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Article

Symmetry and the Nanoscale: Advances in Analytical Modeling in the Perspective of Holistic Unification

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Abstract: Analytical modeling has symmetries and aesthetic-mathematical characteristics that distinguish it from numerical computation. Nanoscience plays an extremely important role in unification efforts that also include holistic aspects of reality. In this paper I present new discovered results about the complete analytical quantum relativistic form of the mean square deviation of position $R^2(t)$ related to a lately appeared Drude-Lorentz-like model (DS model), already performed at classical, quantum and relativistic level. The function $R^2(t)$ gives precise information about the distance crossed by carriers (electrons, ions, etc.) inside a nanostructure, considering both quantum effects and relativistic velocities. The model has a wide scale range of applicability; the nanoscale is considered in this paper, but it holds from sub-pico-level to macro-level because of the existence of a gauge factor, making it applicable to every oscillating processes in Nature. Examples of application and suggestions supplement the paper, as well as interesting developments to be studied related to the model and to one of the basic elements of a current unified holistic approach based on the vacuum energy.

Keywords: symmetry; nanoscale; mean square deviation of position; quantum-relativistic effects; analytical modeling; unification; holism; vacuum energy

1. Introduction

The reflection on many contemporary theories dealing with techno-science has underlined how the scientific theory normally precedes subjective and social conditions; the techno-scientific theory does not tend to deal with reality in a global way, but only with the relationship between man and reality, considering the concept of reality as limited by a scientific-phenomenological and non-holistic anthropocentrism.

A holistic view of reality is such if it goes beyond the study and imitation of some characteristics of a particular organism, involving the incorporation of the general characteristics of life in its design and the application of these characteristics on multiple spatial, temporal and organizational scales of global influence.

Science and technology continue to advance, tending towards a complete understanding of the structure and behavior of matter; the nanoscale plays an important role in this evolution, through the study of complex systems (multiscale space-time structures) up to the most complex system, the human brain [1].

In the first decades of the twenty-first century, a concentrated effort is bringing together nanotechnology, biotechnology, information technology and new technologies based on cognitive science. With proper attention to ethical issues and societal needs, the result can be a huge improvement in human capabilities, new products, social outcomes and improved quality of life.

The correlation between different scales is one of the focal points of complexity science, where the correlation between scales is established by analyzing the compromise among the dominant mechanisms [2–4]. Multiscale analytical methods present mathematical characteristics and properties not visible in numerical approaches, and are interesting approaches to complex systems linking reductionism and holism.

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Nanomaterials are extremely interesting as they provide the building blocks for the construction of nanobiomaterials, which can be both biocompatible and non-toxic. This trend has emerged in nanomedicine and is commonly associated with engineered nanoparticles in the context of bioimaging, drug delivery systems, diagnostic tools and therapeutic modalities [5].

A fruitful combination concerns nanomaterials with biophotonics, i.e. nanobiophotonics; the aspects of electromagnetic nature are related to quantum reality and to approaches of holistic nature, therefore the question concerns how complete is our global understanding of reality without the need of a holistic view of science [6].

The accurate control and handling of individual atoms have recently made possible the production of artificial structures of nanometer size, with new high interesting properties. These nanostructures represent the frontier of progress in materials innovative technology.

It is of primary significance to obtain small objects, but also to have the possibility of checking the nanometric dimensions with careful construction processes, in order to reveal special and peculiar properties.

Physical low-dimensional systems (one or two dimensions) attracted significant attention in recent years, both for the great number of experimental realizations (thanks to the use of nanotechnology and to techniques of laser light), and for the available theoretical techniques. To this class belong the mesoscopic systems, materials with strong electronic correlations and systems of atoms confined in optical lattices.

The reduction of at least one of the dimensions of physical systems to dimensionality of order or less than micron (μm) raised the fame of mesoscopic physics, in which microscopic quantum effects and macroscopic ones play an equal role. The great interest in this field is due to the recent experimental accessibility to sub-micro-dimensions, that brought to the birth of nanotechnology and nanoelectronics [7].

About electronics at mesoscopic scale, a new captivating class of phenomena has been brought to the forefront by the use of quantum physics as the electron spins for the information transport (spintronics) and the realization of macroscopic quantum states, opening interesting prospects in the technology of quantum computation.

About materials with strong electronic correlations, the concourse of different interactions and of different microscopic degrees of freedom originates interesting complex macroscopic behaviors [8].

The deep understanding of the conduct of these systems represents a challenge for fundamental physics, which has a huge technological potential and has already produced significant results. The research activity in this area will improve our understanding of the basic mechanisms underlying the specified materials and systems, suggesting new features, which lead to the creation of "new concept devices" and suggest the importance of the study of new materials.

It is therefore pivotal the theoretical study and mathematical modeling, searching to create new models which integrate the characteristics of the two pillars of modern physics, namely quantum mechanics and relativity. This paper deals with the quantum-relativistic extension of a model which describes analytically the charge transport, appeared in literature in the classical, quantum and relativistic versions [9–11]. In detail, the study of the mean square deviation of position will be done.

2. A recently appeared Drude-Lorentz-type Model

A recent theoretical analytical formulation is showing to fit very well with observed scientific data and also provides stimulating new predictions of several characteristics in nanostructures. It contains a gauge factor which allows its use for the study of dynamics processes from sub-pico-level to macro-level.

The model is based on the total Fourier transform of the frequency-dependent complex conductivity $\sigma(\omega)$, which can be inferred by the linear response theory [12–16].

The presence of a (0, +∞) integration in the Green-Kubo formula is an obstacle for the analytical inversion, but in this model it is overcome by evaluation of the integral on the whole time axis (-∞, +∞), considering the real part of the complex conductivity. The new introduced key idea is the

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integration on the entire time axis (-∞, +∞), not on the half time axis (0, +∞), as ordinarily considered in literature [17].

The model expands the Drude-Lorentz relation entailing the complex conductivity and rests on the reversal of the total Fourier transform in the complex plane considering the whole time axis:

$$<\vec{v}^{\alpha}(0)\,\vec{v}^{\beta}(t)>_T = \frac{k_B T V}{\pi e^2}\int_{-\infty}^{+\infty} d\omega\, \mathrm{Re}\,\sigma_{\beta\alpha}(\omega)\, e^{i\omega t}\,. \tag{1}$$

The mean square deviation of position of particles at equilibrium $R^2(t)$ is defined as:

$$R^2(t) = \left\langle [\vec{R}(t) - \vec{R}(0)]^2 \right\rangle, \tag{2}$$

where $R^2(t)$ is the position vector at the time *t*. Considering that it holds:

$$\vec{R}(t) - \vec{R}(0) = \int_0^t \vec{v}(t')\, dt'\,, \tag{3}$$

where $\vec{v}(t)$ is the velocity, and considering that for a homogeneous system in equilibrium the invariance property for time translation holds, it is possible to rewrite Equation (2) considering a transformation of coordinates relative to the region of integration. We get:

$$R^2(t) = 2\int_0^t dt'(t-t')\left\langle \vec{v}(t')\cdot\vec{v}(0)\right\rangle. \tag{4}$$

$R^2(t)$ can then be evaluated using the velocities correlation function.

Considering the Cauchy integration in the complex plane, $R^2(t)$ is precisely evaluated by the residue theorem. The analytical expressions of $\left\langle \vec{v}(t)\cdot\vec{v}(0)\right\rangle_T$, $R^2(t)$ and the diffusion coefficient $D(t)$ have been obtained for the classical, quantum and relativistic case [9–11].

3. Analytical Expression of the Mean Square Deviation of Position in the Quantum-Relativistic Case

a) About the *quantum behaviour*, I considered the consequence of a frequency-dependent electric field of the form $\vec{E} = \vec{E}_0\, e^{-i\omega t}$; going after the time-dependent perturbation theory, I considered the factor $e\,\vec{E}\cdot\vec{r}$ as perturbing potential (in this context $\vec{r}$ is the position vector of the particle).

The matrix elements of the dipole moment of the charge in the direction of the electric field, between the initial Φ_0 and the excited Φ_j states, are given by:

$$x_{j0} = \int \Phi_j^*\, e\, x\, \Phi_0\, dr. \tag{5}$$

Defining the oscillator strength of the *j*-th transition as:

$$f_j = \frac{2m}{\hbar^2}\sum_j \hbar\,\omega_j \left|x_{0j}\right|^2, \tag{6}$$

and keeping into account of the relation between permittivity and conductivity of the system:

$$\varepsilon(\omega) = 1 + 4\pi i\frac{\sigma(\omega)}{\omega} = 1 + \frac{4\pi N e^2}{m}\sum_j \frac{f_j}{\omega_j^2 - \omega^2 + i\,\omega\,\Gamma_j}, \tag{7}$$

we obtain the relation:

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$$\frac{i\,\sigma(\omega)}{\omega}=\frac{1}{4\pi}\sum_i\frac{\omega_{p_i}^2}{(\omega_i^2-\omega^2)-i\,\omega\,\Gamma_i}\,, \tag{8}$$

with:

$$\omega_{p_i}{}^2=\frac{4\pi\,N\,e^2}{m}\,f_i\,. \tag{9}$$

In previous equations it is:

- $\omega_i=(E_i-E_0)/\hbar$;
- E_i, E_0 energies of the excited and the ground states respectively;
- $\Gamma_i=1/\tau_i$ inverse of the decay time of every mode;
- N density of carriers.

As in the classical case, the analytical calculation formally brings to Equation (1), but with the fundamental difference that the real part of conductivity inside the integral is calculated at quantum level. The key factors including the quantum behaviour are the weights f_i. The relaxation times can be obtained by $\tau_i=1/\Gamma_i$ and weights f_i (Equation (9)). The calculation of *N* can be exactly obtained by Equation (9), considering that it holds: $\sum_i f_i=1$ [1].

b) About the *relativistic behaviour*, starting by the dynamics law:

$$\frac{d}{dt}(m_{part}\,\vec{v})=\sum_i\vec{F}_i\,, \tag{10}$$

I studied the relativistic variation of the mass along an *x*-axis in the fixed ground reference frame. Regarding the forces acting on the generic carrier, I considered an outer passive elastic-type force of the type $F_{el}=K\,x$, a passive friction-type force of the type $F_{fr}=\lambda\,\dot{x}$, depending by the velocity and with $\lambda=m_{part}/\tau$, and an outer oscillating electric field $E=e\,E_0\,e^{-i\omega t}$, considering solutions of the type $x=x_0\,e^{-i\omega t}$. The analytical calculation brings to three sets of results, in relation to the value of the quantity Δ, as in the classical and quantum case [7].

The expressions of $R^2(t)$ in the quantum (Q) and relativistic (R) cases are respectively [18]:

(Q) *Quantum case:*

Q1) Case $\Delta_{R_{quant}}>0$

$$R^2(t)=2\left(\frac{k_B\,T}{m^*}\right)\sum_i\Big(\frac{f_i}{\omega_i^2}\Big(-\frac{1}{\alpha_{iR}}\sin\left(\frac{\alpha_{iR}}{2}\frac{t}{\tau_i}\right)\exp\left(-\frac{t}{2\tau_i}\right)-\cos\left(\frac{\alpha_{iR}}{2}\frac{t}{\tau_i}\right)\exp\left(-\frac{t}{2\tau_i}\right)+1\Big)\Big), \tag{11}$$

with:

$$\alpha_{iR}=\sqrt{4\,\tau_i^2\,\omega_i^2-1} \tag{12}$$

$\in\Re^+$ (positive real numbers);

Q2) Case $\Delta_{I_{quant}}<0$

$$R^2(t)=4\left(\frac{k_B\,T}{m^*}\right)\sum_i\Big(f_i\,\tau_i^2\Big(\frac{1}{\alpha_{iI}\,(1+\alpha_{iI})}\exp\left(-\frac{(1+\alpha_{iI})}{2}\frac{t}{\tau_i}\right)-\frac{1}{\alpha_{iI}\,(1-\alpha_{iI})}\exp\left(-\frac{(1-\alpha_{iI})}{2}\frac{t}{\tau_i}\right)+\frac{2}{1-\alpha_{iI}^2}\Big)\Big), \tag{13}$$

with:

$$\alpha_{iI}=\sqrt{1-4\,\tau_i^2\,\omega_i^2} \tag{14}$$

$\in(0,1)\subset\Re$.

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m_0 is the rest mass, m^* the effective mass, K the Boltzmann's constant, T the system's temperature, ω_i and τ_i frequencies and decaying times of each mode.

(R) *Relativistic case*:

R1) Case $\Delta_{R_{rel}} > 0$

$$R^2(t) = 2\left(\frac{k_B T}{m_0}\right)\left(\frac{1}{\omega_0{}^2}\right)\left[-\frac{1}{\alpha_{R_{rel}}}\sin\left(\frac{\alpha_{R_{rel}}}{2\rho}\frac{t}{\tau}\right)\exp\left(-\frac{t}{2\tau\rho}\right) - \cos\left(\frac{\alpha_{R_{rel}}}{2\rho}\frac{t}{\tau}\right)\exp\left(-\frac{t}{2\tau\rho}\right) + 1\right], \quad (15)$$

with:

$$\alpha_{R_{rel}} = \sqrt{4\gamma\omega_0^2\tau^2 - 1} \quad (16)$$

$\in \Re^+$ (positive real numbers);

R2) Case $\Delta_{I_{rel}} < 0$

$$R^2(t) = 4\left(\frac{k_B T}{m_0}\right)\left(\tau^2\right)(\gamma)\left[\frac{1}{\alpha_{I_{rel}}\left(1+\alpha_{I_{rel}}\right)}\exp\left(-\frac{(1+\alpha_{I_{rel}})}{2\rho}\frac{t}{\tau}\right) - \frac{1}{\alpha_{I_{rel}}\left(1-\alpha_{I_{rel}}\right)}\exp\left(-\frac{(1-\alpha_{I_{rel}})}{2\rho}\frac{t}{\tau}\right) + \frac{2}{\left(1-\alpha_{I_{rel}}{}^2\right)}\right], \quad (17)$$

with:

$$\alpha_{I_{rel}} = \sqrt{1 - 4\gamma\omega_0^2\tau^2} \quad (18)$$

$\in (0,1) \subset \Re$.

It holds: $\alpha_{I_{rel}} = \sqrt{\Delta_{I_{rel}}}$, $\alpha_{R_{rel}} = \sqrt{\Delta_{R_{rel}}}$, $\gamma = 1\big/\sqrt{1-\beta^2}$, $\beta = v/c$, $\rho = 1 + \beta^2\gamma^2 = \gamma^2$.

With the procedure used for obtaining the expressions of the velocities correlation function in the quantum-relativistic case [18], the new results for the mean square deviation of position $R^2(t) = \left\langle [\vec{R}(t) - \vec{R}(0)]^2 \right\rangle$ have the following analytical form:

(Q-R) *Quantum-Relativistic case*:

Q-R1) Case $\Delta_{iR_{Q-R}} > 0$

$$R^2(t) = 2\left(\frac{k_B T}{m_0}\right)\sum_i\left[\left(\frac{f_i}{\omega_i{}^2}\right)\left(-\frac{1}{\alpha_{iR_{Q-R}}}\sin\left(\frac{\alpha_{iR_{Q-R}}}{2\rho}\frac{t}{\tau_i}\right)\exp\left(-\frac{t}{2\tau_i\rho}\right) - \cos\left(\frac{\alpha_{iR_{Q-R}}}{2\rho}\frac{t}{\tau_i}\right)\exp\left(-\frac{t}{2\tau_i\rho}\right) + 1\right)\right], \quad (19)$$

with:

$$\alpha_{iR_{Q-R}} = \sqrt{4\gamma\omega_i^2\tau_i{}^2 - 1} \quad (20)$$

$\in \Re^+$ (positive real numbers);

Q-R2) Case $\Delta_{iI_{Q-R}} < 0$

$$R^2(t) = 4\left(\frac{k_B T}{m_0}\right)(\gamma)\sum_i\left\{\left(f_i\tau_i{}^2\right)\left[\frac{1}{\alpha_{iI_{Q-R}}\left(1+\alpha_{iI_{Q-R}}\right)}\exp\left(-\frac{(1+\alpha_{iI_{Q-R}})}{2\rho}\frac{t}{\tau_i}\right) - \frac{1}{\alpha_{iI_{Q-R}}\left(1-\alpha_{iI_{Q-R}}\right)}\exp\left(-\frac{(1-\alpha_{iI_{Q-R}})}{2\rho}\frac{t}{\tau_i}\right) + \frac{2}{\left(1-\alpha^2_{iI_{Q-R}}\right)}\right]\right\}, \quad (21)$$

with:

$$\alpha_{iI_{Q-R}} = \sqrt{1 - 4\gamma\omega_i^2\tau_i{}^2} \quad (22)$$

$\in (0,1) \subset \Re$.

It holds: $\alpha_{iI_{Q-R}} = \sqrt{\Delta_{iI_{Q-R}}}$, $\alpha_{iR_{Q-R}} = \sqrt{\Delta_{iR_{Q-R}}}$, $\gamma = 1\big/\sqrt{1-\beta^2}$, $\beta = v/c$, $\rho = 1 + \beta^2\gamma^2 = \gamma^2$.

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4. Results, Discussion and Examples of Application

The achieved results in the classical case explain the extreme short times and elevated mobilities, with which charges diffuse in mesoporous systems, of broad interest in all systems involving photocatalysis and photovoltaics. The small times of few τ show facile charge diffusion inside the nanoparticles. The undescribed experimental fact of ultrashort injection of charge carriers, particularly in Grätzel's cells, can be referred to this phenomenon. Deviations by the Drude model become powerful in nanostructured materials, such as photoexcited TiO_2 nanoparticles, ZnO films, InP nanoparticles, semiconducting polymer molecules and carbon NTs [19–22].

The quantum and relativistic models provided newsworthy additional features and added novel ones. The quantum model considers the weight of varied modes with the own ω_i and τ_i values, the relativistic model meets extreme high velocities of carriers inside nanostructures. The quantum-relativistic case connects both characters.

As examples of application, Figure 1 illustrates the evolution of $R^2(t)$ vs time for the fixed value $\alpha_{R_{Q-R}}=10$ ($\Delta>0$) in the case of one only mode (not quantum behaviour) for ZnO ($m^*=0.24\,m_e$; $\tau=0.84\cdot10^{-13}\,s$) [20]; I considered as velocities of carriers $v=10^7\,cm/s$ (blue solid line), $v=10^{10}\,cm/s$ (red dashed line) and $v=2.5\cdot10^{10}\,cm/s$ (green dot-dashed line) (Table 1). From Equation (20) it follows: $\omega=5.98\cdot10^{13}\,s^{-1}$.

Table 1. Computed data for the three examined velocities of carriers.

v (cm/s)	β^2	$1/\rho$	γ
10^7	$0.11\cdot10^{-6}$	0.998	1.001
10^{10}	0.11	0.888	1.061
$2.5\cdot10^{10}$	0.69	0.31	1.796

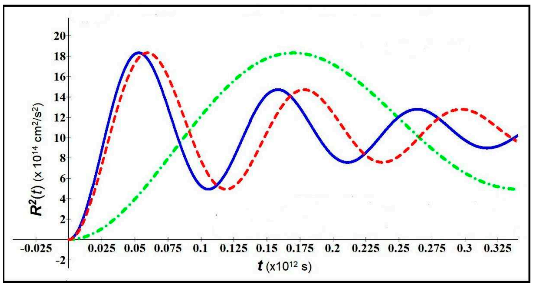

Figure 1. $R^2(t)$ vs t with $\alpha_{R_{Q-R}}=10$, $T=300\,K$; $v=10^7\,cm/s$ (blue solid line), $v=10^{10}\,cm/s$ (red dashed line) and $v=2.5\cdot10^{10}\,cm/s$ (green dot-dashed line).

In Figure 2, I considered the case $\Delta<0$ with $\alpha_{I_{Q-R}}=0.5$ in relation to one only mode (not quantum behaviour); from Equation (22) it follows: $\omega=0.52\cdot10^{13}\,s^{-1}$ [20].

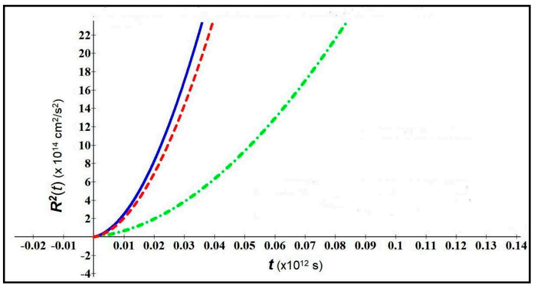

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Figure 2. $R^2(t)$ vs t with $\alpha_{I_{Q-R}}=0.5$, $T=300\,K$; $v=10^7\,cm/s$ (blue solid line), $v=10^{10}\,cm/s$ (red dashed line) and $v=2.5\cdot10^{10}\,cm/s$ (green dot-dashed line).

Then I considered experimental data by literature, from which three states were extracted; Table 2 summarizes the calculated used values. Data are related to single-walled carbon nanotube films at the temperature of 300 K [23–25].

Thanks to Equations (20) and (22), the values of α for the three states result: $\alpha_{I_1}=0.998$; $\alpha_{R_2}=8.57$; $\alpha_{R_3}=5.51$. These values must be calculated for each considered velocity.

Figures 3–5 show the behaviour of $R^2(t)$ vs time considering data of Table 2.

Table 2. Values of each examined state.

States	ω_i (x 10^{-12} Hz)	τ_i (x 10^{12} Hz)	f_i
1	6.59	0.0042	0.312
2	1166.01	0.0037	0.176
3	2000.05	0.0014	0.512

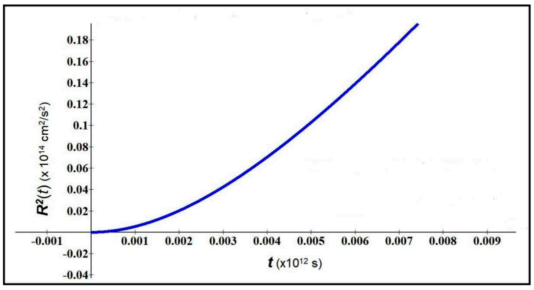

Figure 3. $R^2(t)$ vs t for the state 1 with data from Table 2.

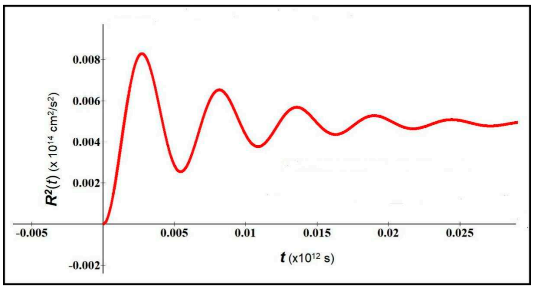

Figure 4. $R^2(t)$ vs t for the state 2 with data from Table 2.

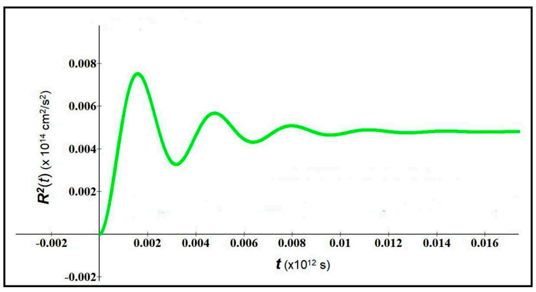

Figure 5. $R^2(t)$ vs t for the state 3 with data from Table 2.

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Figure 6 reproduces what we obtain considering the global state as sum of the previous three states.

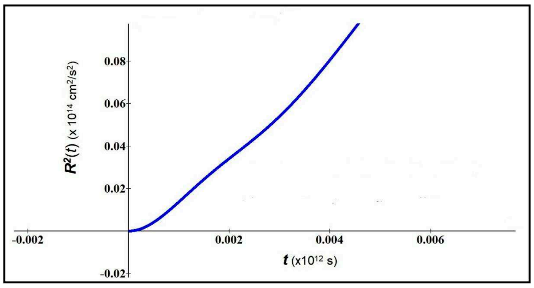

Figure 6. $R^2(t)$ vs t for the sum of the previous three states (data from Table 2).

In Figure 7 we see the general quantum-relativistic case, i.e. considering the quantum behaviour related to the presence of more than one state and the variation of velocity (increasing in our case).

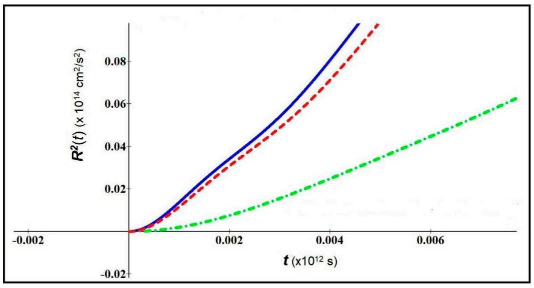

Figure 7. $R^2(t)$ vs t for a state given by the sum of the previous three states (data from Table 2), considering three different velocities ($v=10^7\ cm/s$ (blue solid line), $v=10^{10}\ cm/s$ (red dashed line) and $v=2.5\cdot10^{10}\ cm/s$ (green dot-dashed line)).

From these examples we conclude that it is possible to perform a careful and general quantum-relativistic study of the carrier transport in nanostructures.

The assumption of relativistic speeds and quantum features inside a nanostructure is an attractive present topic; quantum processes with extreme high speeds of carriers are pertinent for current science and technology and are elected by a key point for the future in theoretical and phenomenological (nano)physics.

What debated here can be thought like the starting point of a research pathway referred to physical suppositions, namely how scattering, ballistic transport, influence of temperature [24], size effect, etc., can impact on the studied phenomenon. A challenge arises for phenomenological scientists whose insight could fruitfully match these theoretical efforts.

About the present phenomenological investigation, two ways are suggested:

1) *Photon-Induced Near-Field Electron Microscopy*: this inspection technique connects the spatial resolution at the nanoscale of the electron microscopy with the femto-second temporal resolution of extreme fast light impulses; it can be used to check very fast occurrences present at very small length scales. A way for raising the electron-light interactions in very short intervals consists in enlarging the light field through two synchronized femto-second light impulses. Variations of the time delay among the exciting light impulses and the electronic imaging ones allow to obtain snapshots of the evanescent field whereas it evolves on femto-second intervals. The application of still shorter pulses can allow to keep trace of the extreme fast processes happening in photonic and plasmonic devices [26,27].

2) *Graphene based Plasmonics*: the not linear optical properties of a plasma expected in the relativistic movement of electrons subjected to a high laser field are of central significance for present research. Recently there was a fast progress in the sector of graphene plasmonics, considering the

graphene's special global properties. The application of graphene plasmonics will give stimulating results in the little exploited terahertz to mid-infrared regime.

Graphene and plasmonics strongly overlap, both for the inherent plasmons of graphene and for its mixture with noble metal nanostructures. Graphene based plasmonics can allow the creation of novel optical devices that work in various frequency ranges, from terahertz to visible light, associated to ultra-high speed [28–31].

5. Conclusions

In this paper I introduced new formulas relating to a quantum-relativistic model for the study of transport dynamics at the nanoscale. The obtained results allow the accurate study of transport dynamics contemplating quantum and relativistic effects. In particular, the quantum-relativistic formulas for the function $R^2(t)$ have been introduced.

At phenomenological level, the extreme fast carrier injection is suggested, in relation to manageable technical needs related to a raise of the wavelength of oscillations and a reduction of the amplitude of $R^2(t)$.

The quantum-relativistic version of the model is mathematically very elegant, because of its analytical formulation, and provides an attractive novelty, which can be fruitfully object of experimental investigation through time-resolved techniques, like TRTS, PINEM, Graphene based Plasmonics [32–34].

Keeping into account all parameters which can influence the system at chemical, physical, structural level, so as the intrinsic in the model ones, like the system's temperature *T*, the values of $\alpha_{iI_{Q-R}}$, $\alpha_{iR_{Q-R}}$, τ_i and ω_i, possible variations of the effective mass $m*$, changes of the chiral vector, the quantum weights of modes, the carrier density *N*, the speed of carriers, it is possible to carry out a careful tuning of $\langle \vec{v}(t) \cdot \vec{v}(0) \rangle_T$, $R^2(t)$, and $D(t)$.

From quantum physics we know that everything is energy, vibration, and this is connected with the energy of the vacuum. The attempts to define the structure and the global intrinsic properties of the space constitute a rigorous modern line of research.

In this direction, interesting results were obtained in relation to the ontological interpretation of quantum theory [35–37], with the Fourier transform playing a very important role; this last is one of the core elements of the described analytical model of transport dynamics at the nanometric level, which has a gauge factor allowing its use in a wider range than nanoscale only [38].

Other related concepts connected to this project concern how particular symmetries of $\sigma(\omega)$ are reflected on the solutions of the model, the possible involvement of advanced and delayed waves, the "intrinsic quantum ergonomics", the concept of "extended self".

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