

Technical Note

Not peer-reviewed version

Inverse Gas Dynamics Tables for the Isentropic Expansion of a Gas Due to Area Change and Flow Deflection

[Anil Lal Sadasivan](#)*

Posted Date: 24 July 2023

doi: 10.20944/preprints202307.1518.v1

Keywords: Inverse gas dynamics tables; Taylor's series; Leibnitz theorem; Prandtl-Meyer angle; Isentropic expansion



Preprints.org is a free multidiscipline platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This is an open access article distributed under the Creative Commons Attribution License which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Technical Note

Inverse Gas Dynamics Tables for the Isentropic Expansion of a Gas Due to Area Change and Flow Deflection

Anil Lal S

Associate Professor, Department of Mechanical Engineering, National Institute of Technology Sikkim, Namchi, District, Sikkim, India, anillal65@gmail.com (Responsible Author) ORCID: 0000-0001-8597-9905, +919447007935

Abstract: The available direct gas dynamic tables provide the area ratio and the Prandtl-Meyer(P-M) angle of isentropic expansion of gas as functions of the Mach number. But finding the value of the Mach number in terms of area ratio or P-M angle requires an inverse function. Currently, the Mach number is determined by either interpolation from the direct gas dynamics tables, or the application of non-linear equation solvers using numerical methods. Both methods are approximate. In this paper inverse functions for Mach number in the form of Taylor's series expansions are reported to develop the so-called inverse tables. The advantage of this approach is that the Mach number with a greater accuracy, referred to as nearly exact can be obtained by reading from the inverse table. In the presentation, the necessary equations and new tables that can be used by the scientific community will be disseminated. The data and computer programs are made available on GitHub through <https://github.com/anillals/Inverse-gas-dynamic-tables-and-codes>. A paper describing the complete mathematical steps in the derivation of the equations can be found at <https://rdcu.be/dgrlf>.

Keywords: inverse gas dynamics tables; Taylor's series; leibnitz theorem; Prandtl-Meyer angle; isentropic expansion

Introduction

Gas dynamic tables are available for finding the area ratio and the Prandtl-Meyer(P-M) angle of isentropic expansion as functions of the Mach number. But finding the value of the Mach number in terms of area ratio or P-M angle requires an inverse function. Currently, the Mach number is determined by either interpolation from the direct gas dynamics tables, or the application of non-linear equation solvers using numerical methods. Both methods are approximate. In this report, inverse functions for the Mach number are derived in the form of Taylor's series expansions and used to compute nearly exact values of the Mach number.

The Taylor's series expansion for Mach number as a function of the area ratio is

$$M(\varepsilon) = \sum_{k=0}^n F(k; \varepsilon_o, M_o)(\varepsilon - \varepsilon_o)^k \quad (1)$$

where ε stands for the area ratio or the P-M angle, (ε_o, M_o) denotes the coordinates of the center of expansion.

The Taylor's series expansion for x related to M as $x = \sqrt{M^2 - 1}$

$$x(v) = \sum_{k=0}^n F(k; v_o, M_o)(v - v_o)^k \quad (2)$$

where v represents the Prandtl-Meyer angle and (v_o, M_o) the coordinates of the center of expansion.

Equations (1) and (2) are explicit formulae that can be evaluated at high accuracy levels by a systematic procedure described in the next section on methods.

Methods

The method involves finding the Taylor series coefficients (Yadav, 2023)and a leaping series approach (Lal, 2022) for the summation. A brief explanation of the two steps is given in the following.

Taylor Series Coefficients

The equations for the area ratio $\varepsilon(M)$ and the Prandtl-Meyer angle $\nu(x)$ with definitions of all the involved variables can be found in (Yadav, 2023). These equations are nonlinear, which have been inverted in the form of equation(1) and (2), where $F(k)$ and $X(k)$ is the Taylor series coefficients. For these the original equations of $\varepsilon(M)$ and $\nu(x)$ are differentiated once to obtain first-order ordinary differential equations. Then the terms of the differential equations are expressed as Taylor series sums. During this process, the non-linear terms become convolution sums (Lal and Neeraj, 2014). Finally, recursive relations for the Taylor series coefficients are obtained by equating the coefficients on both sides. The relations can be found in equations (10)-(14) for the area ratio and equations (18)-(20) for the Prandtl-Meyer inverses in Yadav and Lal (2023).

Leaping Series Approach

A single series expanded with the center $\varepsilon_0 = 0$ will be converging only up to a certain value for ε . This is due to instability problems arising from truncation error. In other words, there is a radius of convergence for the expansion of both functions. This problem is overcome by using different series expansions developed having progressively shifted centers of expansion. This approach is called a leaping series method, which was reported by Lal and Milin (2022). The centers of expansions used for generating this inverse Tables 1–3 are available for downloading from (<https://github.com/anillals/Inverse-gas-dynamic-tables-and-codes>). The names of the files containing the data are sub-centers, sup-centers, and mp-center for the subsonic area ratio, supersonic area ratio, and Prandtl-Meyer function tables, respectively.

Table 1. Mach number as a function of area ratio in the isentropic expansion of subsonic airflow ($\gamma = 1.4$).

A/A*	M	A/A*	M	A/A*	M	A/A*	M
50.00	0.115750049049E-01	45.00	0.128613591854E-01	40.00	0.144694107077E-01	35.00	0.165371056078E-01
49.90	0.115982050497E-01	44.90	0.128900093452E-01	39.90	0.145056840441E-01	34.90	0.165845054890E-01
49.80	0.116214983904E-01	44.80	0.129187874458E-01	39.80	0.145421397278E-01	34.80	0.166321779186E-01
49.70	0.116448854898E-01	44.70	0.129476943463E-01	39.70	0.145787791375E-01	34.70	0.166801252546E-01
49.60	0.116683669150E-01	44.60	0.129767309134E-01	39.60	0.146156036657E-01	34.60	0.167283498821E-01
49.50	0.116919432380E-01	44.50	0.130058980216E-01	39.50	0.146526147191E-01	34.50	0.167768542139E-01
49.40	0.117156150353E-01	44.40	0.130351965532E-01	39.40	0.146898137188E-01	34.40	0.168256406911E-01
49.30	0.117393828879E-01	44.30	0.130646273987E-01	39.30	0.147272021000E-01	34.30	0.168747117830E-01
49.20	0.117632473819E-01	44.20	0.130941914565E-01	39.20	0.147647813131E-01	34.20	0.169240699879E-01
49.10	0.117872091078E-01	44.10	0.131238896331E-01	39.10	0.148025528228E-01	34.10	0.169737178336E-01
49.00	0.118112686613E-01	44.00	0.131537228434E-01	39.00	0.148405181092E-01	34.00	0.170236578774E-01
48.90	0.118354266426E-01	43.90	0.131836920105E-01	38.90	0.148786786675E-01	33.90	0.170738927070E-01
48.80	0.118596836570E-01	43.80	0.132137980661E-01	38.80	0.149170360083E-01	33.80	0.171244249406E-01
48.70	0.118840403148E-01	43.70	0.132440419503E-01	38.70	0.149555916579E-01	33.70	0.171752572276E-01
48.60	0.119084972314E-01	43.60	0.132744246118E-01	38.60	0.149943471584E-01	33.60	0.172263922491E-01
48.50	0.119330550270E-01	43.50	0.133049470081E-01	38.50	0.150333040679E-01	33.50	0.172778327179E-01
48.40	0.119577143272E-01	43.40	0.133356101056E-01	38.40	0.150724639608E-01	33.40	0.173295813795E-01
48.30	0.119824757626E-01	43.30	0.133664148794E-01	38.30	0.151118284279E-01	33.30	0.173816410126E-01
48.20	0.120073399693E-01	43.20	0.133973623140E-01	38.20	0.151513990766E-01	33.20	0.174340144291E-01
48.10	0.120323075882E-01	43.10	0.134284534027E-01	38.10	0.151911775314E-01	33.10	0.174867044750E-01
48.00	0.120573792661E-01	43.00	0.134596891481E-01	38.00	0.152311654337E-01	33.00	0.175397140310E-01
47.90	0.120825556549E-01	42.90	0.134910705624E-01	37.90	0.152713644424E-01	32.90	0.175930460126E-01
47.80	0.121078374120E-01	42.80	0.135225986669E-01	37.80	0.153117762338E-01	32.80	0.176467033710E-01
47.70	0.121332252003E-01	42.70	0.135542744928E-01	37.70	0.153524025021E-01	32.70	0.177006890937E-01

47.60	0.121587196883E-01	42.60	0.135860990809E-01	37.60	0.153932449595E-01	32.60	0.177550062046E-01
47.50	0.121843215501E-01	42.50	0.136180734816E-01	37.50	0.154343053367E-01	32.50	0.178096577650E-01
47.40	0.122100314657E-01	42.40	0.136501987554E-01	37.40	0.154755853824E-01	32.40	0.178646468740E-01
47.30	0.122358501205E-01	42.30	0.136824759729E-01	37.30	0.155170868647E-01	32.30	0.179199766693E-01
47.20	0.122617782059E-01	42.20	0.137149062147E-01	37.20	0.155588115702E-01	32.20	0.179756503272E-01
47.10	0.122878164192E-01	42.10	0.137474905716E-01	37.10	0.156007613050E-01	32.10	0.180316710641E-01
47.00	0.123139654636E-01	42.00	0.137802301452E-01	37.00	0.156429378948E-01	32.00	0.180880421362E-01
46.90	0.123402260483E-01	41.90	0.138131260471E-01	36.90	0.156853431851E-01	31.90	0.181447668409E-01
46.80	0.123665988886E-01	41.80	0.138461793998E-01	36.80	0.157279790413E-01	31.80	0.182018485170E-01
46.70	0.123930847058E-01	41.70	0.138793913368E-01	36.70	0.157708473493E-01	31.70	0.182592905454E-01
46.60	0.124196842276E-01	41.60	0.139127630020E-01	36.60	0.158139500155E-01	31.60	0.183170963501E-01
46.50	0.124463981877E-01	41.50	0.139462955507E-01	36.50	0.158572889674E-01	31.50	0.183752693982E-01
46.40	0.124732273265E-01	41.40	0.139799901492E-01	36.40	0.159008661536E-01	31.40	0.184338132013E-01
46.30	0.125001723904E-01	41.30	0.140138479754E-01	36.30	0.159446835440E-01	31.30	0.184927313159E-01
46.20	0.125272341324E-01	41.20	0.140478702181E-01	36.20	0.159887431307E-01	31.20	0.185520273442E-01
46.10	0.125544133123E-01	41.10	0.140820580782E-01	36.10	0.160330469275E-01	31.10	0.186117049346E-01
46.00	0.125817106961E-01	41.00	0.141164127681E-01	36.00	0.160775969707E-01	31.00	0.186717677827E-01
45.90	0.126091270567E-01	40.90	0.141509355120E-01	35.90	0.161223953195E-01	30.90	0.187322196320E-01
45.80	0.126366631738E-01	40.80	0.141856275462E-01	35.80	0.161674440559E-01	30.80	0.187930642747E-01
45.70	0.126643198338E-01	40.70	0.142204901191E-01	35.70	0.162127452853E-01	30.70	0.188543055524E-01
45.60	0.126920978300E-01	40.60	0.142555244916E-01	35.60	0.162583011368E-01	30.60	0.189159473571E-01
45.50	0.127199979628E-01	40.50	0.142907319368E-01	35.50	0.163041137635E-01	30.50	0.189779936316E-01
45.40	0.127480210396E-01	40.40	0.143261137405E-01	35.40	0.163501853429E-01	30.40	0.190404483711E-01
45.30	0.127761678749E-01	40.30	0.143616712014E-01	35.30	0.163965180771E-01	30.30	0.191033156232E-01
45.20	0.128044392903E-01	40.20	0.143974056309E-01	35.20	0.164431141932E-01	30.20	0.191665994893E-01
45.10	0.128328361150E-01	40.10	0.144333183537E-01	35.10	0.164899759440E-01	30.10	0.192303041253E-01

Table 2. Mach number as a function of area ratio in the isentropic expansion of supersonic airflow ($\gamma = 1.4$).

A/A^*	M	A/A^*	M	A/A^*	M	A/A^*	M
30.00	0.192944337428E-01	25.00	0.231555977356E-01	20.00	0.289497406882E-01	15.00	0.386147788141E-01
29.90	0.193589926096E-01	24.90	0.232486523403E-01	19.90	0.290953643247E-01	14.90	0.388744075543E-01
29.80	0.194239850508E-01	24.80	0.233424581192E-01	19.80	0.292424611632E-01	14.80	0.391375544245E-01
29.70	0.194894154500E-01	24.70	0.234370242078E-01	19.70	0.293910536845E-01	14.70	0.394042914891E-01
29.60	0.195552882501E-01	24.60	0.235323598904E-01	19.60	0.295411648292E-01	14.60	0.396746927961E-01
29.50	0.196216079540E-01	24.50	0.236284746032E-01	19.50	0.296928180101E-01	14.50	0.399488344456E-01
29.40	0.196883791264E-01	24.40	0.237253779373E-01	19.40	0.298460371237E-01	14.40	0.402267946622E-01
29.30	0.197556063940E-01	24.30	0.238230796420E-01	19.30	0.300008465632E-01	14.30	0.405086538690E-01
29.20	0.198232944473E-01	24.20	0.239215896280E-01	19.20	0.301572712314E-01	14.20	0.407944947657E-01
29.10	0.198914480409E-01	24.10	0.240209179706E-01	19.10	0.303153365540E-01	14.10	0.410844024101E-01
29.00	0.199600719955E-01	24.00	0.241210749137E-01	19.00	0.304750684935E-01	14.00	0.413784643024E-01
28.90	0.200291711984E-01	23.90	0.242220708725E-01	18.90	0.306364935634E-01	13.90	0.416767704738E-01
28.80	0.200987506048E-01	23.80	0.243239164380E-01	18.80	0.307996388429E-01	13.80	0.419794135789E-01
28.70	0.201688152392E-01	23.70	0.244266223798E-01	18.70	0.309645319921E-01	13.70	0.422864889916E-01
28.60	0.202393701964E-01	23.60	0.245301996508E-01	18.60	0.311312012675E-01	13.60	0.425980949063E-01
28.50	0.203104206428E-01	23.50	0.246346593903E-01	18.50	0.312996755386E-01	13.50	0.429143324428E-01
28.40	0.203819718175E-01	23.40	0.247400129287E-01	18.40	0.314699843042E-01	13.40	0.432353057561E-01
28.30	0.204540290340E-01	23.30	0.248462717910E-01	18.30	0.316421577097E-01	13.30	0.435611221512E-01
28.20	0.205265976811E-01	23.20	0.249534477014E-01	18.20	0.318162265653E-01	13.20	0.438918922035E-01
28.10	0.205996832245E-01	23.10	0.250615525875E-01	18.10	0.319922223640E-01	13.10	0.442277298845E-01
28.00	0.206732912080E-01	23.00	0.251705985848E-01	18.00	0.321701773008E-01	13.00	0.445687526928E-01
27.90	0.207474272550E-01	22.90	0.252805980409E-01	17.90	0.323501242923E-01	12.90	0.449150817927E-01
27.80	0.208220970699E-01	22.80	0.253915635207E-01	17.80	0.325320969974E-01	12.80	0.452668421575E-01
27.70	0.208973064397E-01	22.70	0.255035078109E-01	17.70	0.327161298377E-01	12.70	0.456241627213E-01
27.60	0.209730612353E-01	22.60	0.256164439248E-01	17.60	0.329022580200E-01	12.60	0.459871765368E-01

27.50	0.210493674129E-01	22.50	0.257303851078E-01	17.50	0.330905175581E-01	12.50	0.463560209417E-01
27.40	0.211262310162E-01	22.40	0.258453448420E-01	17.40	0.332809452967E-01	12.40	0.467308377328E-01
27.30	0.212036581773E-01	22.30	0.259613368520E-01	17.30	0.334735789352E-01	12.30	0.471117733482E-01
27.20	0.212816551185E-01	22.20	0.260783751100E-01	17.20	0.336684570526E-01	12.20	0.474989790597E-01
27.10	0.213602281543E-01	22.10	0.261964738419E-01	17.10	0.338656191334E-01	12.10	0.478926111739E-01
27.00	0.214393836928E-01	22.00	0.263156475325E-01	17.00	0.340651055943E-01	12.00	0.482928312435E-01
26.90	0.215191282375E-01	21.90	0.264359109318E-01	16.90	0.342669578123E-01	11.90	0.486998062898E-01
26.80	0.215994683890E-01	21.80	0.265572790611E-01	16.80	0.344712181526E-01	11.80	0.491137090366E-01
26.70	0.216804108472E-01	21.70	0.266797672192E-01	16.70	0.346779299990E-01	11.70	0.495347181555E-01
26.60	0.217619624125E-01	21.60	0.268033909887E-01	16.60	0.348871377844E-01	11.60	0.499630185251E-01
26.50	0.218441299885E-01	21.50	0.269281662426E-01	16.50	0.350988870229E-01	11.50	0.503988015029E-01
26.40	0.219269205834E-01	21.40	0.270541091516E-01	16.40	0.353132243428E-01	11.40	0.508422652122E-01
26.30	0.220103413119E-01	21.30	0.271812361905E-01	16.30	0.355301975208E-01	11.30	0.512936148444E-01
26.20	0.220943993978E-01	21.20	0.273095641457E-01	16.20	0.357498555179E-01	11.20	0.517530629772E-01
26.10	0.221791021756E-01	21.10	0.274391101225E-01	16.10	0.359722485162E-01	11.10	0.522208299110E-01
26.00	0.222644570929E-01	21.00	0.275698915528E-01	16.00	0.361974279571E-01	11.00	0.526971440230E-01
25.90	0.223504717125E-01	20.90	0.277019262029E-01	15.90	0.364254465815E-01	10.90	0.531822421416E-01
25.80	0.224371537146E-01	20.80	0.278352321814E-01	15.80	0.366563584705E-01	10.80	0.536763699416E-01
25.70	0.225245108990E-01	20.70	0.279698279479E-01	15.70	0.368902190889E-01	10.70	0.541797823620E-01
25.60	0.226125511877E-01	20.60	0.281057323210E-01	15.60	0.371270853294E-01	10.60	0.546927440473E-01
25.50	0.227012826271E-01	20.50	0.282429644876E-01	15.50	0.373670155591E-01	10.50	0.552155298151E-01
25.40	0.227907133905E-01	20.40	0.283815440116E-01	15.40	0.376100696673E-01	10.40	0.557484251503E-01
25.30	0.228808517808E-01	20.30	0.285214908433E-01	15.30	0.378563091158E-01	10.30	0.562917267289E-01
25.20	0.229717062325E-01	20.20	0.286628253291E-01	15.20	0.381057969909E-01	10.20	0.568457429727E-01
25.10	0.230632853152E-01	20.10	0.288055682213E-01	15.10	0.383585980572E-01	10.10	0.574107946376E-01
A/A*	M	A/A*	M	A/A*	M	A/A*	M
10.00	0.579872154380E-01	5.00	0.116688946830E+00	1.80	0.345015140724E+00	1.30	0.521962024020E+00
9.90	0.585753527087E-01	4.90	0.119111051791E+00	1.79	0.347258456711E+00	1.29	0.527865284142E+00
9.80	0.591755681089E-01	4.80	0.121636774556E+00	1.78	0.349533817414E+00	1.28	0.533942641803E+00
9.70	0.597882383695E-01	4.70	0.124272974476E+00	1.77	0.351841978038E+00	1.27	0.540204011642E+00
9.60	0.604137560886E-01	4.60	0.127027136168E+00	1.76	0.354183719753E+00	1.26	0.546660228951E+00
9.50	0.610525305776E-01	4.50	0.129907443164E+00	1.75	0.356559850904E+00	1.25	0.553323173642E+00
9.40	0.617049887620E-01	4.40	0.132922862352E+00	1.74	0.358971208297E+00	1.24	0.560205916632E+00
9.30	0.623715761417E-01	4.30	0.136083241091E+00	1.73	0.361418658554E+00	1.23	0.567322893812E+00
9.20	0.630527578145E-01	4.20	0.139399419321E+00	1.72	0.363903099556E+00	1.22	0.574690114243E+00
9.10	0.637490195685E-01	4.10	0.142883359463E+00	1.71	0.366425461978E+00	1.21	0.582325411219E+00
9.00	0.644608690485E-01	4.00	0.146548297536E+00	1.70	0.368986710920E+00	1.20	0.590248747542E+00
8.90	0.651888370025E-01	3.90	0.150408919711E+00	1.69	0.371587847634E+00	1.19	0.598482590061E+00
8.80	0.659334786143E-01	3.80	0.154481569503E+00	1.68	0.374229911378E+00	1.18	0.607052373774E+00
8.70	0.666953749303E-01	3.70	0.158784492083E+00	1.67	0.376913981381E+00	1.17	0.615987083215E+00
8.60	0.674751343873E-01	3.60	0.163338123840E+00	1.66	0.379641178945E+00	1.16	0.625319989599E+00
8.50	0.682733944502E-01	3.50	0.168165437439E+00	1.65	0.382412669683E+00	1.15	0.635089598118E+00
8.40	0.690908233694E-01	3.40	0.173292355421E+00	1.64	0.385229665922E+00	1.14	0.645340883777E+00
8.30	0.699281220669E-01	3.30	0.178748249081E+00	1.63	0.388093429265E+00	1.13	0.656126931275E+00
8.20	0.707860261644E-01	3.20	0.184566544264E+00	1.62	0.391005273338E+00	1.12	0.667511153524E+00
8.10	0.716653081643E-01	3.10	0.190785462420E+00	1.61	0.393966566737E+00	1.11	0.679570360334E+00
8.00	0.725667797969E-01	3.00	0.197448934284E+00	1.60	0.396978736190E+00	1.10	0.692399113730E+00
7.90	0.734912945519E-01	2.90	0.204607736198E+00	1.59	0.400043269956E+00	1.09	0.706116099177E+00
7.80	0.744397504066E-01	2.80	0.212320916707E+00	1.58	0.403161721480E+00	1.08	0.720873787954E+00
7.70	0.754130927741E-01	2.70	0.220657606279E+00	1.57	0.406335713331E+00	1.07	0.736873746440E+00
7.60	0.764123176875E-01	2.60	0.229699339547E+00	1.56	0.409566941452E+00	1.06	0.754392247235E+00
7.50	0.774384752472E-01	2.50	0.239543073464E+00	1.55	0.412857179746E+00	1.05	0.773826196026E+00
7.40	0.784926733533E-01	2.40	0.250305166374E+00	1.54	0.416208285039E+00	1.04	0.795783451065E+00
7.30	0.795760817535E-01	2.30	0.262126709113E+00	1.53	0.419622202452E+00	1.03	0.821285019976E+00
7.20	0.806899364370E-01	2.20	0.275180799672E+00	1.52	0.423100971224E+00	1.02	0.852316924071E+00
7.10	0.818355444102E-01	2.10	0.289682681205E+00	1.51	0.426646731049E+00	1.01	0.893950559348E+00

7.00	0.830142888933E-01	2.00	0.305904220424E+00	1.50	0.430261728952E+00		
6.90	0.842276349818E-01	1.99	0.307633207634E+00	1.49	0.433948326795E+00		
6.80	0.854771358223E-01	1.98	0.309383586049E+00	1.48	0.437709009463E+00		
6.70	0.867644393594E-01	1.97	0.311155381799E+00	1.47	0.441546393816E+00		
6.60	0.880912957144E-01	1.96	0.312949018065E+00	1.46	0.445463238493E+00		
6.50	0.894595652681E-01	1.95	0.314764929995E+00	1.45	0.449462454672E+00		
6.40	0.908712275271E-01	1.94	0.316603565165E+00	1.44	0.453547117903E+00		
6.30	0.923283908637E-01	1.93	0.318465384046E+00	1.43	0.457720481147E+00		
6.20	0.938333032332E-01	1.92	0.320350860503E+00	1.42	0.461985989168E+00		
6.10	0.953883639838E-01	1.91	0.322260482315E+00	1.41	0.466347294477E+00		
6.00	0.969961368938E-01	1.90	0.324194751726E+00	1.40	0.470808275003E+00		
5.90	0.986593645877E-01	1.89	0.326154186013E+00	1.39	0.475373053760E+00		
5.80	0.100380984505E+00	1.88	0.328139318093E+00	1.38	0.480046020771E+00		
5.70	0.102164146623E+00	1.87	0.330150697157E+00	1.37	0.484831857583E+00		
5.60	0.104012233160E+00	1.86	0.332188889338E+00	1.36	0.489735564764E+00		
5.50	0.105928880533E+00	1.85	0.334254478408E+00	1.35	0.494762492824E+00		
5.40	0.107918003865E+00	1.84	0.336348066524E+00	1.34	0.499918377113E+00		
5.30	0.109983824408E+00	1.83	0.338470275002E+00	1.33	0.505209377330E+00		
5.20	0.112130900298E+00	1.82	0.340621745138E+00	1.32	0.510642122427E+00		
5.10	0.114364161116E+00	1.81	0.342803139077E+00	1.31	0.516223761815E+00		
A/A*	M	A/A*	M	A/A*	M	A/A*	M
1.01	0.111264576745E+01	1.51	0.186263238401E+01	2.01	0.220282153484E+01	6.90	0.351731935522E+01
1.02	0.116091806305E+01	1.52	0.187104713759E+01	2.00	0.219719809156E+01	7.00	0.353279049092E+01
1.03	0.119849576567E+01	1.53	0.187937029077E+01	2.10	0.225189335993E+01	7.10	0.354805901419E+01
1.04	0.123049564764E+01	1.54	0.188760425431E+01	2.20	0.230344930284E+01	7.20	0.356313065569E+01
1.05	0.125890481371E+01	1.55	0.189575133388E+01	2.30	0.235224722256E+01	7.30	0.357801089992E+01
1.06	0.128474500411E+01	1.56	0.190381373653E+01	2.40	0.239859927417E+01	7.40	0.359270499934E+01
1.07	0.130862478866E+01	1.57	0.191179357657E+01	2.50	0.244276481495E+01	7.50	0.360721798743E+01
1.08	0.133094176708E+01	1.58	0.191969288112E+01	2.60	0.248496207264E+01	7.60	0.362155469085E+01
1.09	0.135197288913E+01	1.59	0.192751359509E+01	2.70	0.252537668215E+01	7.70	0.363571974079E+01
1.10	0.137192038217E+01	1.60	0.193525758594E+01	2.80	0.256416806538E+01	7.80	0.364971758348E+01
1.11	0.139093736279E+01	1.61	0.194292664793E+01	2.90	0.260147428791E+01	7.90	0.366355249008E+01
1.12	0.140914314663E+01	1.62	0.195052250625E+01	3.00	0.263741581650E+01	8.00	0.367722856584E+01
1.13	0.142663291123E+01	1.63	0.195804682067E+01	3.10	0.267209846771E+01	8.10	0.369074975874E+01
1.14	0.144348406776E+01	1.64	0.196550118908E+01	3.20	0.270561575159E+01	8.20	0.370411986745E+01
1.15	0.145976061673E+01	1.65	0.197288715074E+01	3.30	0.273805075554E+01	8.30	0.371734254892E+01
1.16	0.147551621716E+01	1.66	0.198020618925E+01	3.40	0.276947767441E+01	8.40	0.373042132543E+01
1.17	0.149079640548E+01	1.67	0.198745973544E+01	3.50	0.279996306465E+01	8.50	0.374335959119E+01
1.18	0.150564023582E+01	1.68	0.199464916996E+01	3.60	0.282956688111E+01	8.60	0.375616061854E+01
1.19	0.152008151619E+01	1.69	0.200177582580E+01	3.70	0.285834334077E+01	8.70	0.376882756383E+01
1.20	0.153414975618E+01	1.70	0.200884099055E+01	3.80	0.288634164739E+01	8.80	0.378136347288E+01
1.21	0.154787090447E+01	1.71	0.201584590862E+01	3.90	0.291360660351E+01	8.90	0.379377128615E+01
1.22	0.156126793050E+01	1.72	0.202279178323E+01	4.00	0.294017913040E+01	9.00	0.380605384362E+01
1.23	0.157436128892E+01	1.73	0.202967977836E+01	4.10	0.296609671229E+01	9.10	0.381821388935E+01
1.24	0.158716929446E+01	1.74	0.203651102055E+01	4.20	0.299139377799E+01	9.20	0.383025407582E+01
1.25	0.159970842750E+01	1.75	0.204328660055E+01	4.30	0.301610203011E+01	9.30	0.384217696803E+01
1.26	0.161199358539E+01	1.76	0.205000757495E+01	4.40	0.304025073068E+01	9.40	0.385398504731E+01
1.27	0.162403829097E+01	1.77	0.205667496769E+01	4.50	0.306386694965E+01	9.50	0.386568071501E+01
1.28	0.163585486678E+01	1.78	0.206328977144E+01	4.60	0.308697578230E+01	9.60	0.387726629592E+01
1.29	0.164745458173E+01	1.79	0.206985294897E+01	4.70	0.310960053992E+01	9.70	0.388874404154E+01
1.30	0.165884777527E+01	1.80	0.207636543439E+01	4.80	0.313176291777E+01	9.80	0.390011613317E+01
1.31	0.167004396333E+01	1.81	0.208282813438E+01	4.90	0.315348314350E+01	9.90	0.391138468482E+01
1.32	0.168105192900E+01	1.82	0.208924192928E+01	5.00	0.317478010868E+01	10.00	0.392255174599E+01
1.33	0.169187980073E+01	1.83	0.209560767420E+01	5.10	0.319567148582E+01	10.10	0.393361930432E+01
1.34	0.170253511998E+01	1.84	0.210192619999E+01	5.20	0.321617383265E+01	10.20	0.394458928807E+01
1.35	0.171302490013E+01	1.85	0.210819831426E+01	5.30	0.323630268537E+01	10.30	0.395546356849E+01

1.36	0.172335567784E+01	1.86	0.211442480223E+01	5.40	0.325607264225E+01	10.40	0.396624396208E+01
1.37	0.173353355818E+01	1.87	0.212060642766E+01	5.50	0.327549743879E+01	10.50	0.397693223273E+01
1.38	0.174356425432E+01	1.88	0.212674393360E+01	5.60	0.329459001539E+01	10.60	0.398753009375E+01
1.39	0.175345312261E+01	1.89	0.213283804327E+01	5.70	0.331336257844E+01	10.70	0.399803920983E+01
1.40	0.176320519369E+01	1.90	0.213888946070E+01	5.80	0.333182665558E+01	10.80	0.400846119886E+01
1.41	0.177282520014E+01	1.91	0.214489887152E+01	5.90	0.334999314591E+01	10.90	0.401879763373E+01
1.42	0.178231760117E+01	1.92	0.215086694358E+01	6.00	0.336787236542E+01	11.00	0.402905004394E+01
1.43	0.179168660472E+01	1.93	0.215679432763E+01	6.10	0.338547408852E+01	11.10	0.403921991727E+01
1.44	0.180093618723E+01	1.94	0.216268165793E+01	6.20	0.340280758579E+01	11.20	0.404930870125E+01
1.45	0.181007011154E+01	1.95	0.216852955280E+01	6.30	0.341988165849E+01	11.30	0.405931780467E+01
1.46	0.181909194292E+01	1.96	0.217433861522E+01	6.40	0.343670467021E+01	11.40	0.406924859890E+01
1.47	0.182800506361E+01	1.97	0.218010943334E+01	6.50	0.345328457575E+01	11.50	0.407910241928E+01
1.48	0.183681268604E+01	1.98	0.218584258100E+01	6.60	0.346962894772E+01	11.60	0.408888056635E+01
1.49	0.184551786472E+01	1.99	0.219153861819E+01	6.70	0.348574500098E+01	11.70	0.409858430711E+01
1.50	0.185412350722E+01	2.00	0.219719809156E+01	6.80	0.350163961513E+01	11.80	0.410821487611E+01

Table 3. Mach number as a function of Prandtl-Meyer angle ν in radians for the isentropic expansion of supersonic airflow ($\gamma = 1.4$).

A/A^*	M	A/A^*	M	A/A^*	M	A/A^*	M
11.90	0.411777347661E+01	15.20	0.439924222758E+01	18.50	0.463158566672E+01	21.80	0.483047466409E+01
12.00	0.412726128164E+01	15.30	0.440690109800E+01	18.60	0.463804667576E+01	21.90	0.483608583164E+01
12.10	0.413667943501E+01	15.40	0.441451643074E+01	18.70	0.464447773455E+01	22.00	0.484167504868E+01
12.20	0.414602905226E+01	15.50	0.442208876780E+01	18.80	0.465087914791E+01	22.10	0.484724250411E+01
12.30	0.415531122165E+01	15.60	0.442961864079E+01	18.90	0.465725121593E+01	22.20	0.485278838435E+01
12.40	0.416452700500E+01	15.70	0.443710657128E+01	19.00	0.466359423400E+01	22.30	0.485831287335E+01
12.50	0.417367743858E+01	15.80	0.444455307101E+01	19.10	0.466990849297E+01	22.40	0.486381615268E+01
12.60	0.418276353393E+01	15.90	0.445195864216E+01	19.20	0.467619427921E+01	22.50	0.486929840152E+01
12.70	0.419178627864E+01	16.00	0.445932377758E+01	19.30	0.468245187473E+01	22.60	0.487475979671E+01
12.80	0.420074663711E+01	16.10	0.446664896101E+01	19.40	0.468868155722E+01	22.70	0.488020051283E+01
12.90	0.420964555132E+01	16.20	0.447393466734E+01	19.50	0.469488360020E+01	22.80	0.488562072220E+01
13.00	0.421848394144E+01	16.30	0.448118136278E+01	19.60	0.470105827306E+01	22.90	0.489102059494E+01
13.10	0.422726270662E+01	16.40	0.448838950513E+01	19.70	0.470720584117E+01	23.00	0.489640029899E+01
13.20	0.423598272552E+01	16.50	0.449555954392E+01	19.80	0.471332656593E+01	23.10	0.490176000017E+01
13.30	0.424464485704E+01	16.60	0.450269192069E+01	19.90	0.471942070488E+01	23.20	0.490709986219E+01
13.40	0.425324994083E+01	16.70	0.450978706908E+01	20.00	0.472548851177E+01	23.30	0.491242004673E+01
13.50	0.426179879792E+01	16.80	0.451684541512E+01	20.10	0.473153023663E+01	23.40	0.491772071340E+01
13.60	0.427029223125E+01	16.90	0.452386737733E+01	20.20	0.473754612584E+01	23.50	0.492300201985E+01
13.70	0.427873102624E+01	17.00	0.453085336696E+01	20.30	0.474353642220E+01	23.60	0.492826412178E+01
13.80	0.428711595122E+01	17.10	0.453780378812E+01	20.40	0.474950136502E+01	23.70	0.493350717292E+01
13.90	0.429544775804E+01	17.20	0.454471903795E+01	20.50	0.475544119017E+01	23.80	0.493873132517E+01
14.00	0.430372718246E+01	17.30	0.455159950678E+01	20.60	0.476135613013E+01	23.90	0.494393672851E+01
14.10	0.431195494464E+01	17.40	0.455844557834E+01	20.70	0.476724641409E+01	24.00	0.494912353113E+01
14.20	0.432013174957E+01	17.50	0.456525762982E+01	20.80	0.477311226802E+01	24.10	0.495429187939E+01
14.30	0.432825828755E+01	17.60	0.457203603208E+01	20.90	0.477895391466E+01	24.20	0.495944191791E+01
14.40	0.433633523452E+01	17.70	0.457878114978E+01	21.00	0.478477157367E+01	24.30	0.496457378954E+01
14.50	0.434436325253E+01	17.80	0.458549334152E+01	21.10	0.479056546163E+01	24.40	0.496968763543E+01
14.60	0.435234299008E+01	17.90	0.459217295998E+01	21.20	0.479633579213E+01	24.50	0.497478359504E+01
14.70	0.436027508251E+01	18.00	0.459882035201E+01	21.30	0.480208277579E+01	24.60	0.497986180616E+01
14.80	0.436816015236E+01	18.10	0.460543585884E+01	21.40	0.480780662035E+01	24.70	0.498492240497E+01
14.90	0.437599880971E+01	18.20	0.461201981613E+01	21.50	0.481350753071E+01	24.80	0.498996552603E+01
15.00	0.438379165251E+01	18.30	0.461857255411E+01	21.60	0.481918570899E+01	24.90	0.499499130231E+01
15.10	0.439153926691E+01	18.40	0.462509439773E+01	21.70	0.482484135455E+01	25.00	0.499999986525E+01
N	M	ν	M	ν	M	ν	M
0.0100	0.105584247898E+01	0.3800	0.183642860002E+01	0.7500	0.267016729384E+01	1.1200	0.388058906134E+01
0.0200	0.108993081902E+01	0.3900	0.185635547156E+01	0.7600	0.269647365150E+01	1.1300	0.392245807763E+01

0.0300	0.111920540735E+01	0.4000	0.187645799726E+01	0.7700	0.272311803517E+01	1.1400	0.396497345004E+01
0.0400	0.114599945680E+01	0.4100	0.189666386750E+01	0.7800	0.274988380992E+01	1.1500	0.400823896472E+01
0.0500	0.117114481583E+01	0.4200	0.191697157709E+01	0.7900	0.277707974976E+01	1.1600	0.405181921629E+01
0.0600	0.119509742451E+01	0.4300	0.193737862231E+01	0.8000	0.280427297623E+01	1.1700	0.409635563553E+01
0.0700	0.121819373113E+01	0.4400	0.195788272790E+01	0.8100	0.283198945879E+01	1.1800	0.414167859013E+01
0.0800	0.124059348394E+01	0.4500	0.197840438454E+01	0.8200	0.286000853237E+01	1.1900	0.418788995971E+01
0.0900	0.126240168393E+01	0.4600	0.199917103275E+01	0.8300	0.288832201771E+01	1.2000	0.423468338889E+01
0.1000	0.128380178306E+01	0.4700	0.202002723956E+01	0.8400	0.291703145784E+01	1.2100	0.428242797203E+01
0.1100	0.130484996388E+01	0.4800	0.204096936574E+01	0.8500	0.294590738724E+01	1.2200	0.433039648610E+01
0.1200	0.132555289507E+01	0.4900	0.206207440063E+01	0.8600	0.297504971314E+01	1.2300	0.437978543465E+01
0.1300	0.134605088383E+01	0.5000	0.208334041910E+01	0.8700	0.300467255161E+01	1.2400	0.443000432200E+01
0.1400	0.136630190114E+01	0.5100	0.210460091968E+01	0.8800	0.303465491206E+01	1.2500	0.448100381745E+01
0.1500	0.138635335338E+01	0.5200	0.212618162734E+01	0.8900	0.306498782909E+01	1.2600	0.453301600403E+01
0.1600	0.140625404120E+01	0.5300	0.214783231696E+01	0.9000	0.309565992580E+01	1.2700	0.458599164360E+01
0.1700	0.142605439906E+01	0.5400	0.21696330465E+01	0.9100	0.312665954387E+01	1.2800	0.463945276460E+01
0.1800	0.144575554104E+01	0.5500	0.219158107369E+01	0.9200	0.315785746713E+01	1.2900	0.469433598682E+01
0.1900	0.146535662692E+01	0.5600	0.221375891935E+01	0.9300	0.318958993659E+01	1.3000	0.475030187712E+01
0.2000	0.148490947952E+01	0.5700	0.223590284432E+01	0.9400	0.322184789740E+01	1.3100	0.480729020106E+01
0.2100	0.150430488753E+01	0.5800	0.225835787020E+01	0.9500	0.325438271241E+01	1.3200	0.486538126467E+01
0.2200	0.152375779606E+01	0.5900	0.228103416183E+01	0.9600	0.328741727330E+01	1.3300	0.492465347649E+01
0.2300	0.154315906037E+01	0.6000	0.230384056002E+01	0.9700	0.332093960071E+01	1.3400	0.498459953639E+01
0.2400	0.156256339043E+01	0.6100	0.232686258659E+01	0.9800	0.335457208484E+01	1.3500	0.504616365800E+01
0.2500	0.158191201082E+01	0.6200	0.235000578645E+01	0.9900	0.338890311200E+01	1.3600	0.510912644682E+01
0.2600	0.160131908897E+01	0.6300	0.237326340176E+01	1.0000	0.342367632142E+01	1.3700	0.517326221780E+01
0.2700	0.162060517244E+01	0.6400	0.239681654858E+01	1.0100	0.345887418362E+01	1.3800	0.523877927303E+01
0.2800	0.164000702286E+01	0.6500	0.242057007694E+01	1.0200	0.349460289415E+01	1.3900	0.530558845245E+01
0.2900	0.165946524845E+01	0.6600	0.244461403245E+01	1.0300	0.353084530981E+01	1.4000	0.537344505034E+01
0.3000	0.167891637078E+01	0.6700	0.246875447154E+01	1.0400	0.356720675752E+01	1.4100	0.544314520302E+01
0.3100	0.169842066141E+01	0.6800	0.249317685978E+01	1.0500	0.360454428179E+01	1.4200	0.551458866441E+01
0.3200	0.171797714381E+01	0.6900	0.251768253768E+01	1.0600	0.364221096997E+01	1.4300	0.558751539514E+01
0.3300	0.173745189500E+01	0.7000	0.254246043050E+01	1.0700	0.368056483775E+01	1.4400	0.566196006641E+01
0.3400	0.175710475846E+01	0.7100	0.256760564154E+01	1.0800	0.371945867045E+01	1.4500	0.573825719546E+01
0.3500	0.177687156150E+01	0.7200	0.259291562347E+01	1.0900	0.375886780220E+01	1.4600	0.581566756906E+01
0.3600	0.179661437154E+01	0.7300	0.261848415983E+01	1.1000	0.379863765665E+01	1.4700	0.589542751890E+01
0.3700	0.181646703179E+01	0.7400	0.264410089156E+01	1.1100	0.383926282554E+01	1.4800	0.597724404313E+01

Findings and Discussion

The computer programs sub-table.f90, sup-table.f90, and pm-table.f90 in (<https://github.com/anillals/Inverse-gas-dynamic-tables-and-codes>) are used to create the inverse tables presented in Tables (1)-(3) at the end of this paper. For the area ratio inverse 200 series expansions with a summation of 100 terms are used. The Mean Square Error (MSE) of the reported table of subsonic and supersonic area ratio functions are 1.41×10^{-13} and 1.48×10^{-18} , respectively. The reporting Prandtl-Meyer function inverse table is developed using the 50000 series with summing of 10 terms with an MSE of only 5.31×10^{-10} . Due to the very low values of MSE, we call the inverse tables reported in this work as nearly exact.

Conclusion and Recommendations

This paper reports inverse tables for the Mach number as a function of area ratio Tables 1 and 2 and Prandtl-Meyer angle in Table 3, for expansion of a diatomic gas with $\gamma = 1.4$. These tables help the students and researchers to find the exact values of Mach numbers. The computer programs deposited in (<https://github.com/anillals/Inverse-gas-dynamic-tables-and-codes>) can be downloaded and easily incorporated by researchers in their codes. The advantages of this new approach are low computational effort requirement and better accuracy.

Thanks and Information Note

The author wishes to express his sincere thanks to National Institute of Technology Sikkim for providing the opportunity for conducting this work. I also acknowledge the help received from Mr. Mannu Yadav, Research Scholar, NIT Sikkim in preparing this paper

References

- Yadav, M., & Sadasivan, A. L. (2023). Taylor series solutions for area ratio and Prandtl–Meyer inverses using differential transform method. *International Journal of Advances in Engineering Sciences and Applied Mathematics*, 1-5.
- Lal S, A., & Milin, M. (2022). Accurate benchmark results of Blasius boundary layer problem using a leaping Taylors series that converges for all real values. *arXiv e-prints*, arXiv-2204
- Anil Lal, S., & Neeraj Paul, M. (2014). An accurate taylors series solution with high radius of convergence for the Blasius function and parameters of asymptotic variation. *Journal of Applied Fluid Mechanics*, 7(4), 557-564.