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Article

A Hydroacoustic Adversarial Strategy Based on the Matrix Game Theory

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Abstract: Due to the great threat of torpedo against surface warships, an efficient hydroacoustic countermeasure system must output realtime strategies that can accommodate varied antagonizing scenarios. Aiming at the decision-making problem in the anti-torpedo hydroacoustic countermeasure cases, this paper proposes an intelligent adversarial strategy based on the game theory. By discretizing the strategy space of both sides, a matrix game model is established with the payoff characterized by the capture probability of attacking torpedo. An improved simplex algorithm is then developed to get the mixed-strategy Nash equilibrium of the game model, with which preferred avoiding strategies can be obtained to minimize the risk of being captured by the attacking torpedo. Based on the probabilistic view of preferred adversarial strategy, the launching strategy of floating acoustic decoy can be achieved, and the evasion trajectory of warship can be confirmed simultaneously. Numerical simulation demonstrates that the proposed method can generate realtime intelligent antagonizing strategy, and consequently, can improve the survival rate of surface warships in the of anti-torpedo countermeasure scenarios.

Keywords: hydroacoustic countermeasure; game theory; Gaussian distribution; linear programming; simplex method

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1. Introduction

Due to the great threat of torpedoes, anti-torpedo confrontation is critical to the survival of defensive warships from naval warfares [1]. Hydroacoustic antagonizing is devoted to using hydroacoustic equipments to interfere with the attack of hostile acoustic-guided torpedoes, thereby improving the survival capability of the defensive warships [2]. Since the Second World War, hydroacoustic antagonizing technology has been investigated for more than 60 years, contributing to some representative systems such as the United States / Britain joint Surface Ship Torpedo Defence (SSTD) system, the France Systeme-de Lutte Anti-Torpille (SLAT) system, and the Israel Torpedo Self Defence System (TSDS) system, etc. [3].

Besides the development of various hydroacoustic equipments, it is of supreme importance to propose efficient hydroacoustic counter-measures to address the changing scenarios of anti-torpedo confrontation [4–8]. Given individual antagonizing scenarios, a few researches were reported on the anti-torpedo counter-measures of submarines [9,10]. However, the obtained strategies cannot address the changing adversarial situations of naval warfares. Considering that the counter-measure should be applicable to varied scenarios, Li *et al.* [11] investigated antagonizing results of different manoeuvre evasion strategies based on the Mote-Carlo simulating method. It only provides an evaluation method of counter-measure, and does not propose an intelligent antagonizing strategy that can accommodate the frequently changing confrontation situations.

The intelligent decision-making process for the anti-torpedo antagonizing scenarios need to model and predict the actions of attacking torpedoes, and try to evade to survive from it. Accordingly, modelling how the actions of both sides shape the decision making of each other is the forte of game theory. In past decades, there have been a lot of studies that applied different branches of game

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theory to model a range of defence-related scenarios, including a few research on its applications on researches of naval warfare [12].

Levine [13] studied aspects of naval warfare in the previous centuries using concepts from game theory. Since the powerful nations in the 18th and 19th centuries built warships with cannons positioned along their sides, neither fleet would gain from turning towards the enemy and neither would get ahead. Levine concluded that this strategy—forming a line of battle and sailing parallel to the other fleet—was each fleet's best response, and thus represented a Nash equilibrium.

Maskery *et al.* [14] investigated the problem of deploying counter-measures against anti-ship missiles using a network-enabled operations framework, where multiple ships communicate and coordinate to defend against a missile threat. Here, the missile threats were modelled as a discrete Markov process, and the ships which are equipped with countermeasures were modelled as the players of a transient stochastic game, where the actions of the individual players include the use of countermeasures to maximize their own safety while cooperating with other players which are essentially aiming to achieve the same objective. The optimal strategy of this game-theoretic problem is a correlated equilibrium strategy and is shown to be achieved via an optimization problem with bilinear constraints. In [15], Maskery *et al.* considered the problem of network-centric force protection of a task group against anti-ship missiles. The decision-makers in this model are the ships equipped with hard-kill/soft-kill weapons (counter-measures) and these ships are also considered the players in the formulation of this problem in a game-theoretic setting. Here, the ships play a game with each other instead of with a missile. This approach naturally lends itself to decentralised solutions which may be implemented when full communication is not feasible. Moreover, this formulation leads to an interpretation of the problem as a stochastic shortest past game for which Nash Equilibrium solutions are known to exist.

Bachmann *et al.* [16] analyzed the interaction between radar and jammer using a noncooperative two-player zero-sum game. In their approach, the radar and jammer were considered 'players' with opposing goals: the radar tries to maximize the probability of detection of the target while the jammer attempts to minimize its detection by the radar by jamming it. This game-theoretic formulation was solved by optimizing these utility functions subject to constraints in the control variables (strategies), which for the jammer are jammer power and the spatial extent of jamming while for the radar the available strategies include the threshold parameter and reference window size. The resulting matrix-form games were solved for optimal strategies of both radar and jammer from which they identify conditions under which the radar and jammer are effective in achieving their individual goals.

Considering the scenario of a fleet's anti-torpedo confrontation, Ma *et al.* [17] tried to predict the torpedo's attacking target by the zero-sum game model, which is employed to design target-orient counter-measure for the anti-torpedo warfare. In this research, we would like to investigate how the game theory is helpful to development of realtime counter-measures by investigating the scenario that a surface warship encounters with an hostile submarine, which would like to attacking the warship by wire-guided and acoustic-guided torpedoes. Since a torpedo's speed is much greater than that of a warship, we take it as a antagonizing failure if the warship enter the self-guided sector of a torpedo. Accordingly, the hydroacoustic counter-measure is developed to ensure that the warship never enter the self-guided sector of a torpedo before its range is exhausted.

Due to the high attacking speed and the adjustable attacking direction of torpedo, anti-torpedo antagonizing scenarios are usually emergency situations that requires realtime counter-measures, and it would improve the warship's survival capability by predicting the attacking direction of torpedo, which is addressed in this paper based on a matrix(zero-sum) game model. Taking the evasion angle of the defensive warship and the attacking advance angle of the torpedo as the respective antagonizing strategies of two opposite sides, a matrix game model is constructed by discretizing them in their value ranges, taking the payoff as the warship's probability of being captured by the torpedo. Then, the linear programming (LP) model for matrix game is proposed, and an improved initialization strategy of the simplex method to get the Nash-equilibrium solution to the game model as soon as possible. Starting with the initial information inherited from the Nash equilibrium solution, a strategy of decoy launch can be obtained to get success countermeasure for the anti-torpedo antagonizing scenario. In this way, a realtime antagonizing strategy can be generated to address emergent scenarios of hydroacoustic countermeasure. Contributions of this work are as follows.

- A matrix game model is proposed for the hydroacoustic anti-torpedo antagonizing scenario to accommodate varied confrontation situations.
- An improved initialization strategy is developed to get Nash equilibrium by accelerating the simplex method for linear programming problems.
- Inherited from information included in the Nash equilibrium solution, a preferred evasion strategy of the defensive warship is confirmed to improve its survival probability, and a launching strategy of floating decoy can be obtained to ensure that the warship would not be captured by the attacking torpedo.

Rest of this paper is organized as follows. Section 2 presents a preliminary introduction to the theory of matrix game and the simplex method for LP, and an intelligent decision model of hydroacoustic countermeasure is proposed in Section 3 based on the matrix game theory and simulation of navigation trajectories. Then, numerical simulations are performed in Section 4 to demonstrate the efficiency and practicability of the proposed game model, and finally, Section 5 presents conclusions and discussions.

2. Preliminaries

2.1. Introduction to the Game Theory

• Pure vs. mixed strategies [12]

A game typically consists of two or more players, a set of strategies available to these players, and a corresponding set of payoff values for each player. A pure strategy in a game provides a complete definition of how a player will play a game, and a mixed strategy is a combination of pure strategies where a particular probability p (where $0 \leq p \leq 1$) is associated with each of these pure strategies. Therefore, any pure strategy is actually a degenerate case of a mixed strategy, in which that particular strategy is selected with probability 1 and every other strategy is selected with probability 0.

• Nash equilibrium [12]

The solution to a game model refers to the Nash equilibrium (solution). Let (S, f) be a game with n players, where S_i is the strategy set of a given player i. Thus, the stragety profile S consisting of the strategy sets of all players would be $S = S_1 \times S_2 \times \cdots \times S_n$. Let $f(x) = (f_1(x), \ldots, f_n(x))$ be the payoff function for strategy set $x \in S$. Suppose x_i is the strategy of player i and x_{-i} is the strategy set of all players except player i. Thus, when each player $i \in \{1, \ldots, n\}$ chooses strategy x_i that would result in the strategy set $x = (x_1, \ldots, x_n)$, giving a payoff of $f_i(x)$ to that particular player, which depends on both the strategy chosen by that player (x_i) and the strategies chosen by other players (x_{-i}). A strategy set $x^* \in S$ is in Nash equilibrium if no unilateral deviation in strategy by any single player would return a higher utility for that particular player. Formally put, x^* is in Nash equilibrium if and only if:

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*). \tag{1}$$

x^* is called a pure/mixed-strategy Nash equilibrium when x_i^* is pure/mixed-strategy for any player i, $i \in \{1, \ldots, n\}$.

• Matrix game [12]

The matrix game models, also known as the zero-sum game models, are a class of competitive games where the total of the payoffs of all players is zero. In two-player games, this implies that

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one player's loss in the payoff is equal to another player's gain in the payoff. A maxtrix (two-player zero-sum) game can therefore be represented by a payoff matrix that shows only the payoffs of one player. The mini-max theorem [18] states that in a zero-sum game there is a set of strategies that minimizes the maximum losses (or maximises the minimum payoff) of each player, which can be addressed by solving LP problems. Note that the linear programming problems given for both players are dual to each other. The optimal mixed strategies for both players can be found by solving only one of the linear programming problems because the optimal dual solution is an automatic by-product of the simplex method calculations to find the optimal primal solution. In general, you always can find optimal mixed strategies for both players by choosing just one of the models (either one) and then using the simplex method to solve for both an optimal solution and an optimal dual solution [19].

2.2. The Simplex Method for LP Problems [20]

An LP problem with n' variables and r constraints takes the form

$$\begin{aligned} \max \quad & z' = c'^T x' \\ s.t. \quad & \begin{cases} A'x' \leq b, \\ x' \geq 0, b \geq 0 \end{cases} \end{aligned}$$

where n' and r are positive integers, $c' \in \mathbb{R}$, $x' \in \mathbb{R}^{n'}$, $A' \in \mathbb{R}^{n' \times n'}$, and $b \in \mathbb{R}^r$. Let $N' = \{1, \ldots, n\}$ be the set of variable indices and $R = \{1, \ldots, r\}$ be the set of constraint indices. The feasible region of an LP is denoted by $S' = \{x' \in \mathbb{R}^{n'}_+ : A'x' \leq b\}$ and the optimal solution of an LP is $x'^* \in S$ such that $z'^* = c'^T x'^* \geq c'^T x''$ for all $x'' \in S$.

One of the most popular algorithms to LP problems is the simplex method (SM) created by George Dantzig [21]. Given an LP, the SM requires that all constraints be converted into linear equations by adding r slack variables. This new problem is called the canonical linear program (CLP) and is defined as

$$\begin{aligned} \max \quad & z = c^T x \\ s.t. \quad & \begin{cases} Ax = b, \\ x \geq 0, b \geq 0, \end{cases} \end{aligned}$$

where $x \in \mathbb{R}^{n+r}$, $c \in \mathbb{R}^{n+r}$ is c' augmented with r zeros, and $A \in \mathbb{R}^{r \times (n+r)}$ is A' augmented with an $r \times r$ identity matrix. Let $N = \{1, \ldots, n+r\}$ be the variable indices and $S = \{x \in \mathbb{R}^{n+r}_+ : Ax = b\}$ be the feasible region of an CLP.

Initially, SM requires a starting basic feasible solution. Formally, $BV \subseteq N$ is called a basis if $|BV| = |R|$ and $A_{.BV}$ is nonsigular. The set of nonbasic indices is $NBV = N \backslash BV$. The corresponding vasic and nonbasic variables are x_{BV} and x_{NBV}, respectively. If $A^{-1}_{.BV} b \geq 0$, then BV is a feasible basis with $x_{BV} = A^{-1}_{.BV} b$ and $x_{NBV} = 0$ being the corresponding vasic feasible solution. Here, $A_{.BV}$ represents the columns of A restricted to the indices in BV, and x_{BV} is the x values of the indices in BV.

The input to SM is an CLP and a feasible basise $BV \subseteq N$ that produces a feasible basis solution. The algorithm evaluates each nonbasis variable's reduced cost, which is euql to $c_{BV} A^{-1}_{.BV} A_{.i} - c_i$ for all $i \in NBV$. If all nonbasic reduced costs are nonnegative, then the corresponding vasic feasible solution represents an optimal solution of CLP. Furthermore, BV is said to be an optimal basis. If BV is not optimal, then there exists an entering variable with index $p \in NBV$ such that its reduced cost is negative. In order to determine the leaving variable, define $R^+ = \{j \in R : (A^{-1}_{.BV} A_{.p})_j > 0\}$. If $R^+ = \emptyset$, then the problem is unbounded. If not, SM performs the ratio test and identifies $j^* \in R^+$ such that $\frac{(A^{-1}_{.BV} b)_{j^*}}{(A^{-1}_{.BV} A_{.p})_{j^*}} \leq \frac{(A^{-1}_{.BV} b)_j}{(A^{-1}_{.BV} A_{.p})_j}$ for all $j \in R^+$. SM replaces the j^*th element in BV with p. This process is referred to as a pivot. The algorithm continues pivoting until an optimal basis of CLP is identified or CLP is shown to be unbounded.

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3. A Decision Model of Hydroacoustic Countermeasure Based on the Matrix Game Theory and Trace Simulations

3.1. Flowchart of the Hydroacoustic Antagonizing Model

The proposed hydroacoustic antagonizing model works as follows.

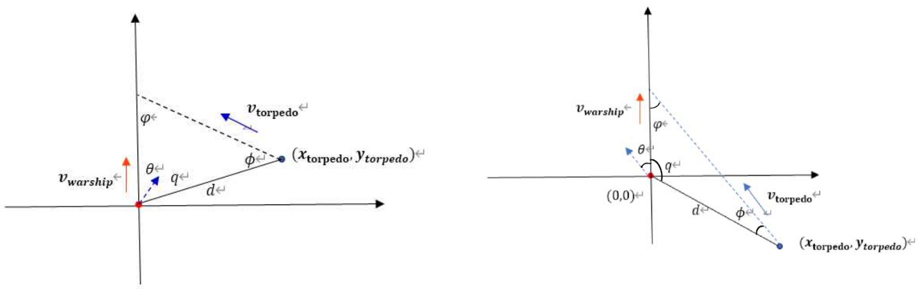
- When the attacking torpedo is detected, a matrix game model is constructed to formulate the countermeasure scenario, and an improved simplex algorithm is developed to address the linear programming model of matrix game.
- By maximizing the expected payoff for all feasible attacking directions, a preferred heading of the torpedo can be forecasted; meanwhile, the promising evasion strategy can be obtained by minimizing the expected payoff for all alternative avoidance directions of the warship.
- The warship executes a circular evasion manoeuvre and launches the floating acoustic decoy to a preferred position that is confirmed according to the attacking trajectory of torpedo.
- The torpedo recognizes the floating decoy as a false target when the distance between them is sufficiently small, and then navigates with a circular trajectory to search the true objective. The warship switches from a circular orbit to a straight trajectory to go as far as possible from the torpedo.
- If the warship does not enter the self-guided sector of the torpedo, a success countermeasure process is reported; otherwise, the anti-torpedo antagonizing operation fails.

3.2. The Matrix Game Model for Hydroacoustic Countermeasure

Knowing that both sides of the hydroacoustic countermeasure would like to maximize their own profits, we construct a matrix game model to simulate the adversarial scenario, where the game strategies of the two opposing sides in the static game are their own sailing/attacking directions. Because the sailing speed of torpedo is much greater than that of the surface ship, we construct the payoff matrix by the maximum capturing probabilities corresponding to pairs of sailing directions. Since the capturing probability depend on relative positions of the opposite objects, we can get the functional relationship between the relative positions of the torpedo and the ship under each game situation as a function of time. Then, a component of the payoff matrix is obtained by taking the maximum capture probability for a pair of sailing settings. If the probability of capture is less than the given threshold, no evasion operation is carried out; otherwise, it is necessary to determine the evasion strategy of the red ship.

To model the scenario of hydroacoustic confrontation, we establish a planar rectangular coordinate system based on the relative positions of the torpedo and the ship. Assuming that the detected position of the torpedo and the location of surface ship obey the normal distribution, we always place the surface ship at the origin of the coordinate system. Then, the position of torpedo can be taken as a two-dimensional random vector obeying a normal distribution.

The hydroacoustic confrontation situations when the sonar system of the red ship detects the incoming torpedo are shown in Figure 1, where the evasion direction angle θ of the red surface ship and the advance angle ϕ of the blue torpedo attack are supposed to be regulated to perform the countermeasure game. For a strategy pair (ϕ, θ), the relative position of the mine-ship can be calculated as a function of time according to the sailing speed of the red ship and the attack advance angle and attack speed of the incoming torpedo. By incorporating a calculation method of capture probability, one can get the capture probability function of sailing time, and take its maximum value as the probability that an attacking torpedo captures the red ship.



(a) Detected torpedo attacks with a small outboard angle.

(b) Detected torpedo attacks with a small outboard angle.

Figure 1. Scenarios of hydroacoustic countermeasures.

Accordingly, the game model of hydroacoustic confrontation between red and blue parties can be constructed, and the game strategy pair is (ϕ, θ). As illustrated in Figure 1, let $\theta \in [-\theta_0, \theta_0]$, $\phi \in [0, \phi_0]$. To construct a discrete matrix game model, intervals $[-\theta_0, \theta_0]$ and $[0, \phi_0]$ are discretized as collections of finite feasible values, that is, we set $\theta \in \{\theta_1, \ldots, \theta_m\}$ and $\phi \in \{\phi_1, \ldots, \phi_n\}$. Then, we get the $n \times m$ payoff matrix $P = (p_{i,j})_{n \times m}$, where the payoff $p_{i,j}$ is the probability that a red ship is captured by a blue torpedo when the adversarial strategy pair is (ϕ_i, θ_j) $(i = 1, \ldots, n, j = 1, \ldots, m)$.

A mixed/pure-strategy Nash equilibrium solution to this matrix game model can be obtained by solving the linear programming model [19]

$$\begin{aligned} &\max \quad z = \sum_{j=1}^{m} r_j \\ &s.t. \begin{cases} \sum\limits_{j=1}^{m} p_{ij} r_j \leq 1, i = 1, \ldots, n, \\ r_j \geq 0, j = 1, \ldots, m. \end{cases} \end{aligned} \tag{2}$$

According to the optimal solution $\overrightarrow{r} = (r_1, \ldots, r_m)$ of model (1), the sum of components $v = \sum_{j=1}^{m} r_j$ is the optimal payment corresponding to the mixed-strategy Nash equilibrium, and the probability distribution $\sigma^* = (\frac{r_1}{v}, \ldots, \frac{r_m}{v})$ is the pure/mixed-strategy Nash equilibrium.

3.2.1. Payoff Computation for a Strategy Pair (ϕ,θ)

- **The Gaussian model for payoff of a strategy pair (ϕ,θ)**

Figure 1 illustrates the confrontation posture at the time of alarm. We establish an inertial coordinate system where the surface ship's position is set as the origin of the coordinate system, denoted by O. The sailing direction of the ship is set as the positive direction of the Y axis, labelled as the due north direction, and the due east direction is taken as the positive direction of the X axis. Denote the sailing speed of the surface ship by $v_{warship}$, and the deflection angle to avoid the attack of torpedo by θ. For the acoustic self-guided torpedo, let q and d_0 be its relative bearing angle and the distance from the surface ship, respectively. While the torpedo attack with the speed $v_{torpedo}$ uniform linear search, the attacking direction can be computed by

$$C = q + \phi + \pi, \tag{3}$$

where ϕ is the favourable advance angle, φ is the angle between the torpedo heading extension line and the Y-axis angle.

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Set $t = 0$ for the moment when the sonar equipment reports attack of the torpedo. We get the position $\mathbf{M}(t) = (x_{warship}(t), y_{warship}(t))$ of the red warship at time t, where

$$x_{warship}(t) = \int_0^t v_{warship} \sin\theta \, dt, \tag{4}$$

$$y_{warship}(t) = \int_0^t v_{warship} \cos\theta \, dt. \tag{5}$$

Similarly, the torpedo's position $\mathbf{T}(t) = (x_{torpedo}(t), y_{torpedo}(t))$ can be identified by

$$x_{torpedo}(t) = d_0 \sin q + \int_0^t v_{torpedo} \sin C \, dt, \tag{6}$$

$$y_{torpedo}(t) = d_0 \cos q + \int_0^t v_{torpedo} \cos C \, dt. \tag{7}$$

The distance between the surface ship and the torpedo is

$$d(t) = \sqrt{(x_{torpedo}(t) - x_{warship}(t))^2 + (y_{torpedo}(t) - y_{warship}(t))^2}. \tag{8}$$

The lead angle is changing with time t by

$$\phi(t) = \pi - \arccos \frac{\mathbf{r}_1(t)\mathbf{r}_2(t)}{\|\mathbf{r}_1(t)\|\|\mathbf{r}_2(t)\|}, \tag{9}$$

where

$$\mathbf{r}_1(t) = \mathbf{T}(0) - \mathbf{T}(t), \mathbf{r}_2(t) = \mathbf{M}(t) - \mathbf{T}(t).$$

Since the positions of both surface ship and torpedo could not be precisely located, it is assumed that their coordinate positions obey the two-dimensional normal distribution:

$$\mathbf{r}_M(t) \sim \mathcal{N}(\mathbf{M}(t), \mathbf{\Sigma}_{warship}), \tag{10}$$

$$\mathbf{r}_T(t) \sim \mathcal{N}(\mathbf{T}(t), \mathbf{\Sigma}_{torpedo}). \tag{11}$$

In case that $\mathbf{r}_M(t)$ and $\mathbf{r}_T(t)$ are independent, we know $\mathbf{r}(t) = \mathbf{r}_M(t) - \mathbf{r}_T(t)$ is a Gaussian random vector with

$$\mathbf{r}(t) \sim \mathcal{N}(\mathbf{r}_0(t), \mathbf{\Sigma}),$$

where

$$\mathbf{r}_0(t) = \mathbf{M}(t) - \mathbf{T}(t), \tag{12}$$

$$\mathbf{\Sigma} = \mathbf{\Sigma}_{warship} + \mathbf{\Sigma}_{torpedo}. \tag{13}$$

The probability density function (PDF) of $\mathbf{r}(t)$ is

$$f(\mathbf{r}, t) = \frac{1}{2\pi|\mathbf{\Sigma}|^{\frac{1}{2}}} \exp[-\frac{1}{2}(\mathbf{r} - \mathbf{r}_0(t))^T \mathbf{\Sigma}^{-1} (\mathbf{r} - \mathbf{r}_0(t))]. \tag{14}$$

While the surface ship is located in the self-guided sector S of the torpedo, it would be captured by the torpedo. Thus, the capture probability $P_c(t)$ can be computed by a double integral

$$P_C(t) = \iint_{S_1} \frac{1}{2\pi|\mathbf{\Sigma}|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(\mathbf{r} - \mathbf{r}_0(t))^T \mathbf{\Sigma}^{-1} (\mathbf{r} - \mathbf{r}_0(t))\right] dxdy \tag{15}$$

where S_1 is the sector illustrated in Figure 2a. Maximizing $P_C(t)$ for time t we can get the payoff (capture probability) corresponding to a strategy pair (θ, ϕ).

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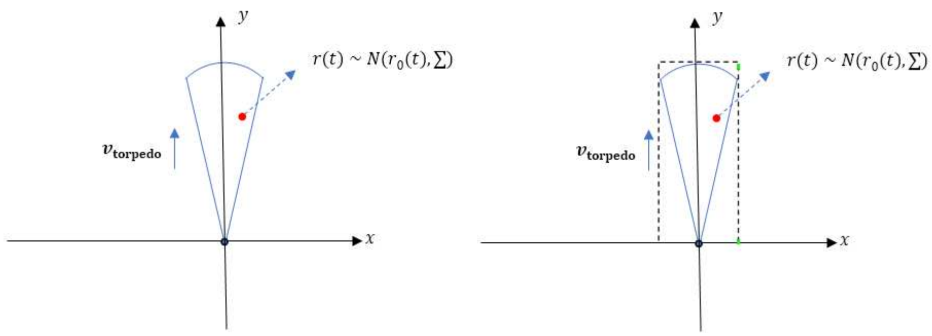

(a) The self-guided sector of the attacking torpedo.

(b) The rectangle approximation of the self-guided sector.

Figure 2. The self-guided sector for calculation of the capture probability.

• **Approximation of the payoff corresponding to a strategy pair** (θ, ϕ)

Since the integral included in (15) cannot be analytically computed, we would like to get an analytical approximation of $P_C(t)$ to get the payoff corresponding to a strategy pair (θ, ϕ).

Assuming that the position error in the x-direction is uncorrelated to the position error in the y-direction, we get a diagonal covariance matrix

$$\mathbf{P} = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}. \tag{16}$$

For any time moment t, denote $\mathbf{r}_0(t) = (x_0(t), y_0(t))$, and Eqn. (15) implies

$$P_C(t) = \frac{1}{2\pi\sigma_x\sigma_y} \iint_{S_1} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)\right] dxdy. \tag{17}$$

Denote the radius and the sector angle of S_1 by L_{max} and λ, respectively. As illustrated in Figure 2a, any $(x, y) \in S_1$ satisfies

$$\begin{cases} x^2 + y^2 \leq L_{max}{}^2, \\ -y\,tan(\frac{\lambda}{2}) \leq x \leq y\,tan(\frac{\lambda}{2}), \\ y \geq 0. \end{cases}$$

Since the self-conducting sector of torpedo is usually a sector with a large radius L_{max} and a comparatively small angle λ, we can approximate $P_C(t)$ by integrating the PDF in a rectangle S_2 that is illustrated in Figure 2b by dashed lines, which is confirmed by

$$S_2 = \{(x, y)| - y\,tan(\frac{\lambda}{2}) \leq x \leq y\,tan(\frac{\lambda}{2}), 0 < y < L_{max}\}. \tag{18}$$

Thus, Equations (17) and (18) implies that the probability can be approximated by

$$P_{C_A}(t) \approx \frac{1}{2\pi\sigma_x\sigma_y} \iint_{S_1} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)\right] dxdy = P_{Cx}(t) \cdot P_{Cy}(t),$$

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where

$$\begin{aligned} P_{Cx}(t) &= \int_{-L_{max}\tan(\frac{\lambda}{2})}^{L_{max}\tan(\frac{\lambda}{2})} \frac{1}{\sqrt{2\pi}\sigma_x} \exp(-\frac{(x-x_0(t))^2}{2\sigma_x^2})dx, \\ &= \Phi(\frac{L_{max}\tan(\frac{\lambda}{2})-x_0(t)}{\sigma_x}) + \Phi(\frac{L_{max}\tan(\frac{\lambda}{2})+x_0(t)}{\sigma_x}) - 1, \end{aligned}$$

$$P_{Cy}(t) = \int_0^{L_{max}} \frac{1}{\sqrt{2\pi}\sigma_y} \exp(-\frac{(y-y_0(t))^2}{2\sigma_y^2})dy = \Phi(\frac{L_{max}-y_0(t)}{\sigma_y}) + \Phi(\frac{y_0(t)}{\sigma_y}) - 1,$$

and $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution. In summary, the capture probability can be approximated by

$$P_C(t) \approx \left[\Phi(\frac{L_{max}\tan(\frac{\lambda}{2})-x_0(t)}{\sigma_x}) + \Phi(\frac{L_{max}\tan(\frac{\lambda}{2})+x_0(t)}{\sigma_x}) - 1\right] \cdot \left[\Phi(\frac{L_{max}-y_0(t)}{\sigma_y}) + \Phi(\frac{y_0(t)}{\sigma_y}) - 1\right]. \tag{19}$$

3.2.2. An improved simplex algorithm with a novel initialization strategy

To get real-time countermeasure strategy for the constructed matrix game model, an improved simplex method algorithm addressing the linear programming model (2) is developed to get a (mixed/pure) Nash equilibrium solution of the game model.

The simplex method can address the canonical form of a linear programming problem

$$\begin{aligned} \max \quad & z = C^T X \\ s.t. & \begin{cases} AX = b, \\ X \geq 0, b \geq 0. \end{cases} \end{aligned} \tag{20}$$

Starting with an initial basic feasible solution, the simplex method iterates successively to get the optimal solution of (20). In this paper, we incorporate a tailored strategy included in Algorithm 1 to accelerate the iteration process of the simplex method. By resorting the decision variables according components of C, problem (20) is transformed to a reformed linear programming problem

$$\begin{aligned} \max \quad & w = {C'}^T X' \\ s.t. & \begin{cases} A'X' = b', \\ X' \geq 0, b \geq 0, \end{cases} \end{aligned}$$

and a promising basis feasible solution X_0 is generated by the Gaussian elimination method with partial pivoting. Accordingly, the initial feasible basis can be obtained by picking up columns of A' that correspond to its leading entries.

The proposed method generates an initial feasible solution with a promising objective value and tries to incorporate in it artificial variables as few as possible, which not only reduces the time complexity of each iteration, but also reduces the number of iterations before the optimal solution is achieved.

3.3. Deployment of the Floating Acoustic Decoy

Floating acoustic decoys can trap active and passive acoustic self-guided torpedoes. With the alarm of the attacking torpedo, the proposed matrix game model output a forecast attacking directions, and the defender (the warship) makes a evasive motion accompanied with the launch of floating decoys. Once the attacking torpedo detects the acoustic decoys, it regulates its sailing direction towards the decoys, which contributes to more avoiding time for the defender warship to sail as far as possible. However, torpedoes can distinguish decoy targets when it approaches the acoustic decoy, then, abandons its present tracking and search for the real target (the warship) with a circular

Algorithm 1 Initialization for the Simplex Method

Input: $A = (a_{j,j})_{n\times m}, C = (c_1, \ldots, c_m)^T, X = (x_1, \ldots, x_m)^T, b = (b_1, \ldots, b_n)^T$.
Output: A', C', X', b', X_0.
1: Sort the components of C in descending order to get the reformed problem

$$\max w = C'^T X' \text{ s.t. } A'X' = b',$$

where $A' = (a_{i,k_j})_{n\times m}, X' = (x_{k_1}, \ldots, x_{k_m})^T, b' = b, C' = (c_{k_1}, \ldots, c_{k_m})^T, c_{k_1} \geq \cdots \geq c_{k_m}$;
2: Perform the Gaussian elimination method with partial pivoting on the linear system $A'X' = b'$, obtaining a linear system $A''X' = b''$, where A'' is a reduced row echelon form of matrix A';
3: Set $A' = A'', b' = b'', X_0 = (b'^T, 0^T)^T$.

trajectory. To successfully escape from the attacking of the torpedo, the warship must manage to escape from the capture region until its engine power is exhausted. Accordingly, success of anti-torpedo countermeasure relies on good deployment of floating acoustic decoy based on simulation of the sailing trajectories of both the warship and the torpedo.

3.3.1. The deploying model based on simulation of sailing trajectories

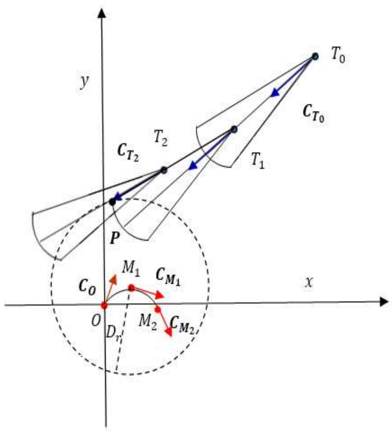

Figure 3. The illustrative scenario for deployment of the floating acoustic decoy.

The principle of acoustic decoy deployment is to guide the attacking direction such that it sails away from the warship. As illustrated in Figure 3, The antagonizing process starts with the alarm of the attacking torpedo, when the warship defender executes a evasion manoeuvre with a circular trajectory. Suppose that the warship is located at the origin O with the heading C_O inherited from the matrix game model, and the attacking torpedo is detected at the position T_0 with the heading C_{T_0}. Simultaneously, the headquarter of the warship makes a decision to launch an acoustic decoy. Due to the time delay of order execution, the warship comes to the point M_1 with a heading C_{M_1} when the floating acoustic decoy is launched to the position P.

Denote the center of the circular trajectory by $O_M = (x_{O_M}, y_{O_M})$. The position of the warship at any moment t can be confirmed by

$$\begin{cases} x_{warship}(t) = x_{O_M} - r_m \times \cos(w_m \times t + C_O), \\ y_{warship}(t) = y_{O_M} + r_m \times sin(w_m \times t + C_O), \end{cases} \tag{21}$$

where

$$\begin{cases} x_{O_M} = r_m \times \cos C_O, \\ y_{O_M} = -r_m \times \sin C_O. \end{cases} \tag{22}$$

Here, r_m and w_m are the radius of the warship's circular trajectory and the warship's angular velocity, respectively.

Assume that the torpedo attacks with a fan-shaped capture area detects the decoy at position P when it is located at point T_1, and then, regulates its sailing direction to attack the fake objective at the position P. While it moves to the position T_2 that is close to P, the acoustic decoy is recognized, and the torpedo will navigate with a circular trajectory to locate the real objective.

To go as far as possible from the attacking torpedo, the warship will regulate to sail with a straight line when it comes to a position M_2, with a heading C_{M_2} perpendicular to C_{T_2}, the attacking direction of the torpedo when it recognizes the floating decoy as a fake objective.

Because the attacking torpedo attacks with a fan-shaped capture area, the acoustic decoy should be launched to a position P, which is the intersection point of the circle centered at M_1 and the edge of the capture sector, where the circle radius D_r is set as the maximum launching distance D_{rmax} to trap the torpedo as far as possible.

3.3.2. Trajectories of the warship and the torpedo

3.3.2.1. Sailing Trajectory of the Warship

The process of evasion manoeuvre can be divided into three consecutive stages.

- Stage 1: The warship sails along the circular track to point M_1 before the acoustic decoy is launched. Since the time delay t_d is known a priori, the coordinate of M_1 can be obtained by setting $t = t_{M_1}$ in Equation (21).
- Stage 2: After the launch of the acoustic decoy, the warship keeps the circular evasion until it arrives at the location M_2. Denoting the arriving time by t_{M_2}, we know

$$t_{M_2} = \frac{C_{M_2} - C_O}{w_m}, \tag{23}$$

  and the coordinate of $M_@$ can be confirmed by setting $t = t_{M_2}$ in Equation (21).
- Step 3: When the warship sails to the point M_2, it switches to a straight strategy, and the location is confirmed by

$$\begin{cases} x_{warship}(t) = x_{warship}(t_{M_2}) + v_{warship} \times \sin C_{M_2} \times (t - t_m), \\ y_{warship}(t) = y_{warship}(t_{M_2}) - v_{warship} \times \cos C_{M_2} \times (t - t_m). \end{cases} \tag{24}$$

3.3.2.2 Attacking Trajectory of the Torpedo

Meanwhile, the trajectory of the attacking torpedo can also be divided into three segments, where the first two segments are straight lines and the last segment is circular.

As illustrated in Figure 3, denote by T_0 the position where the attacking torpedo is detected by the sonar equipment of the warship, and the time moment is $t = 0$. At the first stage, the torpedo attacks in a straight trajectory with a heading C_{T_0}. While it comes to the position T_1 at the time moment t_{T_1}, the torpedo regulate its trajectory to attack the floating decoy, and it comes to the second attacking stage. As soon as it comes to the position T_2 at the time moment t_{T_1}, their distance is equal to a threshold D_L, the torpedo will recognize the decoy as a false target, and it switch to the third stage to perform a target search in a circular orbit.

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- At the first stage, the position of the torpedo at any time moment t ($0 \leq t \leq t_{T_1}$) is confirmed by Equations (6) and (7) with a heading C_{T_0}, which can be input manually or inherited from the Nash equilibrium solution of the game model.
- Assume that the floating decoy is captured by the torpedo when it comes to position $T_1 = (x_{torpedo}(t_{T_1}), y_{torpedo}(t_{T_1}))$ at time moment t_{T_1}. Then, it gradually regulates its heading to attack the fake target, and when it is located at the position $T_2 = (x_{torpedo}(t_{T_2}), y_{torpedo}(t_{T_2}))$ sufficiently close to the floating decoy, it recognizes the decoy as a fake target, and switch to a navigation mode with a circular trajectory. Denoting the headings of torpedo at positions T_1 and T_2 by C_{T_1} and C_{T_2}, respectively, we assume that its attacking trajectory is straight, confirmed by

$$\begin{cases} x_{torpedo}(t) = x_{torpedo}(t_{T_1}) + v_{torpedo} \times \sin C_{T_1} \times (t - t_{T_1}), \\ y_{torpedo}(t) = y_{torpedo}(t_{T_1}) + v_{torpedo} \times \cos C_{T_1} \times (t - t_{T_1}). \end{cases} \quad (t_{T_1} \leq t \leq t_{T_2}) \tag{25}$$

  If the acoustic decoy is deployed to the left of the center line of the sector, the angle increment is positive, and it is negative if the acoustic decoy is located to the right of the center line of the sector. As illustrated in Figure 3, the heading angle at stage 2 is conformed by

$$C_{T_2} = C_{T_1} = C_{T_0} \pm \frac{\lambda}{2}, \tag{26}$$

  where λ is the sector angle of the torpedo.
- While the torpedo is sufficiently close to the decoy, it is recognized as a false target, and the torpedo will navigate in a circular orbit to search the real target. Denoting the distance and the time moment by D_L and t_{T_2}, respectively, we can confirm the value of t_{T_2} by

$$t_{T_2} = \frac{L_{\max} - D_L}{v_{torpedo}} + t_{T_1}, \tag{27}$$

  where L_{max}, D_L and $v_{torpeso}$ are known performance parameters of the torpedo. Accordingly, the torpedo's position can be confirmed by

$$\begin{cases} x_{torpedo}(t) = x_{O_T} - v_{torpedo} \times \cos\left[C_{T_2} - 90 + w_t(t - t_{T_2})\right] \times (t - t_{T_2}), \\ y_{torpedo}(t) = y_{O_T} + v_{torpedo} \times \sin\left[C_{T_2} - 90 + w_t(t - t_{T_2})\right] \times (t - t_{T_2}), \end{cases} \quad (t \geq t_{T_2}) \tag{28}$$

  where the angular velocity is

$$w_t = \frac{v_{torpedo}}{r_t}, \tag{29}$$

  r_t is the radius of the circular trajectory.

3.4. Result of the Hydroacoustic Countermeasure

The success of a hydroacoustic countermeasure process can be confirmed if the target warship never enter the self-guiding sector before the range of the torpedo is exhausted. Denote the torpedo-target distance at the time moment t by d_t, and let B_θ be the angle between the attacking direction and the torpedo-target line. Then, we know

$$\begin{cases} d_t = \sqrt{\left(x_{torpedo}(t) - x_{warship}(t)\right)^2 + \left(y_{torpedo}(t) - y_{warship}(t)\right)^2}, \\ B_\theta = \left| C_T(t) + \arctan\left(\frac{y_{warship}(t) - y_{torpedo}(t)}{x_{warship}(t) - x_{torpedo}(t)}\right) - \frac{\pi}{2} \right|. \end{cases} \tag{30}$$

Here, $C_T(t)$ is the attacking heading of the torpedo.

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The total sailing time of the torpedo is

$$T_{overall} = S_{torpedo}/v_{torpedo}. \tag{31}$$

Then, a successful countermeasure process is achieved if it does not hold that

$$\begin{cases} d_t \leq L_{\max}, \\ B_\theta \leq \dfrac{\lambda}{2}, \end{cases} \quad \forall t \leq T_{overall}. \tag{32}$$

4. Numerical Simulation

In this section, we perform numerical simulations to validate the effectiveness of our proposed method. With the parameters presented in Table 1, we first simulation the adversarial process with given initial position of the attacking torpedo, and then, repeat the adversarial process with random samples of torpedo's initial position to test the success rate of the proposed adversarial strategy.

4.1. Simulation results for given alarm position of the attacking torpedo

Table 1. Parameter settings of simulation.

	Parameter Settings	Remarks
Warship	$v_{warship} = 18kn$	Sailing speed of the warship;
	$\theta_{max} = 35°$	Maximum avoidance angle;
	$t_{M_1} = 30s$	Time delay of order execution;
	$r_m = 200m$	Radius of the warship's circular trajectory;
	$w_m = 2rad/s$	Warship's angular velocity;
Torpedo	$v_{torpedo} = 50kn$	Attacking speed of the torpedo;
	$L_{max} = 1500m$	Maximum self-guiding distance of torpedo;
	$\lambda = 90°$	Self-guiding sector angle;
	$r_t = 300m$	Radius of the torpedo's circular trajectory;
	$w_t = 2rad/s$	Torpedo's angular velocity;
	$D_L = 300m$	Maximum recognizing distance of torpedo;
	$T_M = 160s$	Maximum movement time;
Other	$q = 30°$	Azimuth of the warship on which the torpedo is located;
	$d_0 = 28cab$	Alarm distance of torpedo's attack;
	$\sigma_x = 150m$	Standard deviation of x-coordinate position;
	$\sigma_y = 150m$	Standard deviation of x-coordinate position;
	$D_{rmax} = 3680$	Maximum launching distance of the acoustic decoy;

With an initial position of the torpedo, given by the alarm distance $d_0 = 28cab$ and the azimuth $q = 30^o$, we first simulate the adversarial process to demonstrate the practicability of the proposed method. Once the warship enters the self-guide sector of the torpedo, the warship is locked by the torpedo and the adversarial process fails.

4.1.1. The Nash-Equilibrium of the Matrix Game

Given a pair of game strategy (ϕ, θ), we can maximize $P_C(t)$ for time t to obtain the payoff (capture probability). Since the capture probabilities are very small when ϕ is greater than $40°$, we deleted them from the payoff matrix to reduce the size of the game model. Accordingly, we get a 9×15 payoff matrix presented in Table 2.

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Table 2. The payoff matrix of game model.

ϕ	θ														
	−35°	−30°	−25°	−20°	−15°	−10°	−5°	0°	5°	10°	15°	20°	25°	30°	35°
0°	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
5°	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
10°	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
15°	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
20°	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.94	0.82	0.59
25°	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.96	0.89	0.74	0.52	0.28	0.11	0.02
30°	0.99	0.99	0.98	0.96	0.92	0.85	0.72	0.55	0.35	0.17	0.06	0.01	0.00	0.00	0.00
35°	0.58	0.51	0.42	0.31	0.20	0.11	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
40°	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The linear programming problem (2) is addressed by the simplex method with an initialization strategy presented as Algorithm 1, and we get the Nash equilibrium solution, which demonstrates that the ship-side mixed strategy is

$$S_{warship} = (0.00, 0.00, 0.00, 0.00, 0.72, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.27, 0.00, 0.00, 0.01)$$

and the torpedo-side mixed strategy is

$$S_{torpedo} = (0.01, 0.69, 0.30, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00).$$

Accordingly, the corresponding equilibrium value is $V_G = 0.99$.

The torpedo-side mixed strategy shows that the torpedo would attack the surface ship in the directions characterized the advanced angles $0°$, $5°$ and $10°$, and the corresponding probabilities are 0.01, 0.69 and 0.30, respectively. Meanwhile, the ship-side mixed strategy suggests that the preferred evasion angles of surface ship are $-15°$, $20°$, and $35°$, and the corresponding probabilities are 0.72, 0.27 and 0.01, respectively. The Nash equilibrium solutions could lead to a preliminary decision for the hydroacoustic countermeasure scenario: the advance angle of attacking torpedo could range in $[0° - 15°]$, and the preferred evasion strategy could be turning right for about $-15°, 20°, 35°$, which could provide a suggestive action before the attacking direction of torpedo is addressed, and in turn improve the survival probabilities of surface ship for the emergent scenarios of hydroacoustic countermeasure.

4.1.2. Simulation of the Adversarial Process

By maximizing the expected payoff of the warship, the preferred evasion strategy is a left turn of about $35°$, that is, $C_O = 35^o$. For simulation of the countermeasure process, the preferred advance angle of the attacking torpedo is $5°$, and by (3) we know $C_{T_0} = 215°$.

- Due to the system reaction time, the warship is located at $M_1 = (181.26, 84.52)$ when the acoustic decoy is launched, and its sailing course is $C_{M_1} = 95°$; the torpedo is located at $T_1 = (2150.57, 3859.30)$, and its course is $C_{T_0} = 215°$; the angle between the direction of the torpedo's center line and the torpedo-warship line is $B_\theta = 7.5°$, and the torpedo-warship distance is $d_t = 4257.6(m)$. Accordingly, the acoustic decoys are launched to the place $P = (1873.65, 3352.28)$.
- When the torpedo detects that the acoustic decoy is false, the warship is located on $M_2 = (358.7, -69.73)$, and its course is $C_{M_2} = 257°$; the torpedo is located on $T_2 = (1820.72, 1962.49)$, and its course is $C_{T_2} = 170°$; the angle between the direction of the torpedo's center line and the torpedo-warship line is $B_\theta = 45.35°$,and the torpedo-warship distance is $d_t = 2785.78(m)$.
- When the torpedo's circular range is exhausted, the warship is located at $M_3 = (-228.53, -386.71)$, and its course is $C_{M_3} = -100°$; the torpedo is located at $T_3 = (2196.99, 1725.67)$, and its course is $C_{T_3} = 170°$; the angle between the direction of the torpedo's

center line and the torpedo-warship line is $B_\theta = 73.52°$,and the torpedo-warship distance is $d_t = 3216.41(m)$.

With the given parameters presented in Table 1, the warship never enter the self-guide sector of the torpedo before its sailing range is exhausted, and the adversarial result is reported as "success".

4.2. Simulation of Success Rate with Random Alarming Positions of the Attacking Torpedo

To demonstrate how the proposed adversarial strategy can accommodate varied antagonizing scenarios, we perform multiple simulations with randomly generated initial position of the attacking torpedo. Since the detecting distance depends on the performance of sonar equipments, we fix the alarm distance by setting $d_0 = 28cab$, and sample the values of q in the interval $[30°, 150°]$ with a step size $h = 0.5°$. Then, 240 independent simulations of the adversarial process are performed, and we compute the success rate of the proposed adversarial strategy. The numerical results demonstrate that with the parameters included in Table 1, the proposed adversarial strategy contributes to a 100% success rate, which indicates that it can well address the antagonizing scenarios investigated in this research.

5. Conclusions and Discussions

In order to improve the survival capability of surface warship in the scenarios of anti-torpedo hydroacoustic countermeasure, this paper proposes an intelligent decision strategy based on the matrix game theory and simulation of sailing trajectories. By addressing a matrix game model characterized by a payoff matrix of capture probabilities, our method can output a real-time Nash-equilibrium solution that reports the probabilities for all possible settings of the attacking-evasion direction pair. Accordingly, both possible attacking directions of the hostile torpedoes and preferred evasion directions of the warship can be obtained before the attacking heading is addressed by the sonar system. With this result, the defensive side can take actions before the sonar equipment identifies the attack intention of the hostile torpedoes, and the evasion strategy could be obtained by minimizing the expected capture probability for all possible choices.

Moreover, the attacking direction of torpedo can be confirmed by maximizing the expected capture probability, with which the countermeasure process can be simulated with the assistance of a launch strategy of decoy. Thanks to the wise evasion strategy arising from the game model, successful defensive countermeasure of the warship can be obtained with the given parameter settings, which demonstrates the proposed strategy is both practical and effective.

However, this work does not investigate how the antagonizing result is influenced by changes of both the wire-guided torpedo's attacking direction and the warship's velocity, which lead to countermeasure scenarios that are much more complicated and flexible. Accordingly, our future work will try to propose an improved adversarial model that can accommodate the varied attacking direction of torpedo with an adjustable navigation velocity of warship.

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