Article

Self-Interactions, Self-Energy and the Electromagnetic Contribution to the Anomalous *g*-Factor

Miroslav Georgiev *

Posted Date: 16 August 2023

doi: 10.20944/preprints202308.1136.v1

Keywords: Self-interaction; Anomalous magnetic moment; Electrodynamics

Article

Self-Interactions, Self-Energy and the Electromagnetic Contribution to the Anomalous *g*-Factor

Miroslav Georgiev

Institute of Solid State Physics, Bulgarian Academy of Sciences, Tzarigradsko Chaussée 72, 1784 Sofia, Bulgaria; mgeorgiev@issp.bas.bg

Abstract: The present paper reports an exact approach quantifying the electromagnetic contribution to the anomalous magnetic moment occurring in isolated system comprised of non-composite particle carrying elementary electric charge. Essential averaging procedure and regularization of the electromagnetic field potentials necessary when quantifying the electromagnetic self-interactions and when deriving equations of motion without singularities and obeying the conservation laws are thoroughly discussed. The study shows that the dynamics of the considered system is associated to a unique classical transcendental equations of motion satisfied by the particle's velocity and the electromagnetic contribution to the anomalous *g*-factor known from the quantum electrodynamics. The equations of motion predict a value of the anomalous *g*-factor that agrees with the experimentally measured one reported in the literature and that calculated with the aid of quantum electrodynamics. In the present study the computational accuracy is restricted to match one part in a billion, obtaining $a_e = 0.001159652(23)$, thus reveling the potential of non-perturbative methods in predicting the electron's anomalous *g*-factor.

Keywords: self-interaction; anomalous magnetic moment; electrodynamics

1. Introduction

The anomalous magnetic moment and the intrinsic dynamics of non-composite particles have been considered as unique features of the quantum field theory since the beginning of its elaboration [1–5]. The electron's anomalous magnetic moment and its fundamental properties are the first to be studied and realized. With the aid of quantum electrodynamics the value of corresponding *g*-factor was predicted with a stunning accuracy [6–11], leaving no space for mistrusting its effectiveness. Following, were the properties of the two remaining leptons carrying elementary charge, the muon and tau. Determining with high accuracy the dynamics of both massive leptons is an active field of research. The muon's anomalous magnetic moment is still puzzling the community aiming to reduce the gap between theory and experiment [12–18], recently known to be of about 0.58%. On the other hand, having a very short lifetime and being the massive among all leptons, measuring and predicting tau anomalous magnetic moment is a challenging task requiring great efforts [19–25]. Although, there is a serious discrepancy between theory and experiment, such efforts may have the potential to shed more light on the contribution of high order hadronic terms thus aiding in resolving the inconsistency in the muon data.

The microscopic electrodynamics underlying the occurrence of anomalous magnetic moment is indispensably related to the nature of self-interactions [1,26,27]. The latter are believed to be uniquely addressable by the regularization and renormalization methods of quantum theory [28–32], with no classical analog. Yet, even within the standard methods of quantum mechanics, the evaluation of self-Coulomb energy in multi-electron systems still poses a challenge [33,34].

The quantum and classical theories are believed to have no interconnection pertaining to the occurrence of anomalous magnetic moment and self-energy divergence problem. Therefore, a classical method with the potential to quantify the anomalous magnetic moment and the self-interactions in the absence of singularities and electromagnetic radiation may contribute significantly in establishing a better interrelationship between both theories.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 16 August 2023 doi:10.20944/preprints202308.1136.v1

The present paper propose an exact approach quantifying the electromagnetic contribution to the anomalous magnetic moment and related dynamics of isolated non-composite particle carrying elementary electric charge. The approach build on the methods of classical electrodynamics in studying the microscopic dynamics of self-symmetric systems, like the considered one, by removing all singular points. It incorporates a particular spatial averaging procedure and regularization of the electromagnetic field potentials confining the field to the particle itself. Accordingly, the effect of self-repulsion is accounted for without singularity, or violation of the conservation laws, allowing a complete microscopic description of the system's energy state. Two types of self-interactions are predicted, the self-Coulomb and self-Zeeman ones. Essentially, the considered approach results in a system of transcendental equations of motion satisfied by the particle's velocity and the anomalous g-factor of electromagnetic origin. The derived system of equations ensure fast and accurate computational results and predicts the value of anomalous g-factor with the same success the perturbative approach of quantum electrodynamics does. In the present work, the computations are carried out with accuracy matching one prat in a billion.

The rest of the paper is organized as follows. The mathematical notation of all fundamental physical quantities characterizing the system under consideration along with essential interrelationships between the introduced observables are presented in Section 2. Furthermore, the section discusses the occurrence of self-interactions and their explicit representation. Lagrangian and Hamiltonian density representations are also given. The main results are outlined in Section 3, with computations carried out on Wolfram Mathematica. Section 4 summarizes the used approach and obtained results.

2. Theoretical Background

2.1. General Considerations

In the present study all representations are restricted to the mathematical framework of the classical relativistic mechanics and electrodynamics, overlooking all relevant quantum mechanical representations. For the sake of clarity, all physical quantities and equations of motion are represented within the standard three-dimensional vector formalism. The four-vector convention is omitted, since the representation in Minkowski space with the relevant Lorentz group is straightforward [35–37].

Consider an isolated system composed of single non-composite particle of type q, with rest frame of reference $\mathbf{R}$, rest mass m_q and electric charge $e_q^\pm = \pm e$, where e denotes the elementary charge. Let $r_{cq} = \alpha\bar{\lambda}_{cq}$ be the particle's electromagnetic radius at rest, where α and $\bar{\lambda}_{cq}$ are the fine structure constant and reduced Compton wavelength, respectively. Let $\mathbf{u}_q$, with $\boldsymbol{\kappa} \cdot \mathbf{u}_q = u_q$ and $\dot{u}_q = 0$, be the velocity associated to the considered particle and $\mathbf{p}_q = \gamma_q m_q \mathbf{u}_q$ the corresponding momentum in the observer's rest frame $\mathbf{O}$, where γ_q is the corresponding Lorentz factor. As we will see later in the discussion, for the considered system $u_q = 0 \Leftrightarrow \nexists\, e$. Furthermore, let $r_q = \bar{\lambda}_q$, with $\lambda_q u_q = \lambda_{cq} c\sqrt{1-\beta_q^2}$, be the particle's relative electromagnetic radius, where λ_q is the particle's intrinsic wavelength, $\beta_q = u_q c^{-1}$ and c denotes the light speed in vacuum. Since the particle alone represents the only center of symmetry in the system, r_q and p_q are conjugate intrinsic variables, satisfying $r_q p_q = \bar{\lambda}_{cq} m_q c$.

Let $\rho_{|e_q^\pm|}$ and ρ_{m_q}, with

$$\rho_{\mathrm{s}} = \frac{2}{V_{cq}} \int_0^\infty \mathrm{s}\delta(\bar{\lambda} - \alpha\bar{\lambda}_{cq})\mathrm{d}\bar{\lambda}, \quad \mathrm{s} = \{|e_q^\pm|, m_q\},$$

be the charge and rest mass densities, respectively, such that

$$V_{cq} = 8\pi \int_0^\infty \int_0^{\bar{\lambda}} \delta(\bar{\lambda} - \alpha\bar{\lambda}_{cq}) r^2 \mathrm{d}r\mathrm{d}\bar{\lambda}, \tag{1}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 16 August 2023 doi:10.20944/preprints202308.1136.v1

and

$$e_q^{\pm} = \pm \int_{\Omega_{cq}} \rho_{|e_q^{\pm}|} \mathrm{dv}, \quad m_q = \int_{\Omega_{cq}} \rho_{m_q} \mathrm{dv}, \tag{2}$$

where the spatial domain $\Omega_{cq} \in \mathbb{R}^3$, with a boundary $\partial\Omega_{cq}$, is such that $V_{cq} = \int_{\Omega_{cq}} \mathrm{dv}$.

Let for all q the system is characterized by effective mass given by

$$M_q = \int_{\Omega_{cq}} \rho_{M_q} \mathrm{dv}, \tag{3}$$

where the corresponding density $\rho_{M_q} = \rho_{M_q}(r)$ is a smooth function over $r \in (0, +\infty)$.

The system posses intrinsic angular momentum $\mathbf{f}_q$ and a corresponding magnetic one $\boldsymbol{\mu}_q$, with magnitudes f_q and μ_q, respectively. In accordance to (3), we have

$$\mathbf{f}_q = \int_{\Omega_{cq}} \mathbf{r}_q \times \mathbf{j}_{M_q} \mathrm{dv}, \quad \boldsymbol{\mu}_q = \frac{1}{2} \int_{\Omega_{cq}} \mathbf{r}_q \times G_q \mathbf{j}_{e_q^{\pm}} \mathrm{dv}, \tag{4}$$

where $\mathbf{j}_{M_q} = \gamma_q \rho_{M_q} \mathbf{u}_q$ and $\mathbf{j}_{e_q^{\pm}} = \pm\gamma_q \rho_{|e_q^{\pm}|} \mathbf{u}_q$ are the system's mass and charge density currents, respectively. Here, G_q is the integrand g-factor, where the latter reads

$$g_e = \int_{\Omega_{cq}} G_q \mathrm{dv}, \quad G_q = 2 \frac{|e_q^{\pm}| \rho_{M_q}}{m_q \rho_{|e_q^{\pm}|}}. \tag{5}$$

We further have $g_e = 2(1 + a_e)$, where a_e is the electromagnetic contribution to the anomalous g-factor.

Let us point out that according to (5), the fraction $M_q m_q^{-1}$ is a constant for all q. We would like to stress, furthermore, that $\mathbf{f}_q$ is not an orbital angular momentum, since the system does not posses a center of symmetry independent from the particle. In the vector diagram sketching the dynamics of the considered system, the position vector $\mathbf{r}_q$ associated to the density currents will remain conjugate to the particle's momentum for an infinite time. In other words, the dynamics related to both physical quantities in eq. (4) is not time and space independent event for an observer in $\mathbf{O}$ and hence it cannot be associated to a free spinning sphere of radius r_{cq}. The particle's momentum and corresponding currents are intrinsic, not a consequence of independent external action.

All of the above considered relations and definitions hold for $\mathbf{u}_q = \mathbf{v}_q + \tilde{\mathbf{v}}_q$, where $\mathbf{v}_q$ is the velocity component characterizing a node moving relative to the origin of $\mathbf{O}$ and $\tilde{\mathbf{v}}_q$ is the velocity component associated to an oscillation occurring along an axis perpendicular to $\mathbf{v}_q$, or $\mathbf{v}_q \cdot \tilde{\mathbf{v}}_q = 0$. In this regard, the particle has a standing wave representation that is naturally related to some uncertainty. The corresponding oscillation dynamics can be further studied within the classical wave theory, or with the more rigorous methods of the quantum theory. Such studies lie beyond the goals of the present research and may be discussed elsewhere.

2.2. Field Observables

Since the system is isolated, or $u_q \neq u_q(t)$, and the particle posses elementary electric charge confined within the time independent spatial domain Ω_{cq}, the resulting electric current and its density are time and spatially independent. Accordingly, the electric $\mathbf{E}_q$ and magnetic $\mathbf{B}_q$ components of the corresponding electromagnetic field in $\mathbf{O}$ are time independent and spatially conjugated to the particle's momentum. Thus, under any gauge condition, we have the relations $\mathbf{E}_q = -\nabla\varphi_q$ and $\mathbf{B}_q = \nabla \times \mathbf{A}_q$, where $\varphi_q = \varphi_q(r)$ and $\mathbf{A}_q = \mathbf{A}_q(r)$ are the scalar and vector potentials of the electromagnetic field in $\mathbf{O}$. In particular, we have

$$\varphi_q = \gamma_q \phi_q, \quad \mathbf{A}_q = \gamma_q \frac{\mathbf{u}_q}{c^2} \phi_q, \quad \mathbf{B}_q = \frac{1}{c^2} (\mathbf{u}_q \times \mathbf{E}_q), \tag{6}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 16 August 2023 doi:10.20944/preprints202308.1136.v1

where the function $\phi_q = \phi_q(r)$, given with respect to the observer's rest frame, is regularized to the origin of $\mathbf{R}$ and reads

$$\phi_q(r) = \frac{e_q^{\pm}}{4\pi\varepsilon_o r}\left(1 - e^{-\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}}\right), \tag{7}$$

with ε_o denoting the vacuum permittivity. The function given in eq. (7) is solution to the superposition of Poisson and Helmholtz equations, with minimal wavelength of the electromagnetic field equal to the denominator in the exponent.

Owing to eq. (7), we obtain the analytical expression of the electromagnetic field's energy, with density $\varepsilon_o E_q^2$, for all q. Thus, we get

$$W_q(r) = \alpha c^2 m_q \frac{\bar{\lambda}_{cq}}{2r}\left(2 - 4e^{\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}} + 2e^{2\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}} + \frac{\gamma_q r}{(1+a_e)\lambda_{cq}}\right)e^{-2\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}}. \tag{8}$$

Moreover, from eqs. (7) and (8), we obtain two important limits. We have,

$$\lim_{r\to 0}\varphi_q(r) = \frac{\gamma_q^2 e_q^{\pm}}{4\pi\varepsilon_o(1+a_e)\lambda_{cq}} \quad \text{and} \quad \lim_{r\to 0} W_q(r) = 0,$$

respectively. These results point out that the energy of generated electromagnetic field vanish at $r \to 0$, but the potential energy is a constant ensuring that the self-interactions do not vanish at the given limit and do not associate to singularity points. On the other hand, on the boundary $\partial\Omega_{cq}$, the function given in eq. (8) depends only on the particle's speed. Therefore, for the considered system, the presence of electromagnetic field on $\partial\Omega_{cq}$ appears as a high-energy electromagnetic veil, with corresponding Umov-Poynting vector $\mu_o^{-1}(\mathbf{E}_q \times \mathbf{B}_q) = \varepsilon_o E_q^2 \mathbf{u}_q$, where μ_o is the vacuum magnetic permeability. As an example, at $u_q \to c$, for all q, the value of $W_q(r_{cq})$ equals exactly the rest energy of the particle thus making the latter to appear as a very round object [38].

Note that, the discussed electromagnetic field is consistently confined to the particle. It depends exclusively on the particle's intrinsic characteristics and does not obey the Liénard-Wiechert representation [37,39,40]. Hence, the generated electromagnetic field does not propagate at speed c independently from the particle and does not classify as an on shell coupling between the electric and magnetic field components. We neither observe spontaneous emission nor absorption of photons. The discussed electromagnetic field is a classical analog of the virtual photons presented in quantum electrodynamics.

2.3. Electromagnetic Self-Energy and Self-Interactions

Since the system is closed, with the action of no additional fields, the particle exhibits no exchange of energy and hence momentum. Accordingly, neither external nor net self-forces [37,41,42] are acting on the particle and the energy of the system remains purely kinetic. However, the considered system definitely exhibits two types of self-interactions of electromagnetic nature, with average energy depending only on u_q.

The Hamiltonian describing the total energy of electromagnetically self-interacting particle read

$$H_q = \gamma_q m_q c^2 + \Sigma_q + Z_q, \tag{9}$$

where the energy terms Σ_q and Z_q are associated to the self-Coulomb and self-Zeeman interactions.

The self-Coulomb term is the spatial average over the domain Ω_{cq} of the interaction energy between the particle's charge and its own electromagnetic field. It reads

$$\Sigma_q = \gamma_q c^2 \int_{\Omega_{cq}} \rho_{M_q} - \rho_{m_q} \mathrm{dv}, \tag{10}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 16 August 2023 doi:10.20944/preprints202308.1136.v1

where
$$\rho_{M_q} = \rho_{m_q}\left(1+\eta_q\frac{r_{cq}}{r}\left(1-e^{-\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}}\right)\right), \tag{11}$$
$\eta_q = 1+\beta_q^2$. Here, we have taken into account the relation $\rho_{m_q}c^2 = |e_q^{\pm}|\rho_{|e_q^{\pm}|}(4\pi\varepsilon_o r_{cq})^{-1}$.

The self-Zeeman term represents the spatial average over the domain Ω_{cq} of the interaction energy between the system's intrinsic magnetic moment presented in (4) and the induced magnetic field given in (6). Since the self-Zeeman interaction is time independent, the corresponding integrand satisfies $-\boldsymbol{\mu}_q\cdot\mathbf{B}_q = -\mu_q B_q$, where in accordance to (6), we have $B_q = c^{-2}\gamma_q u_q|\nabla\phi_q|$. Thus, taking into account relations (4), according to which
$$\mu_q = e\bar{\lambda}_{cq}c\frac{M_q}{m_q} \quad \text{and} \quad \mathrm{f}_q = \bar{\lambda}_{cq}M_q c,$$
we obtain
$$Z_q = -\frac{\mathrm{f}_q p_q}{m_q^2\eta_q}\int_{\Omega_{cq}}|\nabla\rho_{M_q}|\mathrm{dv}. \tag{12}$$

The self-energies in eqs. (10) and (12) are intrinsic to the considered particle and hence invariant to an external influence. Therefore, represented in terms of quantum theory they will remain invariant with respect to the particle's orbital state in many-body systems. That may be of benefit to the researchers studying multi-electron systems with the aid of computational methods that fail to account for the self-interactions without generating errors, see for example the case of Kohn-Sham density functional theory [34,43,44].

2.4. *The Lagrangian and Hamiltonian Density*

In Section 2.3 we discuss the two allowed by the conservation laws types of electromagnetic self-interactions that take place only within the domain Ω_{cq} and have particular energy densities.

In general, the Hamiltonian (9) has a corresponding density and Lagrangian. The latter reads
$$\mathcal{L}_q = \mathbf{p}_q\cdot\tilde{\mathbf{u}}_q - \gamma_q c^2\rho_{M_q} + \frac{M_q p_q^3}{m_q^3\eta_q}\boldsymbol{\kappa}\cdot\nabla_p\rho_{M_q}, \tag{13}$$
where
$$\tilde{\mathbf{u}}_q = \boldsymbol{\kappa}\frac{2}{V_{cq}}\int_0^\infty u_q(\bar{\lambda})\delta(\bar{\lambda}-\bar{\lambda}_q)\mathrm{d}\bar{\lambda} \tag{14}$$
is the generalized velocity and
$$\rho_{M_q} = \rho_{m_q}\left(1+\frac{\eta_q}{m_q}\frac{\alpha}{c}p_q\left(1-e^{-\frac{\gamma_q m_q c}{2\pi(1+a_e)p_q}}\right)\right) \tag{15}$$
is the momentum representation of the effective mass density. Here, we take into account that $\boldsymbol{\kappa}\cdot\nabla_p\rho_{M_q}\in\mathbb{R}_+$ for all $u_q\in(0,c)$, such that $|\nabla\rho_{M_q}(r_q)|\to p_q^2(\lambda_{cq}cm_q)^{-1}\boldsymbol{\kappa}\cdot\nabla_p\rho_{M_q}(p_q)$.

For the corresponding Hamiltonian density, we have
$$\mathcal{H}_q = \gamma_q c^2\rho_{M_q} - \frac{M_q p_q^3}{m_q^3\eta_q}\boldsymbol{\kappa}\cdot\nabla_p\rho_{M_q}, \tag{16}$$
where the Hamiltonian's equations read
$$\tilde{\mathbf{u}}_q = \nabla_p\mathcal{H}_q, \quad \text{and} \quad \dot{\mathbf{p}}_q = 0. \tag{17}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 16 August 2023 doi:10.20944/preprints202308.1136.v1

3. Results

3.1. Equations of Motion

Following the Hamiltonian's equations (17), the generalized velocity representation (14) and (5), we obtain the system's equations of motion.

Working with the magnitude of the particle's velocity for convenience, we have

$$u_q = \int_{\Omega_{cq}} \boldsymbol{\kappa} \cdot \nabla_p \mathcal{H}_q \mathrm{dv}. \tag{18}$$

Integrating over the domain Ω_{cq}, we obtain one of the equations. It is a transcendental equation and reads

$$\alpha c \gamma_q \tilde{\eta}_q \left(1 - \left(1 + \frac{c}{2\pi(1+a_e)u_q}\right) e^{-\frac{c}{2\pi(1+a_e)u_q}}\right) - \frac{\alpha c \gamma_q^2}{4\pi^2(1+a_e)} e^{-\frac{c}{2\pi(1+a_e)u_q}} - u_q = 0, \tag{19}$$

where $\tilde{\eta}_q = \eta_q + 3(1+a_e)\gamma_q \beta_q^2$.

On the other hand, taking into account the explicit representation of the effective mass density (11), from (5) we obtain second equation. For all q, we have

$$3\eta_q \left(\frac{1}{2} - \left(\frac{1 - e^{-\frac{\gamma_q \alpha}{2\pi(1+a_e)}}\left(1 + \frac{\gamma_q \alpha}{2\pi(1+a_e)}\right)}{\left(\frac{\gamma_q \alpha}{2\pi(1+a_e)}\right)^2}\right)\right) - a_e = 0. \tag{20}$$

Equations (19) and (20) represent system of transcendental equations of motion, with $u_q = u_e$ for all q and m_q, where $u_e = const.$, for $t \in [0, +\infty)$. The solutions yield to the exact value of the particle's energy represented by the Hamiltonian (9) and essentially to that of the electromagnetic contribution to the anomalous g-factor.

Table 1. Theoretical and experimental (EXP) values of the anomalous g-factor of electromagnetic origin. Second, third and fourth rows show the theoretical results, with prediction of classical electrodynamics (CED) discussed in the present study and some of the recent results based on renormalized quantum electrodynamics (QED). The last row shows the most recent experimental result, with measurements carried out on electrons. The asterisk symbol indicates that the given result is obtained for a value of the fine structure constant taken from NIST [45], see also Section 3.3.

Methods	a_e	Ref.
Exact CED	$0.001159652(23)^*$	(19) & (20)
Perturbative QED	0.00115965218178(77)	[7]
	0.001159652181643(25)	[8]
EXP	0.00115965218059(13)	[11]

3.2. Effective Mass-Energy Equivalence

Taking into account the contribution only of the electric part of the self-interactions in (9), we obtain the effective mass-energy relation. Thus, from the first two terms on the right hand side after the equality symbol, we get

$$\mathcal{E}_q = \gamma_e M_q c^2,$$

where $M_q = m_q(1+a_e)$ and γ_e is a constant for all q. We would like to point out that if $\nexists e$, then $\nexists a_e$ and the particle's speed equals zero. Thus, for the self-Coulomb energy term in (10), we have $\Sigma_q = a_e \gamma_e m_q c^2$.

 doi:10.20944/preprints202308.1136.v1

3.3. The Anomalous g-Factor

The computations of the electromagnetic component of the particle's anomalous g-factor and the corresponding speed are carried out for $\alpha = 137.035999084^{-1}$ [45]. For the particle's speed we obtain $u_e = 2.18821606262 \times 10^6$ ms^{-1}. The corresponding value of a_e is given in the second row of Table 1. Compared to the experimentally obtained [11] and calculated with the aid of quantum electrodynamics [7,8] electron's anomalous g-factor, the numerical value obtained by solving Equations (19) and (20), with the given value of the fine structure constant, is accurate to a one part per billion. To the best of our knowledge, the result given in Table 1 is the most accurate classical evaluation of the electromagnetic contribution to the electron's anomalous g-factor reported to the present days. We would like to point out that the obtained accuracy holds further for $\alpha = 137.035999206^{-1}$ [46], with $u_e = 2.18821616430 \times 10^6$ ms^{-1}. Furthermore, the accuracy can be improved with additional refinement of the fine structure constant after the ninth digit behind the decimal point.

4. Summary

With the aid of essential regularization (Section 2.2) of the electromagnetic field potentials and spatial averaging procedure (Section 2.3), elaborated within the formalism of classical electrodynamics, the present paper reports an exact approach quantifying on a microscopic level the electrodynamics of isolated system comprised of non-composite particle of arbitrary rest mass and possessing elementary electric charge. The proposed approach overcomes all singularities arising in the conventional methods of classical electrodynamics thus uncovering in details the physical nature of electromagnetic self-interactions and the occurrence of anomalous magnetic moment in the system. It quantifies the particle's dynamics exactly through a system of transcendental equations of motion obtained in the absence of any approximations and at the classical limit, see Section 3.1. Essentially, the solutions of these equations give the exact values of the particle's intrinsic velocity and the electromagnetic contribution to the anomalous g-factor (see Section 3.3). The derived transcendental equations ensure fast computations and provide highly accurate results, competitive to the perturbative method of quantum electrodynamics, see the comparison in Table 1.

In general, the proposed approach may be build on and integrated into the mathematical framework of the classical and quantum field theories. In that regard, more contributions to the particle's magnetic moment may be calculated exactly. With the appropriate gauge fields and regularization one can introduce additional self-interactions and generalize the system of transcendental equations discussed in Section 3.1.

In conclusion, it appears that the present study supports a thesis pointing out that the anomalous magnetic moment may not be a unique feature to the quantum theory and its occurrence may have a solid classical description. Moreover, it may be the case that the perturbative method of quantum field theory is not unique to that feature and in addition there exist an exact method for its calculation. On that account, pushing the boundaries of classical or quantum field theories further to test the possibility for the exact calculation of even higher contributions in the anomalous magnetic moment of non-composite particles is worth a shot. The proposed approach to the self-Coulomb and self-Zeeman interactions, furthermore, may significantly facilitate the optimization of some computational methods in solid state physics making them self-consistent. Especially these methods studying multi-electron systems that fail to address the self-interactions without the consideration of additional corrections.

Funding: This work was supported by National Program "Young scientists and postdoctoral researchers-2" approved by DCM 206/07.04.2022.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data generated within this research is included in the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Heisenberg, W. The self-energy of the electron. In *Early Quantum Electrodynamics*, 1 ed.; Miller, A.I., Ed.; Cambridge University Press, 1994; pp. 121–128. https://doi.org/10.1017/CBO9780511608223.011.
2. Roberts, B.L.; Marciano, W.J. *Lepton Dipole Moments*; Vol. 20, *Advanced Series on Directions in High Energy Physics*, World Scientific, 2009. https://doi.org/10.1142/7273.
3. Keshavarzi, A.; Khaw, K.S.; Yoshioka, T. Muon $g-2$: A review. *Nucl. Phys. B* **2022**, *975*, 115675. https://doi.org/10.1016/j.nuclphysb.2022.115675.
4. Schwartz, M.D. *Quantum Field Theory and the Standard Model*; Cambridge University Press, 2013. https://doi.org/10.1017/9781139540940.
5. Melnikov, K.; Vainshtein, A. *Theory of the muon anomalous magnetic moment*; Springer tracts in modern physics, Springer: Berlin : New York, 2006.
6. Vogel, M. The anomalous magnetic moment of the electron. *Contemp. Phys.* **2009**, *50*, 437–452. https://doi.org/10.1080/00107510902765239.
7. Aoyama, T.; Hayakawa, M.; Kinoshita, T.; Nio, M. Tenth-Order QED Contribution to the Electron $g-2$ and an Improved Value of the Fine Structure Constant. *Phys. Rev. Lett.* **2012**, *109*, 111807. https://doi.org/10.1103/PhysRevLett.109.111807.
8. Aoyama, T.; Hayakawa, M.; Kinoshita, T.; Nio, M. Tenth-order electron anomalous magnetic moment: Contribution of diagrams without closed lepton loops. *Phys. Rev. D* **2015**, *91*, 033006. https://doi.org/10.1103/PhysRevD.91.033006.
9. Aoyama, T.; Kinoshita, T.; Nio, M. Theory of the Anomalous Magnetic Moment of the Electron. *Atoms* **2019**, *7*, 28. https://doi.org/10.3390/atoms7010028.
10. Lin, F.; Huang, M. Magnetic correction to the anomalous magnetic moment of electrons. *Commun. Theor. Phys.* **2022**, *74*, 055202. https://doi.org/10.1088/1572-9494/ac5b5e.
11. Fan, X.; Myers, T.; Sukra, B.; Gabrielse, G. Measurement of the Electron Magnetic Moment. *Phys. Rev. Lett.* **2023**, *130*, 071801. https://doi.org/10.1103/PhysRevLett.130.071801.
12. Morte, M.D.; Francis, A.; Gülpers, V.; Herdoíza, G.; Von Hippel, G.; Horch, H.; Jäger, B.; Meyer, H.B.; Nyffeler, A.; Wittig, H. The hadronic vacuum polarization contribution to the muon $g-2$ from lattice QCD. *J. High Energ. Phys.* **2017**, *2017*, 20. https://doi.org/10.1007/JHEP10(2017)020.
13. Jegerlehner, F. *The Anomalous Magnetic Moment of the Muon*; Vol. 274, *Springer Tracts in Modern Physics*, Springer International Publishing: Cham, 2017. https://doi.org/10.1007/978-3-319-63577-4.
14. Logashenko, I.B.; Eidelman, S.I. Anomalous magnetic moment of the muon. *Phys.-Usp.* **2018**, *61*, 480–510. https://doi.org/10.3367/UFNe.2018.02.038312.
15. Westin, A.; Kamleh, W.; Young, R.; Zanotti, J.; Horsley, R.; Nakamura, Y.; Perlt, H.; Rakow, P.; Schierholz, G.; Stüben, H. Anomalous magnetic moment of the muon with dynamical QCD+QED **2020**. *245*, 06035. https://doi.org/10.1051/epjconf/202024506035.
16. Gérardin, A. The anomalous magnetic moment of the muon: status of lattice QCD calculations. *Eur. Phys. J. A* **2021**, *57*, 116. https://doi.org/10.1140/epja/s10050-021-00426-7.
17. Leutgeb, J.; Mager, J.; Rebhan, A. Holographic QCD and the muon anomalous magnetic moment. *Eur. Phys. J. C* **2021**, *81*, 1008. https://doi.org/10.1140/epjc/s10052-021-09780-8.
18. Nedelko, S.; Nikolskii, A.; Voronin, V. Soft gluon fields and anomalous magnetic moment of muon. *J. Phys. G: Nucl. Part. Phys.* **2022**, *49*, 035003. https://doi.org/10.1088/1361-6471/ac4a82.
19. Eidelman, S.; Passera, M. Theory of the τ lepton anomalous magnetic moment. *Mod. Phys. Lett. A* **2007**, *22*, 159–179. https://doi.org/10.1142/S0217732307022694.
20. González-Sprinberg, G.A.; Vidal, J. Tau magnetic moment. *J. Phys.: Conf. Ser.* **2017**, *912*, 012001. https://doi.org/10.1088/1742-6596/912/1/012001.
21. Özgüven, Y.; Billur, A.A.; İnan, S.C.; Bahar, M.K.; Köksal, M. Search for the anomalous electromagnetic moments of tau lepton through electron–photon scattering at CLIC. *Nucl. Phys. B* **2017**, *923*, 475–490. https://doi.org/10.1016/j.nuclphysb.2017.08.008.
22. Chen, X.; Wu, Y. Search for the Electric Dipole Moment and anomalous magnetic moment of the tau lepton at tau factories. *J. High Energ. Phys.* **2019**, *2019*, 89. https://doi.org/10.1007/JHEP10(2019)089.

23. Arroyo-Ureña, M.A.; Hernández-Tomé, G.; Tavares-Velasco, G. Anomalous magnetic and weak magnetic dipole moments of the τ lepton in the simplest little Higgs model. *Eur. Phys. J. C* **2017**, *77*, 227. https://doi.org/10.1140/epjc/s10052-017-4803-z.
24. Dyndał, M.; Kłusek-Gawenda, M.; Szczurek, A.; Schott, M. Anomalous electromagnetic moments of τ lepton in $\gamma\gamma \to \tau+ \tau-$ reaction in Pb+Pb collisions at the LHC. *Phys. Lett. B* **2020**, *809*, 135682. https://doi.org/10.1016/j.physletb.2020.135682.
25. Tran, H.M.; Kurihara, Y. Tau $g-2$ at e^-e^+ colliders with momentum-dependent form factor. *Eur. Phys. J. C* **2021**, *81*, 108. https://doi.org/10.1140/epjc/s10052-021-08846-x.
26. Borges, L.H.C.; Barone, F.A.; Ferrari, A.F. The point-charge self-energy in a nonminimal Lorentz-violating Maxwell electrodynamics. *EPL* **2018**, *122*, 31002. https://doi.org/10.1209/0295-5075/122/31002.
27. Sebens, C.T. Eliminating Electron Self-repulsion. *Found. Phys.* **2023**, *53*, 65. https://doi.org/10.1007/s10701-023-00702-0.
28. Fetter, A.L.; Walecka, J.D. *Quantum theory of many-particle systems*; Dover Publications: Mineola, N.Y, 2003.
29. Weinberg, S. *The Quantum Theory of Fields*; Vol. 1, Cambridge University Press, 1995. https://doi.org/10.1017/CBO9781139644167.
30. Weinberg, S. *The Quantum Theory of Fields*; Vol. 2, Cambridge University Press, 1996. https://doi.org/10.1017/CBO9781139644174.
31. Desai, B.R. *Quantum Mechanics with Basic Field Theory*, 1 ed.; Cambridge University Press, 2009. https://doi.org/10.1017/CBO9780511813979.
32. Collins, J.C. *Renormalization: An Introduction to Renormalization, the Renormalization Group and the Operator-Product Expansion*, 1 ed.; Cambridge University Press, 2023. https://doi.org/10.1017/9781009401807.
33. Perdew, J.P.; Zunger, A. Self-interaction correction to density-functional approximations for many-electron systems. *Phys. Rev. B* **1981**, *23*, 5048–5079. https://doi.org/10.1103/PhysRevB.23.5048.
34. Bao, J.L.; Gagliardi, L.; Truhlar, D.G. Self-Interaction Error in Density Functional Theory: An Appraisal. *J. Phys. Chem. Lett.* **2018**, *9*, 2353–2358. https://doi.org/10.1021/acs.jpclett.8b00242.
35. Hehl, F.W.; Obukhov, Y.N. *Foundations of Classical Electrodynamics: Charge, Flux, and Metric*; Birkhäuser Boston: Boston, MA, 2003. https://doi.org/10.1007/978-1-4612-0051-2.
36. Shanahan, D. A Case for Lorentzian Relativity. *Found. Phys.* **2014**, *44*, 349–367. https://doi.org/10.1007/s10701-013-9765-x.
37. Griffiths, D.J. *Introduction to Electrodynamics*, 4 ed.; Cambridge University Press, 2017. https://doi.org/10.1017/9781108333511.
38. Roussy, T.S.; Caldwell, L.; Wright, T.; Cairncross, W.B.; Shagam, Y.; Ng, K.B.; Schlossberger, N.; Park, S.Y.; Wang, A.; Ye, J.; et al. An improved bound on the electron's electric dipole moment. *Science* **2023**, *381*, 46–50. https://doi.org/10.1126/science.adg4084.
39. Greiner, W. *Classical Electrodynamics*; Theoretical Physics, Springer New York: New York, NY, 1998. https://doi.org/10.1007/978-1-4612-0587-6.
40. Cohen-Tannoudji, C.; Dupont-Roc, J.; Grynberg, G. *Photons and atoms: Introduction to quantum electrodynamics*; Physics textbook, Wiley-VCH: Weinheim, 2004.
41. Rohrlich, F. The self-force and radiation reaction. *Am. J. Phys.* **2000**, *68*, 1109–1112. https://doi.org/10.1119/1.1286430.
42. Gratus, J. Maxwell-Lorentz without self-interactions: conservation of energy and momentum. *J. Phys. A: Math. Theor.* **2022**, *55*, 065202. https://doi.org/10.1088/1751-8121/ac48ee.
43. Lonsdale, D.R.; Goerigk, L. The one-electron self-interaction error in 74 density functional approximations: a case study on hydrogenic mono- and dinuclear systems. *Phys. Chem. Chem. Phys.* **2020**, *22*, 15805–15830. https://doi.org/10.1039/D0CP01275K.
44. Zope, R.R.; Yamamoto, Y.; Diaz, C.M.; Baruah, T.; Peralta, J.E.; Jackson, K.A.; Santra, B.; Perdew, J.P. A step in the direction of resolving the paradox of Perdew-Zunger self-interaction correction. *J. Chem. Phys.* **2019**, *151*, 214108. https://doi.org/10.1063/1.5129533.
45. 2018 CODATA Value: fine-structure constant. The NIST Reference on Constants, Units, and Uncertainty. *NIST* **2019**.
46. Morel, L.; Yao, Z.; Cladé, P.; Guellati-Khélifa, S. Determination of the fine-structure constant with an accuracy of 81 parts per trillion. *Nature* **2020**, *588*, 61–65. https://doi.org/10.1038/s41586-020-2964-7.

 doi:10.20944/preprints202308.1136.v1