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RNN-DBSVM: Optimal Recurrent Neural Network Density Based Support Vector Machine

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Abstract: When implementing SVMs, two major problems are encountered: (a) the number of local minima increases exponentially with the number of samples and (b) the quantity of required computer storage, required for a regular quadratic programming solver, increases by an exponential magnitude as the problem size expands. The Kernel-Adatron family of algorithms gaining attention lately which has allowed it to handle very large classification and regression problems. However, these methods treat different types of samples (Noise, border, and core) in the same manner, which causes searches in unpromising areas and increases the number of iterations. In this work, we introduce a hybrid method to overcome these shortcomings, namely Optimal Recurrent Neural Network Density Based Support Vector Machine (Opt-RNN-DBSVM). This method consists of four steps: (a) characterization of different samples, (b) elimination of samples with a low probability of being a support vector, (c) construction of an appropriate recurrent neural network based on an original energy function, and (d) solution of the system of differential equations, managing the dynamics of the RNN, using the Euler-Cauchy method involving an optimal time step. The RNN remembers the regions explored during the search process thanks to its recurrent architecture. We demonstrated that RNN-SVM converges to feasible support vectors and Opt-RNN-DBSVM has a very low time complexity compared to RNN-SVM with constant time step, and KAs-SVM. Several experiments were performed on academic data sets. We used several classification performances measures to compare Opt-RNN-DBSVM to different classification methods and the results obtained show the good performance of the proposed method.

Keywords: recurrent neural network (RNN); support vector machine (SVM); Kernel-Adatron algorithm (KA); Euler-Cauchy algorithm

MSC: 90C20; 90C29; 90C90; 93E20

1. Introduction

The classification problem is an NP-Hard problem that has many applications in medicine, industry, economics and other fields. Several types of classifiers have been proposed in the literature to solve this problem, including the approach called Support Vector Machine based on Quadratic Programming (QP) [5,6,7]. The difficulty in the implementation of SVMs on massive data comes in the fact that the quantity of required computer storage required for a regular QP solver increases by an exponential magnitude as the problem size expands.

In this paper, we introduce a new version of SVM that implements a preprocessing filter and a recurrent neural network, namely Optimal Recurrent Neural Network Density Based Support Vector Machine (Opt-RNN-DBSVM).

SVMs approaches are based on the existence of a linear separator which can be obtained by exploding the data in a higher dimensional space through appropriate kernel functions. Among all possible hyperplanes, the SVM searches for the one with the most confident separation margin for a good generalization. This issue takes the form of a nonlinear constrained optimization problem that is usually handled using optimization methods. Thanks to the Kuhen-Tuker conditions, all these methods pass to the dual versions and call the optimizations methods to find the support vectors on which the optimal margin is built. Unfortunately, the complexity in time and memory grow exponentially with the size of data sets; in addition, the number of local minima grows too, which influences the location of the separation margin and the quality of the predictions.

A primary area of research in the area of learning from empirical data through support vector machines (SVMs) and addressing classification and regression issues is the development of incremental learning designs when the size of the training dataset is massive \cite{SMOTE}.

Out of many possible candidates, avoiding the usage of regular Quadratic Programming (QP) solvers, the two learning methods gaining attention lately are Iterative Single-Data Algorithms (ISDA) and Sequential Minimal Optimization (SMO) [3,5,7,9]. ISDAs operate from a single sample at hand (pattern-based learning) towards the best fit solution. The Kernel AdaTron (KA) was the primary ISDA for SVMs, using kernel functions to mapping data to the high dimensional character space of SVMs P [2] and conducting AdaTron [1] processing in the character space. Platt's SMO algorithm is an outlier among the so-called decomposition approaches introduced in [4,6], operating on a 2-samples workset of samples at a time. Because the decision for the two-point workset may be determined analytically, the SMO does not require the involvement of standard QP solvers. Due to it being analytically driven, the SMO has been especially wildly popular and is the most commonly utilized, analyzed, and further developed approach. Meanwhile, KA, though yielding somewhat comparable performance (accuracy and computational required time) in resolving classification issues, has not gained as much traction. The reason for this is twofold. First, up till lately [8], KA appeared to be restricted to classification tasks and second, it "missed" the flower of the robust theoretical framework. KA employs a gradient ascent procedure, and that fact also may have caused some researchers to be suspicious of the challenges posed by gradient ascent techniques in the presence of a perhaps ill-conditioned core array. In [10], for a lacking bias parameter b, the authors derive and demonstrate the equality of two apparently dissimilar ISDAs, namely a KA approach and an unbiased variant of the SMO training scheme [9] when constructing SVMs possessing positive definite kernels. The equivalence is applicable to the classification and regression tasks, and sheds additional insight in these apparently dissimilar methods of learning.

Despite the richness of the toolbox set up to solve the quadratic programs from SVM, and with the large amount of data generated by social networks, medical and agricultural fields, etc., the amount of computer memory required for a QP solver from the SVM dual grows hyper-exponentially and additional methods implementing different techniques and strategies are more than necessary.

Classical algorithms, namely ISDAs and SMO, do not distinguish between different types of samples (noise, border, and core) which causes searches in unpromising areas. In this work

we introduce a hybrid method to overcome these shortcoming, namely Optimal Recurrent Neural Network Density Based Support Vector Machine (Opt-RNN-DBSVM). This method proceeds in four steps: (a) characterization of different samples based on the density of the data sets (noise, core, and border), (b) elimination of samples with a low probability of being a support vector, namely core samples that are very far from the borders of different components of different classes, (c) construction of an appropriate recurrent neural network based on an original energy function making balance between the SVM-dual components (constraints and objective function) and insuring the feasibility of the network equilibrium points [3,4], and (d) solution of the system of differential equations, managing the dynamics of the RNN, using the Euler-Cauchy method involving an optimal time step. Due to its recurrent nature, the RNN was able to memorize locations visited during previous explorations. At one hand, two main interesting fundamental results were demonstrated: the convergence of RNN-SVM to feasible solutions and Opt-RNN-DBSVM has a very low time complexity compared to Const-RNN-SVM, SMO-SVM, ISDA-SVM, and L1QP-SVM.

On the other hand, several experimental studies were conducted based on well-known data sets. Based on well-known performance measures (Accuracy F1-score Precision Recal), Opt-RNN-DBSVM outperformed recurrent neural network-SVM with constant time step, Kernel-Adatron family of algorithms-SVM family, and well-known non-kernel models; In fact, Opt-RNN-DBSVM improved the accuracy, the F1Score, the precision, and the recall. Moreover, the proposed method requires a very small number of support vectors. The rest of this paper is organized as follows: In the second section, we give the flowchart of the proposed method. In the third section, we give the outline of our recent SVM versions called Density Based Support Vector Machine. The fourth section presents, in detail, the construction of recurrent neural networks associated with SVM-dual and the Euler-Cauchy algorithm that implements an optimal time step. In the fifth section, we give some experimental results. Finally, we give some conclusions and future extensions of Opt-RNN-DBSVM.

2. The architecture of the proposed method

The Kernel AdaTron (KA) algorithms, namely ISDAs, and SMO, treat different types of samples (noise, border, and core) in the same manner (all samples are considered for several iterations and supposed to be a support candidate with uniform probability), which causes searches in unpromising areas and increase the number of iterations. In this work, we introduce an economic method to overcome these shortcomings, namely Optimal Recurrent Neural Network Density Based Support Vector Machine (Opt-RNN-DBSVM). This method proceeds in four steps (see Figure 1): (1) Characterization of different samples based on the density of the data sets (noise, core, and border); to this end, two parameters are introduced: the size of the neighborhood of the current sample and the threshold that permits such categorization; (2) Elimination of samples with a low probability of being a support vector, namely core samples that are very far from the borders of different components of different classes and the noise samples that contain wrong information about the phenomenon under study. In our previews work [28], we demonstrate that such suppression does not influence the performance of the classifiers; (3) Construction of an appropriate recurrent neural network based on an original energy function making balance between the SVM-dual components (constraints and objective function) and ensuring the feasibility of the network equilibrium points [3,4]; (4) Solution of the system of differential equations, managing the dynamics of the RNN, using the Euler-Cauchy method involving an optimal time step. In this regard, the formula of the coming state of the neurons, of the constructed RNN, is introduced into the energy function, which leads to a 1-dimension quadratic optimization problem whose solution represents the optimal step of the Euler-Cauchy process that ensures a maximum decrease of the energy function [37]. The components of the produced equilibrium point represent the membership degrees of different samples to the support vectors data set.

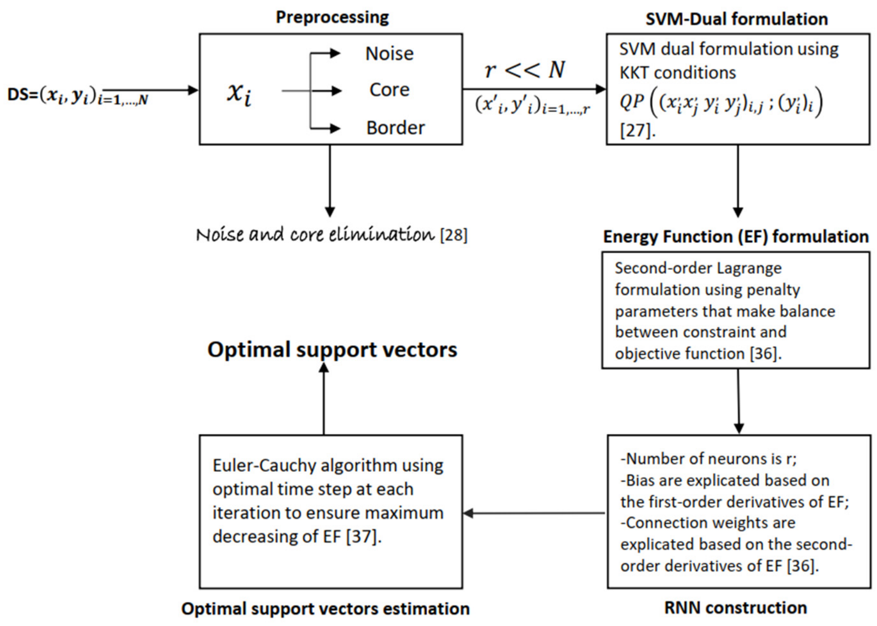

Preprocessing
DS=$(x_i, y_i)_{i=1,\dots,N}$
x_i
Noise
Core
Border
$r << N$
$(x'_i, y'_i)_{i=1,\dots,r}$
SVM-Dual formulation
SVM dual formulation using KKT conditions $QP\left((x'_i x'_j\, y'_i\, y'_j)_{i,j}\,;(y'_i)_i\right)$ [27].
Noise and core elimination [28]
Energy Function (EF) formulation
Second-order Lagrange formulation using penalty parameters that make balance between constraint and objective function [36].
Optimal support vectors
Euler-Cauchy algorithm using optimal time step at each iteration to ensure maximum decreasing of EF [37].
-Number of neurons is r;
-Bias are explicated based on the first-order derivatives of EF;
-Connection weights are explicated based on the second-order derivatives of EF [36].
Optimal support vectors estimation
RNN construction

Figure 1. Opt RNN DBSVM Flowchart.

3. Density based support vector machine

In the following, we denote by BD the set of N samples $x_1, \dots, x_N$ labeled, respectively, by $y_1, \dots, y_N$, distributed via K class $C_1, \dots, C_K$. In our case, $K = 2$ and $y_i \in \{-1, +1\}$.

2.1. Classical support vector machine

The hyperplane we are looking for must satisfy the equation $w.x_i + b = 0$, where w is the weight that defines this SVM separator that satisfies the constraints family given by $\forall i = 1, \dots, N$, $y_i(w.x_i + b) \geq 1$. To ensure a maximum margin, you need to maximize $\frac{2}{\|w\|}$.

As the pattern are not linearly separable, we introduce the kernel functions K (that satisfy the Mercer conditions [9]) to explore the data in appropriate space. By introducing the Lagrange relaxation and writing the Kuhn-Tuker conditions, we obtain a quadratic optimization problem with a single linear constraint that we solve to determine the support vectors [18]. To address the problem of sutured constraints, some researchers have added the notion of a soft margin [8]. They employed N supplementary slack variables $\zeta_i \geq 0$ at every constraint $y_i(w.x_i + b) \geq 1$. The sum of the relaxed variables is weighted and included in the cost function:

$$\begin{cases} Min\ \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\zeta_i \\ subject\ to \\ y_i(\varphi(x_i).w + b) \geq 1 - \zeta_i \\ \zeta_i \geq 0, \forall i = 1, \dots, N \end{cases}$$

Here φ represents the transformation function derived from the function kernel K. We obtain the coming dual problem:

$$\begin{cases} Max\ \sum_{i=1}^{N}\alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{i=1}^{N}\alpha_i\alpha_j y_i y_j K(x_i, x_j) \\ subject\ to \\ \sum_{i=1}^{N}\alpha_i y_i = 0 \\ 0 \leq \alpha_i \leq C, \forall i = 1, \dots, N \end{cases}$$

Several methods can be used to solve this optimization problem: gradient methods, linearization method Frank-Wolf method, generation column method, Newton method applied to the Kuhn system [23], sub-gradient methods, Dantzig algorithm, Uzawa algorithm [23], recursive neural networks [22], hill climbing, simulated annealing, search by calf, A*, genetic algorithm, ant colony [24], and particle swarm method [22] ... etc. Several versions of SVM were proposed in the literature among which we find the Least Squares Support Vector Machine Classifiers (LS-SVM) [12], the generalized support vector machines (G-SVM) [10], Fuzzy support vector machine [2,11], One class Support Vector Machine (OC-SVM) [13,18], Total Support Vector Machine (T-SVM) [14], Weighted Support Vector Machine (W-SVM) [15], Granular Support Vector Machines (G-SVM) [16], Smooth Support Vector Machine(S-SVM) [17], Proximity Support Vector Machine classifiers (P-SVM) [19], Multisurface proximal support vector machine classification via generalized eigenvalues (GEP-SVM) [20], and Twin support vector machines (T-SVM) [21] ... etc.

2.2. Density based support vector machine (DBSVM)

In this section, we give a short description of DBVSM method. We introduce real number r > 0 and the integer mp> 0, called min-points, and we define three types of samples: noise point, border point and interior point (or cord point). We showed that the interior points do not change their nature even when they are projected into another space by the kernel functions. Furthermore, we have established that such points cannot be selected as support vectors [28].

Definition 1. Let $S \subseteq \mathbb{R}^n$. A point $a \in \mathbb{R}^n$ is said to be an Interior Point (or cord point) of S if there $r > 0$ such that $B(a,r) \subseteq S$. The set of all interior point of S is denoted by $Int(S)$.

Definition 2. For a given dataset BD, a non-negative real r and an integer mp, there exist three kind of samples.
1. A simple x is called $C_i - Noise\ Point\ (NP_i)$ if $|C_i \cap B(x,r)| < mp$.
2. A simple x is called $C_i - Cord\ Point\ (CP_i)$ if $|C_i \cap B(x,r)| \geq mp$ and $x \in \overbrace{envol(C_i)}^{o}$
3. A simple x is called $C_i - Border\ Point\ (NP_i)$ if $|C_i \cap B(x,r)| < mp$ and there exists a $C_i - Cord\ Point\ y$ such as $x \in B(y,r)$.

Let K be a kernel function allowing to move from the space $\mathbb{R}^n$ to the space $\mathbb{R}^N$ using the transformation $\phi\ (here\ n < N)$.

Lemma 1. [28] If a is a $C_i - Cord\ point$ for a given ϵ and minpoints (mp), then $\phi(a)$ is also a $C_i - Cord\ point$ with appropriate ϵ' and the same minpoints (mp).

Theorem 1. [28] A cord point is either a noise-point or a Border-point.

Proposition 1. [28] Lets $\epsilon > 0$ be a real number. The cord point set, $CordPoint(minPoint)$, decreasing function for the inclusion operator.

Let $\{\alpha_1, \ldots, \alpha_n\} = BM \cup CM \cup N\,M$ be the set of the Lagrange multipliers where BM, CM, and N M are the Lagrange multipliers of the border samples, cord samples, and noise samples, respectively.

As the elements of NM and CM can not be selected to be support vectors, the reduced dual problem is given by:

$$RD)\begin{cases} Max \displaystyle\sum_{\alpha_i \in BM} \alpha_i - \frac{1}{2} \sum_{\alpha_i \in BM} \sum_{\alpha_i \in BM} \alpha_i \alpha_j y_i y_j K(x_i, x_j) \\ \qquad\qquad subject\ to \\ \qquad\qquad \displaystyle\sum_{\alpha_i \in BM} \alpha_i y_i = 0 \\ 0 \leq \alpha_i \leq C, \ \forall \alpha_i \in BM, \qquad \forall i = 1, \ldots, N \end{cases}$$

In this work, as the (RD) problem is quadratic with linear constraint, we use the continuous Hopfield Network by proposing an original energy function in the coming section [26].

4. Recurrent neural network to optimal support vectors

The continuous Hopfield networks consist of interconnected neurons with a smooth sigmoid activation function (usually a hyperbolic tangent). The differential equation which governs the dynamics of the CHN is:

$$\frac{du}{dt} = -\frac{u}{\tau} + W.\alpha + I$$

where u, α, W and I are, respectively, the vectors of neuron states, the outputs, the weight matrix, and the biases. For a CHN of N neurons, the state u_i and output α_i of the neuron i are relied on by the equation αi For an initial vector state $u_0 \in \mathbb{R}^n$, a vector $u^e \in \mathbb{R}^n$ is called an equilibrium point of the system 1, if and only if, $\exists t^e \in \mathbb{R}^+$ such as $\forall t \geq t^e$, $u(t) = u^e$. It should be noted if the energy function (or Lyapunov function) exists; the equilibrium point exists as well. Hopfield proved that the symmetry of the matrix of the weight is a sufficient condition for the existence of the Lyapunov function [25].

3.1. Continuous Hopfield network based on the original energy function

To solve the obtained dual problem via recurrent neural network [26], we propose the following energy function:

$$E(\alpha_1, \dots, \alpha_{N'}) = \beta_0 \sum_{\alpha_i \in BM} \alpha_i - \frac{\beta_0}{2} \sum_{\alpha_i \in BM} \sum_{\alpha_i \in BM} \alpha_i \alpha_j y_i y_j K(x_i, x_j) + \beta_1 \sum_{\alpha_i \in BM} \alpha_i y_i + \frac{\beta_2}{2} \left(\sum_{\alpha_i \in BM} \alpha_i y_i \right)^2$$

To determine the vector of the neurons biases, we calculate the partial derivatives of E:

$$\frac{\partial E}{\partial \alpha_i}(\alpha_1, \dots, \alpha_{N'}) = \beta_0 - \beta_0 \sum_{\alpha_j \in BM} \alpha_j y_i y_j K(x_i, x_j) + \beta_1 y_i + \beta_2 y_i \sum_{\alpha_i \in BM} \alpha_i y_i$$

The components of the biases vector are given by:

$$I_i = \frac{\partial E}{\partial \alpha_i}(0) = -\beta_0 - \beta_1 y_i \quad , \forall i = 1, \dots, N'$$

To determine the connection weights W between each neurons pair, we calculate the second partial derivatives of E:

$$\frac{\partial^2 E}{\partial \alpha_i \partial \alpha_j}(\alpha_1, \dots, \alpha_{N'}) = -\beta_0 y_i y_j K(x_i, x_j) + \beta_2 y_i y_j$$

The components of the weights W matrix are given by:

$$W_{i,j} = \frac{\partial^2 E}{\partial \alpha_i \partial \alpha_j}(0) = \beta_0 y_i y_j K(x_i, x_j) - \beta_2 y_i y_j$$

To calculate the equilibrium point of the proposed recurrent neural network, we use the Euler-Cauchy iterative method:

(1) Initialization: $\alpha_1^0, \dots, \alpha_{N'}^0$, and the step ρ^0 are randomly chosen;
(2) Given $\alpha_1^t, \dots, \alpha_{N'}^t$ and the step ρ^t, the step ρ^{t+1} is chosen such that E^{t+1} is maximum and we calculate $u_1, \dots, u_{N'}$ using: $\forall i = 1, \dots, N'$, $u_i^{t+1} = u_i^t + \rho^{t+1}(\sum_{j=1,\dots,N'} W_{i,j} \alpha_j^t + I_i)$, then we calculate $\gamma_1, \dots, \gamma_{N'}$ using the activation function $f: \gamma_i = f(u_i^{t+1})$, then the $\alpha_1^{t+1}, \dots, \alpha_{N'}^{t+1}$ are given by: $\alpha^{t+1} = P(\gamma)$, where P is the projection operator on the set $\{\alpha \in \mathbb{R}^{N'}, \ \sum_{i=1}^{N'} \alpha_i y_i = 0 .\}$
(3) Return to 1) until $\|\alpha^{t+1} - \alpha^t\| \leq \varepsilon$, where $\varepsilon > 0$.

The Figure 2 shows the connection weights W between each pair of neurons.

Theorem 2. If $i = j$, $P_{i,j} = 1 - \frac{y_i^2}{N'}$, else $P_{i,j} = -\frac{y_i y_j}{N'}$, where $S = \sum_{i=1,\dots,N'} y_i^2$

Proof of Theorem 2. We have $P = I - A^t.(A.A^t)^{-1}.A$ and $A = [y_1, \dots, y_{N'}]$, then $(A.A^t)^{-1} = ([y_1, \dots, y_{N'}]^t.[y_1, \dots, y_{N'}])^{-1} = \frac{1}{N'}$ because $N' = \sum_{i=1,\dots,N'} y_i^2$
Thus $A^t.(A.A^t)^{-1}.A = \frac{1}{N'}[y_1, \dots, y_{N'}]^t.[y_1, \dots, y_{N'}]$
Finally, for $i = j$, $P_{i,j} = 1 - \frac{y_i^2}{N'}$, and for $i \neq j, P_{i,j} = -\frac{y_i y_j}{N'}$.
Concerning the constraint family satisfaction $0 \leq \alpha_i \leq C, \forall i = 1, \dots, N'$, we use the activation function.
$f(x) = C.tanh\left(\frac{x}{\tau}\right) = C.\frac{e^x - e^{-x}}{e^x + e^{-x}}$, where τ is supposed to be a very large positive real number, which ensures $\forall x, -C \leq f(x) \leq C$.
We consider a kernel function K such that $K(x,x) = C \neq 0$.

Theorem 3. A continuous Hopfield network has an equilibrium points if $W_{i,j} = 0$ and $W_{i,j} = W_{j,i}$.

Theorem 4. If $C = \frac{\beta_2}{\beta_0}$, then CHN-SVM has an equilibrium points.

Proof of Theorem 2. We have $W_{i,j} = (\beta_0 K_{i,j} - \beta_2) y_i y_j$ then $\forall i \, and \, j$ $W_{i,j} = W_{j,i}$ because K is symmetric. In the other hand $W_{i,i} = \beta_0 y_i K(x_i, x_i) - \beta_2 y_i^2 = \beta_0 \times \frac{\beta_2}{\beta_0} - \beta_2 = 0$
Then CHN-SVM has an equilibrium points

3.2. Continuous Hopfield network with optimal time step

In this section, we chose, mathematically, the optimal time size in each iteration of the Euler-Cauchy method to solve the dynamical equation of the recurrent neural network proposed in this paper.

At the end of the k^{th} iteration, we know α^k and let s_k be the next step size permits to calculate α^{k+1} using the formula: $\alpha^{k+1} = \alpha^k + s_k \nabla E(\alpha^k)$ must be chosen such as $E(\alpha^{k+1}) \leq E(\alpha^k)$ at maximum.

As the activation function of the proposed neural network is the $tanh$, then: $\frac{d\alpha_i}{dt} = \frac{2}{\tau}\alpha_i(1 - \alpha_i)\nabla E(\alpha)$

The matrix form of the energy function is:

$$E(\alpha) = \beta_0 U^t \alpha - \frac{\beta_0}{2}(\alpha)^t T \alpha + \beta_1 y^t \alpha + \frac{\beta_2}{2}(y^t \alpha)^2$$

where $U = (1, \dots, 1)^t \in \mathbb{R}^{N'}$, $\alpha = (1, \dots, 1)^t \in \mathbb{R}^{N'}$, $and \, T_{i,j} = y_i y_j K(x_i, x_j) for \, all \, i \, and \, j.$

At the k^{th} iteration, the state α^k is known, then α^{k+1} is calculated by:

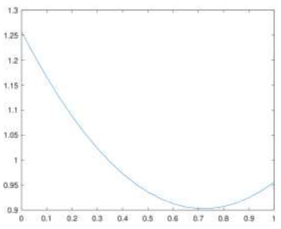
$$\alpha^{k+1} = \alpha^k + s_k \frac{\alpha^k}{dt}$$

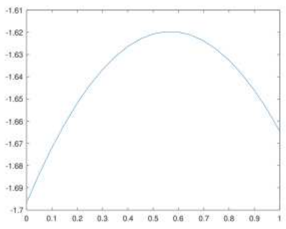
where s_k is the actual time step that must be optimal. To this end, we substitute α^{k+1} by $\alpha^k + s_k \frac{d\alpha^k}{dt}$ in $E(\alpha^{k+1})$:

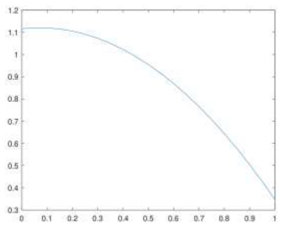
$$e(s_k) = E(\alpha^{k+1}) = 0.5 A_k s_k^2 + B_k s_k + C_k$$

where

$A_k = \beta_2 y^t \frac{d\alpha^k}{dt} - \beta_0 \left(\frac{d\alpha^k}{dt}\right)^t T \frac{d\alpha^k}{dt}$,
$B_k = \beta_0 U^t \frac{d\alpha^k}{dt} - \beta_0 (\alpha_k)^t T \frac{d\alpha^k}{dt} + \beta_1 y^t \frac{d\alpha^k}{dt} + \beta_2 (y^t \alpha^k)\left(y^t \frac{d\alpha^k}{dt}\right)$,
$C_k = \beta_0 U^t \alpha^k - 0.5\beta_0 (\alpha^k)^t T \alpha^k + \beta_1 y^t \alpha^k + 0.5\beta_2 (y^t \alpha^k)^2$.
Thus the best time step is the minimum of $e(s_k)$. The Figure 3a–c gives different cases.


(**a**) RNN-DBSVM 1


(**b**) RNN-DBSVM 2


(**c**) RNN-DBSVM 3

3.3. Opt-RNN-DBSVM algorithm

In this section, the procedures described in Sections 2.2, 3.1, and 3.2 are summarized into Algorithm 1.

Algorithm 1. Opt-RNN-DBSVM.

```
Algorithm 1 Opt-RNN-DBSVM
Require: $mp, r, \beta_0, \beta_1, \beta_2, C, ITER$
Ensure: $Optimal\ support\ vectors$
  % Density based preprocessing:
  $CP \leftarrow \emptyset$;  $BP \leftarrow \emptyset$;  $NP \leftarrow \emptyset$;
  for  all s in DS do
    if $|B(s,r) \cap DS| > mp$ then
      $CP \leftarrow CP \cup \{s\}$
    else if $mp > |B(s,r) \cap DS| > \frac{mp}{2}$ then
      $BP \leftarrow BP \cup \{s\}$
    else
      $NP \leftarrow NP \cup \{s\}$
    end if
  end for
  $RDS \leftarrow DS \setminus \{NP \cup CP\}$
  % RNN-Building:
  $I \ \leftarrow \beta_0 + \beta_1 Y$
  $W \leftarrow \beta_0 K - \beta_2 Y'^2$
  % Optimal Euler-Cauchy to RNN stability:
  $step_0 = rand(0,1)$
  $\alpha'_0 \leftarrow rand(0,1,|RDS|)$
  for k=1,...,ITER do
    $D_k \leftarrow \frac{d\alpha^k}{dt}$
    $A_k \leftarrow \beta_2 y^t D_k - \beta_0 (D_k)^t T D_k$
    $B_k \leftarrow \beta_0 U^t D_k - \beta_0 (\alpha^k)^t T D_k + \beta_1 y^t D_k + \beta_2 (y^t \alpha^k)(y^t D_k)$
    $C_k \leftarrow \beta_0 U^t \alpha^k - 0.5\beta_0 (\alpha^k)^t T D_k + \beta_1 y^t \alpha^k + 0.5\beta_2 (y^t \alpha^k)^2$
    $s_k^* \ \leftarrow argmin_{s \in [0,1]} e(s)$
    $\alpha^{k+1} \leftarrow \alpha^k + s_k^* D_k$
  end for
```

The input of Algorithm 1 is the radius r (the size of the neighborhood of the current sample), the minimum of samples mp into B(Current_sample, r) (that determines the type of this sample), the three lagrangian parameters $\beta_0, \beta_1, and\ \beta_2$ (that makes a compromise between the dual components), the bound C of SVM [28], and the number of iterations (that represents an artificial convergence).

The Algorithm 1 processes in three macro-steps: Data preprocessing, RNN-SVM construction, and RNN-SVM equilibrium point estimation. The input of the first phase is the initial data set with labeled samples. Based on r and mp the algorithm determines the types of different samples based on the number of current sample neighbourhood discrete size. The output of this phase is a reduced sub-data set (The initial data set minus the core samples).

The Input of the second phase are the reduced data set, the Lagrangian parameters $\beta_0, \beta_1, \beta_2$, and the SVM bound C. Based on the energy function built in 3.1 and on the first and second derivatives, the architecture of CHN-SVM is built; the bias and connection weights, which represent the output of this phase, are calculated. These later represent the input of the third phase and the Euler-Cauchy algorithm is used to calculate the degree of membership of different samples to the set of support vectors set; to ensure optimal decreasing of the energy function, at each iteration, an optimal step is determined by solving a quadratic 1-dimension optimization problem; see sub-section 3.2. At the convergence, the proposed algorithm produces the support vectors based on which Opt-RNN-DBSVM can predict the class of unseen samples.

Proposition 2. If N, r, and ITER represent, respectively, the size of a labeled data set BD, the number of the remained samples(output of the preprocessing phase, and the number of iterations, then the complexity of algorithm 1 is $O(r^2 * ITER)$.

Proof of proposition 2. First, in the preprocessing phase, we calculate, for each sample (x^i, x^j), the distance $d(x^i, x^j)$ and we execute N comparisons to determine the type of each sample; thus the complexity of this phase is $O(N^2)$. Second, during ITER iterations, we update the activation of each neuron using the activation of all the other neurons to solve the reduced SVM-dual, thus the third phase has a complexity of $O(r^2 * ITER)$.

Finally, the complexity of algorithm 1 is $O(N^2) + O(r^2 \times ITER)$. Let's denote Const- RNN-SVM the SVM version that implements recurrent neural network based on con- constant time step. Following the same reasoning, the complexity of Const-RNN-SVM is of $O(N^2 \times ITER)$.
Notes: - As Kernel-Adatron Algorithm (KA) is the kernel version of SMO and ISDA and KA implements two embedded N-loops in each iteration, then the complexity of SMO and ISDA is of [40]. In addition, we consider L1QP-SVM that implements numerical linearalgebra Gauss–Seidel method [49], which implements two embedded N-loops in each iteration, then the complexity of SMO and ISDA is of $O(N^2 \times ITER)$. - For a very high size and high-density labeled data set, we have: $r < < N$, thus $ITERour << ITERCHN - SVM$ and $ITERour \ll ITERL1QP - SV\ M$ $ITERour \ll ITERSMO - SV\ M$ and $ITERour << ITERISDA - SV\ M$.
Thus $complexity(our) \prec complexity(CHN - SV\ M)$ and $complexity(our) \prec complexity(L1QP - SVM)$, $complexity(our) \prec complexity(SMO - SV\ M)$ and $complexity(our) \prec complexity(ISDA - SVM)$.

4. Experimentation

In this section, we compare Opt-RNN-DBSVM to several classifiers: Const-RNN-SVM (SVM based on a recurrent neural network using constant Euler-Cauchy time step), SMO- SVM, ISDA-SVM, L1QP-SVM, and some non-kernel classifiers (for example MLP, NB, KNN, Decision Tree...). The classifiers were tested on several data sets: iris, abalone, wine, ecoli, balance, liver, spect, seed, and PIMA (collected from the University of California at Irvine (UCI)[29]). The performance measures, used in this study, are accuracy, F1 score, precision, and recall.

4.1. Opt-RNN-DBSVM vs Const-CHN-SVM

In this section, we compare Opt-RNN-DBSVM to Const-RNN-SVM by considering different values of the Euler-Cauchy time step $s \in \{.1, .2, \dots, .8\}$.

Tables 1 and 2 give different values of accuracy, F1-score, precision, and recall on the considered data sets. The results show the superiority of Opt-RNN-DBSVM over Const-CHN-SVM ($step \in STEP = \{.1, .2, \dots, .9\}$); this superiority is quantified as follow:

$$3.43\% = \max_{(s,d)\in STEP\times DATA} \left(accuracy(Opt - RNN - DBSV\ M(s)) - accuracy(const - RNN - SV\ M)\right)$$

$$2.31\% = \max_{(s,d)\in STEP\times DATA} (\mathrm{F1Score}(\mathrm{Opt - RNN - DBSV\ M(s)}) - \mathrm{F1Score}(\mathrm{const - RNN - SV\ M}))$$

$$7.52\% = \max_{(s,d)\in STEP\times DATA} (\mathrm{precision}(\mathrm{Opt - RNN - DBSV\ M(s)}) - \mathrm{precision}(\mathrm{const - RNN - SV\ M}))$$

$$6.5\% = \max_{(s,d)\in STEP\times DATA} (\mathrm{recall}(\mathrm{Opt - RNN - DBSV\ M(s)}) - \mathrm{recall}(\mathrm{const - RNN - SV\ M}))$$

where DATA is the set of different considering data sets. These results are not strange because Opt-RNN-SVM ensures an optimal decrease of the CHN energy function at each step.

Table 1. Performance of Const-CHN-SVM on different data sets for different values of time step.

	SVM-CHN s = .1				SVM-CHN s = .2			
	Accuracy	**F1-score**	**Precision**	**Recall**	**Accuracy**	**F1-score**	**Precision**	**Recall**
Iris	95.98	96.66	90.52	92.00	96.66	95.98	91.62	92.00
Abalone	80.98	40.38	82.00	27.65	80.66	40.38	81.98	27.65
Wine	79.49	78.26	73.52	74.97	79.49	78.26	73.52	74.97
Ecoli	88.05	96.77	97.83	97.33	88.05	97.77	97.83	97.99
Balance	79.70	70.7	55.60	62.70	79.70	70.7	55.60	62.70
Liver	80.40	77.67	77.90	70.08	80.40	77.67	77.90	70.08
Spect	92.12	90.86	91.33	90.00	97.36	99.60	97.77	1.00
Seed	85.71	83.43	92.70	75.04	85.71	83.43	92.70	75.04
PIMA	79.22	61.90	84.7	49.6	79.22	61.90	83.97	49.6

Table 2. Performance of Const-CHN-SVM on different data sets for different values of time step.

	SVM-CHN s = .3				SVM-CHN s = .4			
	Accuracy	**F1-score**	**Precision**	**Recall**	**Accuracy**	**F1-score**	**Precision**	**Recall**
Iris	94.53	95.86	89.66	98.23	95.96	93.88	89.33	95.32
Abalone	77.99	51.68	83.85	30.88	81.98	41.66	83.56	33.33
Wine	80.23	77.66	74.89	74.97	81.33	77.65	73.11	74.43
Ecoli	88.86	95.65	96.88	97.33	86.77	97.66	97.83	97.95
Balance	79.75	70.89	55.96	62.32	79.66	70.45	56.1	66.23
Liver	80.51	78.33	77.9	70.56	80.40	77.67	77.90	70.08
Spect	97.63	98.99	97.81	98.56	96.40	98.71	96.83	97.79
Seed	85.71	83.43	92.70	75.88	85.71	83.43	92.70	75.61
PIMA	79.22	61.93	84.82	49.86	79.22	61.90	84.98	49.89
	SVM-CHN s = .5				**SVM-CHN s = .6**			
	Accuracy	**F1-score**	**Precision**	**Recall**	**Accuracy**	**F1-score**	**Precision**	**Recall**
Iris	94.53	95.86	89.66	98.23	95.96	93.88	89.33	95.32
Abalone	78.06	51.83	83.96	40.45	82.1	42.15	83.88	38.26
Wine	80.84	78.26	74.91	75.20	81.39	77.86	73.66	74.47
Ecoli	88.97	95.7	96.91	97.43	86.77	97.66	97.83	97.95
Balance	79.89	71.00	56.11	62.72	79.71	70.64	56.33	66.44
Liver	80.66	78.33	77.9	70.56	80.40	77.67	77.90	70.08
Spect	91.36	92.60	91.77	84.33	91.36	92.60	91.77	84.33
Seed	84.67	82.96	92.23	74.18	84.11	83.08	92.63	75.48
PIMA	79.12	61.75	84.62	49.86	79.12	61.33	84.68	48.55

	SVM-CHN s = .7				SVM-CHN s = .8			
	Accuracy	**F1-score**	**Precision**	**Recall**	**Accuracy**	**F1-score**	**Precision**	**Recall**
Iris	94.53	95.86	89.66	98.23	95.96	93.88	89.33	95.32
Abalone	77.99	51.68	83.85	30.88	81.98	41.66	83.56	33.33
Wine	80.23	77.66	74.89	74.97	81.33	77.65	73.11	74.43
Ecoli	88.86	95.65	96.88	97.33	86.77	97.66	97.83	97.95
Balance	79.75	70.89	55.96	62.32	79.66	70.45	56.1	66.23
Liver	80.51	78.33	77.9	70.56	80.40	77.67	77.90	70.08
Spect	94.36	84.60	83.77	85.99	94.36	84.60	83.77	85.99
Seed	85.71	83.43	92.70	75.88	85.71	83.43	92.70	75.61
PIMA	79.22	61.93	84.82	49.86	79.22	61.90	84.98	49.89
	SVM-CHN s = .9				**DBSVM optimal value of s**			
	Accuracy	**F1-score**	**Precision**	**Recall**	**Accuracy**	**F1-score**	**Precision**	**Recall**
Iris	95.96	93.88	89.33	95.32	**97.96**	**96.19**	**95.85**	**98.5**

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Abalone	81.98	41.66	83.56	33.33	**98.38**	**96.07**	**96.18**	**93.19**
Wine	81.33	77.65	73.11	74.43	**96.47**	**95.96**	**96.08**	**96.02**
Ecoli	86.77	88.46	90.77	91.19	**91.82**	**97.66**	**97.83**	**97.95**
Balance	79.66	70.45	56.1	66.23	**91.31**	**90.33**	**89.54**	**90.89**
Liver	80.40	77.67	77.90	70.08	**88.10**	**85.95**	**86.00**	**85.50**
Spect	94.36	84.60	83.77	85.99	**95.55**	**86.20**	**85.28**	**86.31**
Seed	85.71	83.43	86.18	75.61	**88.90**	**84.31**	**92.70**	**84.40**
PIMA	79.22	61.90	75.90	49.89	**79.87**	**68.04**	**84.98**	**62.05**

Figures 4 and A1–A5 give the series of optimal steps generated by Opt-RNN-DBSVM during iterations for different data sets. We remark that all the optimal steps are taken from the interval [3,4] which furnishes an optimal domain for those using a CHN based on constant step.

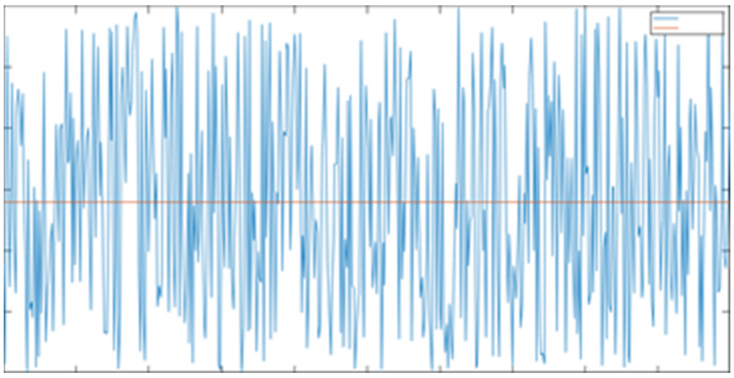

Figure 4. IRIS data set.

4.2. Opt-RNN-DBSVM vs Classical-Optimizer-SVM

In this section, we give the performance of different Classical-Optimizer-SVM (L1QP- SVM, ISDA-SVM, and SMO-SVM) applied to several datasets, and compare the number of support vectors obtained by the different Classical-Optimizer-SVM and Opti-RNN- SVM.

The Table 3 gives the values of the measures of accuracy, F1 score, precision, and recall of Classical-Optimizer-SVM on different datasets. The results show the superiority of Opt-RNN-DBSVM.

Table 3. Performance of Classical-Optimizer-SVM on different data sets.

	L1QP-SVM				**ISDA-SVM**			
	Accuracy	**F1-score**	**Precision**	**Recall**	**Accuracy**	**F1-score**	**Precision**	**Recal**
Iris	71.59	62.02	70.80	55.17	82.00	83.64	76.67	92.00
Abalone	74.15	70.80	71.48	59.30	83.70	68.22	70.00	80.66
Wine	72.90	65.79	75.53	60.11	66.08	65.80	66.00	70.02
Ecoli	66.15	55.89	61.33	41.30	51.60	48.30	33.33	51.39
Balance	65.20	53.01	60.51	41.22	50.44	58.36	68.32	60.20
Liver	64.66	52.06	60.77	40.44	50.00	48.00	62.31	51.22
Spect	70.66	62.02	67.48	50.11	77.60	71.20	75.33	70.11
Seed	70.51	58.98	67.30	45.30	80.66	81.25	79.80	79.30
PIMA	65.18	53.23	60.88	39.48	49.32	44.33	48.90	50.27
		SMO-	**SVM**		**CHN-**	**DBSVM optimal value of s**		
	Accuracy	**F1-score**	**Precision**	**Recall**	**Accuracy**	**F1-score**	**Precision**	**Recal**
Iris	71.59	62.02	70.80	55.17	97.96	96.19	95.85	98.5
Abalone	74.15	70.80	71.48	59.30	98.38	96.07	96.18	93.19
Wine	72.90	65.79	75.53	60.11	96.47	95.96	96.08	96.02

Ecoli	66.15	55.89	61.33	41.30	91.82	88.46	90.77	91.19
Balance	65.20	53.01	60.51	41.22	91.31	90.33	89.54	90.89
Liver	64.66	52.06	60.77	40.44	88.10	85.95	86.00	85.50
Spect	70.66	62.02	67.48	50.11	95.55	86.20	85.28	86.31
Seed	70.51	58.98	67.30	45.30	88.90	84.31	86.18	84.40
PIMA	65.18	53.23	60.88	39.48	79.87	68.04	75.90	62.05

Figures 5–8 illustrates, respectively, the support vectors obtained using L1QP-SVM, L1QP-SVM, SMO-SVM, and Opti-RNN-SVM applied to IRIS data. We note that (a) ISDA considers more than 96% as support vectors, which is really an exaggeration, (b) L1QP and SMO use a reasonable number of samples as support vectors, but most of them are duplicated, and (c) thanks to the preprocessing, Opt-RNN can reduce the number of support vectors by more than 32%, compared to SMO and L1QP, which allows it to overcome the over-learning phenomenon encountered with SMO and L1QP. Figure 8 gives the support vectors obtained by Opt-RNN-DBSVM applied to IRIS data.

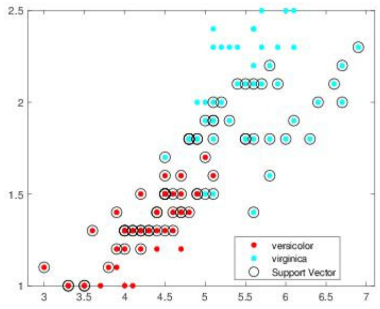

Figure 5. Vectors support obtained by ISDA algorithm.

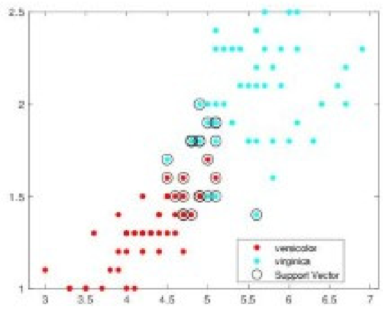

Figure 6. Vectors support obtained by L1QP algorithm.

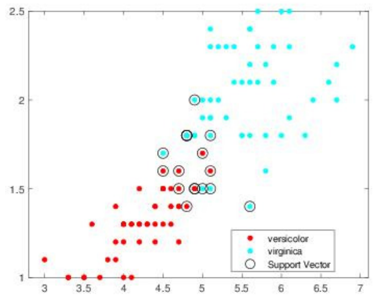

Figure 7. Vectors support obtained by SMO algorithm.

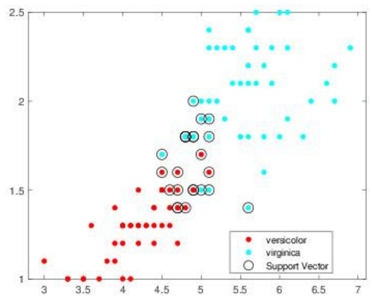

Figure 8. Vectors support obtained by Opt RNN SVM algorithm.

4.3. *Opt-RNN-DBSVM vs non-kernel classifiers*

In this section, we compare Opt-RNN-DBSVM to several non-kernel classifiers, namely NiaveBayes [30], MLP [31], Knn [32], AdaBoostM1 [33], DecisionTree [34], SGDClassifier [35], Nearest Centroid Classifier [35], and Classical SVM [1].

Table 4 give the values of the measures accuracy, F1-score, precision, and recall for the considered data sets. The best performance is reached by Opt-RNN-DBSVM followed by classical methods SVM, Decision Tree, and Adabsot are close to the Opt-RNN-DBSVM method.

Table 4. Comparison between Opt-RNN-DBSVM and different classification methods on the Iris and abalone data sets.

		Iris		
Method	**Accuracy**	**F1-score**	**Precision**	**Recall**
Niave Bayes	90.00	87.99	77.66	1.00
MLP	26.66	0.00	0.00	0.00
Knn	96.66	95.98	91.62	1.00
AdaBoostM1	86.66	83.66	71.77	1.00
Dicision Tree	69.25	76.12	70.01	69.55
SGDClassifier	76.66	46.80	1.00	30.10
Random Forest Classifier	90.00	87.99	77.66	1.00
Nearest Centroid Classifier	96.66	95.98	91.62	1.00
Classical SVM	96.66	95.98	91.62	1.00
Opt-RNN-DBSVM	97.96	92.19	95.85	96.05
		Abalone		
Method	**Accuracy**	**F1-score**	**Precision**	**Recall**
Niave Bayes	68.89	51.19	41.37	67.33
MLP	62.91	47.63	36.32	47.63
Knn	81.93	53.74	70.23	43.02
AdaBoostM1	82.29	55.99	70.56	55.06
Dicision Tree	76.79	51.33	52.06	49.63
SGDClassifier	80.86	64.74	58.08	70.57
Nearest Centroid Classifier	76.07	64.79	62.60	61.15
RandomForestClassifier	82.28	57.56	71.11	48.34
Classical SVM	80.98	40.38	82.00	27.65
Opt-RNN-DBSVM	98.38	96.07	96.18	93.19

Additional comparison studies were performed on PIMA and Germandiabetes data sets and the ROC curves were used to calculate the AUC for the best performance obtained from each non-kernel classifier. Figures A6 and A7 show the comparison of the ROC curves of the classifiers DT, KNN, MLP, NB, and Opt-RNN-DBSVM method, evaluated on the PIMA data set. We point out that Opt-RNN-DBSVM quickly converges to the best results and obtains more true positives for a small number of false positives compared to several classification methods. More comparisons are given in Appendix A; Figures A8 and A9 show the comparison of the ROC curve of the classical SVM and Opt-RNN-DBSVM method, evaluated on the Germany diabetes data set.

5. Conclusions

The main challenges of SVM implementation are: the number of local minima and the amount of computer memory, required for solving the SVM-dual, increase exponentially with respect to the size of the data set. The Kernel-Adatron family of algorithms, ISDA and SMO, has handled very large classification and regression problems. However, these methods treat noise, boundary, and kernel samples in the same way, resulting in a blind search in unpromising areas. In this paper, we have introduced a hybrid approach to deal with these drawbacks, namely Optimal Recurrent Neural Network Density Based SupportVector Machine (Opt-RNN-DBSVM), which performs in four phases: Characterization of different samples, elimination of the samples having a weak probability to be support vector, building an appropriate recurrent neural network based on original energy function, and solving the differential equation system, governing the RNN dynamic, using Euler-Cauchy method implementing an optimal time step. Due to its recurrent nature, the RNN was able to memorize locations visited during previous explorations. On one hand, two main interesting fundamental results were demonstrated: the convergence of RNN-SVM to feasible solutions and Opt-RNN-DBSVM has a very low time complexity compared to Const-RNN-SVM, SMO-SVM, ISDA-SVM, and L1QP-SVM. On the other hand, several experimental studies were conducted based on well-known data sets (iris, abalone, wine, ecoli, balance, liver, spect, seed, pima). Based on credible performance measures (Accuracy F1-score Precision Recal), Opt-RNN-DBSVM outperformed Const-RNN-SVM, KAs- SVM, and some non-kernel models (cited Table 4). In fact, Opt-RNN-DBSVM improved accuracy by up to 3.43%, F1Score by up to 2.31%, precision by up to 7.52%, and recall by up to 6.5%. In addition, compared SMO-SVM, ISDA-SVM, and L1QP-SVM, Opt-RNN-DBSVM a reduction of the number of support vectors by up to 32%, which permits to save of memory for huge applications that implement several machine learning models. The main problem encountered in the implementation of Opt-RNN-DBSVM is the determination of the Lagrange parameters involved in the SVM energy function. In this sense, a genetic strategy will be introduced to determine these parameters considering each data set.

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Data Availability Statement: data are available under request.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

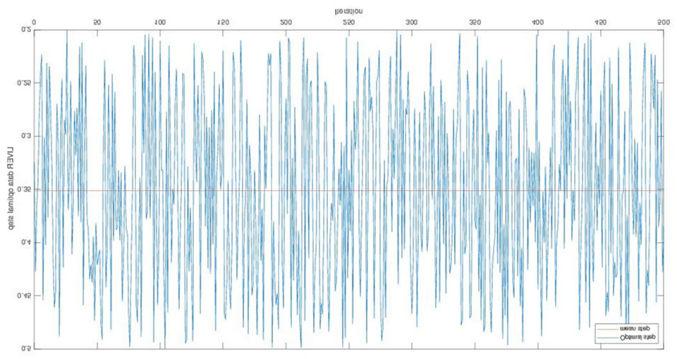

Figure A1. ABALONE data set.

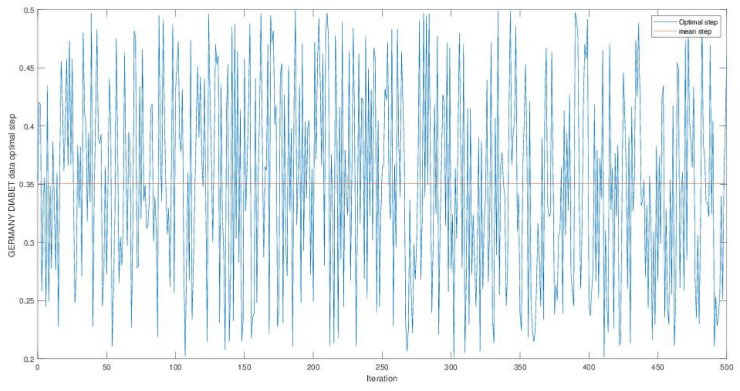

Figure A2. PIMA data set.

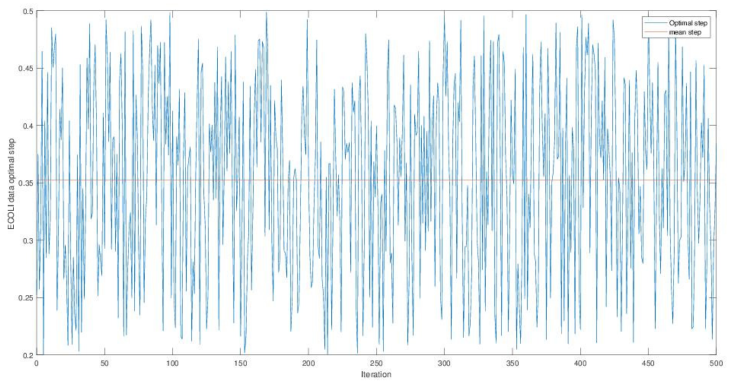

Figure A3. WINE data set

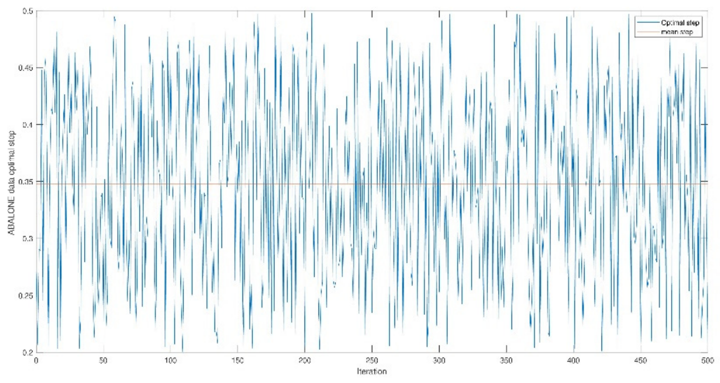

Figure A4. SEED data set.

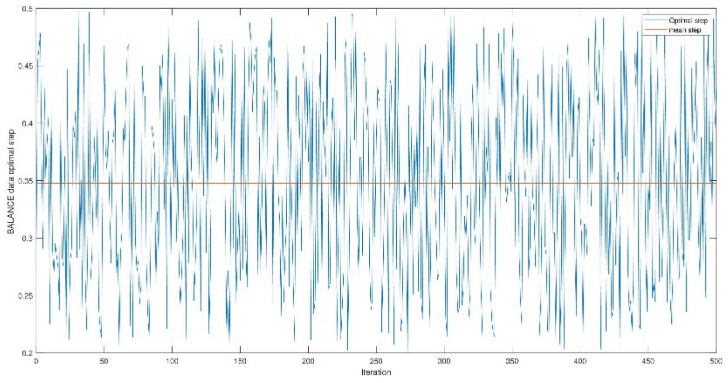

Figure A5. GERMANY data set.

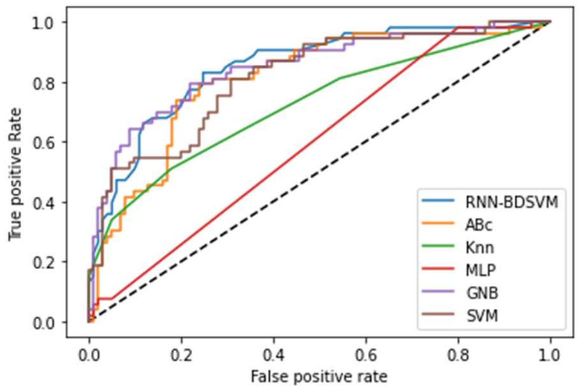

Figure A6. RUC curve for the different classification methods applied to PIMA diabetes data set.

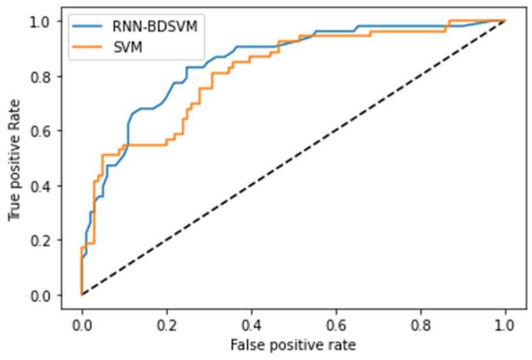

Figure A7. Opt-RNN-DBSVM vs SVM RUC curve applied to PIMA diabetes data set.

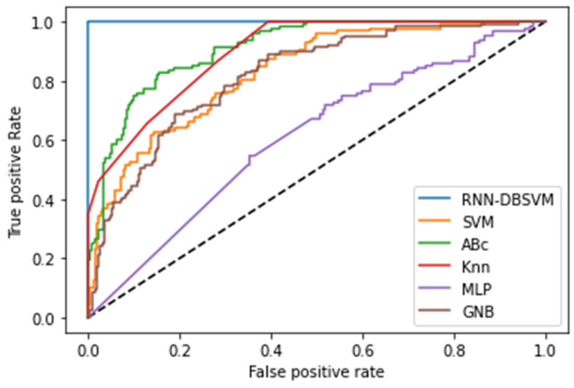

Figure A8. RUC curve for the different classification methods applied to PIMA diabetes data set.

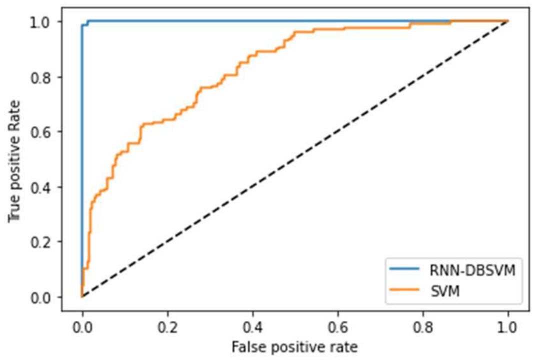

Figure A9. Opt-RNN-DBSVM vs SVM RUC curve applied to PIMA diabetes data set.

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