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Realism *and* Contextuality: Maximal Bell Violations with Non-Invasive Measurements

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Abstract: Predetermined physical properties cannot manifest at the same time if they are mutually exclusive. As part of a single system, they become observable at different times, in sequence. In this case, pairwise detections in a narrow window of coincidence allow for maximal Bell violations (S=4 for the CHSH protocol), without room for doubt about the local nature of the process. This shows that a deeper description of physical reality is possible, as suggested by Einstein, Podolsky and Rosen. Though, a more inclusive definition of Realism is required, in order to account for Bohr's Contextuality.

Keywords: quantum entanglement; EPR paradox; Bell's theorem; realism; contextuality

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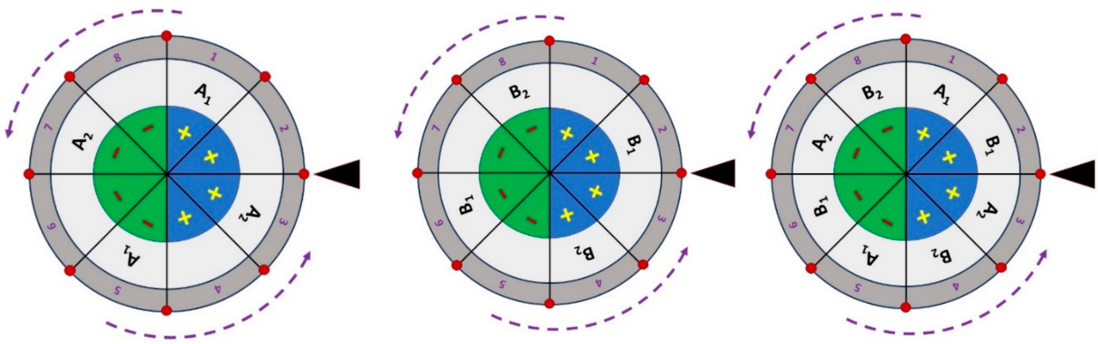
1. Introduction

Quantum theory describes contextual properties. Measurement outcomes are fundamentally conditional in many cases [1]. Does it follow that physical reality is incomplete? Einstein, Podolsky and Rosen (EPR) made a strong argument that *quantum theory* is incomplete (not the Universe). Some outcomes are predictable without direct measurement. Therefore, quantum properties do not require observer interventions to become real [2]. The question is how to interpret Contextuality in this case? EPR did not see it as "physically reasonable" and suggested that predetermined properties are real at the same time, outside of their context. This expectation was later refuted by numerous experiments [3]. Bell's theorem defined a clear boundary between jointly predetermined and contextual properties [4]. Yet, why do we have to choose between Realism and Contextuality? If we assume that physical entities change their qualities objectively from context to context, then predetermined properties can become real at different times. As will be shown below, staggered events produce maximal Bell violations, even if they are predetermined and even if measurements are non-invasive. Therefore, quantum properties can be real and contextual at the same time. Einstein and Bohr can shake hands in agreement.

Many scientists believe that quantum theory is ontologically non-classical. The theoretical and experimental evidence in this regard appears overwhelming. Yet, most claims of support for "quantum weirdness" have one thing in common: they only show that quantum theory is correct. This includes all the loophole-free violations of Bell's inequality, and associated discussions. In contrast, when experts study explicit ontological implications of quantum theory, they often arrive at unexpected conclusions. To list just a few examples, Khrennikov exposed a contradiction between quantum theory and probability theory, if claims of non-locality are taken for granted [5,6]; Cetto and collaborators demonstrated that the bipartite quantum correlation function is local [7,8]; Raymond-Robichaud provided a general proof that any non-signaling process can be predicted by a local realist theory [9,10]. Accordingly, there are good reasons to rethink the nature of quantum interactions. Yet, this raises the question: if Bell violations are possible by local means, how do they actually take place? As it turns out, the mechanism is quite straightforward. A closed chain of 8 event-cards on a spinning table is all that it takes. This solution was inspired by Gill's "wheel-of-fortune" wager [11,12], which is a variant of the quantum Randi challenge [13,14]. Quantum entanglement can be mimicked by making two copies of the wheel and by dividing the observables between them. This is a definitive demonstration that "quantum correlations", "super-quantum correlations" and "quantum monogamy" express classical behavior. "Entangled" qualities are physically independent from each other. Though, such a result is logically impossible for simultaneous properties of a single system. Bell's theorem is not contradicted by this result. Rather, this is used to confirm its limited domain of validity [15].

2. Result: Bel violations with local realism

Let us imagine a typical Bell experiment performed by Alice and Bob. When Alice measures one property A1, while Bob has a choice between B1 and B2, is Alice's event the same in both cases? In practical terms, the answer is hindered by an intuitive obstacle. If Alice's events are the same no matter what Bob does, and if Bob's events are the same no matter what Alice does, then it feels that all the events should be real at the same time. At the very least, it seems that such properties should be compatible with each other. In other words, it should be possible to write them on the same line of an event table. Therefore, Bell's theorem should be universally valid. Surprisingly, this is not the case. A possible solution is shown in Figure 1c below, where four "+" values and four "−" values are placed in sequence on a "wheel of fortune". The nuance here is that coincidences are counted for consecutive values, rather than simultaneous traits. Every Alice event is flanked by two alternative Bob events. Therefore, Alice's events are the same, regardless of Bob's choice of observation. The same is true for Bob. However, it is a necessary feature of this arrangement that B1 and B2 values cannot be adjacent to the same value of A1. Therefore, unusual correlations are possible: A1 is correlated with B1, B1 with A2, and A2 with B2, but B2 and A1 are anti-correlated. The direct consequence of this arrangement is a maximal Bell violation.



(a) (b) (c)

Figure 1. Bell experiment with correlated spinning tables. Two players, Alice and Bob, supervise the flow of events on synchronized "wheel-of-fortune" devices without interfering. The table surface is divided into 8 sectors, with 4 "+" events followed by 4 "−" events, as shown in image (c). Alice's table (a) has empty sectors for Bob's observables, while Bob's table (b) is missing Alice's observables. The tables are hardwired to move and stop in perfect unison. A "game host" can start the synchronous rotation, while a random mechanism forces the wheels to stop together at one of the 8 red dots. The players record the values of the observable adjacent sectors (A_1 or A_2 for Alice, and B_1 or B_2 for Bob). The goal is to analyze the correlation between coincident events, after playing the game long enough for statistical significance. Yet, the outcome is obviously predetermined. Of the 4 pairwise combinations – (A_1,B_1), (B_1,A_2), (A_2,B_2) and (B_2, A_1) – three are always correlated and one is always anti-correlated. The final result is a maximal violation of the CHSH inequality ($S=4$), as shown below. This confirms the equivalence between non-signaling and local phenomena in both classical and quantum physics.

To simulate quantum entanglement, the game requires two copies of the same wheel. Alice may get a version with Bob's values removed, as shown in Figure 1a, while Bob can receive a copy with Alice's values removed (Figure 1b). Alice and Bob can be arbitrarily close or far from each other. The only requirement is that the motion of their wheels is perfectly synchronized. Either player (or a moderator) can start the synchronous motion, while a random mechanism causes the tables to stop in a manner that cannot be predicted by the players. The wheels are also designed to end every turn on identical red dots (on the line between two adjacent sectors). Every time, Alice and Bob must record the nearest observable to the "winning" stop point, with the "+" or "−" value that happens to be in the same sector. After accumulating a sufficiently large number of events, they can reconcile their lists and determine the coefficients of correlation for each of the four possible pairs: (A1, B1), (A1, B2), (A2, B1) and (A2, B2).

A more efficient arrangement would be for Alice and Bob to record all of their events with exact timestamps, as they pass by the pointer on the right side of the table, continuously (i.e., without stopping for each measurement). After accumulating sufficiently large data sets, Alice and Bob can meet to reconcile their events. For example, if they had a fixed rate of rotation of 1 turn per second, they must post-select all the pairs of events that happened within 1/8 of a second from each other. In any version, the game can only have one result: a maximal violation of the CHSH inequality [16]. Barring loss of synchronicity between the tables, the order of events is fixed, and statistical anomalies are not possible even for low numbers of turns. Furthermore, if Alice's table is delayed by exactly half a period from the table of Bob, then all the coefficients of correlation switch polarity (from maximal correlation to maximal anti-correlation, and vice versa). Nonetheless, the final result is the same. The bottom line is that Bell violations are the correct theoretical outcome.

3. Method and explanations

The CHSH inequality [16] for this type of joint measurements is:

$$S = |E(A_1, B_1) - E(A_1, B_2) + E(A_2, B_1) + E(A_2, B_2)| \ , \tag{1}$$

where

$$E(X, Y) = \frac{P(X^+, Y^+) + P(X^-, Y^-) - P(X^+, Y^-) - P(X^-, Y^+)}{P(X^+, Y^+) + P(X^-, Y^-) + P(X^+, Y^-) + P(X^-, Y^+)} = \frac{P(\text{same}) - P(different)}{P(total)} \ . \tag{2}$$

As described in the game, coincidence windows allow exclusively for (+,+) and (-,-) observations between three variables. Hence,

$$E(A_1, B_1) = E(B_1, A_2) = E(A_2, B_2) = \frac{100 - 0}{100} = 1 \ . \tag{3}$$

In contrast, the remaining pair is always anticorrelated, since it can only display (-,+) and (+,-) coincidences:

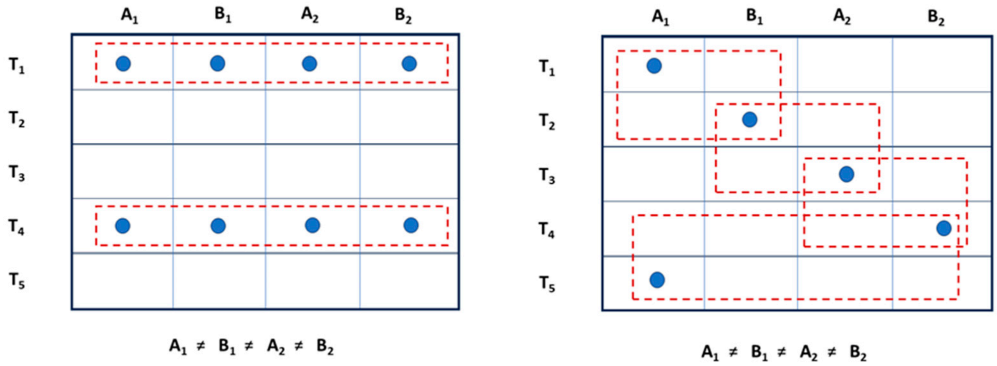
$$E(A_1, B_2) = \frac{0 - 100}{100} = -1. \tag{4}$$

Accordingly, the final tally is:

$$S = |1 - (-1) + 1 + 1| \tag{5}$$

$$S = 4. \tag{6}$$

The reason for this type of behavior is easy to grasp intuitively now. When several properties occur at the same time, they obey the so-called transitive rule. For example, if we have three observables X, Y, and Z, then two correlations automatically determine the third. If X is correlated with Y, and Y is correlated with Z, then X is correlated with Z as well. For illustration, consider a pile of shirts, such that "all the white shirts are made from cotton" and "all the cotton shirts are short-sleeved". It is logically necessary that "all the white shirts are short-sleeved" in the same group. It would be a contradiction to have even one white shirt with long sleeves. Accordingly, Bell violations are logically impossible because they entail contradictory properties for one and the same object (see Figure 2a below). In contrast, when we have mutually exclusive events that are staggered in time, the transitive rule does not apply. Instead, pairwise detections are restricted to adjacent events, preventing the coincidence of B1 and B2 with the same event for A1 (Figure 2b). Therefore, it is natural to have coefficients of correlation that otherwise would be incompatible. Notably, the essence of this relationship is captured by the properties of a single system. Twin systems are only needed to circumvent incompatibility for interpretive reasons.



(a) (b)

Figure 2. Simultaneous and staggered events can have different rules of pairwise combination. Some patterns of coincidence cannot violate Bell-type inequalities. As shown in table (a), simultaneous events are blocked in fixed groups. The same event is paired repeatedly with any other observable. Therefore, corresponding coefficients of correlation do not contradict each other. In contrast, staggered events allow for pairwise combinations outside of the group. As seen in table (b), it is impossible for four different events to be adjacent to each other. Instead, a narrow window of coincidence forces the emergence of incompatible pairings. On the other hand, if four different events are recorded at the same time, this pairwise inconsistency is erased, and pattern (b) switches back to pattern (a). Thus, "quantum correlations", "super-quantum correlations" and "quantum monogamy" are classical phenomena.

This conclusion resonates with an interesting open question in quantum theory. It is well-known that quantum behavior is "non-signaling", but can it be "non-signaling" and "non-local" at the same time? As shown by Popescu and Rohrlich [17], non-signaling events allow for maximal Bell violations (up to S=4), far beyond the known limits of quantum correlations. More recently, Raymond-Robichaud demonstrated the equivalence between local realism and non-signaling behavior [9,10]. In other words, any non-signaling phenomenon can be predicted by at least one local realist theory. The solution described above supports this conclusion, with an important improvement: Bell violations are produced by staggered events in the same universe, as opposed to simultaneous events in parallel universes. Therefore, "super-quantum" correlations are not just "local" and "realist". They are ordinary classical phenomena.

Furthermore, such effects are possible with pairwise joint events, but not necessarily with larger numbers of simultaneous detections. For example, it is possible to make four copies of this wheel-of-fortune, in order to detect all the properties at the same time. In this case, observers end up producing isolated groups of four events, just like in Figure 2a above. The cycle of staggered events is artificially broken into simultaneous bunches. As a result, all the events are forced to become compatible, erasing the effect of temporal mismatch between them. In this case, Bell violations are no longer possible. Therefore, "quantum monogamy" [18] is also an ordinary classical phenomenon.

4. Discussion

As shown above, maximal Bell violations are possible when the phase delay between any two measurements is constant, such that any combination of two observables can fall within the same coincidence window. Furthermore, positive (negative) values have to be grouped together, ensuring maximal coefficients of correlation between adjacent events. Finally, the largest violations are possible when there is a sudden shift from the positive to the negative domain (and vice versa), emphasized with different colors on the "wheel of fortune". There is an instructive difference between the solution presented above and the known features of actual photon polarization (and other spin-like measurements). First of all, there is a gradual increase in phase delay between optical components, as the angle between measurements is increased. Real Bell experiments maintained constant delays by choosing four measurements at equal angular intervals from each other (22.5°). Secondly, rotations of the plane of polarization introduce a gradual (and non-linear) loss of correlation, as dictated by Malus' Law. This explains the gap between the maximal Bell violations that are numerically possible and the ones that are theoretically possible for wave-functions. From the perspective of Bell's argument [4], the apparent problem was that quantum correlations were "too high". Yet, as suggested by Popescu and Rohrlich [17], the true physical puzzle was that they were "too low". In retrospect, there is nothing special or non-classical about Bell violations, be they "quantum" or "super-quantum". Indeed, it is misleading to describe these correlations as "quantum", because they are natural properties of classical beams as well [19].

A favorable condition for the discovery of this solution was the decision to treat "quantum nonlocality" and "quantum monogamy" as a single problem. It would not work to solve just one and hope to patch the other later. Under the influence of real details from loophole-free experiments, an earlier proposal was to consider possible correlations between simultaneous random events, modulo the probability of coincidence [20]. Intuitively, single events are more probable than double events, which are more likely than triple events, and so on. This explained the emergence of incompatible pairs of events, with quantum monogamy as a subset of this pattern. Unfortunately, such a solution could not work for classical projections, where Bell violations and "Bell monogamy" are both possible for ideal measurements, without down-selection or loss of energy. Indeed, as shown above, a better solution was waiting to be found. This shows that quantum experiments are not always the best way to get insights on quantum behavior. Sometimes, the best strategy is to study classical wave mechanics, while keeping in mind that it is still a work in progress.

Surprisingly, this "wheel-of-fortune" protocol does not seem to invalidate the security of quantum encryption, even though it is perfectly deterministic. For instance, if Alice and Bob are asked to produce one value at a time for independently chosen observables (without knowing which red dot is relevant), then they cannot reproduce the described effect. Every possible outcome is used twice in the game. As a result, there is unavoidable uncertainty as to which event (Alice's or Bob's) comes first in a requested combination, following a random point in time. The nuance is that the adequate coincidence window can be sliced in half, and the two players cannot know how to match their pairs in one shot. The two lists of events need to be brought together, in order to reconcile the correct pairings in adequate windows of coincidence. In a way, this is like looking at two beams of entangled quanta, without knowing which entities in Alice's channel are entangled with which entities in Bob's channel. Hence, the quantum Randi challenge cannot be met with the proposed computer code, even after learning that the mechanism is deterministic and local. This suggests that quantum communications can be secure as a result of uncertainty, rather than action at a distance.

5. Conclusions

Bell violations cannot be used as markers for locality. They only serve to differentiate between properties that can be simultaneously real (as part of a single system) and properties that cannot. As shown above, CHSH violations are classically possible for the full range of non-signaling correlations. Obviously, this conclusion does not contradict quantum theory. It only refutes the interpretations that distorted its physical essence. In retrospect, "quantum weirdness" is the result of a single error. Predetermined events were expected to be simultaneous in any "reasonable" description of reality. This assumption was explicit in the EPR paper and implicit in Bell's Theorem. Accordingly, a single correction is required to fix the problem, with all of its complex ramifications.

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References

1. Bohr, N. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Phys. Rev.* **48**, 696 (1935).
2. Einstein, A.; Podolsky, B.; Rosen, N. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Phys. Rev.* **47**, *777* (1935).
3. Rosenfeld, W.; et al. Event-Ready Bell Test Using Entangled Atoms Simultaneously Closing Detection and Locality Loopholes, *Phys. Rev. Lett.* **2017**, *119*, 010402.

4. Bell, J. S. *Speakable and unspeakable in quantum mechanics* (Cambridge UP, 1987).
5. Khrennikov, A. Bell-Boole Inequality: Nonlocality or Probabilistic Incompatibility of Random Variables? *Entropy*, **10**, 19 (2008).
6. Khrennikov, A. Get Rid of Nonlocality from Quantum Physics. *Entropy* **21**, 806 (2019).
7. Cetto, A. M.; Valdés-Hernández, A.; de la Peña, L. On the Spin Projection Operator and the Probabilistic Meaning of the Bipartite Correlation Function. *Found. Phys*. **50**, 27 (2020).
8. Cetto, A.M. Electron Spin Correlations: Probabilistic Description and Geometric Representation. *Entropy* **24**, 1439 (2022).
9. Raymond-Robichaud, P. The Equivalence of Local-Realistic and No-Signalling Theories. *Preprint*: arXiv:quant-ph/1710.01380. https://arxiv.org/abs/1710.01380
10. Raymond-Robichaud, P. *L'équivalence entre le local-réalisme et le principe de non-signalement*. PhD Thesis (Université de Montréal, 2017). https://papyrus.bib.umontreal.ca/xmlui/handle/1866/20497
11. Gill, R.D. The Bell Game Challenge. *Richard Gill Statistics Blog*: https://gill1109.com/2021/12/22/the-bell-game-challenge/ (2021).
12. Gill, R. D. Statistics, Causality and Bell's Theorem. *Statist. Sci.* **29**, 512 (2014).
13. Vongehr, S. Quantum Randi Challenge. Preprint: arXiv:1207.5294 [physics.hist-ph], https://arxiv.org/ftp/arxiv/papers/1207/1207.5294.pdf
14. Vongehr, S. Exploring Inequality Violations by Classical Hidden Variables Numerically. *Annals of Physics* **339**, 81 (2013).
15. Fine, A. Joint Distributions, Quantum Correlations, and Commuting Observables. *J. Math. Phys*. **23**, 1306 (1982).
16. Clauser, J. F.; Horne, M. A.; Shimony, A.; Holt, R. A. Proposed Experiment to Test Local Hidden-Variable Theories. *Phys. Rev. Lett.* **23**, 880 (1969).
17. Popescu, S.; Rohrlich, D. Quantum Nonlocality as an Axiom. *Found. Phys.* **24**, 379 (1994).
18. Dhar, H. S.; Pal, A. K.; Rakshit, D.; Sen (De), A.; Sen, U. "Monogamy of Quantum Correlations - A Review". In: F. Fanchini, D. Soares Pinto, and G. Adesso (eds), *Lectures on General Quantum Correlations and their Applications* (Springer, 2017).
19. Mardari, G. N. Experimental Counterexample to Bell's Locality Criterion. *Entropy* **24**, 1742 (2022).
20. Mardari, G. N. How to Erase Quantum Monogamy? *Quantum Rep* **3**, 53 (2021).