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Article

Hotel Comment Emotion Classification Based on the MF-DFA and Partial Differential Equation Classifier

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Abstract: In this paper, we propose a partial differential equation model based on phase-field for sentiment analysis in the field of hotel comment texts. The comment texts are converted into word vectors using the Word2Vec tool, and then we utilize the multifractal detrended fluctuation analysis (MF-DFA) model to extract the generalized Hurst exponent of the word vector time series to achieve dimensionality reduction of the word vector data. The dimensionality reduced data is represented in a two-dimensional computational domain, and the modified Allen-Cahn (AC) function is used to evolve the phase values of the data to obtain a stable nonlinear boundary, thereby achieving automatic classification of hotel comment texts. The experimental results show that the proposed method can effectively classify positive and negative samples and achieve excellent results in classification indicators. We compared our proposed classifier with traditional machine learning models and the results indicate that our method possesses a better performance.

Keywords: emotion classification; MF-DFA; Hurst exponent; Allen-Cahn

1. Introduction

Natural Language Processing (NLP) is an important direction in computer science and artificial intelligence, aiming to explore various theories and methods that can realize effective communication between humans and computers in natural language [1,2]. NLP has been used by many scholars to excavate emotional bias in texts, especially company press releases and consumer online comments [3–6]. For the detailed development of NLP, please refer to the review [7,8]. Online reviews provide the customers' feelings to the companies and other customers. In order to obtain valuable knowledge about customers, it is significant to collect and analyze the online reviews. Sann and Lai [9] adopted NLP to the hotel complaints about the synthesize specific service failure items, which can help hotels improve service. Jiao and Qu [10] proposed a computerized method which can extract Kansei knowledge from online reviews. Sharma et al. [11] exhibited a hybrid 3-layer NLP framework based on genetic algorithm and ontology, which owned excellent performance of opinion mining patent. Guerrero-Rodriguez et al. [12] proved that online travel reviews can help tourists determine which places are positive and negative. As well, the double language representation model based on NLP has developed rapidly. However, the model can not handle the absence of information. Therefore, a concept of interval-probabilistic two-hierarchy language set combining EDAS was proposed by Wang et al. [13].

Multifractal theories have strong nonlinear processing ability, which has attracted much attention. Therefore, multifractal theories has been widely adopted in the analysis of markets and prices in economics and finance [14,15]. The complexity and uncertainty of the market can be obtained by

analyzing the multifractal characteristics of financial time series. Meanwhile, multifractal theories can be used to investigate the volatility of stock prices and risk management, which can help develop effective risk management strategies [16,17]. In addition, multifractal theory owns excellent dimensionality reduction capability, which has been widely applied to feature extraction [18,19]. In general, Hurst exponent is used to characterize the multifractal features. As well, multifractal method is used in conjunction with other methods to improve classification accuracy. Hurst exponent, Lempel-Ziv information and Shannon entropy were combined as feature vectors into the support vector machine to determine normal and noisy heart sounds [20]. Abbasi et al. [21] extracted Hurst exponent and autoregressive moving average features from EEG signals and put the feature vector into long short-term memory classifier, the accuracy of which was high.

Phase field model is a model in materials science and physics that describes the relationship between the microstructure and macroscopic behavior of a material, and is a mathematical model that can be used to solve the two-phase interface. In 1979, Allen and Cahn first applied the Allen-Cahn (AC) equation in [22] to describe the process of phase separation in a binary alloy, and it has been one of the most widely studied phase field models. In the field of volume reconstruction, Li et al. [23] developed a computational method for weighted 3D volume reconstruction from a series of slice data based on the modified AC equation with a fidelity term. Liu et al. [24] proposed a novel variational model by integrating the AC term with a local binary fitting energy term, aiming to segment images with intensity inhomogeneity and noise. In terms of theoretical research, to ensure unconditional stability of proposed AC equation with logarithmic free energy, Park et al. [25] employed interpolation and finite difference methods to solve the splitting terms. Choi and Kim [26] introduced a novel conservative AC equation with new Lagrange multipliers, and provided corresponding numerical methods that maintain the maximum principle and ensure unconditional stability. Zhang and Yang [27] presented a numerical method for solving the spatial fractional Allen-Cahn equation. Their approach involved utilizing the Crank-Nicolson method for time discretization and employing the second-order weighted and shifted Grünwald difference formula for spatial discretization. Based on the characteristics of phase field phase separation, phase field model can form a interface like hyperplane and the exciting thing is that the interface is nonlinear. Therefore, based on phase field model, novel classifiers were constructed by Wang et al. [28,29], which owned excellent classification performance.

The remainder of the paper is structured as follows. We introduce the methodology in Section 2. Section 3 present the data information. Section 4 contains the empirical results. Section 5 reports the conclusion.

2. Methodology

This study proposes a hotel comment sentiment analysis model in Figure 1 based on the characteristics of hotel comment text. The proposed algorithm mainly includes the modules such as the preprocessing, word vector acquisition, word vector feature extraction, and phase-field classification. Subsequently, we describe the two main methods used in this article.

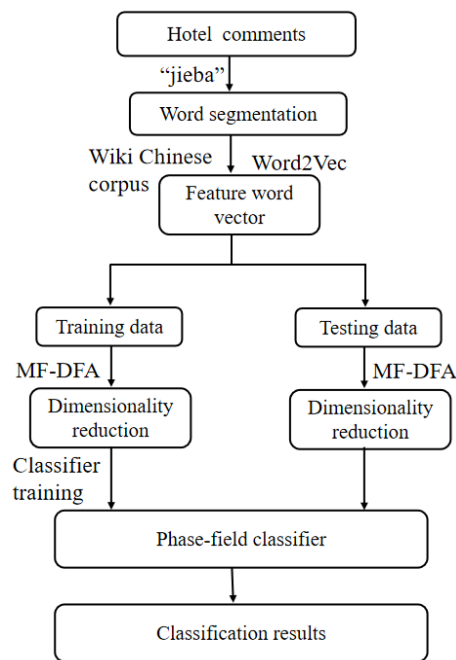


Figure 1. Flowchart of hotel comment emotional analysis model.

2.1. MF-DFA

MF-DFA is adopted to explore the fractal characteristic of the time series, which has been widely applied in finance [30,31], medicine [32,33], etc. The detailed calculation about MF-DFA is presented as follows.

Firstly, construct a mean-removed sum series $T(i)$ for the time series $\{t_k\}$, the length of which is N :

$$T(i) = \sum_{k=1}^i (t_k - \bar{t}), i = 1, 2, \dots, N. \quad (1)$$

where \bar{t} is the mean of the series $\{t_k\}$.

Next, divide the new series $T(i)$ into N disjoint intervals of length s , which aims to change the time scale. Here, N is rounded to N/s . In order to reduce the loss of the information in the process of dividing the sum series $T(i)$, $T(i)$ needs to be divided once according to i from small to large and from large to small, so that a total of $2N_s$ intervals can be obtained.

The following step is performing a polynomial fitting of the series between each interval and the polynomial fitting adopted is the least square method, designed to remove the trend of the series segment:

$$y_v(i) = \alpha_1 i^m + \alpha_2 i^{m-1} + \dots + \alpha_m i + \alpha_{m+1}, i = 1, 2, \dots, s; m = i, 2, \dots \quad (2)$$

where v is the number of the points in each interval.

Then, calculate the mean square error $f^2(s, v)$ of the each interval. When $v = 1, 2, \dots, N_s$,

$$f^2(s, v) = \frac{1}{s} \sum_{i=1}^s \{T[(v-1)s + i] - y_v(i)\}^2. \quad (3)$$

When $v = N_{s+1}, N_{s+2}, \dots, 2N_s$,

$$f^2(s, v) = \frac{1}{s} \sum_{i=1}^s \{T[N - (v - N_s)s + i] - y_v(i)\}^2. \quad (4)$$

Average the $f^2(s, v)$ obtained from all the segment and the q -th wave function F can be obtained as follows:

$$F(s) = \begin{cases} \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [f^2(s, v)]^{q/2} \right\}^{1/q}, & q \neq 0, \\ \exp \frac{1}{2N_s} \sum_{v=1}^{2N_s} \ln[f^2(s, v)], & q = 0. \end{cases} \quad (5)$$

$F(s)$ is the function of the dividing length s and the fractal order q . Here, $F(s) \propto H(q)$, where $H(q)$ is called the generalized Hurst exponents. When q equals 2, $H(q)$, that is, $H(2)$ is the standard Hurst exponent and $F(s)$ is the standard DFA.

2.2. Phase-Field Model

Based on the energy minimization theory, to handle binary classification problems using the modified Allen-Cahn equation, we minimize the following energy equation:

$$\mathcal{E}(\mathbf{s}(\mathbf{x}, t)) = \int_{\Omega} \sum_{k=1}^2 \left[\frac{H(s_k(\mathbf{x}, t))}{\epsilon^2} + \frac{1}{2} |\nabla s_k(\mathbf{x}, t)|^2 + \frac{\lambda}{2} (s_k(\mathbf{x}, t) - h_k(\mathbf{x}))^2 \right] d\mathbf{x}, \quad (6)$$

where $\mathbf{x} = (x, y)$ represents the spatial coordinates, $H(s_k) = 0.25s_k^2(s_k - 1)^2$ denotes the double-well potential energy function, ϵ represents the interface energy parameter, and λ is a fidelity parameter. In this paper, $h_k(\mathbf{x})$ represents a fitting term that conforms to the given data while satisfying the constraint $\sum_{k=1}^2 h_k(\mathbf{x}) = 1$. The vector $\mathbf{s} = (s_1, s_2)$ abides by the constraint $\sum_{k=1}^2 s_k(\mathbf{x}, t) = 1$, where $H'(\mathbf{s}) = (H'(s_1), H'(s_2))$ and $\mathbf{h} = (h_1, h_2)$. After minimizing the free energy functional Equation (6), we can obtain the following modified Allen-Cahn equation in the sense of L^2 , which can be used for binary classification, by employing the gradient flow for $k = 1, 2$:

$$\frac{\partial s_k(\mathbf{x}, t)}{\partial t} = -\frac{H'(s_k(\mathbf{x}, t))}{\epsilon^2} + \Delta s_k(\mathbf{x}, t) - \lambda(s_k(\mathbf{x}, t) - h_k(\mathbf{x})) - \zeta(\mathbf{s}(\mathbf{x}, t)), \quad (7)$$

where $\zeta(\mathbf{s}(\mathbf{x}, t)) = -\sum_{k=1}^2 H'(s_k(\mathbf{x}, t)) / (2\epsilon^2)$. Due to $s_1(\mathbf{x}, t) + s_2(\mathbf{x}, t) = 1$, we utilize zero Neumann boundary conditions to obtain $\mathbf{n} \cdot \nabla s_k(\mathbf{x}, t) = 0$, for $\mathbf{x} \in \partial\Omega$, $t > 0$, where \mathbf{n} is the unit outward normal vector to $\partial\Omega$. In the two-dimensional space $\Omega = (L_x, R_x) \times (L_y, R_y)$, we discretize it to obtain the following discrete computational domain $\Omega_h = \{(x_i, y_j) | x_i = L_x + (i - 0.5)h, y_j = L_y + (j - 0.5)h, 1 \leq i \leq N_x, 1 \leq j \leq N_y\}$, where N_x and N_y are positive integers and h is the uniform mesh size. Let $s_{k(ij)}^n$ denote the numerical approximations of $s_k(x_i, y_j, n\Delta t)$ with a time step of Δt . We divide the modified Allen-Cahn equation into the following two equations for $k = 1, 2$:

$$\frac{\partial s_k(\mathbf{x}, t)}{\partial t} = -\frac{H'(s_k(\mathbf{x}, t))}{\epsilon^2} + \Delta s_k(\mathbf{x}, t) - \zeta(\mathbf{s}(\mathbf{x}, t)), \quad (8)$$

$$\frac{\partial s_k(\mathbf{x}, t)}{\partial t} = \lambda[h_k(\mathbf{x}) - s_k(\mathbf{x}, t)]. \quad (9)$$

To solve Equation (8) using numerical methods, we employ the explicit Euler method:

$$\frac{s_{k(ij)}^{n+\frac{1}{2}} - s_{k(ij)}^n}{\Delta t} = -\frac{H'(s_{k(ij)}^n)}{\epsilon^2} + \Delta_h s_{k(ij)}^n + \sum_{k=1}^2 \frac{H'(s_{k(ij)}^n)}{2\epsilon^2}, \quad (10)$$

where $\Delta_h s_{k(ij)}^n$ represents the Laplace operator. Next, we solve the fidelity equation Equation (9) for $s_{k(ij)}^{n+\frac{1}{2}}$ using the implicit Euler method and we can rewrite it as follows:

$$s_{k(ij)}^{n+1} = \frac{s_{k(ij)}^{n+\frac{1}{2}} + \lambda\Delta t h_{k(ij)}}{1 + \lambda\Delta t}. \quad (11)$$

This scheme is explicit, which means we do not need to solve a system of discrete equations implicitly, resulting in faster computations. However, to ensure the stability of the scheme, we have the constraint $\Delta t < 0.25h^2$ [34].

3. Data Collection

The public sentiment dictionary of hotel comment emotion analysis corpus in this article comes from Tan et al. [35]. Among them, 728 are positive ratings and 772 are negative comments. We take 80% of these samples as the training set and the rest are the test set.

Firstly, we preprocess the hotel text and use the precise word segmentation function "lcut()" from the jieba library in Python to segment the text. In the jieba word segmentation mode, the precise word segmentation mode can accurately separate the text without redundancy. Then, the preprocessed hotel comments are used as a training set, and a given length word vector time series can be trained through the Word2Vec tool. Next, we use the MF-DFA method to extract the generalized Hurst exponents of the word vector time series, and finally utilize the generalized Hurst exponents in the Phase-field classifier model for sentiment classification of hotel texts.

4. Experimental Results

The experimental environment is based on Python 3.8 with the computer by Intel(R) Core(TM) i5-4430 CPU 3.00GHz processor, using the windows 10 operating system.

We first convert 1500 hotel comment texts into 1500 word vectors using Word2Vec model. Figure 2 represent a positive hotel comment word vector time series and a negative word vector sequence, respectively. Both sequences are unstable fluctuation sequences, with a large amount of noise in the data. From the sequence perspective, there is no significant difference between the two types of texts. Therefore, we use the MF-DFA method to extract features from time series to reduce data dimensions and improve classification accuracy.

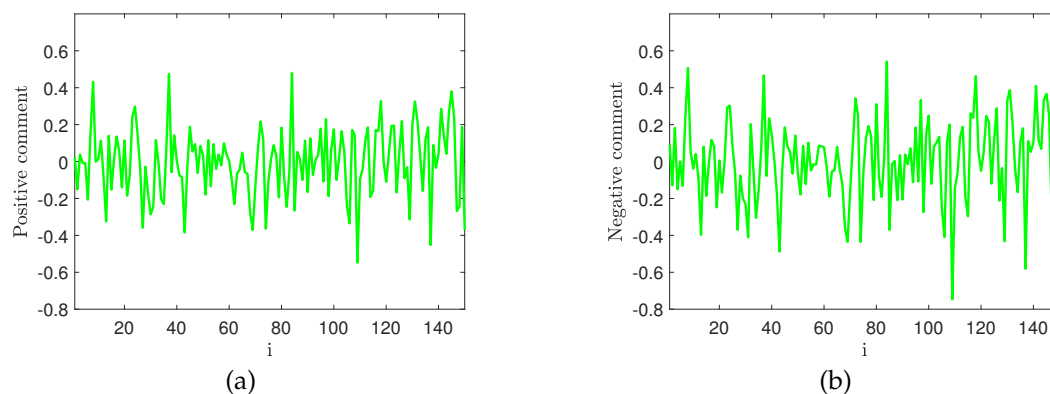


Figure 2. Word vector sequence of (a) positive hotel comment, and (b) negative hotel comment.

We reduce the dimensionality of the data to three dimensions and extracted generalized Hurst exponents $H(-6)$, $H(0)$, and $H(6)$. For ease of representation, we characterize these three feature values in pairs on a two-dimensional plane and regularize the data within the computational domain of $\Omega = (0,1)^2$. Next, we first use the Phase-field model classifier to train the training set, and get a nonlinear dividing line suitable for the data distribution characteristics. The parameter settings here are: $N_x = N_y = 100$, $h = 1/N_x$, $\epsilon = h/(2\sqrt{2}\tanh(0.9))$, $\lambda = 4500$, $\alpha = 0.12$, $\Delta t = \alpha h^2$.

We first investigate the formation of the classification interface under the phase-field model when the Hurst exponent combination is $H(-6)$ and $H(0)$. As shown in Figure 3, the classifier model has strong adaptability to the distribution of initial data. Figure 3(a) shows the data distribution of positive and negative samples of hotel comments, and Figs 3(b)-(d) show the evolution process of the phase of the data points in Figure 3(a) over time, ultimately forming a stable state in Figure 3(d). It can be seen from Figure 3(e) that the dividing line under the phase-field model is nonlinear and can effectively

classify texts belonging to different types. However, traditional machine learning models such as DNN and SVM have a linear classification interface, which makes it easy for text data with unclear boundaries to be misclassified.

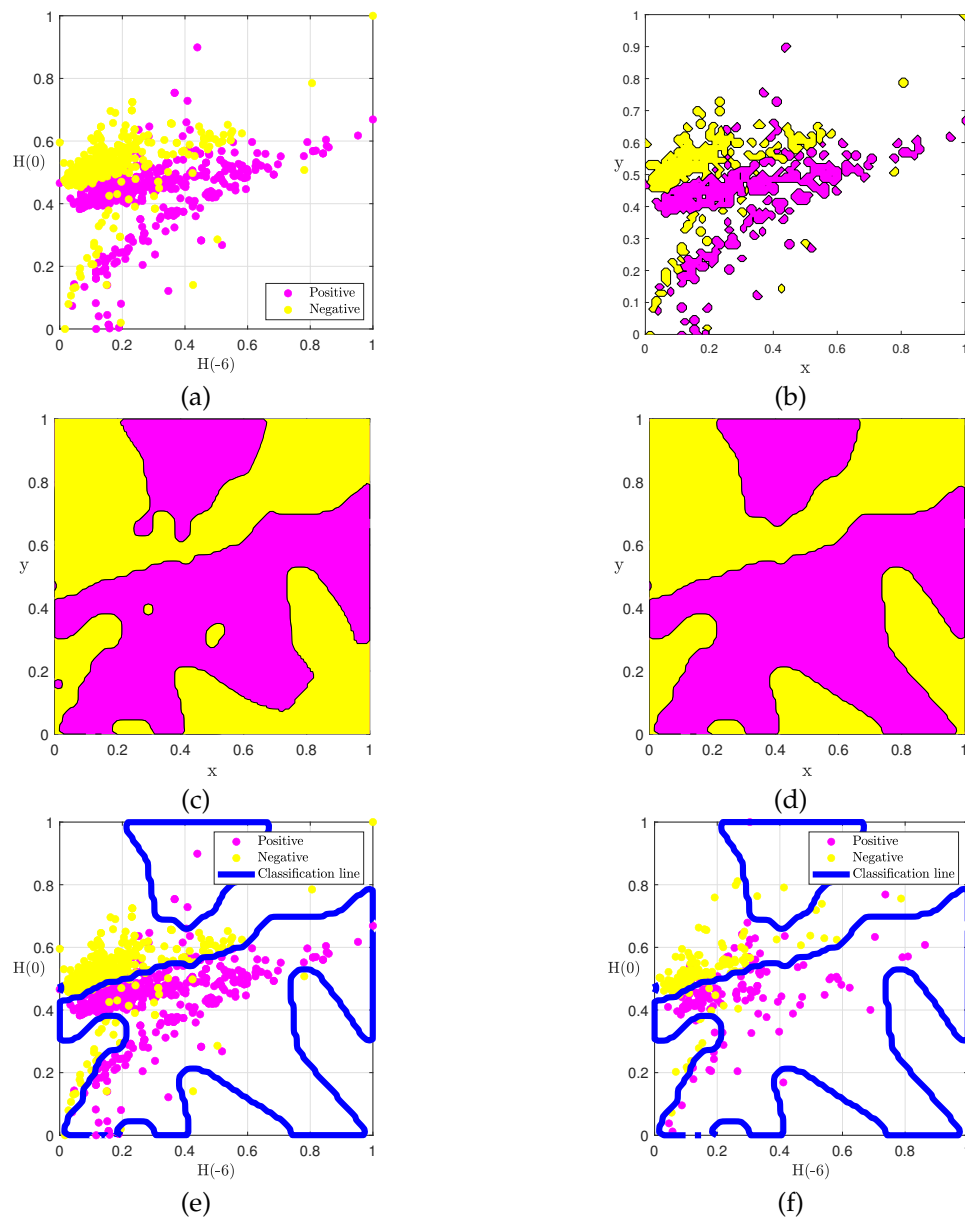


Figure 3. Classification process of Hurst exponent group $H(-6)$ and $H(0)$ of (a) initial training data, (b) phase evolution at $t = 0$, (c) phase evolution at $t = 50\Delta t$, (d) phase evolution at $t = 350\Delta t$, (e) classification interface under training set, and (f) classification results under the test set.

Next, we change the Hurst exponent combination to $H(0)$ and $H(6)$, and use the same parameters as before. As shown in Figure 4, the classifier model can still effectively classify the hotel comment data. Due to differences in the distribution of the initial data compared to Figure 3, the classification interface formed here has also changed.

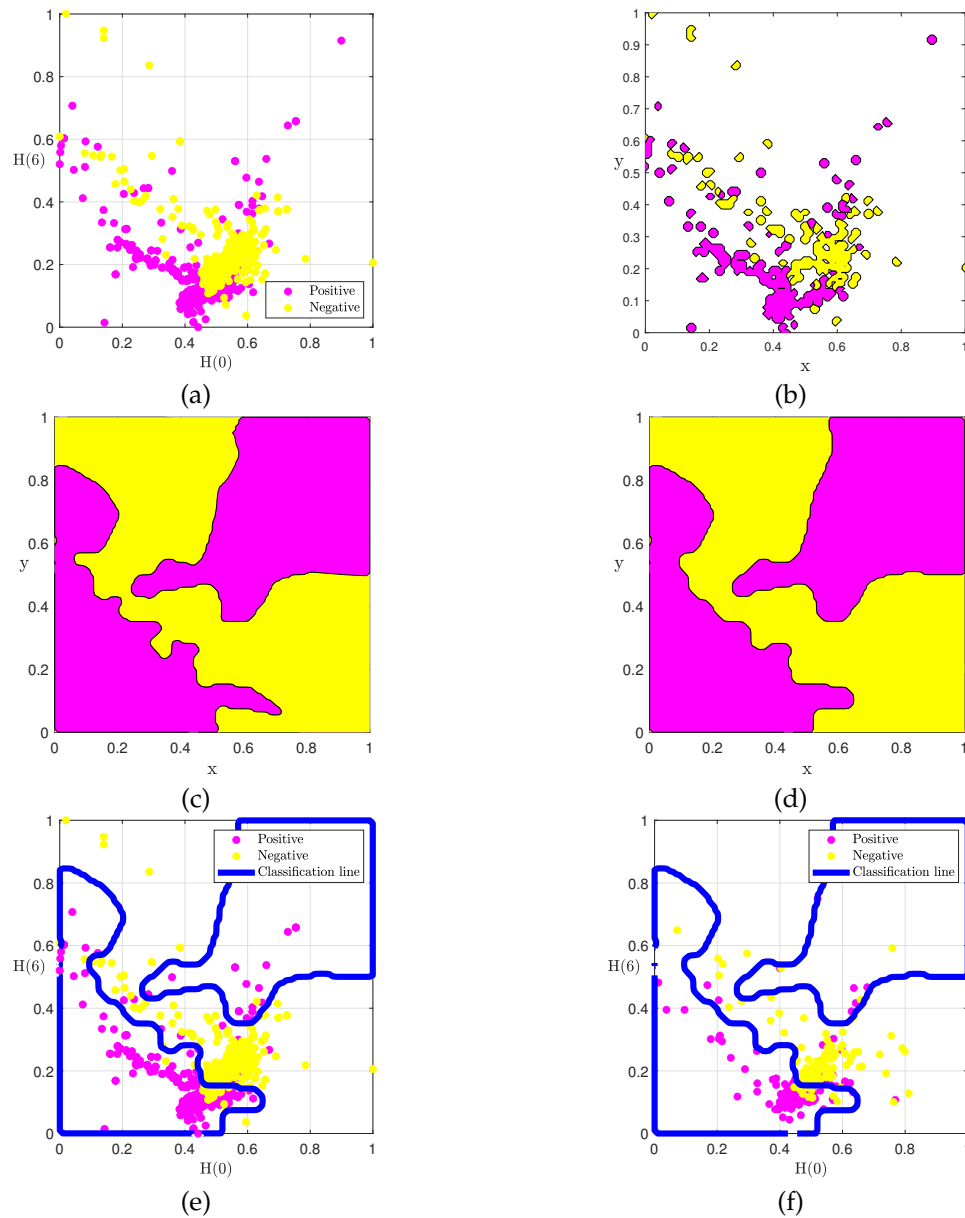


Figure 4. Classification process of Hurst exponent group $H(0)$ and $H(6)$ of (a) initial training data, (b) phase evolution at $t = 0$, (c) phase evolution at $t = 50\Delta t$, (d) phase evolution at $t = 350\Delta t$, (e) classification interface under training set, and (f) classification results under the test set.

Finally, we also observed the phase-field evolution process under the combination of $H(-6)$ and $H(6)$, and the classification process is shown in Figure 5.

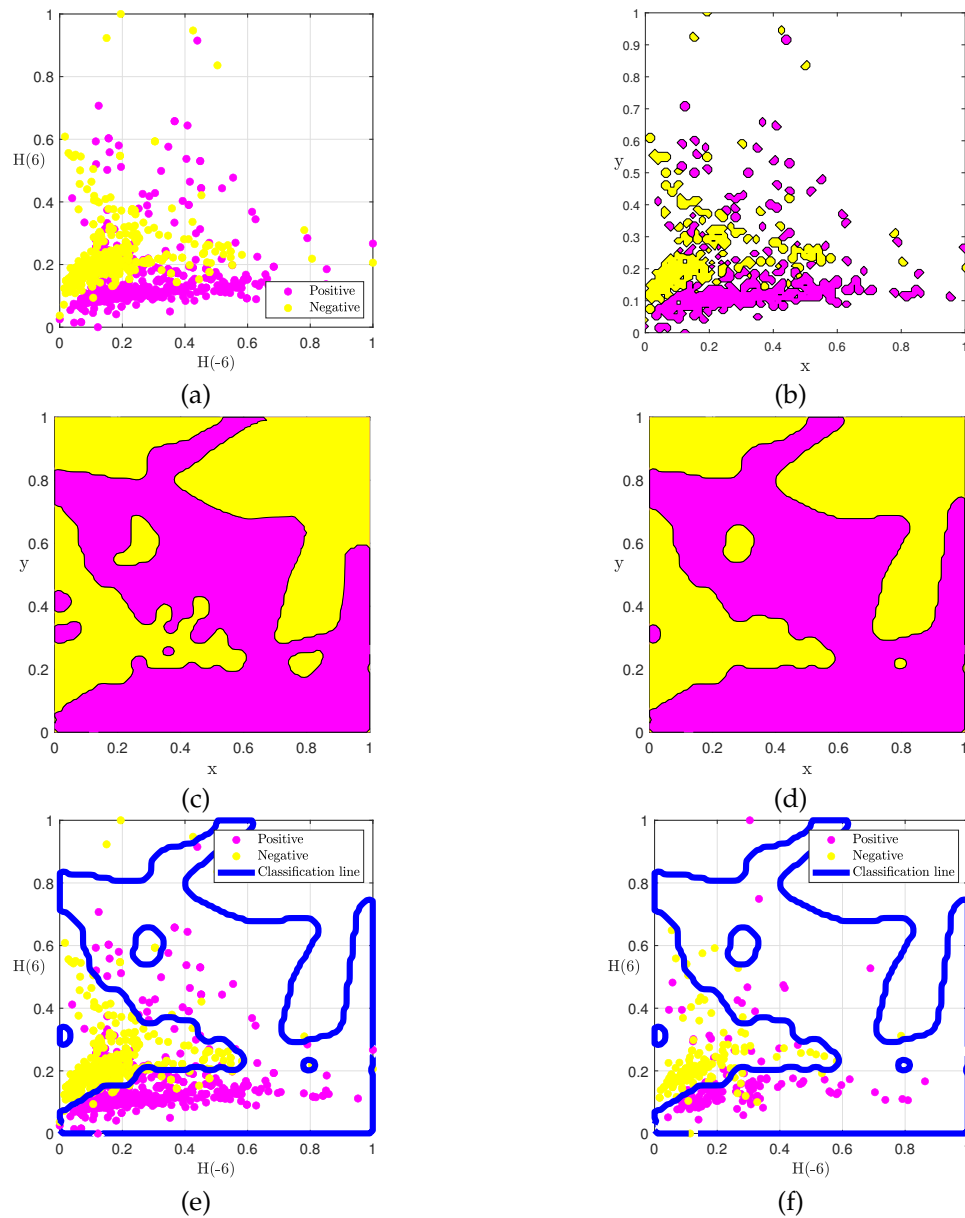


Figure 5. Classification process of Hurst exponent group $H(-6)$ and $H(6)$ of (a) initial training data, (b) phase evolution at $t = 0$, (c) phase evolution at $t = 50\Delta t$, (d) phase evolution at $t = 350\Delta t$, (e) classification interface under training set, and (f) classification results under the test set.

Next, we adopt the calculation of classification metrics to evaluate the classification effectiveness in the above three scenarios. Accuracy, Precision, Recall, and F_1 Score are used as the indicators to evaluate the classification results in this study, and the calculation formulas are as follows:

$$Accuracy = \frac{TP + TN}{TP + FN + FP + TN} \quad (12)$$

$$Precision = \frac{TP}{TP + FP} \quad (13)$$

$$Recall = \frac{TP}{TP + FN} \quad (14)$$

$$F_1 = 2 \times \frac{Precision \times Recall}{(Precision + Recall)} \quad (15)$$

For a binary classification problem, where instances are divided into positive or negative classes, the following four situations may occur in actual classification:

- I. If an instance is positive and predicted to be positive, it is the true class TP (True Positive).
- II. If an instance is positive but predicted to be negative, it is called a false negative class FN (False Negative).
- III. If an instance is negative but predicted to be positive, it is a false positive FP (False Positive).
- IV. If an instance is negative and predicted to be negative, it is true negative class TN (True Negative).

The classification metrics in Table 1 show that there is no significant difference in the classification results of our model under the combination of three different Hurst exponents. This further indicates that the three Hurst indices extracted by MF-DFA retain the main features of the original word vector, and the feature values are all efficient. Excellent results have been achieved on all the indicators of Accuracy, Precision, Recall, and F_1 Score.

Table 1. Classification metrics according to different Hurst exponents group.

Group	TP	TN	FP	FN	Accuracy	Precision	Recall	F_1
H(-6) & H(0)	115	149	10	26	0.88	0.92	0.82	0.87
H(0) & H(6)	118	145	14	23	0.88	0.89	0.84	0.86
H(-6) & H(6)	120	146	13	21	0.89	0.90	0.85	0.87

In order to demonstrate the nonlinear interface formed by our proposed model in hotel comment text classification and achieve the advantage of high accuracy, we compared the model with DNN and SVM classifiers which have linear classification interfaces. As shown in Table 2, the classification indicators of hotel comment texts under the phase-field classifier are superior to the traditional two classifier models. However, our model needs a large number of grids because of the discretization in the required space. Therefore, it is suitable for data classification in low dimensions. For high-dimensional data and is prone to Curse of dimensionality for high-dimensional data.

Table 2. Comparisons between other classifiers and ours.

Method	Accuracy	Precision	Recall	F_1
DNN	0.80	0.79	0.77	0.78
SVM	0.82	0.83	0.79	0.81
Ours	0.89	0.90	0.85	0.87

5. Conclusions

This paper mainly studied the emotional classification problem of hotel comment texts. Firstly, the original text data were preprocessed, and the texts were segmented using the jieba tool. Then, the text was processed into a time series type word vector using Word2Vec. For these unstable sequences with weak features, we adopted the MF-DFA method for feature extraction, reducing the dimensionality of the word vector sequence with a length of 150 to a length of 3. Then, we combine the three reduced feature values in pair in the two-dimensional computing domain. Then we used the phase-field model to evolve the data based phase values, and finally get a nonlinear classification interface that can adapt to the distribution characteristics of data in the computing domain. Through experimental verification on the test set, the results showed that in all three classification scenarios, the classifier can effectively form and classify the positive and negative emotions of hotel comment texts in the test set. In order to demonstrate the advantages of the nonlinear classification interface, we calculated the classification indicators for the same data using DNN and SVM, and the calculation results exhibited that our model was more efficient. Since the phase-field model needs to assign phase values on the discrete grid in space, the data dimensions used for calculation cannot be too large, and the equation discretization in the phase-field model is difficult to occur in a space larger than 3 dimensions. In our future research, our main focus will be on improving the robustness of the

classification algorithm in high-dimensional data, improving the accuracy of classifier classification, and enhancing the applicability of the classification algorithm in other fields of network commentary.

Author Contributions:

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