Article

Does the Universe have its own mass?

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Abstract: Within the framework of the previously proposed formulation of the quantum theory of gravity in terms of world histories, it was suggested that the universe has its own mass. This quantity is analogous to the mass of a particle in relativistic mechanics. The mass of the universe is a distribution of non-zero values of gravitational constraints, which arises and changes in time as a consequence of the initial conditions for fundamental dynamic variables. A formulation of the Euclidean quantum theory of gravity is also proposed to determine the initial state, which can be the source of the universe's own mass. Being unrelated to ordinary matter, the distribution of its own mass affects the geometry of space and forms a dedicated frame of reference. The existence of selected reference systems is taken into account by the corresponding modification of the system of quantum gravitational constraints. A variant of such a modification of the Wheeler-De Witt equation is the operator representation of gravitational constraints, which, together with the state of the universe, determines the parameters of the reference system in the form of a distribution of the spinor field on a spatial section.

Keywords: universe; mass; canonical action; quantization; internal time

1. INTRODUCTION

Speaking about the mass of the universe in this work, we mean the analogy with the mass of a particle in relativistic mechanics, which is included in the well-known relativistic relation between energy and momentum (we assume the speed of light to be unity):

$$H \equiv p_\mu p^\mu = -p_0^2 + p_i^2 + m^2 = 0. \tag{1}$$

This equation has a direct analogy with the constraint equations in the canonical representation of the theory of gravity of Arnowitt, Deser, and Mizner (ADM) [1,2]. The latter will be the basis of our consideration. The question of what is the dynamic nature of the particle mass arose after Dirac and Fock introduced the proper time of the particle as an independent dynamic variable [3,4]. Subsequently, Stõckelberg, Feynman, and Schwinger introduced proper time into quantum electrodynamics [5–7] and considered mass as a dynamic variable conjugate to proper time (see also [8]). In this paper, we ask ourselves the question: should the mass in equation Eq.(1) be considered an independent constant, or is it a consequence of the conditions in the source of the particle that were imposed at its birth? It is clear that the answer to this question should be sought in quantum theory. Another aspect closely related to the concepts of proper time and mass is the proper covariant formulation of quantum theory. The generally accepted formulation of the quantum theory of gravity (QTG) is based on the quantum version of gravitational constraints - the Wheeler-DeWitt (WDW) equations [9,10]. It implements the results of canonical analysis and Dirac's proposals on quantization of covariant theories [11]. Since in the case of a closed universe,

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the Hamiltonian is reduced to a linear combination of constraints, in this formulation it is equal to zero, and the wave function of the universe does not depend on any external time parameter (Kuchař [12]). This is the well-known problem of time in QTG. As we will see later, this also excludes the appearance of the universe's own mass. To describe the evolution of the universe, it is necessary to move from this "frozen" formalism (MTW [1]) in QTG to a formulation in terms of world histories. This will also open up a possibility of the appearance of the universe's own mass. There is also an alternative approach based on the invariant definition of the Batalin-Fradkin-Vilkovysky (BFV) functional integral over world histories [13,14]. It gives an object that can be called the propagator (Green's function) of the constraint operators in the state space of the universe, that includes an additional integration over proper time (see about the BFV theorem in the case of a relativistic particle [15]). Written in the Euclidean representation, the invariant functional integral over all Riemannian $4D$ metrics with one spatial boundary (and "south pole") defines the no-boundary Hartle-Hawking wave function of the universe - the proposed non-singular solution of WDW [16]. After integration over the Euclidean proper time, there are also no sources for the appearance of the universe's own mass.

In this paper, we propose an alternative formulation of the covariant QTG, in which there is an explicit time parameter that describes the evolution of the universe in terms of trajectories in the configuration space (superspace). In the new formulation, the ADM constraints, while remaining canonical generators of dynamics, can be nonzero. Their numerical values are determined by the initial conditions at the time of the birth of the universe. The modified covariant quantum dynamics, after the transition to the Euclidean form, gives a variant of the description of the "subpolar region" or the cosmological vacuum that determines the initial state of the universe. We allow for a natural violation of covariance in this region, so that the initial state can be the source of the universe's own mass. Note that the distribution and motion of this self-mass in space forms a dedicated frame of reference and "spontaneously" breaks the covariance. This circumstance can be considered as one of the examples of spontaneous vacuum symmetry breaking.

However, the description of the evolution of the universe in terms of the external time parameter, although covariant in form and independent of parametrization, is a view "from outside". Since all observers are "inside" the universe, the problem of determining the internal time parameter remains relevant. In this paper, we propose a variant of modification of the ADM constraints, which also includes restrictions on the parameters of the frame of reference. This variant is based on the Witten identity [17,18], that was first obtained in connection with the proof of the gravitational field energy positivity theorem.

In the next section, we briefly formulate the standard approach to the covariant quantum theory of gravity. In the second section, a modification of this quantum theory is proposed, in which a non-zero self-mass of the universe is allowed, and a variant of the initial state of the universe in which this mass arises is also proposed. In the third section, a modification of the WDW system of equations is proposed, which includes the parameters of the selected reference frame. The conditional principle of the extremum of the energy of space is also formulated, which can be used as the basis for determining the internal time of the universe.

2. COVARIANT QUANTUM THEORY OF GRAVITY

Here we briefly formulate the generally accepted approach to the QTG. Consideration begins with the classical action of the general theory of relativity (GR),

$$I[g,\varphi] = \frac{1}{16\pi G}\int R\sqrt{-g}d^4x + I_m[g,\varphi], \tag{2}$$

where the second term is the action functional of matter fields. We will keep in mind the need to add appropriate boundary contributions to obtain a canonical representation and subsequent quantization (see [19]). Variation of the action with respect to the metric tensor $g_{\mu\nu}$ gives the Einstein equations

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$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 16\pi G T^{\mu\nu}, \tag{3}$$

$$T^{\mu\nu} \equiv \frac{\delta I_m}{\delta g_{\mu\nu}}. \tag{4}$$

The transition to the canonical representation is carried out following the ADM [1]. Using the $3+1$ splitting of the $4D$ metric

$$ds^2 = g_{ik}\left(dx^i + N^i dt\right)\left(dx^k + N^k dt\right) - (N dt)^2, \tag{5}$$

where $i, k = 1, 2, 3$, we represent the density of the Lagrange function of the Hilbert-Einstein action in the form

$$16\pi G L = \pi^{ik} \frac{\partial g_{ik}}{\partial t} - NH - N_i H^i, \tag{6}$$

where

$$N_i = g_{ik} N^k, \tag{7}$$

and

$$\pi^{ik} = \sqrt{\det g_{lm}}\left(g^{ik} Tr\mathbf{K} - K^{ik}\right), \tag{8}$$

have the meaning of canonical momenta conjugate to the components of the $3D$ metric g_{ik}, in which

$$K_{ik} = \frac{1}{2N}\left(N_{i|k} + N_{k|i} - \frac{\partial g_{ik}}{\partial t}\right). \tag{9}$$

The density of the Hamilton function is a linear combination of gravitational constraints,

$$H \equiv \frac{1}{\sqrt{\det g_{lm}}}\left(Tr\pi^2 - \frac{1}{2}(Tr\pi)^2\right), \tag{10}$$

$$H^i = -2\pi^{ik}_{|k}. \tag{11}$$

Here and below, for simplicity, we do not include matter fields. The components of the $4D$ metrics N, N_i are called lapse and shift functions and play the role of Lagrange multipliers in the canonical form of the ADM action. A variation of the action with respect to these factors gives the classical constraint equations:

$$H = H^i = 0. \tag{12}$$

It is these equalities that will be the subject of discussion and possible modification in this paper. They are part of the Einstein equations Eq.(3) and a necessary consequence of the general covariance of the theory. Although in the canonical formalism based on the action of the ADM, the dynamic meaning of the components N, N_i of the $4D$ metric differs from the rest of the components, the Euler-Lagrange equations Eq.(12) corresponding to them must be strictly fulfilled.

The quantization of the theory is carried out in the standard way by replacing the canonical momenta with the corresponding Hermitian differentiation operators on the space of wave functions $\psi(g_{ik})$,

$$\widehat{\pi}^{kl}(x_m) = \frac{\hbar}{i} \frac{\delta}{\delta g_{kl}(x_m)}. \tag{13}$$

Their substitution into Eqs.(10),(11) gives the constraint operators $\widehat{H}$,$\widehat{H}^i$ (the problem of ordering noncommuting factors is not discussed here). In the generally accepted version of

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the covariant quantization theory, in accordance with Dirac's proposal [11], the classical constraint equations Eq.(13) are replaced by additional conditions for the wave function. In the case of the QTG, these additional conditions have the form of the WDW equations,

$$\widehat{H}\psi(g_{ik}) = \widehat{H}^i\psi(g_{ik}) = 0. \tag{14}$$

Here we use parentheses to denote the functional dependence of the wave function on the metric, assuming the subsequent introduction of a functional on world histories with a time parameter. In the WDW equations Eq.(14), there is no time parameter, which in the case of a closed universe creates a difficult problem for the interpretation of the theory. The solutions of WDW equations form the set of admissible physical states of the universe in the covariant quantum theory. The construction of a specific solution requires additional conditions for the constraints Eq.(14). One such solution, the no-boundary wave function, was proposed by Hartle and Hawking [16].

To approach the definition of the no-boundary wave function of the universe, it is useful to refer to an analogy with the simplest covariant quantum theory of a relativistic particle based on the Klein-Gordon (KG) equation:

$$\widehat{H}\psi(x) = \left(\hbar^2\partial_\mu\partial^\mu + m^2\right)\psi(x) = 0. \tag{15}$$

In this case, the BFW invariant functional integral reduces to the expression (see [15])

$$G(x,x') = -i\int_0^\infty K(x,x',s)ds, \tag{16}$$

where $K(x,x',s)$ is the solution of the parabolic equation

$$i\hbar\frac{\partial K}{\partial s} = \widehat{H}K \tag{17}$$

with the corresponding initial condition (see also [20])

$$K(x,x',0) = \delta(x-x'). \tag{18}$$

The function $G(x,x')$ is the Feynman propagator of a particle is a singular solution of the KG equation (for $x \to x'$).

Returning to the theory of gravity, we can also start with the universe propagator in the form of a covariant functional integral over all pseudo-Euclidean $4D$ geometries between two fixed spatial sections, which is a singular solution of the WDW equations. In this context, the suggestion of Hartle and Hawking is understandable - to obtain a non-singular solution of the WDW equations by removing one of the boundary surfaces, which is achieved by passing to imaginary time and integrating over all Riemannian $4D$ geometries with one fixed spatial section. In this form of covariant quantum theory of gravity, there is no external parameter of temporal evolution, and there is no place for the universe's own mass. The mass of the universe can be introduced into quantum theory if time is returned there as a parameter of evolution.

3. OWN MASS OF THE UNIVERSE

The interpretation of nonrelativistic quantum mechanics based on the KG equation for a charged particle (for example, a pi meson) is based on the theory of electric charge perturbations [21]. Here the particle and the antiparticle correspond to positive- and negative-frequency solutions of the KG equation. There is also an interpretation of the solutions of the parabolic wave equation with proper time Eq.(17), in which sections of world lines directed backward in time in Minkowski space are compared to antiparticles [5]. Here $K(x,x',s)$ serves as the kernel of the evolution operator for Eq.(17).

Let's give this interpretation in terms of world lines another form, more suitable for generalization. To this end, we start with the action of a relativistic particle in a parametrized form (with an arbitrary parameter τ),

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$$I[x] = \int_0^1 \left(\frac{\dot{x}^2}{4N} - m^2 N \right) d\tau, \tag{19}$$

which has a clear analogy with the ADM representation of the Hilbert-Einstein action. We write the classical equations of motion of a free particle, which follow from (19), in the form

$$\frac{d^2 x^\mu}{ds^2} = 0, \tag{20}$$

where the proper time parameter is explicitly introduced, according to

$$ds = N d\tau. \tag{21}$$

Note that the introduction of the proper time as an evolution parameter in Eq.(19) makes the second term with the mass redundant in the particle dynamics. It is essential for determining proper time using the additional constraint equation,

$$\frac{1}{4}\frac{dx^\mu dx_\mu}{ds^2} = -m^2, \tag{22}$$

that is obtained by varying Eq. (19) with respect to N and taking into account the definition of the proper time Eq. (21). Let us introduce the canonical momenta

$$p_\mu = \frac{1}{2}\dot{x}_\mu, \tag{23}$$

and write the action Eq.(19) in the canonical form

$$I[x] = \int_0^1 \left(p_\mu \dot{x}^\mu - NH \right) d\tau, \tag{24}$$

where H is determined by relation Eq.(1), an analog of the ADM theory of gravity. Relativistic quantum mechanics is obtained by replacing the 4-momentum of the particle by the differentiation operators

$$\widehat{p}_\mu = \frac{\hbar}{i}\frac{\partial}{\partial x^\mu}, \tag{25}$$

substituting which into Eq.(1) we obtain the KG equation. In this quantum theory, the probability measure

$$\left(\frac{i\hbar}{2m} \right) \left(\overline{\psi}\frac{\partial \psi}{\partial t} - \psi \frac{\partial \overline{\psi}}{\partial t} \right) \tag{26}$$

is sign indefinite in accordance with the interpretation of positive- and negative-frequency solutions of the KG equation [21].

Passing to the interpretation in terms of world lines, we will normalize the solutions of Eq.(17) $\psi(x,s)$ by the quadratic form

$$\langle \psi | \psi \rangle = \int d^4 x \overline{\psi}(x,s)\psi(x,s). \tag{27}$$

Let us divide the time interval $[0,S]$ into small segments of length ε by points $s_n = nS/N$, $n = 1,2,\ldots,N$, and approximate an arbitrary world line $x^\mu = x^\mu(s)$ by a polyline with vertices $x_n^\mu = x^\mu(s_n)$. Let us introduce the multiplicative function of the polygonal vertices

$$\Psi(x_n) = \prod_n \psi(x_n, s_n), \tag{28}$$

where $\psi(x,s)$ the solution of equation Eq.(17) on the interval $[0,S]$. We define the norm of this function by the quadratic form

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$$\langle\Psi|\Psi\rangle=\int\prod_n d^4x\overline{\Psi}\Psi. \tag{29}$$

Thus, the function $\Psi(x_n)$ determines the probability of movement along some world line passing through the points of the polyline with vertices x_n, provided that the initial wave function $\psi_0(x)$ is given. As such, we take a wave packet with a given initial 4-momentum $p_{0\mu}$ [22]:

$$\psi_0(x)=A\exp\left[-\sum_\mu\frac{\left(x^\mu-x_0^\mu\right)^2}{2\sigma_\mu^2}+\frac{i}{\hbar}p_{0\nu}x^\nu\right], \tag{30}$$

where we put

$$p_0^2=p_{0\mu}p_0^\mu=-m_0^2. \tag{31}$$

This wave packet obviously describes the state of a particle in a source localized near the point $x_{0\mu}$ of the Minkowski space, which has finite dimensions σ_μ^2 (source coherence parameters). Using function Eq.(28), one can calculate the average acceleration values

$$\begin{aligned}&\left\langle\frac{d^2x^\mu(s)}{ds^2}\right\rangle_\Psi\\ =&\ \frac{\langle x^\mu(s_{n+1})\rangle_\psi-2\langle x^\mu(s_n)\rangle_\psi+\langle x^\mu(s_{n-1})\rangle_\psi}{\varepsilon^2}\\ =&\ 0,\end{aligned} \tag{32}$$

In relation Eq.(32), the Ehrenfest theorem [23] is formulated for a relativistic particle. The proper here remains undefined. In the classical theory, it is determined by the constraint equation Eq.(22) with the kinematic mass m. To determine the proper time in quantum theory, [22] proposed a quantum analogue of the Eq.(22) – the condition for the extremum of the real phase α of the wave function $\psi(x,S)$ with respect to proper time:

$$\frac{\partial\alpha}{\partial S}=0. \tag{33}$$

Now the proper time is determined by the particle mass m_0 in the initial state (with quantum corrections depending on the coherence parameters σ_μ^2), if its kinematic mass m in the KG equation is set equal to zero. In this case, in quantum theory, the particle has a mass entirely determined by the initial state.

We will come to a new formulation of the RQM in terms of world lines of a particle if we pass to the limit $\varepsilon\to 0$, in which the broken line approximates an arbitrary world line $x^\mu=x^\mu(s)$ arbitrarily exactly. In this limit, the function $\Psi(x_n)$ turns into a wave functional $\Psi[x(s)]$ on the space of particle world lines, and the Schrödinger equation Eq.(17) is replaced by the quantum principle of least action [22],[24] ,which is a secular equation for the action operator

$$\begin{aligned}&\widehat{I}\Psi\\ \equiv&\ \int_0^S ds\left[\frac{\widetilde{\hbar}}{i}\dot{x}^\mu(s)\frac{\delta\Psi}{\delta x^\mu(s)}+\widetilde{\hbar}^2\frac{\delta^2\Psi}{\delta x^\mu(s)\delta x_\mu(s)}\right]\\ =&\ \Lambda\Psi,\end{aligned} \tag{34}$$

where

$$\widetilde{\hbar}=\hbar\cdot\varepsilon, \tag{35}$$

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and the limit $\varepsilon \to 0$ is assumed. Based on this form of RQM, we will now present the necessary modification of the QTG, which we will also use to determine the initial state of the universe.

This modification will be based on the parabolic Schrödinger equation for the wave function of the universe $\psi(g_{ik}, t)$

$$i\hbar\frac{\partial\psi}{\partial t} = \int_{\Sigma} d^3x \left(N\widehat{H} + N_k\widehat{H}^k\right)\psi, \tag{36}$$

with coordinate time t, where Σ is a spatial slice in which the lapse and shift functions are arbitrary and fixed. This equation is analogous to Eq.(17). Proceeding further in the same way as in the case of a particle, we divide the time interval on which the dynamics of the universe is considered into small segments of length ε and compose a multiplicative wave function (in the limit $\varepsilon \to 0$, the wave functional)

$$\Psi(g_{ik}(x_l, t_n)) = \prod_n \psi(g_{ik}(x_l, t_n)). \tag{37}$$

With this limit in mind, we define the generalized momentum operator on the space of wave functionals

$$\widehat{\pi}^{kl}(x_m, t) = \frac{\widetilde{\hbar}}{i}\frac{\delta}{\delta g_{kl}(x_m, t)}. \tag{38}$$

Replacing the canonical momenta by operators in the canonical form of the ADM action (we agree to place them on the right in all terms), we obtain the action operator $\widehat{I}$. As in ordinary quantum mechanics, the secular equation for this operator

$$\begin{aligned} \widehat{I}\Psi \quad &\equiv \quad \int d^4x \left[\frac{\widetilde{\hbar}}{i}\frac{\partial g_{kl}(x_m, t)}{\partial t}\frac{\delta}{\delta g_{kl}(x_m, t)} \right. \\ &\quad \left. - N\widehat{H} - N_k\widehat{H}^k\right]\Psi \\ &= \quad \Lambda\Psi, \end{aligned} \tag{39}$$

where the eigenvalue is determined by the boundary values of the metric on the initial and final spatial sections,

$$\frac{i}{\hbar}\Lambda = \ln\psi(g_{ik}, T) - \ln\psi(g_{ik}, 0), \tag{40}$$

(T is considered time interval) is equivalent to the Schrödinger equation Eq.(37). We assume that the eigenfunctional $\Psi[g]$ - the solution of this secular equation is an invariant of transformations of the space-time coordinates that do not affect the boundary surfaces. We emphasize that Ψ is the world history functional $g_{\alpha\beta}\left(x^k, t\right)$, including the dependence of the metric on time. As in the case of a relativistic particle, sections of history are allowed, with backward movement in time, i.e. compression of parts of the universe ($\det g_{ik} \to 0$) with the formation of black holes. We define the invariant norm of the wave functional $\Psi[g(x)]$ by the quadratic form

$$\langle\Psi|\Psi\rangle = \int \prod_x J[g] d^{10}g \overline{\Psi}[g]\Psi[g], \tag{41}$$

where $J[g]$is the corresponding element of the Faddeev-Popov invariant measure [25]. Based on it the probabilistic interpretation of QTG in a new formulation, we can calculate the average values of the Einstein equations Eq.(2) (taking into account at this stage also the matter fields):

$$\left\langle R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - 16\pi G T^{\mu\nu} \right\rangle_{\Psi}. \tag{42}$$

Now the question is whether these averages are equal to zero. For some of them, which determine the $3D$ metric g_{ik}, we assume the validity of the Ehrenfest theorem, and hence their equality to zero. Some of the Einstein equations, which are obtained by varying the lapse and shift functions N, N_i, in the classical theory give the classical constraint equations Eq.(12). In QTG, as in relativistic quantum mechanics, we will replace them with conditions for the extremum of the real phase of the wave function of the universe with respect to N, N_i. Now, these extremum conditions do not mean that the classical constraints $H^a = \left(H, H^i\right)$, or their mean values $\langle H^a \rangle$, are equal to zero. In this case, the constraint algebra determined by the commutation relations

$$\left\{H^a, H^b\right\} = C_d^{ab} H^d, \tag{43}$$

retains its meaning in the new formulation. Here the structural "constants" C_d^{ab} are functions of the $3D$ metric g_{ik} [2], and summation over indices also implies integration over spatial coordinates. It follows from Eq.(42) that the average values $\langle H^a \rangle$, which are scalar and vector densities in space, also depend on time:

$$\frac{\partial}{\partial t} \langle H^a \rangle = \left\langle C_d^{ab} N_b H^d \right\rangle. \tag{44}$$

Thus, the mean values $\langle H^a \rangle$ are always and everywhere equal to zero if they are equal to zero at the beginning. Whether this is so depends on the initial state of the universe.

One of the options for determining the initial state of the universe was proposed in [26]. In its construction, a new representation of the QTG is used in terms of the quantum principle of least action Eq.(39). To do this, we pass to the Euclidean form of action by Wick's rotation of time in the complex plane, $t \to it$, with the simultaneous transformation of the canonical momenta, $\pi \to -i\pi$. The Euclidean representation allows us to formulate the quantum principle of least action for a $4D$ geometry with one "spatial" section. We pay attention to the fact that in Riemannian geometry the specificity of the time coordinate is completely lost and for Euclidean quantization the generalized canonical form of De-Donder-Weil follows [27,28]. This is also allowed by the quantum principle of least action, which for the initial state takes the form (see [26])

$$\begin{aligned} \widehat{I}_E \Psi_0 &\equiv \int d^4x \frac{\widetilde{\hbar}_\alpha}{i} \partial_\alpha g_{\beta\gamma}(x) \frac{\delta \Psi_0}{\delta_\alpha g_{\beta\gamma}(x)} \\ &\quad - \mathcal{H}\left[g_{\beta\gamma}(x), \frac{\delta}{\delta_\alpha g_{\beta\gamma}(x)}\right] \Psi_0 \\ &= \Lambda_0 \Psi_0, \end{aligned} \tag{45}$$

where the integral is taken over a compact domain of a $4D$ dimensional Riemannian space with one boundary. Here

$$\widetilde{\hbar}_\alpha = \hbar \cdot \varepsilon_\alpha \tag{46}$$

where ε_α are spatial lattice constants (see [26]). The eigenvalue of the action Λ_0 now depends only on the boundary values of the $4D$ metric and determines the initial state of the universe at this boundary for the subsequent dynamics of the state in time:

$$\psi_0(g_{ik}) = \exp\left[\frac{i}{\hbar} \Lambda_0(g_{ik})\right]. \tag{47}$$

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Note that the De-Donder-Weil canonical formalism as applied to a metric field requires the fulfillment of an additional condition

$$\det g_{\alpha\beta} = const, \tag{48}$$

which violates the $4D$ covariance of this initial state theory. Thus, there is no reason to attribute the initial state Eq.(47) to the set of solutions for WDW equations Eq.(14). It can be assumed that the subsequent evolution of the universe with such an initial state will include additional dynamic variables in the form of the distribution and motion of its own mass.

Own mass is not part of the matter that fills the universe. However, its presence affects the geometry and thus it interacts with matter through some form of gravitational force. This effect of the presence of its own mass means that it is possible to associate a distinguished frame of reference with it.

4. PROPER TIME OF THE UNIVERSE

Above, we have considered a variant of describing the dynamics of the universe in terms of coordinate time, which can be called external. The principle of covariance is needed precisely in order to ensure the independence of physical laws from the choice of this coordinate time. But this description looks as if there were some external observer for the universe. Therefore, the description in terms of internal time remains relevant, which must be introduced in the dynamic interpretation of solutions of gravitational constraints. There is no single approach to this issue. The general idea is that the picture of motion in time that is familiar to us arises in the semiclassical approximation [29]. When analyzing simple mini-superspace models of the universe, one of the fundamental dynamical variables of the theory is chosen as the time parameter. It seems natural to choose the $3D$ volume of the universe (volume logarithm) as the time parameter (see [1]). This choice is consistent with the hyperbolic signature of the constraint Eq.(10). However, here we again encounter the problem of the cosmological singularity, which in quantum theory takes on the chaotic form of a quantum billiard [30]. In a loop QTG near a cosmological singularity, a massless scalar field is considered [31] as the time parameter.

This section discusses an alternative approach to determining internal time that does not involve fundamental dynamic variables. It also takes into account the presence of the own mass of the universe, with which the selected frame of reference is associated. In classical general relativity, the frame of reference is given by the lapse and shift functions N, N_i, and in the conventional QTG, any dependence on these functions is excluded as a consequence of the WDW equations. These equations must be modified in such a way as to take into account the presence of a dedicated frame of reference. Such a variant of constraints is provided by a construction based on the Witten identity in the theory of gravity [17],[18], which can be written in integral form for the case of a closed universe [32]:

$$\begin{aligned} & \left(\chi, \widehat{W}\eta\right) - \int_\Sigma d^3x \sqrt{\det g_{ik}} [N(\chi,\eta) H \\ & + N_i(\chi,\eta) H^i \Big] \\ = \; & 0, \end{aligned} \tag{49}$$

where χ, η are the Dirac bi-spinors on the spatial section Σ. Parentheses denote the scalar product in the space of bi-spinor fields:

$$(\chi, \eta) = \int_\Sigma d^3x \sqrt{\det g_{ik}} \chi^+ \eta, \tag{50}$$

where the cross denotes the Hermitian conjugation (see [32]). If $\chi = \eta$, identity Eq. (49) gives a representation of the Hamiltonian function of a closed universe in a special gauge [33], so that the bi-spinor field χ completely determines the frame of reference (see also [18]).

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It also follows from identity Eq.(49) that the set of gravitational constraints is equivalent to the operator equality on the space of bi-spinor fields:

$$\widehat{W} \equiv \widehat{\mathcal{D}}^2 - \left(-\widehat{\Delta}\right) = 0, \tag{51}$$

where $\widehat{\mathcal{D}}$ is a Hermitian Dirac operator with respect to the scalar product Eq.(50), and $\widehat{\Delta}$ is also a Hermitian Beltrami-Laplace operator. Note that both operators in this equality are positive-definite: $\widehat{\mathcal{D}} \geqslant 0, -\widehat{\Delta} \geqslant 0$. After quantization, the operator $\widehat{W}$ additionally becomes an operator $\widehat{\widehat{W}}$ on the space of wave functions $\psi(g_{ik})$, which is marked with an additional "lid". The "double" operator $\widehat{\widehat{W}}$ allows us to write the quantum constraint equations for the wave function of the universe $\psi(g_{ik})$ together with the conditions for the frame field χ:

$$\widehat{\widehat{W}}\chi \times \psi = 0. \tag{52}$$

Thus, in this variant of the quantum theory, the physical states of the universe should be referred to a selected frame of reference given by the field χ.

Quantum constraints Eq.(52) form a system of differential equations for the functions $\chi\left(x^k\right)$ and $\psi(g_{ik}, \varphi_A)$ (here we have also added matter fields). By themselves, they do not constitute a specific mathematical problem without additional conditions. For hyperbolic equations, these are usually the initial-boundary conditions in the configuration space. Rejecting the initial-boundary value problem, let us pay attention to another possibility dictated by the structure of the operator equality Eq.(51). It is obvious that the mean value of any of the two positive operators included in it has a minimum on the set of solutions of the relation system Eq.(52). We choose the square of the Dirac operator, which does not explicitly include matter fields, and therefore, as part of the Hamilton function, can be called the energy of space. All physical degrees of freedom, including the transverse components of the gravitational field, are included in the second positive-definite operator. Thus, we come to the conditional extremum principle for the space energy

$$\begin{aligned} E \;=\; & \frac{\langle\psi|\left(\chi, \widehat{\widehat{\mathcal{D}}}^2\chi\right)|\psi\rangle}{\langle\psi|(\chi,\chi)|\psi\rangle} \\ & + \langle\langle\mathcal{N}|\left|\widehat{\widehat{W}}\chi \times \psi\right\rangle\right\rangle \\ & + \left\langle\left\langle\chi \times \psi\widehat{\widehat{W}}\right|\left|\mathcal{N}\rangle\rangle\right., \end{aligned} \tag{53}$$

in which the constraint equations Eq.(52) are taken into account as additional conditions with the corresponding Lagrange multipliers $\mathcal{N}$. The double brackets here mean the scalar product in the composition space $\chi \times \psi$. In addition to the minimum value of the space energy, the conditional extremum problem Eq.(52) determines the entire spectrum of its excitations E_ω, which is described by a certain set of quantum numbers ω. The identification of quantum numbers ω with the internal time of the universe is an alternative to choosing the volume of the universe as a time parameter. An additional reason for such an interpretation is that such a choice of the cosmological arrow of time will be consistent with the thermodynamic arrow (entropy), if the increasing "complexity" of the excited states of space is taken as a measure of disorder in the universe. However, a large element of arbitrariness remains in the proposed interpretation until the structure of the excitation spectrum of space is known.

5. CONCLUSIONS

The description of the quantum dynamics of the universe using the wave functional on the space of world histories $g_{\alpha\beta}\left(x^k, t\right)$ allows us to assume that the average values of gravitational constraints $\langle H^a\rangle$, equal to zero in classical GR, are nonzero in QTG. The difference between these averages from zero in quantum theory can arise in the initial state of the universe. For a new version of the description of dynamics, a quantum principle of least action is proposed, in which the action operator is the central object. This operator is obtained by replacing the canonical momenta in the canonical form of the ADM action by operators of variational differentiation on the space of wave functionals from world histories. According to the quantum principle of least action, the evolution of the universe is described by the eigenfunctions of the action operator. The new formalism also allows for modification to define the initial state of the universe. The quantum principle of least action for the "near polar region" is formulated, in which the action operator is defined on the space of wave functionals of $4D$ Riemannian metrics in a compact region with one boundary. In this case, we are interested in the eigenvalue of the action operator, which is the logarithm of the initial state of the universe. This state can be called the cosmological vacuum, and there is reason to expect that it is the source of the universe's own mass. Thus, the appearance of the universe's own mass is a consequence of quantum theory, and the associated violation of the covariance of the original theory is an example of a spontaneous violation of the symmetry of the cosmological vacuum.

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References

1. Misner, C. W.; Thorne, K. S.; Wheeler, J. A. *Gravitation*, W. H. Freeman and Company, San Francisco (1973).
2. Arowitt R., Deser S., and Misner C.W., The dynamics of general relativity, *in Gravitation: An Introduction to Current Research*, ed. L. Witten, Willey, New York, 1962, p. 227.
3. Dirac, P. A. M. *Proc. Roy. Soc. London* Ser. A **1926**, *110* , 405.
4. Fock, V. A. *Phys. Z. Sovjetunion* **1937**, *12*, 404.
5. Stökelberg, E. C. G. *Helv. Phys. Acta* **1941**, *14*, 322.
6. Feynman, R. P. *Phys. Rev.* **1948** *74*, 939.
7. Schwinger, J. *Phys. Rev.* **1951**, *82*, 664679.
8. Aparicio, J. P.; Gaioli, F. H.; Alvarez, E. T. G. *Phys. Rev. A* **1995**, *51(1)*, 96.
9. Wheeler, J. A. *Einsteins Vision*; Springer-Verlag: Berlin-Heidelberg, Germany, (1968).
10. DeWitt, B.S. *Quantum Theory of Gravity. II. The Manifestly Covariant Theory. Phys. Rev.* **1967**, *162*, 1195.
11. Dirac, P. A. M. *Lectures on quantum mechanics*, Yeshiva Universitry, New York (1964).
12. Kuchař, K. V. *Int. J. of Modern Physics D* **2011**, *20(supp01)*, 3.
13. Fradkin, E. S.; Vilkovisky, G. A. *Phys. Lett.* **1975**, *55B*, 224
14. Batalin, I. A.; Vilkovisky, G. A. *Phys. Lett. 69B*, 309. (1977).
15. Govaerts, *J. Int. J. of Modern Physics A* **1989**, *4(17)*, 4487
16. Hartle,J. B.; Hawking, S. W. *Phys. Rev. D* **1983**, *28*, 2960 .
17. Witten, E. *Commun. Math. Phys.* **1981**, *80*, 381.
18. Faddeev, L. D. *UFN* **1982**, *136*, 435.
19. Hawking, S. W. in *General Relativity*, ed. S.W. Hawking, W. Israel, Cambridge University Press, (1980).
20. B. S. De Witt, in *General Relativity*, ed. S. W. Hawking, W. Israel, Cambridge University Press, (1980).
21. Bjorken, J. D.; Drell, S. D. *Relativistic quantum fields*; Mc Graw-Hill Book Company (1978).
22. Gorobey, N. N.; Lukyanenko, A. S. *Russian Journal Nauchno Tehnicheskie Vedomosti SPbGPU* **2012**, *2(147)*, 169.
23. Ehrenfest, P. *Zeitschrift für Physik* **1927**, *45*, 455.
24. Gorobey, N. N.; Lukyanenko, A. S.; Goltsev, A. V. *Universe* **2021**, *7*, 452.
25. Faddeev, L. D.; Popov, V. N. *Uspehi Fiz. Nauk* **1973**, *111*, 427.
26. Gorobey, N. N.; Lukyanenko, A. S.; Goltsev, A. V. *Universe* **2022**, *8*, 568.

27. De Donder, T. *Theorie invariantive du calcul des variations*; Gauthier-Villars, (1930).
28. Weyl, H. *Geodesic fields in the Calculus of Variation for Multiple Integrals. Ann. Math.* **1935**, *36*, 607.
29. Maniccia, G.; De Angelis, M.; Montani, G. *Universe* **2022**, *8*, 556.
30. Kleinschmidt, A.; Nicolai, H. *Foundations of Space and Time: Reflections on Quantum Gravity* **2009**, *12*, 106.
31. Ashtekar, A., Pawlowski, T.; Singh, P. *Phys. Rev. lett.* **2006**, *96(14)*, 141301.
32. Gorobey, N. N.; Lukyanenko, A. S. *Russian Journal Teoreticheskaya I Matematichescaya Fizika* **1995**, *105*, 503.
33. Ashtekar, A. *Phys. Rev. D* **1987**, *36*, 1587.