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# Gauge Fields and Quantum Dynamics at the Event Horizon and Beyond

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*Article*

# Gauge Fields & Quantum Dynamics at the Event Horizon and Beyond

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**Abstract:** The field dynamics of neutral and charged matter (treated as gauge fields) beyond the event horizon of black holes are investigated. As an extension of the work done in Ganesan (2023) [1], the analysis was done using the combined dynamics of the two-dimensional Yang-Mills and Liouville gravity fields beyond the event horizon. The quantum interactions and symmetry breaking mechanisms of the combined two-dimensional fields are discussed. The second part of this paper explores the quantum dynamics occurring at the event horizon. By considering the structure of the event horizon as a Möbius strip (embedded in three-dimensional space), a formulation for the lower bound of the first energy eigenvalue of a quantum particle (electron) at the event horizon is obtained.

**Keywords:** two-dimensional field theory; gauge fields; event horizon; Möbius strip; electron energy eigenvalue

## 1. Background

The aim of this work is to explore the field dynamics of neutral and charged matter (as gauge fields) beyond the event horizon of black holes; as an extension to the work done in Ganesan (2023) [1]. The second part of this work explores the quantum dynamics at the event horizon. A formulation for the lower bound of the first energy eigenvalue of a quantum particle (electron) at the event horizon is obtained. Gauge theories and symmetries within the context of black holes have been a recent research focus. In Samuel et al., (2022) [2], the authors used theoretical tools from gauge theories and geometric phase for polarimetric very long baseline interferometry performed using the Event Horizon Telescope. In this view, the multiplicative distortion of polarized signals at the individual elements are represented as gauge transformations by general complex matrices. This was done to ensure that the closure traces appear as gauge-invariant quantities. Another interesting work in this direction is observed in Corrigan and Poisson (2018) [3]. In that work, the authors focus their studies on the properties of the EZ gauge from black hole perturbation theory. In Corrigan and Poisson (2018) [3], it was proven that the EZ gauge is necessarily singular at the event horizon of black holes. Research involving the Couch–Torrence (CT) inversion symmetry at the event horizon is seen in the work of Fernandes et al., (2021) [4]. In that work, the probe Maxwell field dynamics was studied on the extreme Reissner–Nordström solution; in the context of the Couch–Torrence (CT) inversion isometry. In relation to the mentioned isometry, the Newman–Penrose and Aretakis like conserved quantities were built along the future null infinity and the future event horizon. In Araújo Filho et al., (2023) [5], the thermodynamics and evaporation of a Schwarzschild black hole was analyzed using non-commutative gauge theory; using gravitational field potentials. In that research, focusing on a static spherically symmetric black hole, the authors determined the thermodynamic properties as well as the modified Hawking temperature. A similar approach for studying thermodynamics properties of black holes is observed in the work of Touati and Zaim (2023) [6]. In that research, the authors employed non-commutative gauge theory to study the thermodynamics and evaporation properties of a deformed Schwarzschild black hole. The primary results of their work show that the parameters of the non-commutative gauge field play a similar role to the thermodynamic variables. In Ogawa and Ishihara, (2023) [7], a gravastar type black hole was studied

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using a field-theoretic framework. In that work, the authors employed computational methods to obtain gravastar solutions as non-topological solitons. These compact gravastar soliton solutions were determined within a system of a $U(1)$ gauge-Higgs model with Einstein gravity and a complex scalar field. In recent times, the advent of the Event Horizon Telescope has unveiled various opportunities in terms of testing theories on black holes. Some tests and analysis on gravitational theories (black hole solutions) based on observational data are presented in Vagnozzi et al., (2022) [8].

This paper is organized as follows: Section 2 provides some background on two-dimensional symmetry breaking beyond the event horizon based on the ideas presented in Ganesan (2023) [1]. Sections 3 and 4 present the field theory of neutral and charged particles respectively (beyond the event horizon). Section 5 discusses the quantum dynamics at the event horizon and the lower bound of the first energy eigenvalue of a quantum particle (electron) is obtained. This paper ends with the Analysis & Conclusions Section which discusses the central concepts and key findings presented in this paper.

## 2. Symmetry Breaking Beyond the Event Horizon

The existence of the two-dimensional Yang-Mills field beyond the event horizon provides a potential avenue to explore its interactions with another mathematically well-developed two-dimensional field: the Liouville gravity field [9,10]. It has been shown that Liouville gravity could be obtained directly from the Einstein field equations in two-dimensions [11]. This work extends the theoretical framework presented in Ganesan (2023) [1] where the following hypothesis was developed: there exists a two-dimensional analogue to the Higgs field beyond the event horizon – existing in the form of a two-dimensional Yang-Mills field. In this work, the region beyond the event horizon of a black hole is assumed to have two-spatial dimensions with no time-dimension. The Lagrangian of the combined two-dimensional Yang-Mills and the Liouville fields are as follows:

$$L = L_{YM} + L_G$$

$$L_{YM} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{1}$$

$$L_G = \frac{1}{4}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + QR\varphi + 4\left(\frac{\Gamma(1-b^2)}{\Gamma(b^2)}\lambda^b\right)e^{2b\varphi}$$

where $$Q = \left(b + \frac{1}{b}\right) \tag{2}$$

where $F^{\mu\nu}$ are the Yang-Mills field strengths, $\Gamma()$is the gamma function, $b \in (0,1)$ is the Liouville field coupling strength, $\varphi$ is the Liouville gravitational real scalar field, $Q$ is the background charge, $R$ is the Ricci scalar curvature and $\lambda$ is the parameter that appears in the Liouville field correlation function. Expanding $e^{2b\varphi}$ to the second order via the Maclaurin series and defining $\alpha = 4\Gamma(1-b^2)/\Gamma(b^2)$, the combined Lagrangian, $L$ is as follows:

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + QR\varphi + \alpha\lambda^b(1 + 2b\varphi + 2b^2\varphi^2)$$

$$= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi) \tag{3}$$

The field potential $V(\varphi)$ is influenced by the balancing of terms $QR\varphi$ and $\alpha\lambda^b(1 + 2b\varphi + 2b^2\varphi^2)$. As the Liouville gravity field coupling strength vanishes, $b \to 0$, the background charge, $Q \to \infty$ and $\alpha \to 0$. On the other hand, if the Liouville gravity field coupling strength, $b \to 1$, the background charge, $Q \to 2$ and $\alpha \to \infty$. The maximum potential at $R > 0$ and $\varphi = 0$ is $V(\varphi = 0) = \alpha\lambda^b$. To access the lower unstable equilibria at $R < 0$ via symmetry breaking, the Liouville gravitational real scalar field is defined with respect to its complex components with a new parameter $v$ defined as $v^2 = R/\lambda^b$:

$$\varphi = \varphi^*\varphi'$$

$$\varphi' = \frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2) \text{ and } \varphi^* = \frac{1}{\sqrt{2}}(v + \phi_1 - i\phi_2) \tag{4}$$

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where $\phi_1, \phi_2 \in R$. The field potential $V(\varphi = \varphi^* \varphi') \to V(\phi_1, \phi_2)$ is obtained as follows:

$$V(\phi_1, \phi_2) = V_1(\phi_1, \phi_2) + V_2(\phi_1, \phi_2) + V_3(\phi_1, \phi_2)$$

$$V_1(\phi_1, \phi_2) = \left(\left(\frac{1}{2}Q + 3b^2\alpha\right)(\lambda^b v^2) + b\alpha\lambda^b\right)\phi_1^2 + \left(\left(\frac{1}{2}Q + b^2\alpha\right)(\lambda^b v^2) + b\alpha\lambda^b\right)\phi_2^2$$

$$V_2(\phi_1, \phi_2) = \left((Q + 2b^2\alpha)(\lambda^b v^3) + (2b\alpha)(\lambda^b v)\right)\phi_1 + (2b^2\alpha)(\lambda^b v)\phi_1^3 +$$

$$(2b^2\alpha)(\lambda^b v)\phi_1\phi_2^2 + (b^2\alpha\lambda^b)\phi_1^2\phi_2^2 + \left(\frac{1}{2}b^2\alpha\lambda^b\right)\phi_1^4 + \left(\frac{1}{2}b^2\alpha\lambda^b\right)\phi_2^4$$

$$V_3(\phi_1, \phi_2) = \frac{1}{2}Q\lambda^b v^4 + \lambda^b\alpha + \frac{1}{2}b^2\alpha\lambda^b v^4 + b\alpha\lambda^b v^2 \tag{5}$$

The field potential in equations (5) is grouped into three terms: $V_1(\phi_1, \phi_2)$ consists of the scalar gravitational bosonic (dilaton) terms, $V_2(\phi_1, \phi_2)$ consists of the bosonic coupling terms and $V_3(\phi_1, \phi_2)$ represents the constants. The field potential $V(\phi_1, \phi_2)$ describes two bosonic quanta $\phi_1$and $\phi_2$. At Liouville field coupling strength, $b \to 0$ (resulting in $Q \to \infty$ and $\alpha \to 0$) the following occurs in $V_1(\phi_1, \phi_2)$:

$$V_1(\phi_1, \phi_2) \to \left(\left(\frac{1}{2}Q\right)\lambda^b v^2\right)\phi_1^2 + \left(\left(\frac{1}{2}Q\right)\lambda^b v^2\right)\phi_2^2 \tag{6}$$

Hence, when the Liouville field coupling strength, $b \to 0$ the bosons $\phi_1$ and $\phi_2$ have Ricci scalar curvature of the magnitude $\frac{1}{2}Q$. In this formulation, a two-dimensional analogue to the three-dimensional Higgs mechanism is presented – where instead of mass, the scalar bosons (or dilatons) $\phi_1$ and $\phi_2$ have Ricci scalar curvature, $R = v^2\lambda^b$ during symmetry breaking. This occurs due to the non-existence of the property of three-dimensional mass in two-dimensional space. Thus, the property of mass is replaced by the two-dimensional Ricci scalar curvature. At $b \to 1$, the bosons $\phi_1$ and $\phi_2$ have the a mixed property of Ricci scalar curvature and $\lambda^b$ with the magnitude of $\left(\frac{1}{2}Q + 3b^2\alpha\right)R + b\alpha\lambda^b$ and $\left(\frac{1}{2}Q + b^2\alpha\right)R + b\alpha\lambda^b$ respectively.

### 3. Gauge Fields with Neutral Matter

In Ganesan (2023) [1], the mechanisms in the previous sections were extended to include neutral particles with mass crossing the event horizon. The was done by considering the particles as local gauge fields with the potentials, $A_\mu$ . The associated Lagrangian is:

$$L' = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}g^{\mu\nu}D_\mu\varphi D_\nu\varphi + V(\varphi)$$

where,

$$\varphi = \partial_\mu\varphi + eA_\mu\varphi \quad \text{and} \quad D_\nu\varphi = \partial_\nu\varphi + eA_\nu\varphi \quad \text{such that} \quad e = \lambda^b$$

$$F_{\mu\nu} = \partial_\mu A_\nu + \partial_\nu A_\mu \tag{7}$$

Consider Lagrangian invariance under the following local gauge transformations:

$$\varphi(x) \to \varphi(x)e^{\beta(x)} \quad \text{with}$$

$$A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\beta(x) \quad \text{and} \quad A_\nu(x) \to A_\nu(x) - \frac{1}{e}\partial_\nu\beta(x) \tag{8}$$

The terms $\frac{1}{4}g^{\mu\nu}D_\mu(\phi_1, \phi_2)D_\nu(\phi_1, \phi_2)$ in the Lagrangian, $L' \to L'(\phi_1, \phi_2)$ would then become:

$$L' = -\frac{1}{4}F^{\mu\nu}F_{\mu\mu} + \frac{1}{4}g^{\mu\nu}D_\mu(\phi_1, \phi_2)D_\nu(\phi_1, \phi_2) + V(\phi_1, \phi_2)$$

$$g^{\mu\nu}(\phi_1, \phi_2)D_\nu(\phi_1, \phi_2)$$

$$= g^{\mu\nu}\left(\frac{1}{2}\partial_\mu\phi_1^2 + \frac{1}{2}\partial_\mu\phi_2^2 + \frac{1}{2}ev^2A_\mu + evA_\mu\phi_1 + \frac{1}{2}eA_\mu\phi_1^2 + \frac{1}{2}eA_\mu\phi_2^2 + v\partial_\mu\phi_1\right)\left(\frac{1}{2}\partial_\nu\phi_1^2 + \frac{1}{2}\partial_\nu\phi_2^2 + \frac{1}{2}ev^2A_\nu + evA_\nu\phi_1 + \frac{1}{2}eA_\nu\phi_1^2 + \frac{1}{2}eA_\nu\phi_2^2 + v\partial_\nu\phi_1\right)$$

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$$g^{\mu\upsilon}(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2) = \frac{1}{4}g^{\mu\upsilon}e^2v^4A_\mu A_\nu + g^{\mu\upsilon}e^2v^2A_\mu A_\nu\phi_1^2 + \frac{1}{4}g^{\mu\upsilon}e^2A_\mu A_\nu\phi_1^4 + \frac{1}{4}g^{\mu\upsilon}e^2A_\mu A_\nu\phi_2^4 + \frac{1}{4}g^{\mu\upsilon}e^2A_\mu A_\nu\phi_1^2\phi_2^2 \quad (9)$$

In equations (9), it can be observed that the gauge fields, $A_\mu A_\nu$ (representing neutral mass) acquire Ricci scalar curvature by the magnitude of $R^2 = e^2v^4$ . The bosonic term, $\phi_1^2$ acquires Ricci scalar curvature as well $R = ev^2$. The bosonic interaction term, $\phi_1^2\phi_2^2$ as well as the self-interaction terms, $\phi_1^4$ and $\phi_2^4$ exists in equation (9). Considering the case where the Liouville field coupling strength is weakened, $b \rightarrow 0$, then the parameter, $e \rightarrow 1$. This then corresponds to the curvature, $R = v^2$. The $\frac{1}{4}g^{\mu\upsilon}D_\mu(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2)$ of the Lagrangian would then take the following form:

$$\begin{aligned} g^{\mu\upsilon}D_\mu(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2) &= g^{\mu\upsilon}\frac{v^4}{4}A_\mu A_\nu + g^{\mu\upsilon}v^2A_\mu A_\nu\phi_1^2 + \frac{1}{4}g^{\mu\upsilon}A_\mu A_\nu\phi_1^4 + \frac{1}{4}g^{\mu\upsilon}A_\mu A_\nu\phi_2^4 + \frac{1}{4}g^{\mu\upsilon}A_\mu A_\nu\phi_1^2\phi_2^2 \dots \\ &= \frac{R^2}{4}g^{\mu\upsilon}A_\mu A_\nu + Rg^{\mu\upsilon}A_\mu A_\nu\phi_1^2 + \frac{1}{4}g^{\mu\upsilon}A_\mu A_\nu\phi_1^4 + \frac{1}{4}g^{\mu\upsilon}A_\mu A_\nu\phi_2^4 + \frac{1}{4}g^{\mu\upsilon}A_\mu A_\nu\phi_1^2\phi_2^2 \dots \end{aligned} \quad (10)$$

In the scenario when the Liouville field coupling strength is weakened, the acquisition of Ricci scalar curvature by the bosonic term, $\phi_1^2$ becomes significant. In addition, the bosonic interaction and self-interaction terms acquire the local gauge fields, $A_\mu A_\nu$. Since the local gauge fields represent particle mass, therefore the interacting bosonic terms acquire mass.

## 4. Gauge Fields with Charged Matter

In this section, the case where particles entering the event horizon have mass and charge is considered. In such cases, the field electrodynamics of a fermion could be obtained using Dirac equation – where the action of the Dirac field is:

$$S = \int \overline{\psi}\left(i\hbar c\gamma^\mu\partial_\mu - mc^2\right)\psi d^4x \quad (11)$$

The associated gauge group for the field is $U(1)$ which is the rotation of the phase angle, $\theta$ of the field. Localizing the global symmetry of the system, $\psi \longmapsto e^{i\theta}\psi$ by defining $\theta = \theta(x)$, the covariant derivative is obtained:

$$D_\mu = \partial_\mu - i\frac{q}{\hbar}A_\mu \quad \text{and} \quad D_\nu = \partial_\nu - i\frac{q}{\hbar}A_\nu \quad (12)$$

To incorporate the curvature parameter, $e = \lambda^b$ into equation (12), the parameter, $k_0$ in units (C/J.s) (electric current per unit energy) is defined and the covariant derivative takes the following form:

$$D_\mu = \partial_\mu + \left(e - ik_0\frac{q}{\hbar}\right)A_\mu \quad \text{and} \quad D_\nu = \partial_\nu + \left(e - ik_0\frac{q}{\hbar}\right)A_\nu \quad (13)$$

The introduction of the parameter, $k_0$ is necessary to maintain the non-dimensional nature of the curvature expression in equation (13). Then under local gauge transformations as in equation (8), The obtained gauge field terms when the operation $D_\mu(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2)$ in the Lagrangian, $L' \rightarrow L'(\phi_1,\phi_2)$ (see equation (7)) is performed for charged massive gauge fields. The $\frac{1}{4}g^{\mu\upsilon}D_\mu(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2)$ part of the Lagrangian is obtained as follows:

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$$\frac{g^{\mu\upsilon}}{4}D_{\mu}(\phi_1,\phi_2)D_{\nu}(\phi_1,\phi_2)$$
$$=\frac{g^{\mu\upsilon}}{16}\left(e^2v^4-\frac{qk_0v^4}{\hbar}-\frac{2iqk_0v^4}{\hbar}\right)A_{\mu}A_{\nu}$$
$$+\frac{g^{\mu\upsilon}}{4}\left(e^2v^2-\frac{qk_0v^2}{\hbar}-\frac{2iqk_0v^2}{\hbar}\right)A_{\mu}A_{\nu}\phi_1^2$$
$$+\frac{g^{\mu\upsilon}}{16}\left(e^2-\frac{qk_0}{\hbar}-\frac{2iqk_0}{\hbar}\right)A_{\mu}A_{\nu}\phi_1^4+\frac{g^{\mu\upsilon}}{16}\left(e^2-\frac{qk_0}{\hbar}-\frac{2iqk_0}{\hbar}\right)A_{\mu}A_{\nu}\phi_2^4$$
$$++\frac{g^{\mu\upsilon}}{16}\left(e^2-\frac{qk_0}{\hbar}-\frac{2iqk_0}{\hbar}\right)A_{\mu}A_{\nu}\phi_1^2\phi_2^2 \tag{14}$$

Similar to the neutral mass case, in equations (14), it can be seen that the gauge field term, $A_{\mu}A_{\nu}$ representing charged mass acquires Ricci scalar curvature on the real part by the magnitude of $R^2=e^2v^4$. If the Liouville field coupling strength is weakened, $b\rightarrow 0$, then the parameter, $e\rightarrow 1$. This would then correspond to the Ricci scalar curvature, $R=v^2$. Equation (14) is then expressed as follows:

$$\frac{g^{\mu\upsilon}}{4}D_{\mu}(\phi_1,\phi_2)D_{\nu}(\phi_1,\phi_2)$$
$$=\frac{g^{\mu\upsilon}}{16}R^2\left(1-\frac{qk_0}{\hbar}-\frac{2iqk_0}{\hbar}\right)A_{\mu}A_{\nu}+\frac{g^{\mu\upsilon}}{4}R\left(1-\frac{2iqk_0}{\hbar}-\frac{qk_0}{\hbar}\right)A_{\mu}A_{\nu}\phi_1^2$$
$$+\frac{g^{\mu\upsilon}}{16}\left(1-\frac{qk_0}{\hbar}-\frac{2iqk_0}{\hbar}\right)A_{\mu}A_{\nu}\phi_1^4+\frac{g^{\mu\upsilon}}{16}\left(1-\frac{qk_0}{\hbar}-\frac{2iqk_0}{\hbar}\right)A_{\mu}A_{\nu}\phi_2^4$$
$$++\frac{g^{\mu\upsilon}}{16}\left(1-\frac{qk_0}{\hbar}-\frac{2iqk_0}{\hbar}\right)A_{\mu}A_{\nu}\phi_1^2\phi_2^2 \tag{15}$$

The overall mass term represented by the gauge fields, $A_{\mu}A_{\nu}$ is split into individual real and complex terms. It can be observed that there are two real field terms with Ricci curvature, $\frac{g^{\mu\upsilon}}{16}R^2A_{\mu}A_{\nu}$ and $-\frac{g^{\mu\upsilon}qk_0}{16\hbar}R^2A_{\mu}A_{\nu}$ as well as a complex term (with Ricci curvature): $-\frac{g^{\mu\upsilon}iqk_0}{8\hbar}R^2A_{\mu}A_{\nu}$. One of the real field terms do not acquire charge and hence is a neutral particle: $\frac{g^{\mu\upsilon}}{16}R^2A_{\mu}A_{\nu}$ as opposed to the other two terms. This can be observed in the bosonic terms, $\phi_1^2$ as well as the bosonic interaction terms: $\phi_1^4,\phi_2^4$ and $\phi_1^2\phi_2^2$. A possible interpretation of this result is that the real part of the terms contributes directly to the acquired Ricci curvature, $R$ and charge $q$, while the contribution of the complex term is solely dependent and proportional to the magnitude of the electric charge, $q$ of the particle.

In current and previous sections, the region beyond the event horizon of black holes is assumed to have two-spatial dimensions with no time-dimension. In this picture, matter and energy pass the event horizon travel to a zone of 'no return' becoming non-measurable to an external observer. Additionally with the assumption no time dimension, the field dynamics beyond the event horizon exists in a complicated 'frozen' state with respect to the external observer. The event horizon acts as a boundary between this two-dimensional region and conventional three-dimensional space. The next section theoretically discusses the quantum dynamics at this boundary.

## 5. Quantum Dynamics at the Event Horizon

The previous sections describe symmetry breaking mechanisms occurring beyond the event horizon. In this section, the quantum dynamics occurring exactly at the at the event horizon is theoretically explored. Thus far the region beyond the event horizon is considered as a two-dimensional surface (spatial dimensions) with no time-dimension. One possible two-dimensional structure that can exist and be embedded into three-dimensional space is the Möbius strip. The idea of a spatial two-dimensional framework existing beyond the event horizon is extended to the event horizon itself. The event horizon is then assumed to be an Möbius strip with two spatial dimensions and one parametric time dimension. In addition, it is conjectured that this Möbius strip structure has multiple twists. It is important to note that at this boundary, matter or energy has not yet crossed the event horizon – i.e., it has not entered the zone of 'no return'. Hence unlike the previous sections, the effects of the quantum dynamics exactly at the event horizon are theoretically measurable (since this boundary has not been passed). The following projection of formulas is formulated for the event horizon parametrized as a Möbius strip with multiple twists in three-dimensional space:

$$x(s,t) = \left(r + s\cos\left(\frac{t}{2}\right)\right)\cos t$$
$$y(s,t) = \left(r + s\cos\left(\frac{t}{2}\right)\right)\sin t$$
$$z(s,t) = s\sin\left(\frac{t}{2}\right) \quad (16)$$

where $x, y, z$ represent three-dimensional space, $t \in [0, 2n\pi]$ and $s \in [-d, d]$ are the parametric positions along he Möbius strip. $r > 0$ is the radius of the Möbius strip and $n \in \mathbb{R}[1, \infty)$ is the parameter for the number of twists. The configuration of the Möbius strip could be modified using the two parameters: $r$ and $n$. In this viewpoint, $r$ is the Schwarzschild radius (i.e., radius of the event horizon (Möbius strip) determined using: $r = 2GM/c^2$; where $G$ is the gravitational constant, $M$ is the mass of the black hole and $c$ is the speed of light. The expression for the Gaussian and mean curvatures of the event horizon are [12]:

$$K = \frac{-4r^2}{\left[4r^2+3s^2+2s\left(4r\cos\left(\frac{1}{2}t\right)+s\cos t\right)\right]^2}$$
$$H = \frac{2\left[2(r^2+s^2)+4rs\cos\left(\frac{1}{2}t\right)+s^2\cos t\right]\sin\left(\frac{1}{2}t\right)}{\left[4r^2+3s^2+2s\left(4r\cos\left(\frac{1}{2}t\right)+s\cos t\right)\right]^2} \quad (17)$$

Using the Lichnerowicz–Obata theorem, the curvatures are then employed to obtain the first energy eigenvalue, $\lambda_1$ for a quantum particle (electron) at the event horizon [13,14]. In this scenario, the gravitational potential of the black hole presents itself as space-time curvature at the event horizon. Hence, the electron's interactions with potentials other than this gravitational field are deemed insignificant in comparison (i.e., considered negligible). Thus, an electron on the event horizon can be treated as a free particle. Considered as a Möbius strip, the structure of the event horizon is defined as a compact Riemannian manifold without boundary, $M$. Taking the electron as a free particle, the dimensionless Schrödinger equation with zero potential is considered for this analysis:

$$-\frac{\hbar}{2m}\partial_x^2\psi = i\partial_t\psi \quad (18)$$

where $\hbar$ is the reduced Planck constant and $m$ is the electron mass. Using the Laplace-Beltrami operator, equation (18) is represented as an eigenvalue equation:

$-\Delta\psi = \lambda\psi$ with the Laplace-Beltrami operator:

$$\Delta = \frac{1}{\sqrt{|g|}}\frac{\partial}{\partial x^j}\left(\sqrt{|g|}\frac{\partial}{\partial x^j}\right) \quad (19)$$

with the spatial dimensions, $x^j : j \in [1,3]$ and real-valued the energy eigenvalue, $\lambda$. Therefore, the energy eigenvalue, $\lambda \in \mathbb{R}$ is represented as follows: $\lambda = 2mi\lambda'\hbar^{-1}$. The auxiliary wavelength, $\lambda' \in \mathbb{C}$ is a complex term to enable real-valued energy eigenvalues, $\lambda$. The energy eigenvalues, $\lambda$ could be proved to be real valued using the self-adjointness property and the compactness of the manifold, $M$ could be employed to prove the discreet and finite nature of the eigenvalues. Using integration by parts in consideration of the Laplace-Beltrami operator, $-\Delta$, it could be shown that the eigenvalue, $\lambda \geq 0$:

$$-\int_M \Delta\psi\,\psi\,dV = \lambda\int_M \psi^2\,dV = \int_M |\nabla\psi|^2\,dV$$

where

$$dV = \mathrm{vol}_n\,, \quad \therefore \lambda \geq 0 \quad (20)$$

For a given compact $n$-dimensional manifold with no boundary where $n \geq 2$, the Ricci curvature is assumed to satisfy the lower bound for a tangent vector, $X$, a metric tensor, $g(.,.)$ and the curvature, $\kappa$: $R(X,X) \geq \kappa g(X,X)$ where $\kappa \geq 0$ [13]. Following this assumption, the first positive energy eigenvalue with respect to equations (18) and (19) for a Möbius strip:

$$\lambda_1 \geq \frac{n}{n-1}\kappa \qquad \text{where} \qquad n = 2 \quad (21)$$

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The expressions if the curvature is taken as the Gaussian curvature, $\kappa = K$ and the mean curvature, $\kappa = H$ are respectively as follows:

$$\lambda'_1 \geq \frac{4i\hbar r^2}{m\left[4r^2 + 3s^2 + 2s\left(4r\cos\left(\frac{1}{2}t\right) + s\cos t\right)\right]^2}$$

$$\lambda'_1 \geq -\frac{2\hbar i\left[2(r^2+s^2)+4rs\cos\left(\frac{1}{2}t\right)+s^2\cos t\right]\sin\left(\frac{1}{2}t\right)}{m\left[4r^2+3s^2+2s\left(4r\cos\left(\frac{1}{2}t\right)+s\cos t\right)\right]^2} \quad (22)$$

where $r = 2GM/c^2$ is the Schwarzschild radius.

## 6. Analysis & Conclusions

This manuscript investigates quantum fields at regions beyond the event horizon and quantum dynamics at the event horizon. Figure 1 presents a schematic diagram of the regions around the black hole discussed in this work:

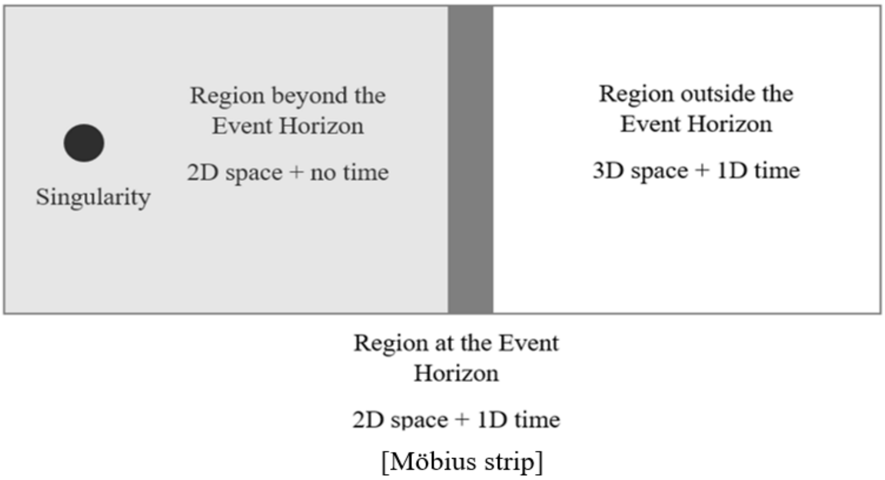


**Figure 1.** Schematic diagram of the regions around the black hole.

The first part of this work concerns the physics of fields in the region beyond the event horizon of black holes. The mathematical framework presented in Ganesan (2023) [1] was extended to describe the field dynamics of neutral and charged matter (as gauge fields) beyond the event horizon. This is carried out by studying the two-dimensional symmetry breaking outcomes. In can be observed for the case of neutral matter, Ricci scalar curvature is acquired by the bosonic term while the interacting bosonic terms acquire mass (equation (10)). As for the scenario involving charged matter, three massive field terms with Ricci curvature are obtained. In each group of terms: massive field, bosonic (and bosonic interaction terms), one real field term is a neutral particle where all the other terms acquire charge by the magnitude of $q$. These results may help elucidate the field mechanisms involving particle dynamics beyond the event horizon of black holes.

The second part of this work explores the quantum dynamics at the event horizon by treating it as a Möbius strip with multiple twists, $n$. As mentioned in the previous section, exactly at this region certain physical properties are measurable to the external observer. The central outcome of this analysis is the expressions for the Gaussian and mean curvatures of the event horizon. This then results in the formulation for determining the lower bound of the first energy eigenvalue of a quantum particle (electron) at the event horizon. Due to intense gravitational forces and high temperatures, material in the form of plasma exists as an accretion disk at the event horizon and regions surrounding it [15,16]. This plasma emits radiation in a wide range of wavelengths including X-rays and gamma rays. In plasma phase, the electrons are stripped away from their nucleus and completely under the influence of the gravitational field (representing as space-time curvature). The first energy eigenvalue solution to the Schrödinger equation with zero potential (and the Laplace-Beltrami operator) within the context of the Lichnerowicz–Obata theorem provides a theoretical

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estimate of plasma characteristics near the event horizon (accretion disk). Considering the event horizon as a Möbius strip, two parameters come into play: the width of the strip, $d$ (where $s \in [-d, d]$) and the number of twists, $n$. The Schwarzschild radius, $r$ is determined by the mass of the black hole, $M$. It may be conjectured that the width of the strip, $d$ (where $s \in [-d, d]$) and the number of twists, $n$ are respectively correlated to the linear momentum and angular momentum of a rotating black hole. This conjecture would require further empirical validation with observatory data. If such a correlation is established, then the angular momentum and linear momentum of rotating black holes could be computed by obtaining observatory data related to wavelength ranges of radiation emission (X-rays and gamma rays) of the plasma at the accretion disk near the event horizon. Future research work in this direction will be carried out by the author of this work. In addition, considering the event horizon as an Möbius strip may also provide a possible explanation as to the non-existence of magnetic fields at black holes due to the no-hair theorem [17]. Conductive material around the event horizon in this view causes it to act as Möbius resistor – where it cancels its own self-inductance. Therefore, despite having electrical charge, the region around the event horizon of black holes has no measurable magnetic property.

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