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Article

An Evolutionary Analysis of Higher-Order Interaction Collaborative Innovation Networks in China's New Energy Vehicle Industry

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Abstract: As a manifestation of technological innovation achievements, patents reflect the frontier of technological development in the field. This paper constructs a multi-entity patent collaboration network and analyzes its evolution patterns to provide a reference for innovation organizations to grasp the general direction of innovation development and select partners. Based on the data of collaboration patents of two and three organizations in China's new energy vehicles (NEV) industry from 2008 to 2021, this paper constructs higher-order interaction collaborative innovation networks in different periods to explore the spatial evolution of networks and patent cooperation activities. The results show that the generalized degree distribution of nodes and edges conforms to the power-law distribution. The two-dimensional simplex networks gradually form triangle groups centered on the core nodes, and the importance of key edges is beginning to emerge. Key organizations and partners, including State Grid Corporation of China, Tsinghua University, and China Electric Power Research Institute, will largely determine the direction for the future development of cooperation innovation. The study also reveals that Beijing, Jiangsu, Zhejiang, Guangzhou, and Shanghai are in central positions in the cooperative innovation network, based on their cooperation breadth and depth measures. Additionally, The key links established by Beijing as hubs not only serve as centers for information exchange but also act as bridges connecting to other regions.

Keywords: new energy vehicle; higher-order interaction network; generalized degree; simplicial complex

1. Introduction

Under the general goal of the country's innovation-driven development strategy, efficient innovative collaboration alliances play an important role in promoting economic and technological development. Inter-organizational cooperation depends on industrial innovation networks. The industrial innovation network, as an important carrier for the diffusion of innovation resources can provide support for members to complement advantages and share risks. Since patents contain knowledge, technology, economic, legal, and other information, they have become important information that reflects the trends of technological innovation [1–3]. According to the report of WIPO, patents are the largest sources of information technology and cover over 90% of global research and development (R&D) outputs. Compared with other information resources, patent information is characterized by novel content, similar systems, standardized format, and scientific classification [4].

Patent cooperation application means that two or more individuals or organizations apply to the National Patent Office for a patent for the same invention on the same date. It is determined through consultation that they jointly enjoy the right to apply for the patent. A patent cooperation network formed by entities through joint patent application in the process of cooperative innovation is the most important way to evaluate the innovation cooperation and expansion of knowledge. Entities can reduce the cost of knowledge acquisition, share research and development risks, and improve the probability of innovation success through a patent cooperation network [5]. Recently, studies of patent cooperation networks are receiving considerable attention. Focusing on the wind

energy industry in China, Liu [6] constructed and analyzed the cooperation network using patent cooperation data. Wang and Zhang [7] utilize Chinese biomedical collaborative patent data among entities to establish inter-regional cooperation innovation networks. Previous studies have analyzed collaborative innovation networks based on complex network theory. However, they have a limitation in capturing complex interactions among multiple elements of a complex system. In the paper, we attempt to build higher-order interaction cooperation innovation networks to describe many-body interactions.

Developing the new energy vehicle (NEV) industry contributes to reducing air pollution and decreasing dependence on oil resources [8,9]. Recently, the NEV industry has emerged as a national strategic focus in major automobile countries such as the United States, Germany, Japan and China [10]. To accelerate the development of the NEV industry, a wide range of policies has been formulated and implemented since 2010, covering macro-level policies, R&D initiatives, management systems, and financial measures [11]. The "Development Plan of the New Energy Automobile Industry (2021-2035)" was officially enacted in 2020. In terms of China, it is vital to promote the NEV industry transformation and advance the development of the NEV industry [10]. Given the current situation, it has become an important goal to monitor and analyze the development trend of the NEV industry. Nevertheless, rare prior research pays attention to the NEV industry's cooperation innovation from the perspective of higher-order networks. This research seeks to construct higher-order interaction cooperation innovation networks within the NEV field in China, with the goal of identifying key innovative organizations and their cooperative relationships. By analyzing the trend of patent collaboration and exploring regional distribution patterns, this paper aims to explore the potential of technological innovation in the NEV industry and provide valuable insights for decision-making.

The remaining sections of the article are arranged as follows: Section 2 presents a comprehensive overview of previous studies. In Section 3, we represent some basic properties of higher-order interaction collaborative innovation networks and our methodology. Section 4 describes the data and divides network phases. The higher-order network is constructed and analyzed in Section 5. In Section 6, we identify key nodes and key partnerships. Section 7 analyses regional cooperation networks. Section 8 gives conclusions and some ideas for future work.

2. Literature Review

2.1. New energy vehicle industry

The transportation sector is responsible for one-third of the world's total energy consumption, making energy conservation in this sector a matter of widespread concern [12]. Among various modes of transportation, road transport is the primary contributor to global warming and air pollution [13]. Notably, vehicle exhaust is widely recognized as one of the major sources of severe air pollution in China. Mainstream countries formulate and implement strategic plans to encourage the development of the domestic NEV industry [14]. For the Chinese government and original equipment manufacturers (referred to as OEMs), the NEV industry provides a historic opportunity [15]. The goal is to enhance innovation capability and accomplish the transformation of the automotive industry [16]. However, since the majority of energy storage devices are still in the early phases of development in China [17], more efforts should be paid to enhance the development of NEV from the technological perspective [9]. Therefore, it is necessary to implement corresponding technical innovation measures to address these technological challenges.

Obviously, the NEV industry has become a significant research focus, and numerous scholars have conducted extensive research on related topics. While most studies focus on the interpretation and evaluation of policies, environmental benefits and user needs. Few scholars study China's NEV from the perspective of organizational cooperation. The exploring of collaboration innovation for NEV can promote technological cooperation between organizations. Furthermore, it can achieve technology sharing and collaborative innovation, thus improving the level of NEV technology.

2.2. Patent collaboration network

Evaluating collaborative patents helps researchers understand the process of expansion of knowledge [18-20]. With the development of the social network analysis method, more research regards network science as an important tool for studying technological collaborations [21]. Ramos [22] demonstrated that participating in national or regional collaboration networks positively impacts R&D activities. Xiang et al. revealed that international innovation collaboration has facilitated the transfer of technology and the sharing of knowledge among countries [23-24]. For emerging industries such as the NEV industry, it is of great importance to form a collaborative innovation network for promoting sustainable development. Therefore, this study aims to construct and visualize a patent cooperation network in China's NEV industry to gain comprehensive insights into the patterns and dynamics of collaborative innovation.

Clarifying the geographical location of patent applicants helps to understand the distribution and flow of knowledge, providing a basis for researchers to conduct technical management and formulate policies [25]. Sun and Cao [26] constructed a patent collaboration network model of intra-regions and inter-regions in China during the period 1985-2008. Sun [27] also discovered that colleges and state-owned enterprises served as central entities in regional cooperation networks. These studies concentrate on the types of innovative entities and the distribution of patent cooperation networks, revealing the position of different organizational types and regions in the network [6]. Plainly, it is important to investigate the patent cooperation network in China's NEV industry, considering the network's components and regional distribution [28].

A patent cooperation network serves as a communication platform for patent inventors to connect with relevant experts and collaborative innovation partners, facilitating the exchange of external knowledge [1]. Scholars have extensively researched collaboration types, characteristics, motivations, and other aspects from different perspectives [29]. However, limited attention has been given to the patent cooperation network within the context of the NEV industry.

2.3. Social network analysis

Social network analysis (SNA) provides a framework for studying and analyzing the relationships and interactions among entities. It is extensively utilized in various disciplines such as sociology, management science, statistics, graph theory, and many others [30]. With the development of social analysis tool visualization, the research of the patent cooperation application network receives increased attention from scholars. Liu et al. [31] analyze how new organizations join the network and how existing organizations form connections based on preferential attachment. Liu et al. [29] utilize the complex network theory and social network analysis (SNA) method to examine the characteristics and structure of the patent cooperation network in the field of SG in China. Yin et al. [32] study technological collaborations and analyze CCS patents through a patent cooperation network all over the world. Liu et al. [33] analyze the characteristics of the international technological collaboration network in the ICM industry.

Collaborative innovation research often employs complex network analysis as a major social network analysis (SNA) method. In this method, if a patent has two applicants, there will be an edge between the two applicants. However, due to the characteristics of the complex network structure, there are limitations in accurately depicting the interactions among members. For instance, when three or more enterprises jointly apply for a patent, it can be challenging to express the cooperative relationship between multiple enterprises using complex network analysis. Some scholars began to use hypergraphs to describe cooperative networks [34]. A hyperedge is a collection of nodes that can connect any number of nodes in a hypergraph [35]. The properties of nodes can be different, and nodes in the same hyperedge are completely connected. Zhao et al. [36] constructed a dynamic model that includes an enterprise hypergraph and a knowledge hypergraph. They used multiple entities to simulate the process of knowledge creation and diffusion.

Taken together, there have been significant achievements in using SNA to study patent cooperation in the industrial field. However, existing research on cooperative networks typically involves selecting a specific number of patent collaborators, whereas the number of patent

collaborators in the real world is often random [37]. In this paper, we attempt to explore and investigate higher-order interaction cooperation innovation networks for China's NEV.

3. Construction of Higher-Order Interaction Cooperation Innovation Networks

3.1. Basic Properties of Higher-Order Networks

A complex system can be represented by a network, which is essentially a graph denoted as $G(V, E)$, where the set of nodes V represents the system's elements, and the set of links E represents their interactions [38]. Networks are pivotal to capturing the architecture of complex systems. However, they cannot capture higher-order interactions. In a complex system, to encode the many-body interactions between the elements, higher-order networks need to be used. A powerful mathematical framework adopted to describe higher-order networks is provided by simplicial complexes [39]. Simplicial complexes are formed by a set of simplices. The simplices indicate the interactions existing between two or more nodes and are defined as follows:

A d -dimensional simplex α (also indicated as a d -simplex) is formed by a set of $(d+1)$ interacting nodes [40]

$$\alpha = [v_0, v_1, \dots, v_d] \quad (1)$$

It describes a many-body interaction among the nodes, enabling both a topological and a geometrical interpretation of the simplex. For instance, a node is a 0-simplex, a link is a 1-simplex, a triangle is a 2-simplex a tetrahedron is a 3-simplex and so on [41] (see Figure 1).

A face of a d -dimensional simplex α is a simplex α' formed by a proper subset of nodes the simplex, i.e. $\alpha' \subset \alpha$. For instance, the faces of a 2-simplex $[v_0, v_1, v_2]$ include three nodes $[v_0], [v_1], [v_2]$ and three links $[v_0, v_1], [v_0, v_2], [v_1, v_2]$. Similarly, we characterize the faces of a tetrahedron in Figure 2 [40].

This paper studies the cooperation relationship on the patent application networks, taking the patent applicants as the network nodes. The 1-simplex means that two organizations apply for a patent jointly, and the 2-simplex means that three organizations apply for a patent jointly. According to the cooperation relationship between the innovation entities, the pure 1-simplex cooperation network composed of only 1-simplex and the pure 2-simplex cooperation network composed of only 2-simplex are constructed respectively [42].

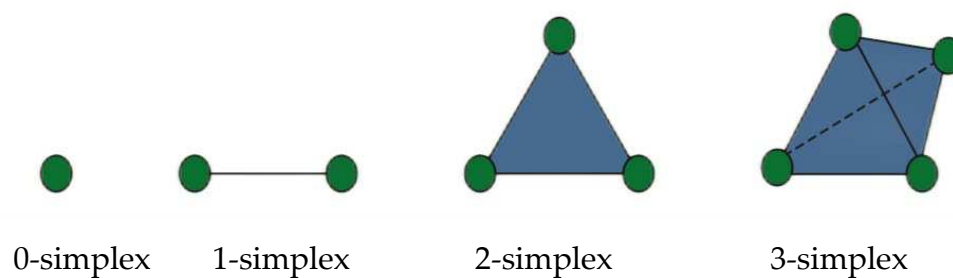


Figure 1. Simplexes with different dimensions.

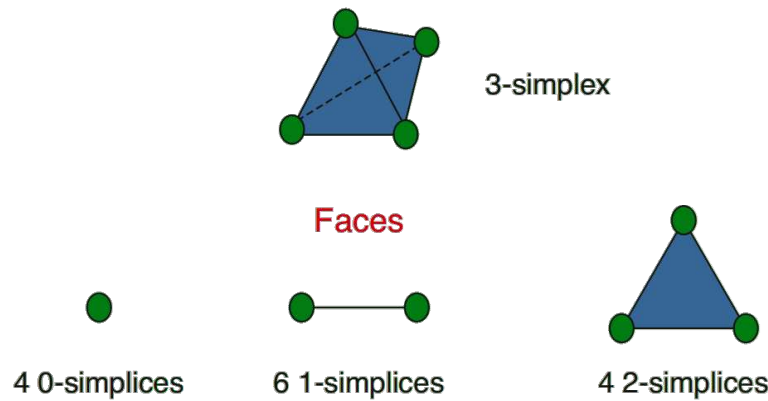


Figure 2. Different faces of a 3-simplex (tetrahedron).

The number of nodes reflects the scale of the higher-order interaction cooperation innovation networks. The weight of each 1-dimensional simplex represents the number of patents jointly applied by the two organizations. The number of 1-dimensional simplexes on the 1-dimensional simplex network indicates the number of links formed by two innovators. Similarly, the weight of each 2-dimensional simplex represents the number of patents jointly applied by the three applicants represented by these three nodes. The number of 2-dimensional simplexes indicates the number of triangles formed by three applicants, and so on for higher-order interactions.

3.2. Generalized Degree

Inter-organizational differences in innovation resources, innovation capabilities, and innovation environment vary the importance of members in the patent application cooperative innovation network. The key nodes and key cooperation relationships that occupy an important position in the network often play a crucial role in the development of the innovation network. Degree centrality as the most direct indicator of node centrality in complex networks can describe node importance to identify significant nodes. The degree centrality of nodes in a collaboration network positively affects their innovation performance. The greater the degree of a node, the higher its degree centrality, and the more partners. It is natural to desire to extend the concept of degrees to simplicial complexes.

Additionally, the generalized degree $k_{d,m}(\alpha)$ of an m -dimensional simplex α indicates the number of d -dimensional simplexes incident to the m -simplex α . The higher-order networks constructed in this paper are pure d -dimensional simplex networks. The generalized degrees $k_{d,m}(\alpha)$ of a pure d -dimensional simplicial complex can be represented in terms of the adjacency tensor $a^{[d]}$ as [43- 44]:

$$k_{d,m}(\alpha) = \sum_{\alpha' \in Q_d(N) | \alpha \supset \alpha'} a_{\alpha'}^{[d]} \quad (2)$$

Where $Q_d(N)$ is the set of all possible and distinct d -dimensional simplexes including N nodes. α is a simplex composed of a subset of nodes of the simplex α' ($\alpha' \supset \alpha$). The adjacency tensor $a^{[d]}$ has elements $a_{\alpha'}^{[d]} \in \{0,1\}$ indicating, for every possible d -dimensional simplex $\alpha' \in Q_d(N)$, if the simplex is present ($a_{\alpha'} = 1$) or absent ($a_{\alpha'} = 0$).

The generalized degree of the node on the 1-dimensional simplex $k_{1,0}([r])$ represents the number of times that the organization r participated in applications of the patents jointly applied by two innovation organizations. $k_{2,0}([r])$ represents the number of times that the organization r participated in the joint patent applications involving three organizations. The higher the centrality of key nodes in the network, the more significant their position will hold. Establishing cooperative relationships with more innovative organizations is conducive to resource accumulation and promotes the flow and allocation of knowledge, information, technology, and other innovation elements between nodes. Key nodes with high centrality play a leading and organizational role in innovation aggregation, utilizing their resource advantages. A pure 2-dimensional simplicial

complex patent application network is formed by a set of triangles, links, and nodes. The importance of innovation entities in pure 2-complex patent application networks can be described as generalized degree $k_{2,1}([r,s])$ of the 2-dimensional simplicial complex. $k_{2,1}([r,s])$ represents times of organization r and organization s joint patent application. The larger the $k_{2,1}([r,s])$, the more organizations that innovation organization r and organization s cooperate with. Therefore the generalized degree can be an indicator to identify critical partners in the 2-simplex patent cooperation application network [45]. To identify key nodes and cooperative partners, this paper extends the notion of degrees in complex networks to the generalized degree in the simplicial complex.

3.3. Spatial analysis of patent cooperation activities

We analyze the spatio-temporal evolution characteristics of industrial patent cooperation application activities from the view of pure simplicial complexes. To investigate the influence of regional boundaries on patent cooperation, the study first counts the number and proportion of patents applied through internal cooperation in different regions (provinces, cities). Secondly, the evolutionary network of interregional cooperation applying for patents is constructed to analyze spatial patterns of cross-regional patent cooperation activities at different stages. Finally, we introduce the "cooperation breadth - cooperation depth" two-dimensional matrix to classify patent applicant regions into four categories. This classification can provide a theoretical basis for the government to formulate regional industrial innovation policies and for enterprises to make informed decisions.

4. Data and Life Cycle Division

In this study, the data utilized in this study were sourced from the website of the State Intellectual Property Office of the People's Republic of China (SIPO)¹[46]. This article enters the search formula "Abstract=new energy vehicle OR instructions=new energy vehicle" on the SIPO database. The patent application time is before December 31, 2021. The patent applicants include two or more keywords: university, college, school, enterprise, group, factory, research institute, and research institute. Then, filter all results to determine whether they belong to the new energy vehicle industry, and carry out missing value processing and manual proofreading. Finally, this paper obtains a total number of 1484 collaborative patents in the NEV field. The curves representing the increasing number of patents and collaborative patents, as well as the cumulative number of patents and collaborative patents, are plotted in chronological order, as depicted in Figure 3.

The earliest retrieved data on collaborative patents in the NEV industry was in 2008. From 2008 to 2012, the number of patents applied and jointly applied patents were small, and the growth was relatively slow. The increasing rate of patents and collaborative patents accelerated from 2013 to 2016. It is obvious that the growth rate of the cumulative number of patents and collaborative patents further accelerated from 2017 to 2020. Therefore, this paper divides the patent cooperation network of China's NEV industry into three phases: 2008-2012, 2013-2016, and 2017-2021.

¹<http://www.sipo.gov.cn>.

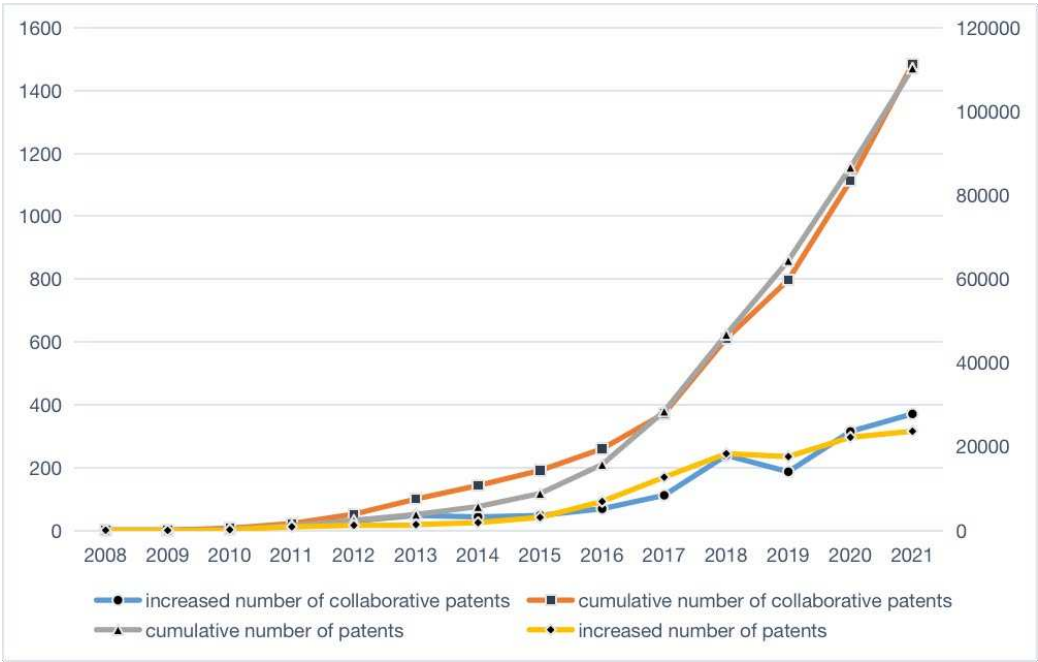


Figure 3. Patent application trend from 2008 to 2021.

5. Evolution Analysis of Higher-order Network

5.1. Higher-order collaboration network dynamics

If innovation organizations are regarded as network nodes and two innovation organizations cooperate to apply for a patent, the two nodes form a one-dimensional simplex (link). If three innovation organizations cooperate to apply for a patent, the three nodes form a two-dimensional simplex (triangle). The applicants for a patent include four organizations and these four nodes form a three-dimensional simplex (tetrahedron), and so on. The weight of simplex represents the number of collaborative patents by the organizations, and thus the sum of all simplex weights represents the total number of collaborative patents. Table 1 shows collaborative patents of each dimension simplex on the higher-order network in the NEV industry constructed. Collaborative patents on one-dimensional simplex and two-dimensional simplex account for 77. 2% and 16.7% of the higher-order collaborative network respectively. The number of three-dimensional simplexes and higher-dimensional simplexes is relatively small, indicating that patent cooperation in the NEV industry is highly competitive and exclusive. It is difficult to find statistical rules from the small number of simplexes. Therefore, this paper mainly focuses on studying the characteristics of patent cooperation on one-dimensional and two-dimensional simplexes.

Table 1. Collaborative patents in each dimension simplex.

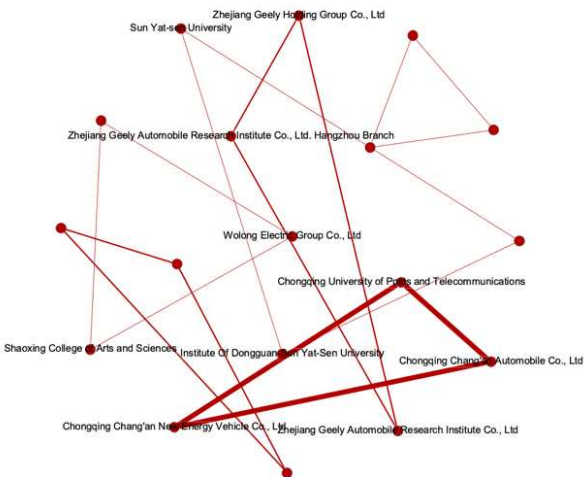
Dimension	1	2	3	4	5	6	7
Patent applications	1145	247	66	21	2	1	1

This section utilizes the Gephi software to generate patent collaboration network maps of the NEV industry, specifically for applicants on one-dimensional and two-dimensional simplex networks during three time periods: 2008-2012, 2013-2016, and 2017-2021. The resulting network maps are displayed in Figure 4. The nodes represent patent applicants, which are composed of a variety of organizations including enterprises, universities, and scientific research institutes. The links between the two nodes indicate represent the collaborative relationship between two patent applicants. Similarly, the triangles represent the cooperative application relationship of three patent applicants. In the patent collaboration network maps shown in Figure 4, the thickness of the edges or

triangles between nodes represents the frequency of collaboration between two or three patent applicants. The thicker the edge or triangle, the higher the frequency and intensity of collaboration between the respective patent applicants.



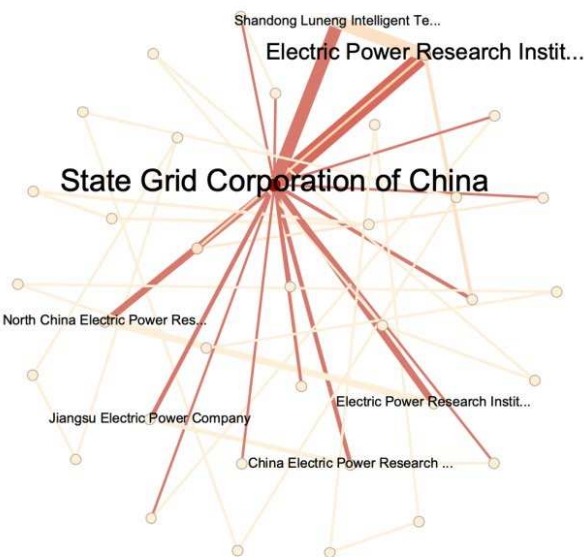
(a). 1-simplex network in 2008-2012



(b). 2-simplex network in 2008-2012



(c). 1-simplex network in 2013-2016



(d). 1-simplex network in 2013-2016

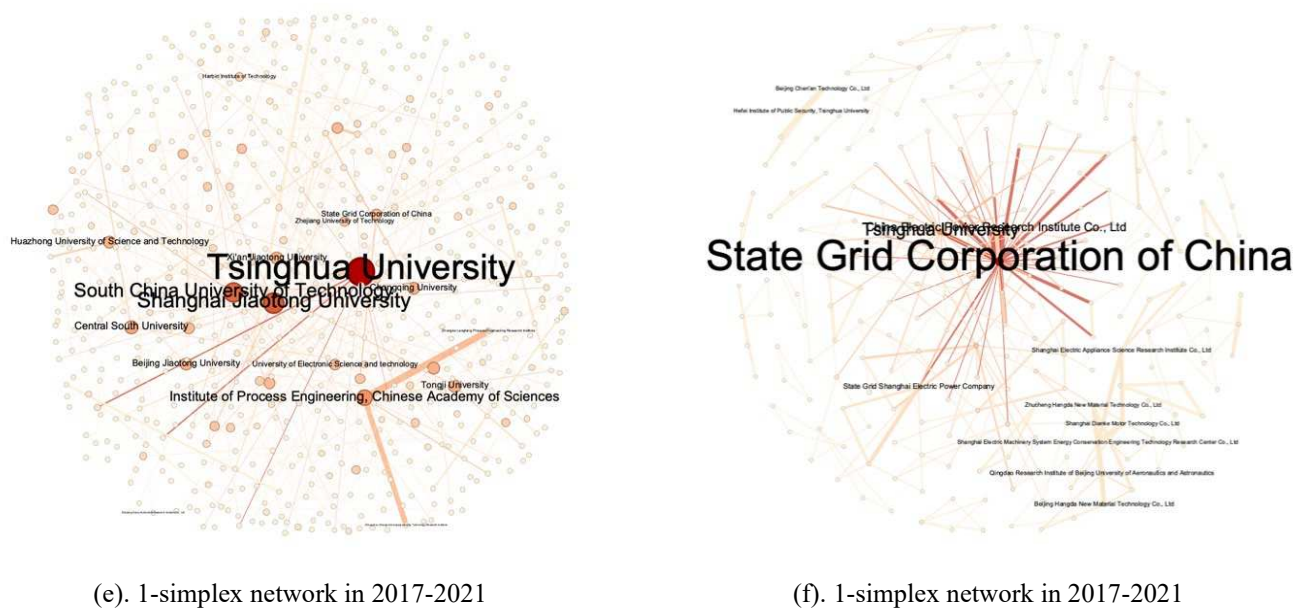


Figure 4. Network maps of one-dimensional and two-dimensional simplex collaboration in the NEV industry from 2008 to 2021.

Figure 4(a) illustrates that during the period from 2008 to 2012, there were relatively few cooperative patent applications. From 2008-2012 to 2013-2016, the scale of the patent collaboration network in the NEV industry showed a significant increase. In the one-dimensional simplex patent collaboration network of 2013-2016, Tsinghua University and State Grid Corporation emerged as the core nodes. On the two-dimensional simplex patent collaboration network during the same period, a two-dimensional simplex group centered on the State Grid Corporation of China was formed, and the majority of collaborative patents involved either research institutes or universities.

During 2017-2021, the patent collaboration network in the NEV industry experienced a significant increase in scale, with innovation hubs rapidly emerging in the one-dimensional simplex network during this period. These hubs included several enterprises such as State Grid Corporation of China, Zhejiang Geely Holding Group Co., Ltd., Beijing Science and Technology Xinyuan Information Technology Co., Ltd., and Chongqing Golden Queen New Energy Automobile Manufacturing Co., Ltd. Moreover, the network also included some research institutes such as the Chinese Academy of Sciences Process Engineering Research Institute, China Science Langfang Process Engineering Research Institute, and Zhengzhou China Science Emerging Industry Technology Research Institute, as well as universities such as Tsinghua University, Beijing University of Technology, and the South China University of Technology. In the two-dimensional simplex patent collaboration network depicted in Figure 4(f), it is worth noting that there were several key edges in the network, in addition to the two-dimensional simplex groups with State Grid Corporation of China and Tsinghua University as central nodes.

5.2. Higher-order collaboration network structure

To further analyze the evolution characteristics of the three-level interaction on patent collaboration networks in the NEV industry, this section introduces some structural indicators of the collaboration network. According to Table 2, the scale of patent applicants in the one-dimensional simplex collaborative innovation network in the NEV industry has increased significantly from 35 in the period of 2008-2012 to 727 in the period of 2017-2021. The number of collaboration patents in the NEV industry has increased significantly from 39 to 962, indicating that an increasing number of organizations are willing to participate in the research and development (R&D) of the NEV industry through patent collaboration. In addition, the maximum node generalized degree has increased from 3 to 12, which suggests that core members are attracting newly embedded nodes to participate in patent collaboration activities. This pivotal role played by core members in the innovation network helps to promote the involvement of more organizations in collaborative activities.

The number of nodes in the two-dimensional simplex collaboration network has significantly increased from 9 to 236 in the period of 2017-2021, along with an increase in the number of simplexes from 2 to 119. This trend indicates that the mode of collaboration innovation among three organizations is becoming more and more popular over time. Moreover, the increase in the maximum node generalized degree and the maximum edge generalized degree from the second stage to the third stage is greater than the change from the first stage to the second stage on the two-dimensional simplex network. This suggests that the importance of the three-organization collaboration mode in the patent collaboration network is beginning to highlight.

Table 2. Structural characteristics of higher-order patent collaboration network in China's NEV industry from 2008 to 2021.

period dimension	2008-2012		2013-2016		2017-2021	
	d=1	d=2	d=1	d=2	d=1	d=2
nodes	35	9	132	36	727	236
Number of simplexes	21	2	82	20	527	119
Number of patents	39	12	144	47	962	188
Maximum node Generalized degree	3	1	6	13	12	36
Maximum link Generalized degree	-	1	-	1	-	5

6. Evolution characteristics of nodes and edges

6.1. Generalized degree distribution of nodes and edges

In the previous section, we observe several core nodes and edges in the atlas of higher-order patent collaboration networks in China's NEV industry from 2008 to 2021. To further explore the distribution characteristics of nodes and edges, Figure 5 shows the generalized degree distribution curve of nodes and edges in the one-dimensional simplex network and two-dimensional simplex network of China's NEV industry from 2008 to 2021. Nodes in both the one-dimensional and two-dimensional simplex networks follow a power-law distribution, with the majority of nodes having only a few edges and a small number of nodes having a large number of edges. This implies that in the NEV industry, most innovative organizations participate in a limited number of joint patent applications, while a select few organizations collaborate frequently and apply for most of the patents. These highly connected nodes occupy critical positions in the collaboration network and play a significant role in driving innovation and development. The generalized degree distribution of edges in the two-dimensional simplex network also follows a power-law distribution, indicating that a small number of edges contribute to a large number of triangles, while most edges only form a small number of two-dimensional simplices. This suggests that the two-dimensional simplex network includes some critical edges that play a leading role in the patent collaboration network of the three organizations. The heterogeneity of both nodes and edges in the network indicates that innovation organizations and their partners exhibit significant diversity in terms of their connectivity and collaboration patterns.

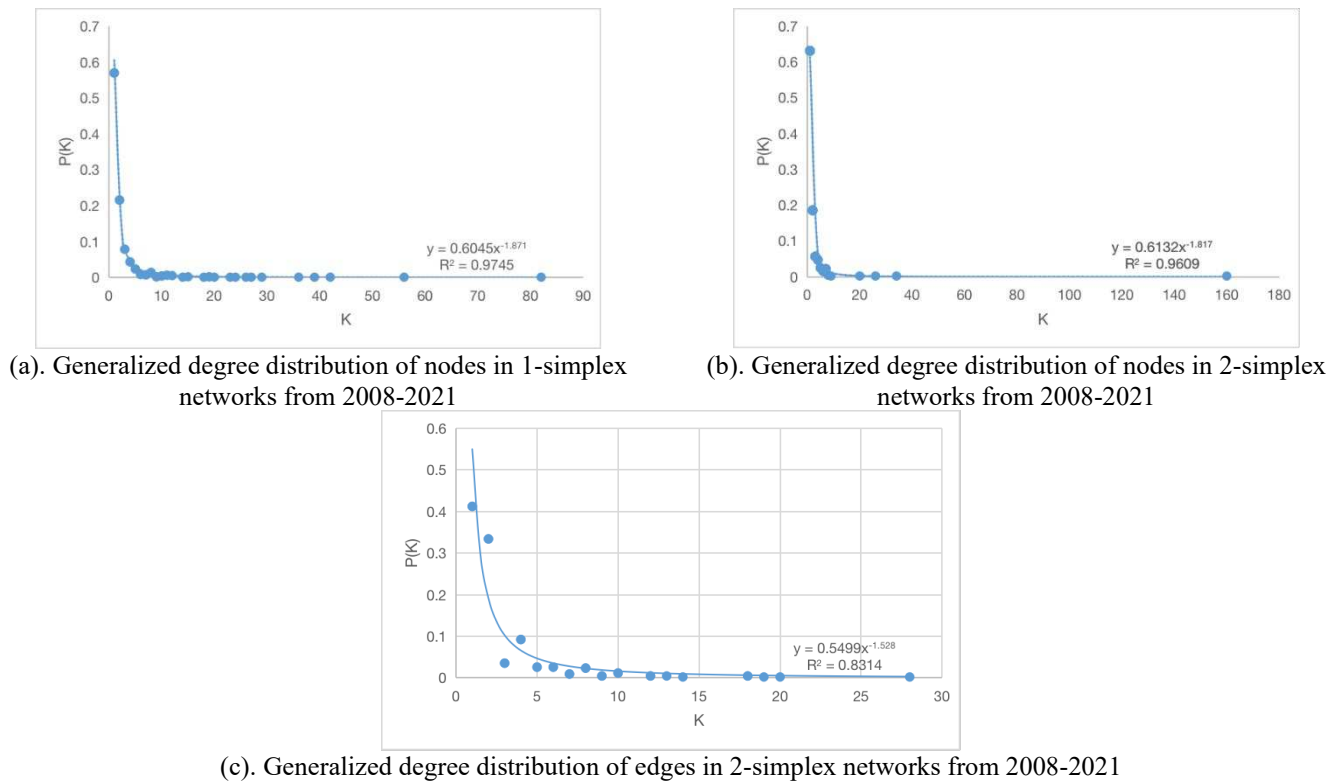


Figure 5. Generalized degree distribution of nodes and edges of 1-simplex and 2-simplex networks from 2008 to 2021.

6.2. Collaboration breadth and depth of nodes and edges

Figure 5 highlights that the higher-order interaction patent cooperation network of China's NEV industry conforms to a power-law distribution for both nodes and edges, as indicated by the generalized degree distribution. In order to holistically evaluate the roles played by nodes and edges in innovation networks, this section presents the "cooperation breadth - cooperation depth" framework. This framework is employed to analyze the collaboration breadth and depth distribution of innovation organizations and innovation partners on the one-dimensional and two-dimensional simplex networks during different periods. The collaboration breadth of nodes and edges is reflected by the node generalized degree and edge generalized degree respectively. The theory of the Matthew effect suggests [47] that nodes and edges with a wider collaboration breadth have a greater capacity to facilitate connections with others. Broadening the scope of collaboration results in increased access to innovative resources, thereby incentivizing more organizations to join the collaboration and assume central roles in the network as key hubs. This section evaluates the collaboration breadth of innovation individuals and partners by analyzing the values of $k_{1,0}([r]), k_{2,0}([r])$ and $k_{2,1}([r,s])$. The collaboration depth is evaluated by unit weight, where the unit weight equals the number of links divided by the generalized degree. This indicates the average weight of nodes or edges within the network. That is, the higher the average weight, the stronger the cooperative relationship with other organizations becomes, indicating a more robust collaboration. Figure 6 depicts the cooperation breadth and cooperation depth of nodes and edges in both the 1-simplex and 2-simplex networks in China's NEV industry.

Figures 6(a) and 6(b) demonstrate that the average breadth and depth of both nodes and edges were relatively low between 2008 and 2012. In the 1-simplex patent collaboration network, China Electric Power Research Institute and Hongfujin Precision Industry (Shenzhen) Co., Ltd. exhibited a low breadth-high depth position, while Shanghai Jiaotong University held a high breadth-low depth position. State Grid Corporation of China and Tsinghua University occupied a high breadth-high depth position, indicating their significant importance in the networks. On the two-dimensional simplex patent collaboration network of China's NEV industry, Zhejiang Geely Automotive Research

Institute Co., Ltd., Zhejiang Geely Holding Group Co., Ltd., and Zhejiang Geely Automotive Research Institute Co., Ltd. Hangzhou Branch, along with the connections between them, were in a low breadth-high depth position, while other nodes and connections exhibited a low breadth-low depth position. In general, during this period, most nodes and edges were characterized by a low breadth-low depth position, suggesting that the network was still in its infancy.

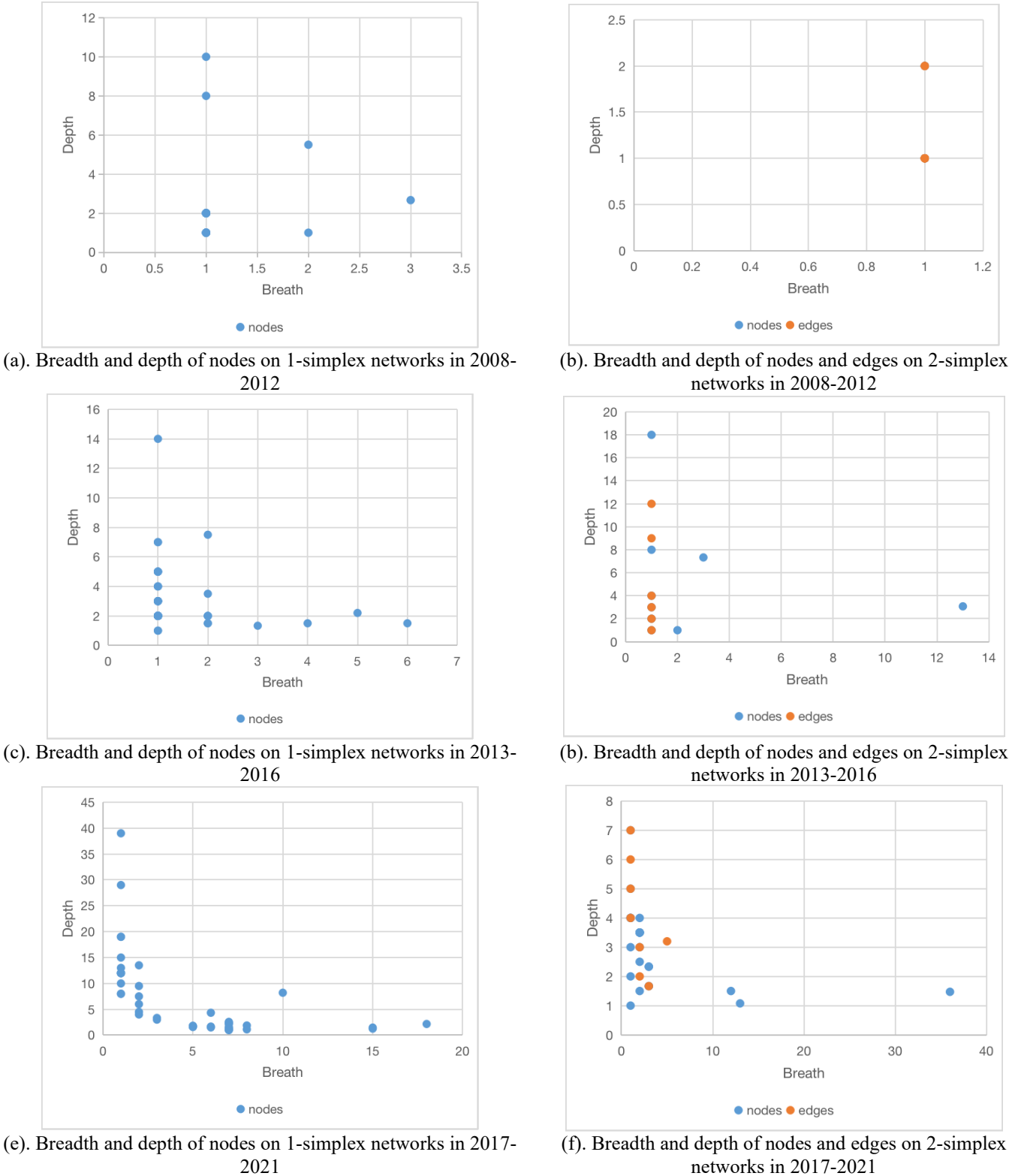


Figure 6. Collaboration breadth-depth matrix.

In the 1-simplex patent collaboration network between 2013 and 2016 (refer to Figure 6(c)), Tsinghua University and Shanghai Jiaotong University were positioned at high breadth-low depth, indicating the presence of numerous targets for knowledge exchange. Zhejiang Geely Holding Group Co., Ltd. and Dongguan Sanxin Electric Vehicle Technology Co., Ltd. were located in the low breadth-high depth position, suggesting that they had established relatively stable collaborative relationships with other organizations. The Institute of Dongguan-Sun Yat-Sen University, State Grid Corporation of China, and Zhejiang Normal University held positions of high breadth and high depth, indicating their significant influence as dominant players in the collaboration networks during 2013-2016.

In the 2-dimensional simplex patent cooperation network of China's NEV industry in 2013-2016 (as shown in Figure 6(d)), most nodes occupied low breadth-low depth positions, except for State Grid Corporation of China and Electric Power Research Institute of State Grid Shandong Electric Power Company, which were in high breadth-high depth positions. Shandong Luneng Intelligent Technology Co., Ltd. and Electric Power Research Institute of State Grid Jibei Electric Power Co., Ltd. held low breadth-high depth positions. All edges had a generalized degree of 1, indicating that the collaborative relationships among the three organizations were just beginning to emerge. The connection between State Grid Corporation of China, Shandong Luneng Intelligent Technology Co., Ltd. and Electric Power Research Institute of State Grid Shandong Electric Power Company was in the low breadth-high depth position, indicating a solid collaborative relationship between these organizations. Overall, the majority of nodes and edges occupied low breadth-low depth positions, indicating a stage of rapid expansion.

During the period of 2017 to 2021, the distribution of nodes on the breadth-depth matrix displays significant variation. The majority of nodes are located in the low width-low depth position. As demonstrated in Figure 6(e), universities such as Tsinghua University, South China University of Technology, and Shanghai Jiaotong University hold a prominent position in the network, situated in a high breadth-low depth position. This suggests that they possess abundant innovation resources and have the capability to cooperate effectively with other organizations. Zhongke Langfang Process Engineering Research Institute, Zhengzhou Zhongke Emerging Industry Technology Research Institute, Chongqing Jiaotong Vocational College, and Chongqing Golden Queen New Energy Automobile Manufacturing Co., Ltd. are situated in a low breadth-high depth position, indicating that they have established stable collaborative relationships. The Institute of Process Engineering, Chinese Academy of Sciences is situated in a high breadth-high depth position, indicating that it possesses ample knowledge resources, innovative abilities, and advantages in establishing stable relationships with other entities, and generates a vital role in the evolution of the 1-dimensional simplex cooperation network in China's NEV industry.

In the two-dimensional simplex patent collaboration network of the NEV industry, most nodes and links occupy the low breadth-low depth position, indicating the network is in a phase of significant expansion (as shown in Figure 6(f)). Large enterprises and universities, such as State Grid Corporation of China, China Electric Power Research Institute Co., Ltd., and Tsinghua University, hold a high breadth and low depth position, indicating their important role and abundant innovation resources in the network. On the other hand, Anhui Chenkong Intelligent Technology Co., Ltd. and Shanghai Dianke Motor Technology Co., Ltd. are in position of low breadth and high depth, indicating that these enterprises have formed stable cooperation relationships. Some edges are situated in the low breadth-high depth position, suggesting that a cooperative innovation relationship between the three organizations has been established. The partnership between State Grid Corporation of China and China Electric Power Research Institute Co., Ltd. forms a high breadth-high depth connection, indicating a strong and stable collaboration between two organizations with significant innovation capabilities. This type of partnership is critical to the success of a multi-entity cooperative innovation network, as it allows for deep cooperation between the involved organizations and effective collaboration with other entities.

7. Evolutionary Analysis of Regional Cooperation

To investigate the influence of regional factors on collaborative invention patents in China's NEV industry, this section examines the spatial evolution of the higher-order patent collaboration network from 2008 to 2021. Based on the registered places of applicants with collaboration relationships, the provinces to which they belong can be identified. These provinces can then be used as nodes to construct a regional patent collaboration network. The nodes representing the provinces are connected by edges that represent patent collaboration relationships between the provinces. We can measure the strength of the connections between two nodes in the regional patent collaboration network by assessing the number of patents that applicants from the two provinces have filed together. Similarly, the strength of the triangles between the three nodes can be determined by the number of patents filed jointly by applicants from the three provinces.

7.1. Evolution analysis of intra-regional cooperation

Based on the analysis of the intra-regional distribution of the patent collaboration network, we can explore the impact of regional boundaries on patent collaboration. Figure 7 presents the number of patent applications for intra-regional cooperation in China's NEV industry. Figure 7(a) shows that, during the initial stage of the one-dimensional simplex patent application network in the NEV industry, the body of patent applications was relatively low and the proportion was not statistically significant. Therefore, the following section mainly focuses on analyzing the two stages from 2013-2016 and 2017-2021. By comparing the average proportions of internal cooperation in the NEV industry between these two periods, the regions where internal cooperation exists have been classified into four distinct categories. The proportion of intra-regional cooperation in the first category remains high in both 2013-2016 and 2017-2021 (higher than 60%). In both stages, the proportion of collaboration in Anhui, Shanghai, and Chongqing is higher than 60%, which suggests that these regions prefer to collaborate with partners within their own region and rely on knowledge exchange within the region. This also indicates that regional boundaries have a significant impact on their patent collaboration behavior. The second category of regions consistently exhibits low levels of internal cooperation (below 60%) during both the 2013-2016 and 2017-2021 periods. Specifically, Beijing, Hunan, Jiangsu, Shaanxi, Fujian, and Hubei all exhibit values below 60%, but greater than 0. This indicates that these regions prefer to collaborate with partners from outside their own region and rely on knowledge exchange across regions, thereby suggesting that regional boundaries have a relatively minor influence on their patent cooperation behavior. The third category of regions includes those where the proportion of internal cooperation was higher than 60% during 2013-2016, but lower than 60% during 2017-2021. The proportion of internal cooperation shows that the proportion of internal cooperation decreases significantly in Guangdong, Zhejiang, and Shandong in two periods. The decreasing trend from higher than the average value to lower than the average value indicates that the influence of regional boundaries on patent cooperation decreases significantly. The fourth category comprises regions where internal cooperation is negligible, as exemplified by Yunnan, Tianjin, Sichuan, Shanxi, Inner Mongolia, Liaoning, Jiangxi, Jilin, Heilongjiang, Hebei, Guizhou, Guangxi, and Gansu, all of which recorded 0 internal cooperation ratio during 2013-2016. This indicates that these regions face a scarcity of internal resources and therefore, seek out partners from other regions with richer resources. Based on the information provided, it seems that the proportion of intra-regional collaborative patents in the one-dimensional simplex patent application network has increased from 0.56 to 0.63 over three stages. This indicates that organizations are more inclined to collaborate with partners within their own region when pursuing innovation. Geographical proximity can play a critical role in reducing search costs and improving innovation efficiency. When organizations are physically closer to each other, it can be easier and less expensive for them to communicate, exchange ideas, and collaborate. Furthermore, being located in the same region can provide access to similar resources, such as skilled workers or specialized equipment, which can further facilitate innovation. The rise in intra-regional collaborative patents implies that organizations are increasingly recognizing the advantages of collaborating with local partners and actively seeking out innovation opportunities within their region.

Since the number of collaboration patents by the three organizations of the NEV industry in 2008-2012 and 2013-2016 is relatively small, this paper counts the number and proportion of Intra-regional collaboration patents in the two-dimensional simplex network between 2008 and 2021 and does not count by stages. In the two-dimensional simplex network, the three organizations also prefer internal cooperation, with an overall internal cooperation ratio of 0.59. Collaborating on patents among multiple entities in a region can lead to cost savings and risk mitigation. By pooling resources and knowledge, the entities can reduce expenses and share the risks involved in the patent development process. The proportion of intra-regional patent applications in Hunan, Chongqing, Shanghai, Liaoning, Shaanxi, Jilin, Guangxi, Gansu, and Xinjiang regions exceeds 60%, suggesting that these regions rely heavily on the accumulation of knowledge, technology, and information within their own region. Regional boundaries play a significant role in patent cooperation within these regions. On the other hand, the number of internal patent applications in Tianjin, Guangdong, Shandong, Jiangsu, Zhejiang, Beijing, Inner Mongolia, and Jiangxi is less than 60%, indicating that regional boundaries have a less significant impact on patent cooperation in these regions. Organizations within these regions tend to collaborate with partners from different regions and make the most of the advantages of other areas, such as combining different production elements.

Overall, the findings suggest that the impact of regional boundaries on patent cooperation varies across different regions, and it is essential to understand regional differences when developing strategies to promote innovation and collaboration.

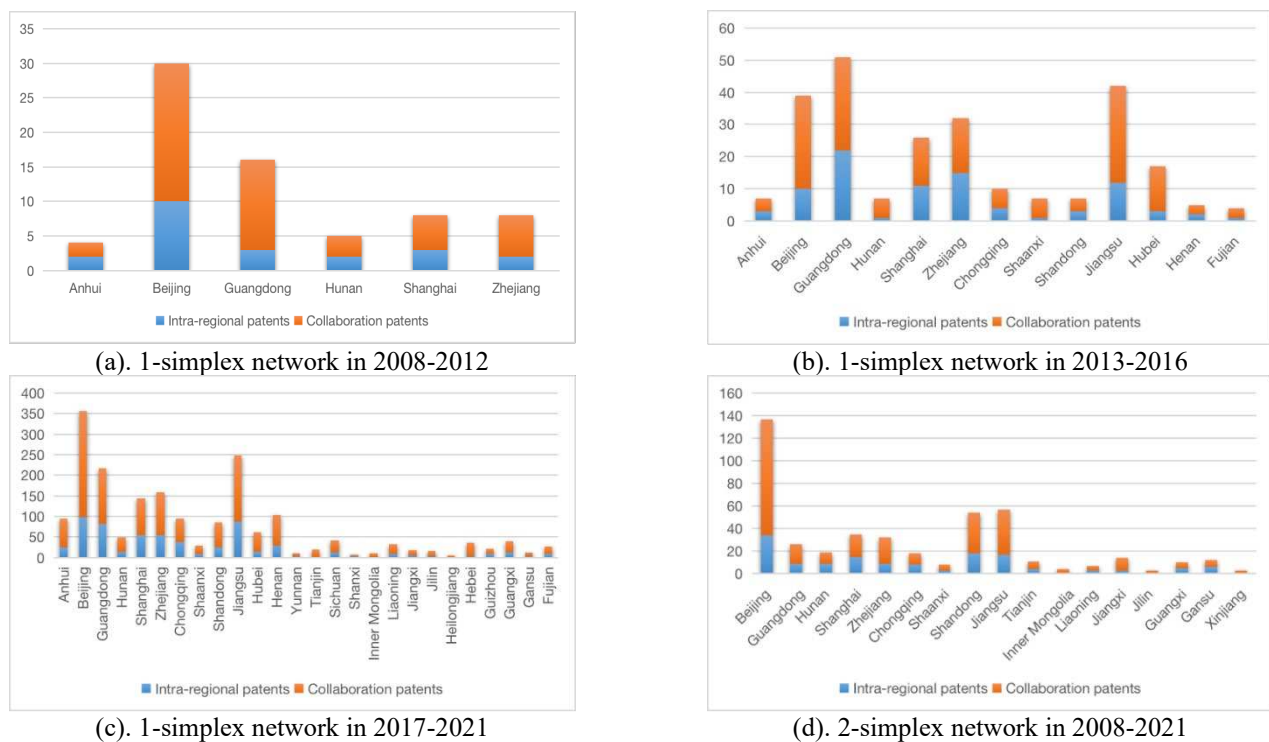


Figure 7. Number of intra-regional and regional collaboration patents.

7.2. Evolution analysis of cross-regional cooperation

In this part of the analysis, we use Gephi software to visualize the patterns of patent collaborations across regions in China's NEV industry. The section presents spatial evolution maps of 1-dimensional simplex patent collaboration networks from 2008-2012, 2013-2016, and 2017-2021, as well as a spatial evolution map of 2-dimensional patent collaboration networks from 2008-2021 (see Figure 8). These visualizations provide insights into the changing dynamics of patent collaborations across regions and over time in China's NEV industry. Figure 8 indicates that, the nodes represent provincial administrative regions or municipalities directly under the Central Government in China. The links between nodes on the map represent cross-regional collaboration

relationships, with thicker edges indicating more frequent collaborations. The thickness of the edges represents the number of patents filed for cross-regional collaborations in China's NEV industry. The visualization clearly shows that the scale of cross-regional patents in China's NEV industry has been expanding continuously. Beijing has always been at the center of the patent collaboration network, while Jiangsu, Zhejiang, Guangdong, and Shanghai are at the sub-center of the network. These regions have rapid economic development, well-established infrastructure, and a strong awareness of environmental protection and clean consumption, making them ideal partners for R&D collaboration in the NEV industry. They have actively sought out partners from different regions to establish patent collaboration relationships, leading to their prominent position in the network. Chongqing, Shandong, Hubei, Hebei, and Henan are also emerging as important players in the network. On the other hand, Inner Mongolia, Jilin, Shaanxi, Ningxia, Hunan, Qinghai, Anhui, and Guizhou have a relatively marginal position in the network and show weak patent collaboration with other regions.

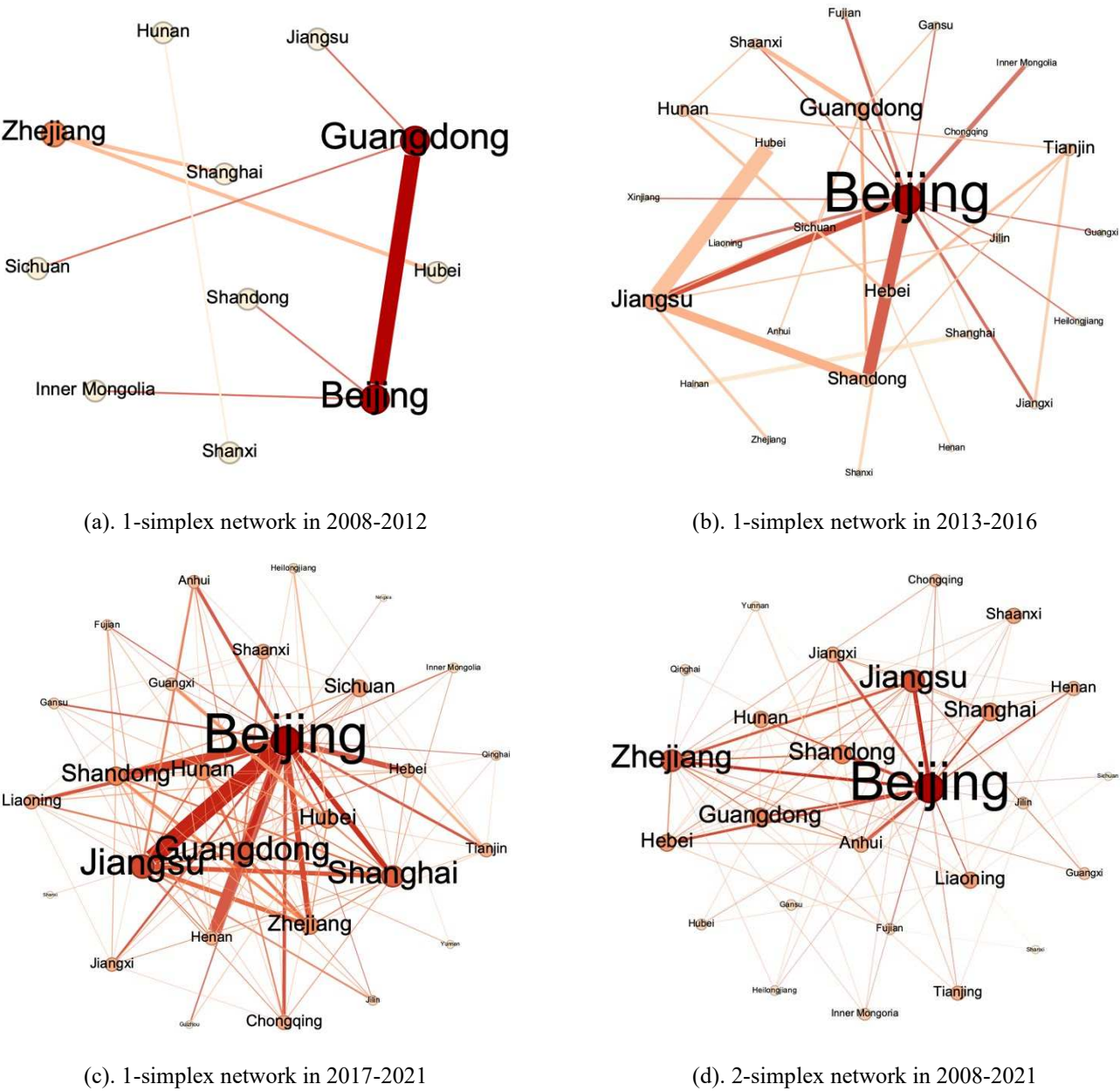
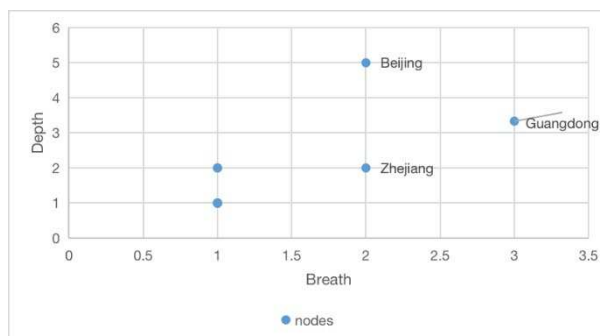


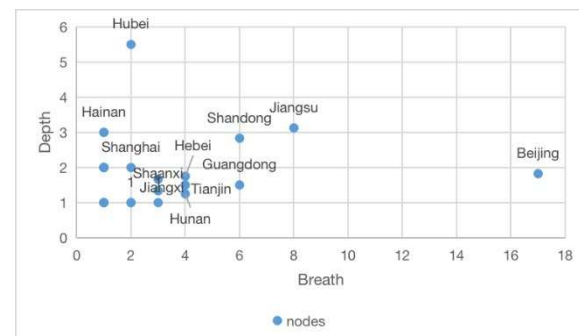
Figure 8. Evolution map of cross-regional patent collaboration in China's NEV industry.

Additionally, this section introduces a "breadth-depth" two-dimensional matrix to analyze the spatial distribution pattern of cross-regional patent collaboration in China's NEV industry [48]. As illustrated in Figure 9, the breadth of cross-regional cooperation is depicted by the generalized degree of regional nodes and cross-regional edges. The average weight of the generalized degree of nodes and edges is an indicator of the depth of cross-regional collaboration. Figure 9(a) illustrates that Beijing, Zhejiang, and Guangzhou were in a high breadth-high depth position from 2008 to 2012, indicating that they functioned as the primary hubs of cross-regional information exchange during this period. Between 2013 and 2016, Jiangsu was in a high breadth and high depth position, whereas Hebei, Beijing, Jiangxi, Guangdong, Tianjin, Shaanxi, and Hunan held a high breadth and low depth position. Compared to the period of 2013-2016, the average collaboration breadth significantly increased from 2017-2021, and the number of regions in the high breadth-high depth position also increased during this period. Jiangsu, Beijing, Zhejiang, Shandong, Guangdong, Henan, and Hubei all held a high breadth-high depth position, indicating a high level of knowledge exchange with other regions. On the other hand, Hunan, Sichuan, Shanghai, and Anhui were in a high breadth-low depth position, also indicating significant knowledge exchange but with room for deeper collaboration. Finally, Hebei, Liaoning, and Guangxi held a low breadth-high depth position, indicating relatively stable collaboration relationships.

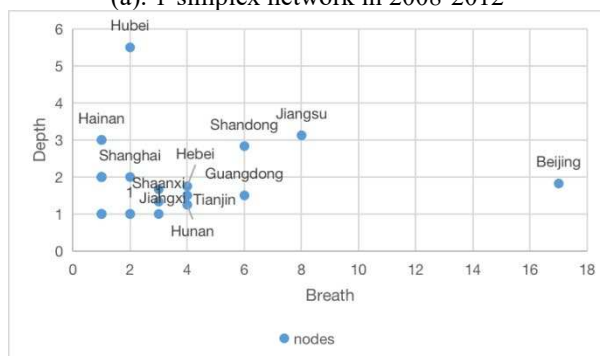
In the two-dimensional simplex patent collaboration network, Beijing occupies a high breadth-high depth position, and the Beijing-Anhui, Beijing-Hebei, Beijing-Shanghai, Beijing-Zhejiang, and Beijing-Henan links also hold a high breadth-high depth position. This indicates that Beijing is not only the core of the three-entities cross-regional collaboration innovation network but also serves as a bridge for collaboration and innovation with other regions. Additionally, the link between Zhejiang and Jiangsu also occupies a high breadth-high depth position in the 2-dimensional simplex patent collaboration network.



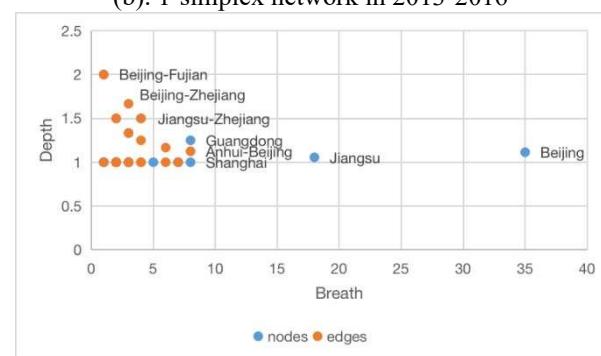
(a). 1-simplex network in 2008-2012



(b). 1-simplex network in 2013-2016



(c). 1-simplex network in 2017-2021



(d). 2-simplex network in 2008-2021

Figure 9. Distribution pattern of cross-regional patent collaboration in China's NEV industry.

8. Conclusion and future work

8.1. Conclusion

Nowadays, collaboration and teamwork are crucial for national innovation systems, with growing research focusing on collaborative innovation networks. However, little effort has been made to study higher-order interaction collaboration networks. This study aims to fill the gap that many-body interactions of a complex system can be captured by simplicial complexes, expanding the limitations of SNA methods. Based on patent collaboration data on SIPO patents granted from 2008 to 2021 in China's NEV industry, the 1-dimensional simplex network and 2-dimensional simplex network are constructed in different periods. Then, this paper identifies the key innovation organizations and key cooperative relationships on the patent collaboration network by introducing the generalized degree index of nodes and edges in a high-order network. Furthermore, the "cooperation breadth - cooperation depth" framework is used to analyze the impact of the organizations and partners on the collaboration network and the evolution characteristics of the cross-regional patent collaboration network. The following research conclusions are obtained.

(1) From 2008 to 2021, the number of patent collaborative organizations in China's NEV industry increased rapidly, and patent collaboration networks are becoming increasingly dense. The one-dimensional simplex network and two-dimensional simplex network showed significant differences in different stages. Some innovative organizations on the one-dimensional simplex network have established a stable cooperative relationship, and the patent cooperation network has developed towards complexity and diversification. The two-dimensional simplex network gradually forms a triangle group with the core node as the center, and the position of the key links begins to highlight in the network.

(2) A few core nodes and key partnerships play an important role in different stages of the patent cooperation network in the NEV industry. The nodes and edges of the higher-order interaction patent cooperation network of China's NEV industry conform to the power-law distribution, indicating a large number of links and triangles are formed by a few nodes and links. The study identifies State Grid Corporation, Tsinghua University, South China University of Technology, Beijing University of Technology, China Electric Power Research Institute, Hong Fujin Precision Industry (Shenzhen) Co., Ltd, Chongqing Chang'an Automobile Co., Ltd, Chongqing Chang'an New Energy Automobile Co., Ltd, Zhejiang Geely Holding Group Co., and other patent applicants as key innovation organizations that have formed stable cooperative relationships. These organizations and their key relationships act as bridges for cooperation with other organizations and significantly impact the cooperative innovation behavior of the entire network.

(3) Intra-regional and cross-regional patent cooperation shows different spatial evolution patterns. The number of provinces involved in patent cooperation in China's new energy vehicle industry is increasing, and patent cooperation is spreading from local regions all over the country. However, the cooperation intensity of different provinces varies greatly, indicating that some regions have more active and intense cooperation compared to others. Anhui, Shanghai, and Chongqing rely more on intra-regional knowledge exchange. They are willing to choose partners in the same region. The proportion of internal cooperation in Guangdong, Zhejiang, and Shandong decreased significantly. The impact of regional boundaries on patent cooperation in these regions decreased. In the cross-regional patent cooperation network, Beijing has always been at the core of the patent cooperation network. Jiangsu, Zhejiang, Guangzhou, and Shanghai are at the sub-center of the cooperative innovation network. The core status of Shandong, Hunan, Sichuan, Anhui, Henan, Hubei, and other regions has risen significantly. The core regions drive other regional innovation organizations to increase their participation. Beijing is at the core of the two-dimensional simplex patent cooperation network. Beijing-Anhui, Beijing-Hebei, Beijing-Shanghai, Beijing-Zhejiang, Beijing-Henan, and Zhejiang-Jiangsu play an intermediary role in multi-entity and cross-regional collaboration innovation activities. The ability to acquire knowledge in Inner Mongolia, Jilin, Shaanxi, Ningxia, Hunan, Qinghai, Anhui, Guizhou, and Guangxi still needs to be strengthened.

8.2. Future work

The following future research directions are identified in this paper.

(1) This article does not provide a detailed analysis of the characteristics of patent applicants. In the future, more detailed research can focus on the nature and innovation assets of innovation organizations and investigate how these characteristics influence innovation cooperation behavior. Identifying the characteristics that have a significant impact on the network can provide valuable insights for policymakers and practitioners to facilitate and promote innovation collaboration more effectively.

(2) A higher-order network model can capture the multi-entity interaction behaviors that exist in the real world. Further research can use topological clustering of higher-order networks to explore communication and cooperation characteristics of different communities, promoting overall connectivity and openness of the cooperation network.

(3) This paper uses cooperative patent application data to construct a cooperative innovation network, which focuses on formal innovation. However, there may be other forms of informal innovation that are not considered in this study. It would be interesting to investigate the impact of informal innovation information on the collaborative innovation network of the NEV industry.

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