Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 May 2023
doi:10.20944/preprints202305.0897.v1

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

The Quantization of Mass

José Garrigues-Baixauli[1]

[1] Higher Technical School of Telecommunication Engineers, Universitat Politècnica de València, Camino de Vera s/n, 46022 Valencia, Spain. Retired professor from 2015
E-mail: jgarrigu@eln.upv.es

Abstract The value of the mass of the particles is one more mystery that current theories cannot predict, so they have to be determined experimentally. In this paper we will see how the quantization of 4D space gives rise to the quantization of mass and, in turn, the quantization of mass implies that space-time is discrete.

Key words: Quantization Mass; Planck Length

1 Introduction

All physical magnitudes need, to give a number, a unit of measure. The unit of measure for length is the meter and it can be artificial (from A to B) or natural (using a fundamental constant) depending on which gives greater precision. In any case, it is an arbitrary definition. Currently the meter is defined in terms of the speed of light and the second.

In the case of charge, it is quantized and current theory predicts the charges of elementary particles from a unit, for example that of the electron. However, it does not predict the value of the charge of the electron. In the case of mass, it does not seem to be quantized or the quantization rule has not been found yet, so it is an open problem. However, the standard model explains the origin of mass through interaction with the Higgs field. Simply, it is only able to predict the relationship between the different masses of the particles and their couplings to the Higgs Boson. Coupling that is given by a constant, which cannot be calculated but can be measured, and by the vacuum expectation value of the Higgs field, a value that depends on the Fermi constant. In short, all you have is that the mass of elementary particles is proportional to a given mass, which is obvious and even shocking. However, the Higgs mechanism was highly celebrated by the scientific community and the media when the Higgs Boson was discovered at CERN on July 4, 2012, so for most physicists that is the correct mechanism.

In recent years, theoretical physics has focused on the unification of the four fundamental interactions. The electro-weak model unites the weak interaction with the electromagnetic one. In turn, the grand unification theory or standard model is a quantum field theory that unites the electro-weak interaction with the strong nuclear interaction. However, a theory of everything is needed that unifies gravity with quantum field theory, also known as quantum gravity.

Associated with each fundamental interaction is one or more virtual force-bearing particles, called bosons, which are field quanta.

© 2023 by the author(s). Distributed under a Creative Commons CC BY license.

- The mediators of the electromagnetic interaction are the virtual photons. (Griffiths, 2008)
- Strong nuclear interaction mediators are gluons. (Griffiths, 2008)
- The mediators of the weak interaction are the $W^{\pm}$ and Z bosons. (Griffiths, 2008)

The quantum of the gravitational field is the graviton, proposed in the theories of quantum gravity. The graviton is not part of the standard model because it has not been found experimentally. The graviton is a spin 2 boson, because it is associated with a second order classical tensor field. On the other hand, since the range of the graviton is infinite, in principle, the mass of the graviton could solve the dark matter problem. Hence, different methods have been proposed to limit the mass of the graviton.

Gravitational waves can impose a direct limit on the mass of the graviton, and this mass can also be related to the existence of new polarizations of gravitational waves. The mass of the graviton can arise from a pole or from a resonance (Rham et al., 2017).

The predictions of dynamical general relativity agree with the decay rates of binary pulsars. Finn and Sutton (Finn & Sutton, 2002) analyzed the differences in the data from the binary Hulse-Taylor pulsar and from the PSR B1534+12 pulsar and from the analysis of the data provided a limit for the graviton mass of

$$m_g < 7{,}6 \times 10^{-20}\ \mathrm{eV/c^2} = 1{,}4 \times 10^{-55}\ \mathrm{kg} \quad (1)$$

$$\lambda_g = \frac{h}{m_g c} > 1{,}6 \times 10^{13} m$$

Baskaran et al. have considered the navigation effect that occurs when the phase velocity of gravitational waves is less than the phase velocity of electromagnetic waves (Baskaran et al., 2008). From the limits of this deviation they find a limit for the mass of the graviton of

$$m_g < 3{,}6 \times 10^{-25}\ \mathrm{eV} = 6{,}4 \times 10^{-61}\ \mathrm{kg} \quad (3)$$

Which gives a Compton wavelength for the graviton of

$$\lambda_g = \frac{h}{m_g c} > 4 \times 10^{18} m \quad (4)$$

Zakharov et al., compare observed data and simulations of stellar orbit S2 (or Source 2) at the Galactic Center (Zakharov, 2016). From which they obtain a limit for the mass of the graviton of

$$m_g < 2.9 \times 10{-}21\ \mathrm{eV} = 5{,}2 \times 10^{-57}\ \mathrm{kg} \quad (5)$$

Which gives a Compton wavelength for the graviton of

$$\lambda g > 6.9 \times 10^{13}\ \mathrm{m} \quad (6)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 May 2023 doi:10.20944/preprints202305.0897.v1

2 Discrete Space-time

Space is a fundamental quantity in physics, because it cannot be defined through other fundamental physical quantities. In classical physics, time is an absolute fundamental quantity. For Newton, space and time are independent and absolute entities. For Einstein, space and time are instead united in a 4D structure called space-time.

In quantum gravity models, space-time is discrete, that is, it has a fundamental length that cannot be divided into smaller ones. The discrete space hypothesis, in principle, collides with Einstein's theory of special relativity, because for an observer moving at constant speed, this fundamental length would be shorter.

Can space and time be divided into smaller and smaller units, or is there a limit? Are space and time a continuum or are they composed of indivisible discrete units? These and similar questions were raised by Greek and medieval philosophers, such as Zeno of Elea presents in the Paradox of Plurality (Hagar, 2014) and Maimonides (Maimónides, 1190) in the Guide for the Perplexed.

Heisenberg showed that the mass of these particles must be derived from a fundamental length, together with the Planck constant *h* and the speed of light (Heisenberg, 1943 & 1957).

The Planck length has been considered as the shortest distance that has any physical significance. Planck assumed that Newton's gravitational constant, Planck's constant and the speed of light were the most important universal constants. Using a dimensional analysis, he obtained the Planck mass, length, time and energy (Planck, 1899 & 1906).

"… a fundamental (minimal) length scale naturally emerges in any quantum theory in the presence of gravitational effects that accounts for a limited resolution of space-time. As there is only one natural length scale we can obtain by combining gravity (G), quantum mechanics (h) and special relativity (c), this minimal length is expected to appear at the Planck scale" (Sprenger, Nicolini, & Bleicher, 2012).

Minimum values of volume, length and area are measured in Planck units (Smolin, 2004). Padmanabhan shows that the Planck length provides a lower limit of length in any suitable physical (Padmanabhan, 1985a). "*It is impossible to construct an apparatus which will measure length scales smaller than Planck length. These effects exist even in flat space-time because of vacuum fluctuations of gravity* (Padmanabhan, 1985b).

Haug proposes different methods of measuring the Planck length independently of the gravitational constant *G*. The Planck length is both a physical measurement and the diameter of the true fundamental particle: *"The gravitational constant is a composite (derived) constant, while the Planck length represents something physical; it is the shortest reduced Compton wavelength possible. According to recent developments in mathematical atomism, there are also strong indications that the Planck length is the*

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 May 2023 doi:10.20944/preprints202305.0897.v1

diameter of the only truly fundamental particle, namely an indivisible particle that together with void is making up all matter and energy" (Haug, 2017).

There are several theories that predict the existence of a minimum length (Maggiore 1993, 1994). In quantum gravity models, space-time is discrete, that is, it has a fundamental length that cannot be divided into smaller ones.

Interest in discrete spacetime has increased in recent years due to the appearance of loop quantum gravity. (Rovelli & Speziale, 2003; Collins, Perez, Sudarsky, Urrutia &Vucetich, 2004; Gambini & Pullin, 2011)

"A Planck-scale minimal observable length appears in many approaches to quantum gravity. It is sometimes argued that this minimal length might conflict with Lorentz invariance, because a boosted observer can see the minimal length further Lorentz contracted. We show that this is not the case within loop quantum gravity. In loop quantum gravity the minimal length (more precisely, minimal area) does not appear as a fixed property of geometry, but rather as the minimal (nonzero) eigenvalue of a quantum observable". (Rovelli & Speziale, 2003).

"Trying to combine standard quantum field theories with gravity leads to a breakdown of the usual structure of space time at around the Planck length, 1.6x10(-35) m, with possible violations of Lorentz invariance". (Collins et al., 2004)

For Santilli, space must be a solid and an incompressible medium. Being also the means of transmission of waves and forces. Matter is a dynamic modification of space. "*Space, that must transmit waves and forces, must be full, and matter, which must be a dynamic state of this space – because it interferes and generates forces – must be 'empty in relation to common concepts'. If we could stop all its movements for a moment, matter would completely disappear, as it actually does, whenever corpuscular radiation interferes*" (Santilli, 1956).

3 Four-Dimensional (4D) Discrete Space-Time

The starting hypothesis is that the universe is made up of spheres of 4 spatial dimensions whose diameter is the Planck length. Of the 4 spatial dimensions, 3 are observed as space and the fourth spatial dimension is observed as time ($u=ct$). This is because we lack references to the fourth dimension due to the expansion of the universe.

Smolin states that space is formed from atoms of space and time: *"If we could probe to size scales that were small enough, would we see atoms of space, irreducible pieces of volume that cannot be broken into anything smaller?"* that he calls *"Atoms of Space and Time"* (Smolin, 2004). Planck's 4D spheres are the atoms of space and time that Smolin comments on. To simplify the drawing, let us consider only three dimensions r(x,y) and u.

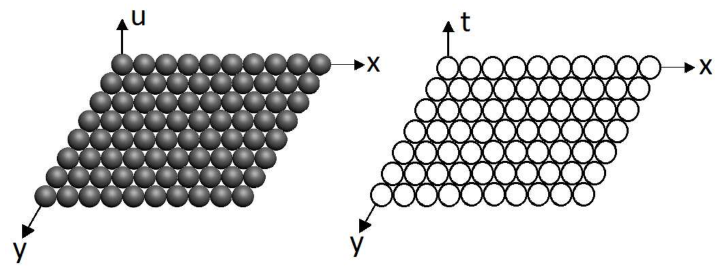

Figure 1 An expanding flat 3D universe is observed as 2D and t

If space is made up of 4D spheres the size of Planck, each Planck sphere can only be at rest or rotating on itself. In addition the Planck sphere has two rotations, one in three-dimensional space and one in the fourth dimension. Rotation in the fourth dimension (ω_u) rotates the u-axis and another spatial axis around any two axes. For example, the u-y axes rotate around the x-z axes. In the rotation in space (ω_e) it is rotated around the u-axis and another spatial axis. For example, the x-z axes rotate around the u-y axes.

Each Planck 4D sphere can rotate both in 3D space and in the fourth dimension ($u = ct$, Figure 2), resulting in the following possible combinations (Baixauli, 2016; Garrigues-Baixauli, 2016, 2017a, b & 2019):

- zero rotations (vacuum space);
- one spatial rotation, ω_e (photons);
- one rotation in the fourth dimension, ω_u (neutrinos);
- two rotations i.e. one spatial rotation, ω_e, and one rotation in the fourth dimension, ω_u (first-generation electrons and quarks).

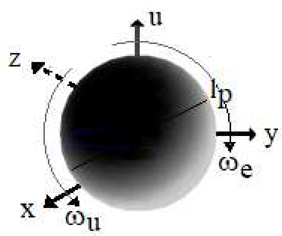

Figure 2. Rotations of a 4D Planck sphere

Static spatial spheres are not observed; it is what we call empty space. We can observe the spheres that rotate on themselves as elementary particles, such as electrons, photons and the first generation of quarks and neutrinos.

These spheres constitute a solid and incompressible space that transmits the waves. *"Since space transmits waves and forces, it is absurd to assume that it is empty, hereas it must be a solid, incompressible medium. And the elementary particles, since they interfere and generate forces, can by no means have a ponderable nature, but must be active energy states determined by a* dynamic state of space points". (Santilli, 1956).

Those spheres are the fundamentally grainy nature of space that Das et al. discuss. *"We again arrive at quantization of box length, area and volume and an indication of the fundamentally grainy nature of space"*. (Das, Vagenas & Ali, 2010).

In the theory of causal sets, space-time is made up of identical space-time atoms with no internal structure. (Bombelli, 1972; Sorkin, 1991, 2005; Dowker, 2014) "*According to this proposal, spacetime is comprised of discrete "spacetime atoms" at the Planck scale."*(Dowker, 2014**).** Also T. Padmanabhan describes gravity from space-time atoms. (Padmanabhan, 2017).

Tejinder P. Sing proposes a New Quantum Theory of Gravity based on space-time atoms in such a way that each atom has its own matter content. "*In our theory, the universe is made up of enormously many such atoms of space-time matter (STM). We do not know at this stage as to what determines the total number of such STM atoms in the universe, and whether this total number is finite or infinite. Each STM atom carries its own space-time and its own matter content"*(Sing, 2019). It also uses the Planck length and speed of light as constants. "*The theory is in fact a classical matrix dynamics with only two fundamental constants-the square of the Planck length and the speed of light, along with the two string tensions as parameters"*(Sing, 2019).

4 Quantum Black Holes

Since gravitational quantum effects are very small, quantum gravity proposals are difficult to test in practice. However, some authors make proposals to test quantum gravity in the laboratory. "*We propose to witness quantum-like features in the gravitational field, by probing it with two masses, each in a superposition of two locations.*" (Marletto & Vedral, 2017). Instead, other authors analyze the proposal to measure a quantum gravity phenomenon in the laboratory by gravitationally entwining two nanoparticles (Christodoulou & Rovelli, 2019).

The fundamental problem of Quantum Gravity lies in reconciling quantum mechanics, applicable at the atomic level, with gravity, which is applicable to the dimensions of the Universe. It is about describing the evolution of the Universe starting from elementary particles. Recent research focuses on quantum black holes that exhibit a discrete mass spectrum. Because the gravity generated is so large, black holes are the most suitable environments to study the effects of quantum gravity (Gibbons & Hawking, 1993; Nicolini, 2009).

Already in 1972, Bekenstein sees a similarity in the behavior of the area of the black hole and of the entropy, since both quantities tend to increase irreversibly. He concludes that the entropy of the black hole is equal to the ratio of the area of the black hole to the square of the Planck length multiplied by a dimensionless constant (Bekenstein, 1973). He considers that black holes possess some quantum characteristics, thus he addresses the possibility that the black hole mass spectrum is discrete (Bekenstein, 1974), and derives a formula for the allowable mass levels.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 May 2023 doi:10.20944/preprints202305.0897.v1

Subsequently, Bekenstein and Mukhanov consider that the modification of the Hawking radiation spectrum is due to the quantization of mass. (Bekenstein and Mujanov, 1995). These investigations have given rise, in the last decade, to new treatises on quantum black holes (Banerjee et al., 2010; Lochan & Chakraborty, 2016; Pourhassan et al., 2013).

If the mass of the black holes is quantized, superposition states of different masses of the black hole can be considered. "*Here we construct a spacetime generated by a BTZ black hole in a superposition of masses, using the notion of nonlocal corre*7 *lations and automorphic fields in curved spacetime. This allows us to couple a particle detector to the black hole mass superposition. We show that the detector's dynamics exhibits signatures of quantum-gravitational effects arising from the black hole mass superposition in support of and in extension to Bekenstein's original conjecture*." (Foo et al., 2021)

Cartas et al. study the Higgs mechanism dynamically along flows defined in the parameter space and in the field space and conclude that the mass spectrum is discrete (Cartas-Fuentevilla et al., 2021). Also, Dvali et al. from the Poincaré invariance of the asymptotic background they show that the mass of black holes must be quantified. (Dvali, Gomez & Mukhanov, 2011)

5 Mass Quantization

In quantum mechanics, microscopic objects are defined as point particles with properties, such as: mass, charge, spin, etc. Real physical objects cannot have zero volume, as this would result in infinite mass or charge density. But since the size of the particle is below the resolving power of the instruments used to measure that size, the concept of a "point particle" can be used mathematically.

The starting hypothesis is that the universe is made up of four-dimensional spheres of space, whose diameter is the Planck length. Some of these spheres rotate, giving rise to the particles that are observed. Therefore, the Planck scale defines two regions in terms of Planck mass or Planck length.

1) $m<m_P$, this is the length scale where quantum effects are dominant, and is characterized by its reduced Compton wavelength:

$$\lambda\!\!\!^{-}_C = \frac{\hbar}{mc} \qquad (7)$$

Where quantum mechanics is applied so that particles are described by their wave function.

2) $m>m_P$,which corresponds to the gravitational length scale, where the theories of gravitation are applied. It is characterized by the radius**:**

$$r = \frac{Gm}{v^2} \qquad (8)$$

Where G is the gravitational constant, v is the orbital velocity on the surface of mass m, and r is the radius of mass m. If it is now assumed that the radius of the mass m decreases until the orbital speed is that of light, it results:

$$r_g = \frac{Gm}{c^2} \qquad (9)$$

"*In this range of length gravitation is "strong" and closed trapping surfaces, colloquially referred to as "horizons", can appear.*" (Spallucci & Smailagic, 2021).

If the escape velocity c is considered, instead of the orbital velocity, then:

$$r_S = \frac{2Gm}{c^2} = 2r_g \qquad (10)$$

Which corresponds to the Schwarzschild radius. On the other hand, it can also be assumed that any particle of mass $m<m_P$ can be reduced in size until it becomes a black hole. Similarly, any mass m such that: $m>m_P$ has a reduced Compton wavelength. This length will be less than the diameter of the mass m and even less than the Planck length and therefore, not measurable it. Consequently, we have two length scales, such that:

- Gravitational scale: $r_g > \lambdabar_C$ with $m > m_p$
- Quantum scale: $\lambdabar_C > r_g$ with $m < m_p$

The boundary between the two scales is determined by the Planck mass m_p and the Planck length l_P. If the two lengths are now multiplied, it results:

$$r_g \lambdabar_C = \frac{Gm}{c^2}\frac{\hbar}{mc} = l_p^2 \qquad (11)$$

Then, for any mass m, it is verified that the product of its reduced Compton wavelength by its gravitational radius is equal to the square of the Planck length. If instead of the gravitational radius, the Schwarzschild radius is considered, we have:

$$r_s \lambdabar_C = \frac{2Gm}{c^2}\frac{\hbar}{mc} = 2l_p^2 \qquad (12)$$

Quantum Scale

If spacetime is discrete, which is the starting hypothesis, then the diameter of the particle coincides with its reduced Compton wavelength, then:

$$\lambdabar_C = nl_p = \frac{\hbar}{mc}; \quad \text{n = 1, 2, 3…….} \qquad (13)$$

From where:

$$m = \frac{\hbar}{nl_p c} = \frac{1}{n} m_p \qquad (14)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 May 2023 doi:10.20944/preprints202305.0897.v1

Therefore, the mass of any particle is quantized in values of m_p/n, where n is a positive integer.

Gravitational Scale

If spacetime is discrete, with a diameter equal to the Planck length, the diameter of the mass m will be:

$$2r_g = \frac{2Gm}{c^2} = nl_p\,; \quad \text{n} = 1, 2, 3\ldots\ldots \quad (15)$$

Which corresponds to the Schwarzschild radius of a black hole.

$$r_S = \frac{2Gm}{c^2} = nl_p \quad (16)$$

From where:

$$m = n\frac{l_p c^2}{2G} = n\frac{l_p c^2}{2\hbar c} m_p^2 = n\frac{m_p}{2} \quad (17)$$

Therefore, any mass m, greater than the Planck mass, is quantified in integer or semi-integer values of the Planck mass, as long as it is a black hole, since we start from the gravitational radius (r_g) that corresponds to a black hole of mass m. But the mass m does not have to be a black hole, however, in discrete space-time, the speed is also quantized, so

$$c = N\,v\,; \quad N = 1, 2, 3\ldots\ldots \quad (18)$$

The orbital speed can only be an integer submultiple of the speed of light, then:

$$\frac{r}{r_g} = \frac{c^2}{v^2} = N^2 \quad (19)$$

On the other hand, the diameter of the mass m can only be an integer number of Planck lengths, then:

$$2r = \frac{2Gm}{v^2} = nl_p \quad (20)$$

From where:

$$m = nl_p\frac{v^2}{2G} = \frac{n}{2N^2}\frac{c^2 l_P}{G} = \frac{n}{2N^2} m_P \quad (21)$$

Where n and N are positive integers. In short, if space-time is discrete, the mass is quantized.

"For any finite N and finite size, the energy of any such configuration must be quantized. This is true about a hydrogen atom, a neutron star and a black hole." (Dvali, Gómez & Mukhanov, 2011). Where N ≫ 1 is the number of constituent quantum particles,

For Spallucci & Smailagic, quantum black holes (QBH) can only exist above the Planck mass, so that the mass-energy spectrum is discrete. "*In spite of this simple looking form, the corresponding metric ensures that QBH can exist only above the Planck mass. On the thermodynamical side, QBH cannot evaporate completely, but rather ends up as zero temperature Planckian remnant.*" (Spallucci & Smailagic, 2021).

In the last two decades, the development of a quantum theory of gravity starts from a quantification of the area of black holes and it is concluded that they may have a discrete mass spectrum. From equation (16), the area of the black hole will be:

$$A_n = 4\pi n^2 l_p^2 \quad (22)$$

Kastrup quantifies the area of the black hole. "… *another possible approach to the problem, which starts from the quantum theory of an isolated spherically symmetric gravitational system and then studies the coupling of such a quantum system to (quantised) matter, at least approximately*." " (Kastrup, 1996). With what you get:

$$A_n = 4\pi n l_p^2 \quad (23)$$

$$M_n = \frac{1}{2}\sqrt{n} m_p \quad (24)$$

Ashtekar and Lewandowski also quantify the area and get a minimum area. "*Regulated operators corresponding to areas of 2-surfaces are introduced and shown to be self-adjoint on the underlying (kinematical) Hilbert space of state."* (Ashtekar & Lewandowski, 1997).

$$A_1 = \frac{\sqrt{3}}{4} l_p^2 \quad (25)$$

Also Bekenstein and Mukhanov "*develop the idea that, in quantum gravity where the horizon fluctuates, a black hole should have a discrete mass spectrum with concomitant line emission*" (Bekenstein, & Mukhanov, 1995). Get a minimum area:

$$A_1 = 4\ln 2 l_p^2 \quad (26)$$

In recent years, this quantization of the area has been taken into account by various authors. (Ahluwalia, 1999; Garattini, 2000; Bojowald & Kastrup, 2000).

6 Quantization of space-time

In this section it will be assumed that the mass is quantized, which will give rise to a discrete space-time, which is precisely the starting hypothesis

Particles

If the mass is quantized, then:

$$m_P = nm = n\frac{\hbar}{\lambdabar_C c}; \qquad n = 1, 2, 3.... \quad (27)$$

From where:

$$\lambdabar_C = n\frac{\hbar}{m_P c} = n\lambdabar_P \quad (28)$$

Therefore, the reduced Compton wavelength is an integer number of Planck lengths. Thus, a wave moving at the speed of light can only travel one Planck length each Planck time.

Black holes

If the mass is quantized, then a black hole of mass *m* verifies:

$$m = \frac{r_g c^2}{G} = n\frac{m_P}{2} \;; \quad n = 1, 2, 3.... \quad (29)$$

The 2 in the denominator is because the Planck length corresponds to the diameter of the Planck mass. From where:

$$2r_g = n\frac{Gm_P}{c^2} = nl_p \quad (30)$$

Therefore, the diameter of a black hole is an integer number of Planck lengths. In short, if the mass is quantized, space-time is discrete. Therefore, discrete spacetime is a necessary and sufficient condition for mass to be quantized.

7 Natural Units

The units of length, mass and time have been chosen arbitrarily, and consequently the fundamental constants have the value they have. But you can choose any system of units. If the Planck units are chosen as units of length, mass and time: Planck length (l_P=1), Planck mass (m_P=*1*) and Planck time (t_P=*1*), it turns out that $\hbar$=1 , *G*=1 and *c*=1. This system of units was first proposed in 1899 by Max Planck.

Particles

The reduced Compton wavelength will be:

$$\lambdabar_C = \frac{\hbar}{mc} = \frac{1}{m} \qquad (31)$$

If the mass is quantized, $m = m_P/n$, with which:

$$\bar{\lambda}_C = \frac{n}{m_P} = nl_p \qquad (32)$$

Therefore, in the Planck system of units, it is evident that the quantification of the mass of the particles implies the quantification of space-time and vice versa. For masses less than the Planck mass, it turns out that the wavelength or diameter of the particle is inversely proportional to its mass.

Black holes

The gravitational radius will be:

$$r_g = \frac{Gm}{c^2} = m \qquad (33)$$

Therefore, in the Planck system of units, the quantification of the gravitational radius implies the quantification of the mass and vice versa. For masses greater than the Planck mass, the mass of the black hole is proportional to the radius.

8 Conclusion

The hypothesis that the universe is made up of four-dimensional spheres of space, with a diameter equal to the Planck length, quantifies spacetime, which in turn quantifies mass. Furthermore, the Planck length defines two distinct regions. On the one hand, there is the subatomic scale, where particles are measured in terms of their reduced Compton wavelength, which in turn is an integer number of Planck lengths. The second region, corresponding to the macroscopic scale, where the diameter of the mass also corresponds to an integer number of Planck lengths. However, in this region what is measured is the concept called mass.

References

Ahluwalia, D.V (1999). *On Quantum Nature Of Black-Hole Spacetime: A Possible New Source of Intense Radiation,* Int. J. Mod. Phys. D 8, 651

Ashtekar, A. &. Lewandowski, J. (1997). *Quantum Theory of Geometry I: Area Operators,* Class. Quantum Gr*av.* 14 A55.

Baixauli, J. G. (2016). *Relation Between the Gravitational and Magnetic Fields*, J. Phys. Math., *7*, 169.

Banerjee. R., Majhi, B.R. & Vagenas, E.C. (2010). *Quantum tunneling and black hole spectroscopy*, Physics Letters B 686, 279

Baskaran, D., Polnarev, A.G., Pshirkov, M.S. & Postnov, K.A. (2008). *Limits on the Speed of Gravitational Waves from Pulsar Timing*, Phys. Rev. D 78, 044018/1-11; E: 78, 089901. ArXiv: 0805.3103

Bekenstein, J.D. (1973). *Black holes and entropy*, Phys. Rev. D 7, 2333

Bekenstein, J.D. (1974). *The quantum mass spectrum of the kerr black hole*. World Scientific. Jacob Bekenstein: 331–33 (2019). Reprinted by permission from Springer Nature, J. D. Bekenstein, *Lettere al Nuovo Cimento* 11 467470. (1974). https://doi.org/10.1007/BF02762768

Bekenstein, J.D. & V. Mukhanov, V. (1995). *Spectroscopy of the quantum black hole*, Physics Letters B 360, 7.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 May 2023 doi:10.20944/preprints202305.0897.v1

Bojowald, M & Kastrup, H.A. (2000). *The Area Operator in the Spherically Symmetric Sector of Loop Quantum Gravity,* Class.Quant.Grav. 17, 3009.
Bombelli, L. Lee, J., Meyer, D. & Sorkin, R.D. *Space-time as a causal set*, Phys. Rev. Lett 59 (1987) 521.
Cartas-Fuentevilla, R.,Herrera-Aguila, A. & Berra-Montiel, J.(2021). *The Higgs mechanism and geometrical flows for two-manifolds*, World Scientific.
Christodoulou, M. & Rovelli, C. *On the possibility of laboratory evidence for quantum superposition of geometries*, Physics Letters B 792, 64–68 (2019).

Collins' J., Perez, A., Sudarsky, D., Urrutia, L. Vucetich, H. (2004). *Lorentz invariance and quantum gravity: an additional fine-tuning problem?***,** Phys.Rev.Lett. 93
Das, S., Vagenas, E. C. & Ali, A. F. (2010). *Discreteness of space from GUP II: relativistic wave equations*, Phys. Lett. B 690 407
Dvali, G., Gómez, C. & Mukhanov, S. (2011). *Black Hole Masses are Quantize,* arXiv:1106.5894 **[hep-ph]**
Dowker F. (2014). *The birth of spacetime atoms as the passage of time.* Ann. N.Y. Acad. Sci., 1326: 18-25
Finn L.S. & Sutton, P.J. (2002). *Bounding the Mass of the Graviton Using Binary Pulsar Observations*, Phys. Rev. D 65, 044022/1-7.
Foo, J., Arabaci, C. S., Zych, M. & Mann R.B. (2021). *Quantum signatures of black hole mass superpositions*, arXiv:2111.13315.
Gambini, R. & Pullin, J. (2001). *Consistent discretizations as a road to quantum gravity,* arXiv:gr-qc/0512065
Garattini, R. (2000). En*tropy and the cosmological constant: A spacetime-foam approach,* Nucl. Phys. Proc. Suppl. 88, 297-300.
Garrigues-Baixauli, J. (2016). Derivation *of the Schrodinger Equation from Classical Physics*, *Int. J. Sci. Res., 5*, 77-80.
Garrigues-Baixauli, J. (2017a). *Classic 4D Model of the Discrete Universe*. I: *Quantu,*. Scholars' Press.
Garrigues-Baixauli, J. (2017b). *Wave-Particle Duality as a Classic Phenomenon*, Int. Sci. Res. Org. J. (ISROJ),
Garrigues-Baixauli1, J. (2019). *Discrete Model of Electro,*. Applied Physics Research; Vol. 11, No. 6; 2019
Gibbons G.W, &S. W. Hawking, S.W. (1993). *Euclidean quantum gravity*, World Scientific
Griffiths D. (2008). *Introduction To Elementary Particles*, Wiley-VCH Verlag GmbH; (ed. 2). 47
Hagar, A. (2014). *Discrete or continuous?: the quest for fundamental length in modern physics*, Cambridge University Press., 69
Haug, E. G. (2017). *Can the Planck length be found independent of big G?,* Applied Physics Research, 9(6)
Heisenberg, W. (1943). *Die Leobachtbaren Grossen in der Theorie der Elemntarteilchen. Z. Phys., 120*, 513-538.
Heisenberg, W. (1957). *Quantum Theory of Fields and Elementary Particles*. Rev. Mod. Phys., 29, 269-278.
Kastrup, H.A. (1996). O*n the Quantum Levels of Isolated Spherically Symmetric Gravitational System,*. Phys. Lett. B 385, 75.
Lochan, K. & S. Chakraborty, S. (2016). *Discrete quantum spectrum of black holes*, Phys. Lett. B 755, 37

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 May 2023 doi:10.20944/preprints202305.0897.v1

Maggiore, M. (1993. *The Algebraic Structure of the Generalized Uncertainty Principle*, Phys. Lett. B, 319, 83-86

Maggiore, M. (1994). *Quantum Groups, Gravity, and the Generalized Uncertainty Principle*, Phys. Rev. D, 49(10), 5182-5187

Maimonides, M. (1190). *The Guide for the Perplexed*, translated from the original arabic text by M. Friedlander, Ph.D (1904)

Marletto, C. & V. Vedral*, V.* (2017) *Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity*, Phys. Rev. Lett. 119, 240402

Nicolini, P. (2009). *Noncommutative black holes, the final appeal to quantum gravity: A review,* Int. J. of Mod. Phys. A 2487, 1229-1308. https://doi.org/10.1142/S0217751X09043353

Padmanabhan, T. (1985a). *Planck Length As The Lower Bound To All Physical Length Scales*, Gen. Rel. Grav., 17(3), 215-221

Padmanabhan, T. (1985b). *Physical Significance of Planck Length*, Ann. Phys., 165(1), 38-58

Padmanabhan T. (2017). *The atoms of spacetime and the cosmological constant,* J. Phys.: Conf. Ser. 880 012008

Planck, M. (1899). *Naturlische Masseinheiten*. Der Königlich Preussischen Akademie Der Wissenschaften, 479.

Planck, M. (1906). *Vorlesungen über die Theorie der Wärmestrahlung* (p. 163). Leipzig: J.A. Barth.

Pourhasan, R. Niayesh Afshordi, N. & Manna. R.B. (2013). *Out of the White Hole:A Holographic Origin for the Big Bang,* arXiv:1309.1487(v2).

de Rham, C,Deskins, J.T., Tolley, A. J & Zhou, S. (2017). *Graviton Mass Bound,* Rev. Mod. Phys. 89, 025004.

Rovelli, C. and Speziale, S. (2003). *Reconcile Planck-scale discreteness and the Lorentz-Fitzgerald contraction*, Phys. Rev. D 67, 064019

Santill, R.M. (1956). *Percé lo spazio é rigido (Why space is rigid)*, Il Pungolo Verde, Campobasso, Italy (1956).

Singh, Tejinder P. (2019). *Proposal for a New Quantum Theory of Gravity*, Zeitschrift für Naturforschung *A* 74 (7), 617-633. https://doi.org/10.1515/zna-2019-0079

Smolin, L. (2004). *Atoms of Space and Time* Sci. Am., *290*(1), 66-75.

Sorkin, R.D. (1991). *Space-time and causal sets, in Relativity and Gravitation: Classical and Quantum*, Proceedings of the SILARG VII Conference, Cocoyoc, Mexico, December 1990 (Published in: J. C. D'Olivo, E. Nahmad-Achar, M. Rosenbaum, M. P. Ryan, L. F. Urrutia, and F. Zertuche, (eds.)), pp. 150–173, World Scientific, Singapore.

Sorkin,R.D. (2005). *Ten Theses on Black Hole Entropy,* Stud. Hist. Phil. Sci. B 36 (2005) 291-301

Spallucci, E. & Smailagic, A. (2021). *Horizons and the Wave Function of Planckian Quantum black holes*, Physics Letters B 816

Sprenger, M., Nicolini, P., & Bleicher, M. (2012). *Physics on the Smallest Scales – An Introduction to Minimal Length Phenomenology*, Eur. J. Phys., *33*(4), 853-862.

Zakharov, A. F., Jovanovic, P., Borka, D. & Jovanovic, V.B. (2016), *Constraining the range of Yukawa gravity interaction from S2 star orbits II: Bounds on graviton mass*, JCAP 1605 (05), 045, arXiv:1605.00913 [gr-qc].