a given set of actions. The ELECTRE family is one of the most powerful MCDM techniques based on outranking relations. Fundamentally, the ELECTRE method eliminate options that are worse than other options by a specified degree (Akram et al., 2019). Because ELECTRE allows combining qualitative and quantitative information, it is considered a flexible method that requires less complicated information (Tolga, 2012). Moreover, the ELECTRE method enables rating the alternatives for each criterion independently without aggregating the score of the alternatives for all criteria (Çalı and Balaman, 2019). Among the methods in the ELECTRE family, ELECTRE I is one of the most widely used version. The ELECTRE I method is applied to selection problems (Adeel et al., 2019) and its complexity can be easily increased through combination with other methods (Govindan and Jepsen, 2016). However, the ELECTRE method lacks precise measurements for producing criteria weights and performance ratings (Hatami-Marbini and Tavana, 2011) because exact (or crisp) numbers are often inadequate for describing real-life situations. Fuzzy set theory (Zadeh, 1965) is an ideal solution for overcoming this problem in that it resembles human reasoning in its use of approximate information and uncertainty to generate decisions (Belbag et al., 2016). The core advantages of the fuzzy-ELECTRE method can be summarized that it is very applicable and non-compensatory when criteria are described as in the ordinal scale (Chhipi-Shrestha et al., 2017). Therefore, this paper combined the ELECTRE I method and fuzzy set theory to select the best demand forecasting method. Fuzzy ELECTRE has been investigated by researchers in various fields. Belbag et al. (2016) used fuzzy ELECTRE to rank four smart phone brands on the basis of a survey of 250 students. Akram et al. (2020) indicated that Pythagorean fuzzy set model can effectively capture the vagueness in human evaluations and thus proposed a Pythagorean fuzzy ELECTRE I method. Ayyildiz et al. (2020) proposed integrating the AHP and the ELECTRE method using interval type-2 trapezoidal fuzzy ELECTRE to evaluate individual credit. Chen (2020) developed an extension of the ELECTRE method by using novel Chebyshev distance measures as Pythagorean membership grades and applied it to bridge-superstructure construction methods for validating feasibility and applicability. Wang and Chen (2021) used a T-spherical fuzzy ELECTRE approach to select potential companies for extending the scope of a business. However, fuzzy ELECTRE I has yet to be applied to select demand forecasting methods for sustainable manufacturing. To fill this gap, the study proposes an extension of fuzzy ELECTRE I for selecting the most suitable demand forecasting method.

In fuzzy ELECTRE I, the division of two fuzzy numbers is needed to produce a discordance matrix; however, the membership function produced by this division has not been precisely defined. Thus, a proper defuzzification method is necessary to produce the discordance matrix. Numerous ranking/defuzzification methods have been investigated. Peddi (2019) proposed a defuzzification method for ranking fuzzy numbers based on centroids and maximizing and minimizing sets. The literature on defuzzification methods has a long history which can be seen in Kataria (2010), Kumar (2017) and Talon and Curt (2017). Recently published works are described in the articles by Arman et al. (2021) and Menìz (2021). Each method has advantages and disadvantages. In this paper, the signed distance (Yao and Wu, 2000) method is used because it is simple and can be applied to both negative and positive fuzzy numbers. Moreover, this paper derives defuzzification formulas based on signed distance (Yao and Wu, 2000) to derive an ELECTRE I model that can assist in decision-making. Zhang et al. (2018) used the ELECTRE method to determine the ranking order of substrate nodes for resolving a virtual network embedding problem. They obtained the modified weighted summation matrix for ranking alternatives by using the Hadamard product to combine the

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concordance and modified discordance matrices. Despite the merits of the method proposed by Zhang et al. (2018), it may have information loss that could lead to the production an incorrect ranking order. Nghiem and Chu (2021) suggested ranking sustainable conceptual designs by using a total net dominance value based on Nijkamp and Van Delft's (1977) net concordance dominance value and net modified discordance dominance value in order to avoid information loss in the method of Zhang et al. (2018) method. Moreover, Ke and Chen (2012) suggested an ELECTRE method to for selecting e-services; the Hadamard product of the concordance matrix and modified discordance matrix was used to obtain the modified total matrix for ranking alternatives. Despite the merits of their method, it also can produce an incorrect ranking due to information loss resulting from zero values in the modified discordance matrix when the Hadamard product is used. To resolve this problem in Ke and Chen (2012), Nghiem and Chu (2022) proposed subtracting discordance values from concordance values to obtain the total dominance matrix to produce the Boolean matrix to obtain ranking result, and further applied it to develop a BWM-based fuzzy ELECTRE I method to evaluate lean facility layout designs. Nevertheless, the two suggested methods still exhibit the problem of information loss when the Hadamard product is used. To resolve this problem, the present study adopts a closeness coefficient based on an extended modified discordance matrix. Herein, the proposed extension is compared with the methods of Ke and Chen (2012) and Zhang et al. (2018) to demonstrate its advantages. Finally, a numerical example is used to show the feasibility of the proposed method; furthermore, a numerical comparison is conducted with some other methods to display the advantages of the proposed fuzzy ELECTRE I method.

3. Fuzzy set theory

3.1. Fuzzy sets

A fuzzy set $\tilde{A}$ can be denoted as $\tilde{A}=\{(x, f_{\tilde{A}}(x)) | x \in X\}$, where X is the universe of discourse. The fuzzy set $\tilde{A}$ in the universe of discourse X is characterized by a membership function $f_{\tilde{A}}(x)$, $\forall x \in X$, $f_{\tilde{A}}(x) \in [0,1]$ (Kaufmann and Gupta, 1991). The lager $f_{\tilde{A}}(x)$, the stronger the grade of membership for x in $\tilde{A}$. A fuzzy number is a fuzzy set. Definition of a fuzzy number can be seen in Dubois and Prade (1978).

3.2. Fuzzy numbers

A triangular fuzzy number is denoted as $\tilde{A}=(a^l, a^\lambda, a^u)$, with the membership function $f_{\tilde{A}}(x)$ is presented by (Laarhoven and Pedrycz, 1983), and $\tilde{A}$ can be denoted as $[a^l, a^\lambda, a^u]$ if $f_{\tilde{A}}(x)$ is nonlinear.

$$f_{\tilde{A}}(x)=\begin{cases} \dfrac{x-a^l}{a^\lambda-a^l}, & a^l \le x \le a^\lambda, \\ \dfrac{x-a^u}{a^\lambda-a^u}, & a^\lambda \le x \le a^u, \\ 0, & otherwise, \end{cases} \tag{1}$$

3.3. Arithmetic operations on fuzzy numbers

Assume two fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$, with the α- cut $(0 \le \alpha \le 1)$, the closed interval can be defined as $\tilde{A}_1^{\alpha} = \left[A_1^L, A_1^R\right]$, $\tilde{A}_2^{\alpha} = \left[A_2^L, A_2^R\right]$. Some main operations of two fuzzy numbers can be determined as follows (Kaufmann and Gupta, 1991).

$$\left(\tilde{A}_1 \oplus \tilde{A}_2\right)^{\alpha} = \left[A_1^L + A_2^L, A_1^R + A_2^R\right] \tag{2}$$

$$\left(\tilde{A}_1 \ominus \tilde{A}_2\right)^{\alpha} = \left[A_1^L - A_2^R, A_1^R - A_2^L\right] \tag{3}$$

$$\left(\tilde{A}_1 \otimes \tilde{A}_2\right)^{\alpha} = \left[A_1^L \times A_2^L, A_1^R \times A_2^R\right] \tag{4}$$

$$z \otimes \tilde{A}_1^{\alpha} = \left[z \times A_1^L, z \times A_1^R\right],\ z \in R^+ \tag{5}$$

3.4. Ranking fuzzy numbers by signed distance

Fuzzy number ranking is one of the steps in a fuzzy MCDM model. Scholars have investigated various ranking methods, including Chen and Hong (2014), Yu et al. (2017), Nayagam et al. (2018), De et al. (2020), Aguilera et al. (2021), and Hop (2022). Yao and Wu (2000) proposed the use of the signed distance to rank fuzzy numbers; this method has been used by many researchers and can be applied to both negative and positive fuzzy numbers. The present study applies the signed distance because of its intuitive nature. The signed distance calculates the distance of the middle point of the left and right end point from the y-axis. If a fuzzy number with the middle point is further away from the y-axis, the fuzzy number receives the larger value. The signed distance of two fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ is calculated as follows:

$$d\left(\tilde{A}_1, \tilde{A}_2\right) = d\left(\tilde{A}_1, 0_1\right) - d\left(\tilde{A}_1, 0_1\right) = \frac{1}{2}\int_0^1 \left[A_1^L + A_1^R - A_2^L - A_2^R\right] d\alpha \tag{6}$$

3.5. Linguistic values

A linguistic variable is a one whose values are not numbers but are instead words or linguistic terms. The concept of a linguistic variable is useful in complex situations (Zadeh, 1975); linguistic variables can be used to describe the degrees of a criterion if crisp data are inefficient for modeling a real situation in MCDM (Wang and Lee, 2009). For example, the ratings of alternative versus qualitative criteria constitute a linguistic variable whose values can be defined as VL (very low), L (low), M (medium), H (high), and VH (very high). These linguistic variables can be represented by triangular fuzzy numbers such as the following: VL = (0, 0.1, 0.3), L = (0.1, 0.3, 0.5), M = (0.3, 0.5, 0.7), H = (0.5, 0.7, 0.9), and VH = (0.7, 0.9, 1).

4. Model establishment

Assume that a committee of *k* experts (i.e. E_t, $t = 1 \sim k$) is responsible for the evaluation of *m* alternatives (i.e. $O_i, i = 1 \sim m$) under *n* criteria ($C_j, j = 1 \sim n$). The criteria can be categorized to quantitative and qualitative. Quantitative criteria can be further classified to benefit (B), for which larger-is-better, and cost (C), for which smaller-is-better.

Step 1. Develop decision matrix

Assume $\tilde{p}_{ijt} = \left(p_{ijt}^{l}, p_{ijt}^{\lambda}, p_{ijt}^{u}\right)$, $\tilde{p}_{ijt} \in R^{+}, i = 1 \sim m, j = 1 \sim n$, $t = 1 \sim k$, are the ratings of O_i with respect to C_j criterion given by the expert E_t. Further assume that the ratings of alternative versus qualitative criteria are provided by the experts employing the linguistic terms with equivalent fuzzy numbers as displayed in section 3.5, which can be aggregated by Eq. (7).

$$\tilde{p}_{ij} = \left(\frac{1}{k}\right) \times (\tilde{p}_{ij1} \oplus \ldots \oplus \tilde{p}_{ijt} \oplus \ldots \oplus \tilde{p}_{ijk}), j \text{ is the qualitative criterion} \tag{7}$$

Step 2. Normalization of values under quantitative criteria

The normalized decision matrix is necessary to ensure that all the performance ratings of alternative versus quantitative criteria have homogeneous and comparable scale. Assume that $\tilde{r}_{ij} = \left(r_{ij}^{l}, r_{ij}^{\lambda}, r_{ij}^{u}\right)$ is the normalized value of $\tilde{p}_{ij}$. The formulas of normalizing the values under quantitative criteria are presented in Eq. (8)-(9).

$$\tilde{r}_{ij} = \left(r_{ij}^{l}, r_{ij}^{\lambda}, r_{ij}^{u}\right) = \left(\frac{p_{ij}^{l}}{b_j^{+}}, \frac{p_{ij}^{\lambda}}{b_j^{+}}, \frac{p_{ij}^{u}}{b_j^{+}}\right), j \in B \tag{8}$$

$$\tilde{r}_{ij} = \left(r_{ij}^{l}, r_{ij}^{\lambda}, r_{ij}^{u}\right) = \left(\frac{a_j^{-}}{p_{ij}^{u}}, \frac{a_j^{-}}{p_{ij}^{\lambda}}, \frac{a_j^{-}}{p_{ij}^{l}}\right), j \in C \quad \text{where} \\ b_j^{+} = \max_i \left(p_{ij}^{u}\right), a_j^{-} = \min_i \left(p_{ij}^{l}\right) \tag{9}$$

Step 3. Determine the criteria weights

Suppose that $\tilde{w}_{jt} = \left(w_{jt}^{l}, w_{jt}^{\lambda}, w_{jt}^{u}\right)$ denotes the weight of criterion C_j given by the expert E_t. The average value of criteria weight is produced by the committee of experts by Eq. (10).

$$\tilde{w}_j = \left(\frac{1}{k}\right) \times \left(\tilde{w}_{j1} + \ldots + \tilde{w}_{jt} + \ldots + \tilde{w}_{jk}\right) \tag{10}$$

where $w_j^{l} = \sum_{t=1}^{k} \frac{w_{jt}^{l}}{k}, w_j^{\lambda} = \sum_{t=1}^{k} \frac{w_{jt}^{\lambda}}{k}, w_j^{u} = \sum_{t=1}^{k} \frac{w_{jt}^{u}}{k}$

Step 4. Weighted normalization matrix

The weighted normalized value, $\tilde{v}_{ij} = \left[v_{ij}^{l}, v_{ij}^{\lambda}, v_{ij}^{u}\right]$, in decision matrix is obtained by the following equation.

$$\tilde{v}_{ij} = \left(r_{ij}^{l}, r_{ij}^{\lambda}, r_{ij}^{u}\right) \otimes \left(w_j^{l}, w_j^{\lambda}, w_j^{u}\right) \tag{11}$$

Suppose the $\alpha -$cut of $\tilde{v}_{ij}$ is denoted as $v_{ij}^{\alpha} = \left[v_{ij}^{L}, v_{ij}^{R}\right]$. Formulas for v_{ij}^{α} can be developed by Eqs. (2)-(5) as shown in the following equation (Kaufmann and Gupta, 1991).

$$v_{ij}^{\alpha} = \left[r_{ij}^{L} \cdot w_j^{L},\ r_{ij}^{R} \cdot w_j^{R}\right] = \left[\alpha^2 \cdot G_{ij1} + \alpha \cdot H_{ij1} + L_{ij1},\ \alpha^2 \cdot G_{ij2} + \alpha \cdot H_{ij2} + L_{ij2}\right] \tag{12}$$

where $G_{ij1} = \left(r_{ij}^{\lambda} - r_{ij}^{l}\right) \cdot \left(w_j^{\lambda} - w_j^{l}\right),\ H_{ij1} = r_{ij}^{l} \cdot \left(w_j^{\lambda} - w_j^{l}\right) + w_j^{l} \cdot \left(r_{ij}^{\lambda} - r_{ij}^{l}\right),\ L_{ij1} = r_{ij}^{l} \cdot w_j^{l}$

$$G_{ij2} = \left(r_{ij}^{\lambda} - r_{ij}^{u}\right)\cdot\left(w_j^{\lambda} - w_j^{u}\right),\ H_{ij2} = r_{ij}^{u}\cdot\left(w_j^{\lambda} - w_j^{u}\right) + w_j^{u}\cdot\left(r_{ij}^{\lambda} - r_{ij}^{u}\right),\ L_{ij2} = r_{ij}^{u}\cdot w_j^{u}$$

Step 5. Defuzzification

The signed distance of two fuzzy numbers, $\tilde{v}_{ij}$ and $\tilde{v}_{sj}$, is obtained by the by Yao and Wu (2000) as in Eq. (13).

$$d\left(\tilde{v}_{ij},\tilde{v}_{sj}\right) = \frac{1}{2}\int_0^1 \left(v_{ij}^{L} + v_{ij}^{R} - v_{sj}^{L} - v_{sj}^{R}\right)d\alpha$$

$$= \frac{1}{2}\int_0^1 \left(\alpha^2\left(G_{ij1}+G_{ij2}\right) + \alpha\left(H_{ij1}+H_{ij2}\right) + L_{ij1} + L_{ij2} - \alpha^2\left(G_{sj1}+G_{sj2}\right) - \alpha\left(H_{sj1}+H_{sj2}\right) - L_{sj1} - L_{sj2}\right)d\alpha \tag{13}$$

$$= \frac{G_{ij1}+G_{ij2}-G_{sj1}-G_{sj2}}{6} + \frac{H_{ij1}+H_{ij2}-H_{sj1}-H_{sj2}}{4} + \frac{L_{ij1}+L_{ij2}-L_{sj1}-L_{sj2}}{2}$$

Step 6. Identify the concordance and discordance sets

By Yao and Wu (2000), $\tilde{v}_{ij} > \tilde{v}_{sj}$ iff $d\left(\tilde{v}_{ij},\tilde{v}_{sj}\right) > 0$; $\tilde{v}_{ij} < \tilde{v}_{sj}$ iff $d\left(\tilde{v}_{ij},\tilde{v}_{sj}\right) < 0$; $\tilde{v}_{ij} = \tilde{v}_{sj}$ iff $d\left(\tilde{v}_{ij},\tilde{v}_{sj}\right) = 0$. The concordance and discordance sets can be determined as:

$$C_{is} = \left\{j, \tilde{v}_{ij} \geq \tilde{v}_{sj}\right\} \tag{14}$$

$$D_{is} = \left\{j, \tilde{v}_{ij} < \tilde{v}_{sj}\right\} \tag{15}$$

Step 7. Produce concordance and discordance matrices

First, the fuzzy weight $\tilde{w}_j$ is defuzzified by the signed distance as shown in Eq. (16) and is normalized to obtain the crisp weight w_j as shown in Eq. (17).

$$w_j^{'} = d\left(\tilde{w}_j, 0\right) = \frac{1}{2}\int_0^1 (w_j^{L} + w_j^{R})d\alpha = \frac{1}{4}\left(2w_j^{\lambda} + w_j^{l} + w_j^{u}\right) \tag{16}$$

$$w_j = \frac{w_j^{'}}{\sum_{j=1}^{n} w_j^{'}} \quad \text{where} \sum_{j=1}^{n} w_j = 1 \tag{17}$$

The concordance matrix is produced by aggregating the criteria weights in the concordance set. The formula for concordance matrix *Con* can be obtained by Eq. (18).

$$Con = \left[c_{is}\right]_{m\times m},\ c_{is} = \frac{\sum_{j\in C_{is}} w_j}{\sum_{j=1}^{n} w_j} \tag{18}$$

The discordance matrix is produced by Eq. (19), in which the $d\left(\tilde{v}_{ij},\tilde{v}_{sj}\right)$ is obtained by the signed distance (Yao and Wu, 2000).

$$D=\left[d_{is}\right]_{m\times m}\ ,\ d_{is}=\frac{\max\limits_{j\in D_{is}}\left\{\left|d\left(\tilde{v}_{ij},\tilde{v}_{sj}\right)\right|\right\}}{\max\limits_{j\in J}\left\{\left|d\left(\tilde{v}_{ij},\tilde{v}_{sj}\right)\right|\right\}},\ J=\{1,2,.....,n\} \quad (19)$$

Step 8. An extended modified discordance matrix

Ke and Chen (2012) introduced the Hadamard product of c_{is} and $d_{is}^{'}$, where $d_{is}^{'}=1-d_{is}$, to obtain a modified total matrix for ranking alternatives. Despite the merits of this method, it can result in information loss because if a value of $d_{is}^{'}$ is zero; the corresponding value in the modified total matrix will be zero owing to the nature of multiplication, which can influence the ranking order. Zhang et al. (2018) used the Hadamard product to obtain a modified weighted summation matrix to produce net dominating values for ranking alternatives. Despite the merits, their method could also result in information loss that influences the ranking order. To resolve this problem, we propose an extended modified discordance matrix as follows.

$$D^{'}=\left[d_{is}^{'}\right]_{m\times m}+\left[1_{is,i\neq s}\right]_{m\times m},\ \text{where } d_{is}^{'}=1-d_{is},\ i,s=1\sim m \quad (20)$$

Because "1" is the maximum value in the discordance matrix according to Eq. (19), adding "1" to the corresponding $d_{is}^{'}$ can avoid zero value in the modified discordance matrix and avoid losing information when calculating the Hadamard product. By contrast, the concordance matrix $\left[c_{is}\right]_{m\times m}$ has a zero, no information is lost because its corresponding value in the modified discordance matrix will also be zero according to Eqs. (19)-(20).

Step 9. Closeness coefficient for ranking alternatives

The closeness coefficient proposed by Hwang and Yoon (1981) is used to rank alternatives based on the Hadamard product. The extended modified total matrix is obtained as presented in Eq. (21). The closeness coefficient index cc_i can be derived as shown in Eq. (22).

$$G=\left[e_{is}\right]_{m\times m},\ \text{where } e_{is}=c_{ij}\circ\left(d_{is}^{'}+1_{is}\right)\ i,s=1\sim m \quad (21)$$

$$cc_i=\frac{\sum\limits_{s=1\wedge s\neq i}^{m}e_{is}}{\sum\limits_{s=1\wedge s\neq i}^{m}e_{is}+\sum\limits_{i=1\wedge i\neq s}^{m}e_{si}},\ i,s=1\sim m \quad (22)$$

According to the concept of net dominance value presented by Nijkamp and Van Delft (1977), if $\sum\limits_{s=1\wedge s\neq i}^{m}e_{is}$ is larger or if $\sum\limits_{i=1\wedge i\neq s}^{m}e_{si}$ is smaller, the ranking order of the corresponding alternative is higher. Therefore, a larger cc_i value indicates that the corresponding alternative has a higher ranking order. Accordingly, the closeness coefficient index used in this study is effective for ranking alternatives.

4.1. Comparison with similar methods

The proposed extension is compared with the methods presented by Ke and Chen (2012) and Zhang et al. (2018) to demonstrate its advantages. Assume that three alternatives (A_1,A_2,A_3) under four benefit criteria (C_1,C_2,C_3,C_4) must be evaluated by decision-makers who have determined the performance ratings of the alternatives, $p_{ij}, i=1\sim 3, j=1\sim 4$, under four criteria and the

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criteria weights, $w_j, j=1\sim 4$, as listed in Table 1. The normalization matrix can be obtained using $r_{ij}=\dfrac{p_{ij}}{\sqrt{\sum_{i=1}^{3} p_{ij}^2}}$, as presented in Table 2. The weighted normalized matrix can be produced using $v_{ij}=r_{ij}\times w_j$, $i=1\sim 3, j=1\sim 4$, as displayed in Table 3. The concordance matrix can be obtained using $c_{is}=\dfrac{\sum_{j\in C_{is}} w_j}{\sum_{j=1}^{n} w_j}$, $C_{is}=\{j, v_{ij}\ge v_{sj}\}$, as shown in Table 4. The discordance matrix can be obtained using $d_{is}=\dfrac{\max_{j\in D_{is}}\{|v_{ij}-v_{sj}|\}}{\max_{j\in J}\{|v_{ij}-v_{sj}|\}}$, $J=\{1,2,.....,n\}$, $D_{is}=\{j, v_{ij}<v_{hj}\}$, as presented in Table 5, and the modified discordance matrix can then be obtained as indicated in Table 6. Through the use of the Hadamard product, the modified total matrix can obtained using $f_{is}=c_{is}\times d_{is}^{'}$ as shown in Table 7. According to Ke and Chen (2012), the Boolean matrix $Q=[q_{is}]_{m\times m}$, $\begin{cases} q_{is}=1, f_{is}\ge \overline{f} \\ q_{is}=1, f_{is}<\overline{f} \end{cases}$ can be obtained on the basis of Table 7, as presented in Table 8, in which the threshold $\overline{f}$ is set between the smallest value f_1 and the next smallest value f_2, $f_s=\max\{f_{is}\,|i=1\sim m\}, s=1\sim m$. The net dominating values in the method of Zhang et al. (2018) with $H_i=\sum_{s=1\wedge s\neq i}^{m} f_{is}-\sum_{i=1\wedge i\neq s}^{m} f_{is}$ can be obtained on the basis of Table 7, as presented in Table 11.

The extended modified discordance matrix $d_{is}^{'}$ is obtained using Eq. (20), as presented in Table 9. The extended modified total matrix can be obtained using Eq. (21), as listed in Table 10. The closeness coefficients can then be obtained using Eq. (22), as presented in Table 11. According to the values in Table 3, $v_{21}(0.286)>v_{11}(0.245)$, $v_{22}(0.137)>v_{12}(0.116)$, $v_{24}(0.075)=v_{14}(0.075)$ and $v_{23}(0.130)<v_{13}(0.187)$; thus, the alternative A_2 is clearly preferable to A_1. However, the Boolean matrix derived by the method of Ke and Chen (2012) in Table 8 produces the ranking order $A_1>A_2>A_3$ (Table 12); the values in Table 11 derived by the method of Zhang et al. (2018) also produce the ranking order $A_1>A_2>A_3$ (Table 12). These ranking results contradict the values presented in Table 3. However, the proposed extension using the closeness coefficient (Table 11) correctly obtains the ranking order $A_2>A_1>A_3$ (Table 12), which is consistent with the values in Table 3. Therefore, the proposed extension can overcome the limitations of the methods of Ke and Chen (2012) and Zhang et al. (2018). Moreover, if the concordance matrix has a zero value, such as $c_{32}=0$ in Table 4, the corresponding value in the modified discordance matrix also becomes zero, such as $d_{32}=0$ in Table 6, because if Eq. (18) produces a zero value, Eq. (19) produces a value of 1, resulting in a zero value in the modified discordance matrix. Thus, no information is lost using the Hadamard product if there is (or are) a zero value (or zero values) in the concordance matrix.

Table 1. The performance ratings of alternatives versus criteria.

	C_1	C_2	C_3	C_4
A_1	6	5.5	6.5	8
A_2	7	6.5	4.5	8
A_3	6	2	4	3
Weight	0.450	0.185	0.255	0.110

Table 2. Normalization matrix.

	C_1	C_2	C_3	C_4
A_1	0.545	0.629	0.734	0.683
A_2	0.636	0.743	0.508	0.683
A_3	0.545	0.229	0.451	0.256

Table 3. Weighted normalization matrix.

	C_1	C_2	C_3	C_4
A_1	0.245	0.116	0.187	0.075
A_2	0.286	0.137	0.130	0.075
A_3	0.245	0.042	0.115	0.028

Table 4. Concordance matrix.

	C_1	C_2	C_3
A_1	-	0.365	1.000
A_2	0.745	-	1.000
A_3	0.450	0.000	-

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Table 5. Discordance matrix.

	A_1	A_2	A_3
A_1	-	0.711	0.000
A_2	1.000	-	0.000
A_3	1.000	1.000	-

Table 6. Modified discordance matrix.

	A_1	A_2	A_3
A_1	-	0.289	1.000
A_2	0.000	-	1.000
A_3	0.000	0.000	-

Table 7. The modified total matrix.

	A_1	A_2	A_3
A_1	-	0.106	1.000
A_2	0.000	-	1.000
A_3	0.000	0.000	-

Table 8. Boolean matrix of Ke & Chen (2012).

	A_1	A_2	A_3
A_1	-	1.000	1.000
A_2	0.000	-	1.000
A_3	0.000	0.000	-

Table 9. Extended modified discordance matrix.

	A_1	A_2	A_3

A_1	-	1.289	2.000
A_2	1.000	-	2.000
A_3	1.000	1.000	-

Table 10. The extended modified total matrix.

	A_1	A_2	A_3
A_1	-	0.471	2.000
A_2	0.745	-	2.000
A_3	0.450	0.000	-

Table 11. Ranking values.

	Zhang et al.'s values	Closeness coefficients
A_1	1.106	0.674
A_2	0.894	0.854
A_3	-2.000	0.101

Table 12. Ranking order.

	Ranking
Ke & Chen (2012)	$A_1 > A_2 > A_3$
Zhang et al. (2018)	$A_1 > A_2 > A_3$
Proposed method	$A_2 > A_1 > A_3$

5. Numerical example

Assume that an industrial company intends to select a suitable demand product forecasting method to establish a production plan and achieve its sustainable manufacturing goals. A committee of four decision-makers has been formed to evaluate four forecasting techniques A_i, $i = 1 \sim 4$. Furthermore, assume that eleven qualitative and quantitative criteria as listed in Table 13, including benefit and cost, are determined by the committee to rank the options. Assume that the qualitative criteria are benefit criteria. The solution can be obtained by the following steps.

Table 13. List of criteria.

Symbol	Criteria	Quantitative	Qualitative
C_1	Data availability (B)		x
C_2	Data validity (B)		x
C_3	Technology development predictability (B)		x
C_4	Techology similarity (B)		x
C_5	Method adaptability (B)		x
C_6	Ease of operation (B)		x
C_7	Implementation cost (C, UDS)	x	
C_8	Maintenance cost (C, USD)	x	
C_9	Accuracy (C, %)	x	
C_{10}	Timeliness in providing forecasts (C, month)	x	
C_{11}	Ease of interpretation (B)		x

Step 1. The decision-makers provided the performance ratings of four options versus criteria by using Eq. (7), as displayed in Table 14. The linguistic terms converted into triangular fuzzy numbers in section 3.5 are employed to provide performance ratings of options versus qualitative criteria, as also presented in Table 14.

Step 2. The ratings of options versus quantitative criteria are normalized by Eqs. (8)-(9) as shown in Table 15.

Step 3. Suppose the criteria weights are assigned by decision-makers and the average weights can be obtained by the Eq. (10) as also displayed in Table 15.

Step 4. The weighted normalization values are calculated by Eqs. (11)-(12) as presented in Table 16.

Step 5. The defuzzified values can be obtained by Eq. (13) as also displayed in Table 17.

Step 6. The concordance and discordance sets can be determined by Eqs. (14)-(15) as presented in Table 18.

Step 7. The crisp weights of criteria can be obtained by Eqs. (16)-(17) as also displayed in Table 17. The concordance matrix can be obtained By Eq. (18) as shown in Table 19, and the discordance matrix can be obtained by Eq. (19) as displayed in Table 20.

Table 14. Performance ratings of alternatives versus criteria.

	C_1			C_2			C_3			C_4			C_5			C_6		
	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u
A_1	0	0.1	0.3	0	0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.5	0.7	0.9	0.7	0.9	1
A_2	0.1	0.3	0.5	0.3	0.5	0.7	0.5	0.7	0.9	0.5	0.7	0.9	0.3	0.5	0.7	0.7	0.9	1
A_3	0.3	0.5	0.7	0.3	0.5	0.7	0.5	0.7	0.9	0.3	0.5	0.7	0.3	0.5	0.7	0.5	0.7	0.9
A_4	0.7	0.9	1	0.7	0.9	1	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	1	0	0.1	0.3

	C_7			C_8			C_9			C_{10}			C_{11}		
	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u
A_1	348	348	348	50	50	50	35	35	35	1	1	1	0.7	0.9	1

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A_2	340	340	340	60	60	60	20	20	20	2	2	2	0.3	0.5	0.7
A_3	350	350	350	60	60	60	23	23	23	3	3	3	0.5	0.7	0.9
A_4	360	360	360	50	50	50	10	10	10	2	2	2	0.5	0.7	0.9

Table 15. Normalization matrix.

	C_1			C_2			C_3			C_4		
	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u
A_1	0.000	0.100	0.300	0.000	0.100	0.300	0.500	0.700	0.900	0.300	0.500	0.700
A_2	0.300	0.500	0.700	0.300	0.500	0.700	0.100	0.300	0.500	0.500	0.700	0.900
A_3	0.300	0.500	0.700	0.300	0.500	0.700	0.500	0.700	0.900	0.300	0.500	0.700
A_4	0.700	0.900	1.000	0.700	0.900	1.000	0.300	0.500	0.700	0.500	0.700	0.900
Weight	0.300	0.500	0.700	0.300	0.500	0.700	0.000	0.100	0.300	0.000	0.100	0.300
	C_5			C_6			C_7			C_8		
	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u
A_1	0.500	0.700	0.900	0.700	0.900	1.000	0.977	0.977	0.977	1.000	1.000	1.000
A_2	0.300	0.500	0.700	0.700	0.900	1.000	1.000	1.000	1.000	0.833	0.833	0.833
A_3	0.300	0.500	0.700	0.500	0.700	0.900	0.971	0.971	0.971	0.833	0.833	0.833
A_4	0.700	0.900	1.000	0.000	0.100	0.300	0.944	0.944	0.944	1.000	1.000	1.000
Weight	0.000	0.100	0.300	0.300	0.500	0.700	0.500	0.700	0.900	0.300	0.500	0.700
	C_9			C_{10}			C_{11}					
	x^l	x^γ	x^u	x^l	x^γ	x^u	x^l	x^γ	x^u			
A_1	0.286	0.286	0.286	1.000	1.000	1.000	0.700	0.900	1.000			
A_2	0.500	0.500	0.500	0.500	0.500	0.500	0.300	0.500	0.700			
A_3	0.435	0.435	0.435	0.333	0.333	0.333	0.500	0.700	0.900			
A_4	1.000	1.000	1.000	0.500	0.500	0.500	0.500	0.700	0.900			
Weight	0.700	0.900	1.000	0.300	0.500	0.700	0.300	0.500	0.700			

Table 16. The weighted normalization matrix

	C_1						C_2					
	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}
A_1	0.020	0.030	0.000	0.040	-0.200	0.210	0.020	0.030	0.000	0.040	-0.200	0.210
A_2	0.040	0.120	0.090	0.040	-0.280	0.490	0.040	0.120	0.090	0.040	-0.280	0.490
A_3	0.040	0.120	0.090	0.040	-0.280	0.490	0.040	0.120	0.090	0.040	-0.280	0.490

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A_4	0.040	0.200	0.210	0.020	-0.270	0.700	0.040	0.200	0.210	0.020	-0.270	0.700
	C_3						C_4					
	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}
A_1	0.020	0.050	0.000	0.040	-0.240	0.270	0.020	0.030	0.000	0.040	-0.200	0.210
A_2	0.020	0.010	0.000	0.040	-0.160	0.150	0.020	0.050	0.000	0.040	-0.240	0.270
A_3	0.020	0.050	0.000	0.040	-0.240	0.270	0.020	0.030	0.000	0.040	-0.200	0.210
A_4	0.020	0.030	0.000	0.040	-0.200	0.210	0.020	0.050	0.000	0.040	-0.240	0.270
	C_5						C_6					
	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}
A_1	0.02	0.05	0.00	0.04	-0.24	0.27	0.04	0.20	0.21	0.02	-0.27	0.70
A_2	0.02	0.03	0.00	0.04	-0.20	0.21	0.04	0.20	0.21	0.02	-0.27	0.70
A_3	0.02	0.03	0.00	0.04	-0.20	0.21	0.04	0.16	0.15	0.04	-0.32	0.63
A_4	0.02	0.07	0.00	0.02	-0.23	0.30	0.02	0.03	0.00	0.04	-0.20	0.21
	C_7						C_8					
	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}
A_1	0.00	0.20	0.49	0.00	-0.20	0.88	0.00	0.20	0.30	0.00	-0.20	0.70
A_2	0.00	0.20	0.50	0.00	-0.20	0.90	0.00	0.17	0.25	0.00	-0.17	0.58
A_3	0.00	0.19	0.49	0.00	-0.19	0.87	0.00	0.17	0.25	0.00	-0.17	0.58
A_4	0.00	0.19	0.47	0.00	-0.19	0.85	0.00	0.20	0.30	0.00	-0.20	0.70
	C_9						C_{10}					
	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}
A_1	0.00	0.06	0.20	0.00	-0.03	0.29	0.00	0.20	0.30	0.00	-0.20	0.70
A_2	0.00	0.10	0.35	0.00	-0.05	0.50	0.00	0.10	0.15	0.00	-0.10	0.35
A_3	0.00	0.09	0.30	0.00	-0.04	0.43	0.00	0.07	0.10	0.00	-0.07	0.23
A_4	0.00	0.20	0.70	0.00	-0.10	1.00	0.00	0.10	0.15	0.00	-0.10	0.35
	C_{11}											
	G_{ij1}	H_{ij1}	L_{ij1}	G_{ij2}	H_{ij2}	L_{ij2}						
A_1	0.04	0.20	0.21	0.02	-0.27	0.70						
A_2	0.04	0.12	0.09	0.04	-0.28	0.49						
A_3	0.04	0.16	0.15	0.04	-0.32	0.63						
A_4	0.04	0.16	0.15	0.04	-0.32	0.63						

Table 17. Defuzzification

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_{12}	-0.191	-0.191	0.050	-0.025	0.025	0.000	-0.016	0.083	-0.188	0.250	0.184
A_{13}	-0.191	-0.191	0.000	0.000	0.025	0.084	0.004	0.083	-0.130	0.333	0.084

A_{14}	-0.375	-0.375	0.025	-0.025	-0.019	0.375	0.023	0.000	-0.625	0.250	0.084
A_{21}	0.191	0.191	-0.050	0.025	-0.025	0.000	0.016	-0.083	0.188	-0.250	-0.184
A_{23}	0.000	0.000	-0.050	0.025	0.000	0.084	0.020	0.000	0.057	0.083	-0.100
A_{24}	-0.184	-0.184	-0.025	0.000	-0.044	0.375	0.039	-0.083	-0.438	0.000	-0.100
A_{31}	0.191	0.191	0.000	0.000	-0.025	-0.084	-0.004	-0.083	0.130	-0.333	-0.084
A_{32}	0.000	0.000	0.050	-0.025	0.000	-0.084	-0.020	0.000	-0.057	-0.083	0.100
A_{34}	-0.184	-0.184	0.025	-0.025	-0.044	0.291	0.019	-0.083	-0.495	-0.083	0.000
A_{41}	0.375	0.375	-0.025	0.025	0.019	-0.375	-0.023	0.000	0.625	-0.250	-0.084
A_{42}	0.184	0.184	0.025	0.000	0.044	-0.375	-0.039	0.083	0.438	0.000	0.100
A_{43}	0.184	0.184	-0.025	0.025	0.044	-0.291	-0.019	0.083	0.495	0.083	0.000
Weight	0.101	0.101	0.025	0.025	0.025	0.101	0.141	0.101	0.177	0.101	0.101

Table 18. Concordance and discordance sets

Concordance = 1, Discordance = 0

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_{12}	0	0	1	0	1	1	0	1	0	1	1
A_{13}	0	0	1	1	1	1	1	1	0	1	1
A_{14}	0	0	1	0	0	1	1	1	0	1	1
A_{21}	1	1	0	1	0	1	1	0	1	0	0
A_{23}	1	1	0	1	1	1	1	1	1	1	0
A_{24}	0	0	0	1	0	1	1	0	0	1	0
A_{31}	1	1	1	1	0	0	0	0	1	0	0
A_{32}	1	1	1	0	1	0	0	1	0	0	1
A_{34}	0	0	1	0	0	1	1	0	0	0	1
A_{41}	1	1	0	1	1	0	0	1	1	0	0
A_{42}	1	1	1	1	1	0	0	1	1	1	1
A_{43}	1	1	0	1	1	0	0	1	1	1	1

Table 19. Concordance matrix.

	A_1	A_2	A_3	A_4
A_1	-	0.455	0.621	0.571
A_2	0.646	-	0.000	0.369

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A_3	0.429	0.455	-	0.369
A_4	0.530	0.758	0.268	-

Table 20. Discordance matrix.

	A_1	A_2	A_3	A_4
A_1	-	0.763	0.573	1.000
A_2	1.000	-	1.000	1.000
A_3	1.000	0.842	-	1.000
A_4	0.600	0.857	0.588	-

Step 8. The modified discordance matric can be easily produced as shown in Table 21. The extended modified discordance matrix can then be obtained by Eq. (20) as shown in Table 22.

Table 21. Modified discordance matrix.

	A_1	A_2	A_3	A_4
A_1	-	0.237	0.428	0.000
A_2	0.000	-	0.000	0.000
A_3	0.000	0.158	-	0.000
A_4	0.400	0.143	0.412	-

Table 22. Extended modified discordance matrix.

	A_1	A_2	A_3	A_4
A_1	-	1.237	1.428	1.000
A_2	1.000	-	1.000	1.000
A_3	1.000	1.1583	-	1.000
A_4	1.400	1.1429	1.412	-

Step 9. The extended modified total matrix can be obtained by Eq. (21) as shown in Table 23. The closeness coefficient cc_i can then be obtained via Eq. (22) as shown in Table 24. According to the closeness coefficients in Table 24, the ranking order is $A_4 > A_1 > A_3 > A_2$ because $0.603 > 0.526 > 0.512 > 0.342$. Therefore, the forecasting method A_4 should be selected as the best solution.

Table 23. Extended modified total matrix.

	A_1	A_2	A_3	A_4
A_1	-	0.562	0.887	0.571
A_2	0.6465	-	0.000	0.369
A_3	0.4293	0.527	-	0.369
A_4	0.7424	0.866	0.378	-

Table 24. Final ranking result.

	Closeness coefficients	Ranking
A_1	0.526	2
A_2	0.342	4

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A_3	0.512	3
A_4	0.603	1

5.1. Numerical comparison

The proposed method is compared with those of Ke and Chen (2012), Zhang et al. (2018), Nghiem and Chu (2021), and Nghiem and Chu (2022) by using the numerical example presented in Section 5. According to Ke and Chen (2012) and Zhang et al. (2018), the modified total matrix (Table 25) can be derived using the Hadamard product based on Tables 19 and 21. The Boolean matrix derived using the method of Ke and Chen (2012) is presented in Table 26. The net dominating values derived by the method of Zhang et al. (2018) are listed in Table 30. The total dominance matrix based on subtracting discordance values from concordance values by Nghiem and Chu (2022) method can be obtained through the equation $G=[e_{is}]_{m\times m}$, $e_{is}=c_{is}-d_{is}$, as presented in Table 27. The Boolean matrix (Table 28) can be derived by the method of Nghiem and Chu (2022) based on Table 27. In the method of Nghiem and Chu (2021), the net concordance and net modified discordance values can be derived using $c_i=\sum_{s=1\wedge s\neq i}^{m} c_{is}-\sum_{i=1\wedge i\neq s}^{m} c_{is}$ and $d_i^{'}=\sum_{s=1\wedge s\neq i}^{m} d_{is}^{'}-\sum_{i=1\wedge i\neq s}^{m} d_{is}^{'}$, respectively, as shown in Table 29. Their net total dominance values can be obtained using $U_i=c_i\oplus d_i^{'},\ \ i=1\sim m$, as listed in Table 30.

According to the Boolean matrix in Table 26, the method of Ke and Chen (2012) produces the following ranking order: $A_4>A_1>A_3=A_2$ (Table 30). According to the net dominating values in Table 30, the method of Zhang et al. (2018) yields the following ranking order: $A_4>A_1>A_2>A_3$ (Table 30). Their methods are inconsistent with the ranking order $A_4>A_1>A_3>A_2$, presented in Table 24, obtained by the proposed method. The methods of Ke and Chen (2012) and Zhang et al. (2018) can lose information when using the Hadamard product to generate the modified total matrix, leading to incorrect ranking result. The proposed method can resolve the problem of information loss in the methods of Ke and Chen (2012) and Zhang et al. (2018) where the Hadamard product is used. The method of Nghiem and Chu (2021) uses the total net dominance value (Table 30) and yields the ranking order of $A_4>A_1>A_3>A_2$ (Table 30), which is consistent with that derived by the proposed method as shown in Table 24. In addition, the method of Nghiem and Chu (2022) uses the total dominance matrix method based on subtracting discordance values from concordance values to produce the Boolean matrix and obtains the ranking order of $A_4>A_1>A_3>A_2$ (Table 30), which is also consistent with that derived by the proposed method as shown in Table 24. The comparison with the methods of Nghiem and Chu (2021, 2022) further indicates the effectiveness of the prosed method. However, in the methods of Nghiem and Chu (2021, 2022), the problem of information loss remains if the Hadamard product is used. The proposed method in this study overcomes this problem.

Table 25. Modified total matrix.

	A_1	A_2	A_3	A_4
A_1	-	0.108	0.266	0.000
A_2	0.000	-	0.000	0.000
A_3	0.000	0.072	-	0.000
A_4	0.212	0.108	0.110	-

Table 26. Boolean matrix of Ke & Chen (2012).

	A_1	A_2	A_3	A_4

A_1	-	0	1	0
A_2	0	-	0	0
A_3	0	0	-	0
A_4	1	0	1	-

Table 27. Total dominance matrix.

	A_1	A_2	A_3	A_4
A_1	-	-0.309	0.049	-0.429
A_2	-0.354	-	-1	-0.631
A_3	-0.571	-0.387	-	-0.631
A_4	-0.070	-0.100	-0.320	-

Table 28. Boolean matrix of Nghiem & Chu (2022).

	A_1	A_2	A_3	A_4
A_1	-	1	1	0
A_2	1	-	0	0
A_3	0	1	-	0
A_4	1	1	1	-

Table 29. Net concordance and net modified discordance matric.

	Net concordance	Net modified discordance
A_1	0.040	0.264
A_2	-0.652	-0.538
A_3	0.364	-0.681
A_4	0.247	0.955

Table 30. Final ranking result.

	Ke & Chen (2012)	Zhang et al. (2018)		Nghiem & Chu (2022)	Nghiem & Chu (2021)	
	Ranking	Values	Ranking	Ranking	Net total dominance	Ranking
A_1	2	0.161	2	2	0.305	2
A_2	3	-0.288	3	4	-1.189	4
A_3	3	-0.304	4	3	-0.317	3
A_4	1	0.431	1	1	1.202	1

Conclusion

Forecasting method selection plays a key role in sustainable development for manufacturing companies; selecting a suitable forecasting method can help companies avoid overproduction or shortages. Therefore, evaluating forecasting methods has attracted the attention of numerous scholars and practitioners. More than 200 forecasting methods are described in the economic literature; thus, companies must compare forecasting methods and select the most suitable one to improve their production process.

Evaluating forecasting methods is a fuzzy MCDM problem. To the best of our knowledge, this study is the first to apply fuzzy ELECTRE I to the evaluation and selection of demand forecasting methods. Specifically, this study proposes an extension of fuzzy ELECTRE I for selection the most suitable demand forecasting method. In the proposed method, the fuzzy weighted ratings are defuzzified by applying the signed distance (Yao and Wu, 2000) to form a crisp ELECTRE I model. The defuzzification formulas can yield a complete model that can assist in decision-making. Moreover, the proposed extension uses a closeness coefficient based on the extended modified discordance matrix to rank alternatives; the use of this extension avoids the problem of information loss when calculating the Hadamard product. Herein, a comparison with the methods of Ke and Chen (2012) and Zhang et al. (2018) has demonstrated advantages of the proposed extension. Moreover, a numerical example has been used to present the feasibility of the proposed fuzzy ELECTRE I method, and a numerical comparison of the proposed method with other methods has been conducted to reveal the advantages of the proposed method.

In future studies, the proposed method could be used in other fuzzy MCDM problems and could be investigated under uncertain environments with other types of fuzzy numbers, such as interval type-2 fuzzy numbers, to expand its applicability. However, there are several factors that can affect the final results, such as the weight derivation method, defuzzification method, normalization method, or number of criteria, etc.; these factors could be further investigated in further research.

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