

*Article*

# Traffic Flow Techniques Applied to the Modeling of the Intersection of East 53rd and East 56th Streets in Panama City: A Practical Case

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**Abstract:** Intersections within urban centers do not allow all citizens to move in a fluid manner. In this sense, this paper proposes a queuing network model based on Kendall's notation for the intersection of two urban streets (East 53rd Street and East 56th Street) in Panama City. A set of traffic lights operating under a fixed time scheme controls the vehicular flow at the intersection. The objective is to optimize the vehicular flow at the intersection by minimizing the waiting time of the vehicles using mathematical models, Linear Regressor Model, and simulations performed in MATLAB evaluating the waiting time experienced by the vehicles. Based on the results obtained, the optimal signal timing of the traffic lights is determined to minimize the waiting time for both streets.

**Keywords:** Vehicular traffic flow; queuing theory; optimal traffic light control; optimization; simulation; simulation

## 1. Introduction

A significant problem within cities is the transit of citizens. Roads and intersections within urban centers do not allow all citizens to move fluidly, showing severe traffic congestion problems on their roads caused by the increasingly high traffic demand in the face of insufficient road capacity [1]. Cities manage the different flows of cars at intersections in a static manner, which means that the times assigned to traffic lights are constant and do not adapt to the changing traffic conditions at any given time, resulting in sub-optimal performance. Given that a city has many intersections regulated by these systems, the inefficiency of these systems results in hundreds of hours lost by citizens [2,3].

In this sense, this article proposes a mathematical model based on queuing theories for the analysis of vehicular flow in one of the most congested arteries in Panama City. After the application of the method (for a large number of cases and under real traffic restriction systems), it is sought to obtain time sequences for the traffic lights that allow them to work in a coordinated manner and to allow a large number of vehicles to pass the intersection, thus reducing waiting times and the length of queues, allowing the proposed model to serve as a basis for approaching similar problems and for creating better methods. The paper's focus is on the issue of vehicular congestion in the city of Panama and proposes a mathematical model to optimize traffic flow at a specific intersection. While the findings may interest researchers and practitioners in the field of transportation engineering. The paper's methodology and approach may serve as a valuable reference for similar studies in other cities facing similar issues.

The rest of the paper is as follows: Section 2 Related Research, Section 3 Theoretical Framework, Section 4 Problem Definition and Motivation, Section 5 Mathematical Model, Section 6 Results, and Discussions, and finally in section 7 Conclusions and Future Work.

## 2. Related Research

This section discusses related research in the field of traffic flow optimization.

Traffic flow management (TFM) allocates various airports, airspace, and other resources to maintain efficient and safety-consistent traffic flow. TFM is a complex research area involving operations research, guidance and control, human factors, and software engineering. Hundreds of human operators make TFM decisions involving thousands of aircraft, in route air traffic control centers, the Federal Aviation Administration System Command Center, and many airline operations centers. This research provides an overview of how TFM decisions are made today, the system's future challenges, and the reviews of the modeling and optimization approaches to facilitate system-wide modeling and performance evaluations [4].

In the research of Liu et al. [5] proposes the development of a system that allows the visualization of information posted on social networks about traffic incidents. Feature engineering methods, such as vector counting and TF-IDF, were applied to process tweets into structured data. Machine Learning models were created for traffic-related tweet classification using SVM, Naïve Bayes, Random Forest, and XGBoost. The prediction models resulted in two: a classification model that detects incident or non-incident tweets and a categorization model that determines the type of incident (accident, hazard, or obstacle). This system has advantages such as speeding up the detection and visualization of traffic incidents, which can significantly help the country's traffic authorities and the public.

Walraven et al. [6] propose a new method for traffic flow optimization based on reinforcement learning. They show that a traffic flow optimization problem can be formulated as a Markov decision process. They use Q-learning to learn policies that dictate the maximum driving speed allowed to reduce traffic congestion.

Neukart et al. [7] present a real-world application using quantum technologies. Specifically, they show how to map certain parts of a real-world traffic flow optimization problem to be suitable for quantum annealing. They present that time-critical optimization tasks, such as continuous redistribution of position data for automobiles in dense road networks, are suitable for quantum computing.

In the research of Garcia & Larraga [8], the objective was to increase the number of vehicles that can circulate without long queues. For this purpose, two entry flow situations were analyzed with regard to the two streets considered: symmetric entry flow and asymmetric entry flow. The results were obtained from an average of 50 independent experiments for each simulation. For both cases, analytical approximations that agree very well with the simulation results were obtained.

On the other hand, the study of Arpi et al. [9], aimed to analyze the possible cause of vehicular congestion of queues in the existing traffic lights within the 25 de junio avenue from the distributor of El Bananero to the traffic circle of El Cambio, in the city of Machala-Ecuador. The results obtained employing the Queue Theory indicated that there is no kind of vehicle oversaturation concerning each traffic light. A modeling was performed in the SYNCHRO 8.0 program. The data obtained indicate that the service levels at each point are not very high, and the capacity volume ratio is slightly greater than 1. The program optimized these results and generated improved service levels and capacity volume ratios.

## 3. Theoretical Context

Over the last fifty years, a wide range of traffic flow theories and models have been developed as tools to solve the economic and social problems arising from high vehicular demand. Research aims to optimize the efficiency of existing traffic systems, thereby increasing vehicle capacity [10,11].

### 3.1. Traffic and vehicular flow

Vehicular flow is the phenomenon caused by the flow of vehicles on a road, street, or highway. It also has many similarities in other phenomena, such as the flow of particles (liquids, gases, or solids) and pedestrians. In large cities, vehicular flow is present in almost all spheres of people's daily activities, and causes numerous phenomena, among which congestion stands out [12].

### 3.2. Queuing Theory

Queuing theory is the study of a technique based on operations research to solve problems that arise in situations where waiting for shifts or queues are formed for the provision of a service or execution of a job [13].

Among the most important terms that comprise the queuing theory [14] are "Customers," which refer to the entity that arrives at the system, such as: Cars waiting at a traffic light, machines waiting to be repaired, airplanes waiting to land, among others. "Arrivals" which refers to the number of customers arriving at the service facility. "Service Rate" which is used to designate the service capacity, which can be provided by one server or by multiple servers. "The Arrival Rate" which describes in units of time the feeding of the system. "The Server," which oversees providing the respective service to the client. "The Queue Capacity," which can be infinite or finite. Given the above, different queue models can be presented, with corresponding efficiency measures that characterize the system [15-17].

### 3.3. Kendall's notation

D.G. Kendall suggested a valuable notation for classifying the vast diversity of different wait-line models that have been developed [18,19]. Kendall's notation of three symbols is as follows: A/B/K, where A: indicates the probability distribution of arrivals, B: indicates the probability distribution of service times, and K: indicates the number of channels. Various waiting line systems can be described depending on which letter appears in the A or B position.

Commonly used letters are M: designates a Poisson probability distribution for arrivals or exponential probability distribution for service time. D: designates the fact that arrivals or service time is deterministic or constant. G: indicates that arrivals or service time have a general probability distribution, with known mean and variance [13,20].

### 3.4. Artificial Intelligence (AI) Application

Artificial intelligence (AI) can be key in optimizing vehicular flow. For example, intelligent traffic control systems (ITS) use AI techniques to monitor traffic and adjust traffic lights and signals in real-time [6]. These systems can help reduce waiting times and improve traffic flow at intersections. Intelligent transportation systems (ITS) can use technologies such as traffic sensors, surveillance cameras, and navigation systems to collect real-time traffic data and analyze it using AI techniques. ITS systems can automatically adjust traffic lights, direct traffic to alternate routes, and provide real-time information to drivers and pedestrians [21].

In recent years, intelligent transportation systems (ITS) has received considerable attention due to increased road safety and efficiency demands in highly interconnected road networks. As an essential part of ITS, traffic forecasting can provide support in many aspects, such as road routing, traffic congestion control, applications, etc., and analyze how traffic forecasting can improve the performance of these applications[22].

AI to predict traffic behavior and vehicular flow under different conditions. For example, machine learning models can analyze extensive traffic data sets to identify patterns and trends. These models can be used to predict traffic demand at different times of the day and in different weather conditions[23,24].

Some practical cases of the use of artificial intelligence in the optimization of vehicular traffic can be observed in the work of Guo y Yan [25]. They propose an intelligent network control architecture based on SDN and artificial intelligence. The proposed architecture consists of three modules: a network state collection/perception module, an AI intelligent analysis module, and an SDN controller module. The experimental results demonstrate that using SDN and artificial intelligence in operator networks can do intelligent network control and traffic optimization more intelligently.

Chin et al. [26] propose an artificial intelligence algorithm in the traffic signal synchronization scheme to enable the learning capability of traffic management systems. The Q-Learning algorithm is a learning mechanism for traffic signal intersections to be released from traffic congestion situations. Adjacent traffic signal intersections will operate independently and cooperate with the common goal of ensuring smooth traffic flows within the traffic network. Experimental results show that the Q-Learning algorithm can learn from dynamic traffic flow and optimize the traffic flow.

Nam Bui y Jung [27], a game-theoretic approach of cooperative games among agents is proposed to improve traffic flow within a large network. For this purpose, a distributed merge-and-split algorithm for coalition formation is presented. This algorithm is applied to discover how to incorporate cooperation among agents to dynamically control the traffic light at intersections. In addition, a traffic simulation framework is constructed to evaluate our approach. With various parameters for traffic density, the proposed system can effectively improve both uniform and non-uniform traffic flow. Through inter-controller coordination, the waiting time of vehicles at intersections can be reduced by 15% to 25% compared with previous methods (e.g., Green Wave coordination).

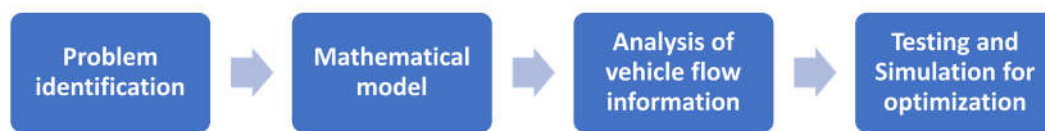
Yang et al. [28] propose multiagent reinforcement learning for traffic signals (MARL4TS) to support traffic signal control and deployment. First, information about traffic flows and multiple intersections is formalized as input environments for reinforcement learning. Second, they design a new reward function to continuously select the most appropriate strategy as control during multi-agent learning to track traffic signal actions. Finally, they use a supporting tool, Simulation of Urban Mobility (SUMO), to simulate the proposed traffic signal control process and compare it with other methods. Experimental results show that our proposed MARL4TS method is superior to the baselines.

Li et al. [29] propose a deep feature learning approach using supervised learning techniques to predict the short-term traffic flow in the next multiple steps. To achieve next-day traffic flow forecasting, an advanced multi-objective particle swarm optimization algorithm is applied to optimize some parameters in deep belief networks.

In the work of Shengdong et al. [30], it is aimed to discuss problems such as complex object types, large amounts of data collection, high transmission and computation demand, and weak real-time control and scheduling capability in constructing modern intelligent traffic information, physical fusion networks and cloud-based control. The system theory, modern intelligent traffic control network as the research object and the physical design of intelligent transportation information fusions cloud control system scheme. The scheme includes intelligent transportation edge control technology and intelligent transportation network virtualization technology. Based on intelligent traffic flow data, in the center of the cloud control management server using deep learning and overrun learning machine intelligence study methods to predict urban road short-term traffic flow and congestion.

### 3.5. Methodology

The following is a description of the development process of this mathematical model to optimize the vehicular flow of Panama's 50th Street:



**Figure 1.** Block diagram of the model.

The project begins by identifying the problem and the motivation for the study. This step is essential, as it provides a clear understanding of the problem and helps create the variables needed to develop the mathematical model.

The mathematical model is based on queuing networks, a mathematical tool used to analyze and optimize the system's performance involving queues or waiting lines. In the context of the study, the queuing network is used to model the flow of vehicles. The system's performance is evaluated as a function of the waiting time experienced by the vehicles.

To check and verify the effectiveness of the mathematical model, simulations, and tests are performed. These simulations create a computer model of the vehicle flow, which allows for observing the system's behavior under different conditions. By varying different parameters, such as traffic volume, road capacity, and traffic control mechanisms, the impact of each parameter on the waiting time experienced by vehicles can be evaluated.

The research aims to optimize vehicle waiting time and improve traffic flow. This can bring several benefits, such as reduced travel time, increased safety, and improved fuel efficiency. The research can also help traffic planners and policymakers make more informed decisions and implement more effective traffic management strategies.

- **Summary:** The problem and motivation of the study are identified; this will allow us to create the necessary variables to develop the mathematical model based on queuing networks and thus be able to perform the information analysis of the vehicular flow. Finally, we perform tests and simulations to verify the optimization of the waiting time experienced by vehicles.

#### **4. Problem definition and purpose**

Every time there is population growth, vehicular traffic also increases. Currently, Panama City does not escape from this problem. This situation can be evidenced in the rush hours of working days, which include when people go to work, at lunchtime, and at the time of departure of workers. The city center becomes chaotic, with people spending more time in a traffic jam than at home. Many ways have been sought to improve this problem, including creating exclusive lanes for the metro bus, implementing lines 1 and 2 of the Panama subway; still, there is no difference [31].

In this sense, this research seeks to develop mathematical models that allow working on the current problem of vehicular congestion in Panama, specifically on Calle 50 (the most used route for vehicular flow) (see Figure 2). The model to be proposed seeks to reduce the time in which an individual is involved daily in these vehicular congestions employing queuing theory, by modeling the current situation of vehicular congestion on Calle 50, at the intersections of Calle 53 East and Calle 56 East, to establish then the optimal traffic light times, which will help to decongest the section.





Figure 2. Calle 50, the most used route in Panama (Source: via Google Earth).

## 5. Mathematical Model

The mathematical model consists of implementing an M/M/1 queuing system in which the waiting system is characterized by the fact that the arrival times and service times are exponentially distributed and have a single server. The queue discipline is FIFO, and the size of the input population is infinite, i.e., the number of customers in the system does not affect the arrival rate (see Figure 3) [32–34].

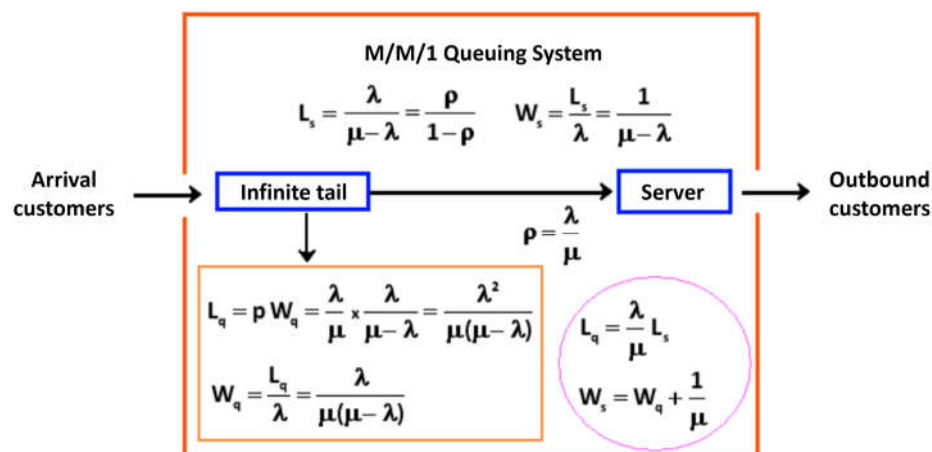
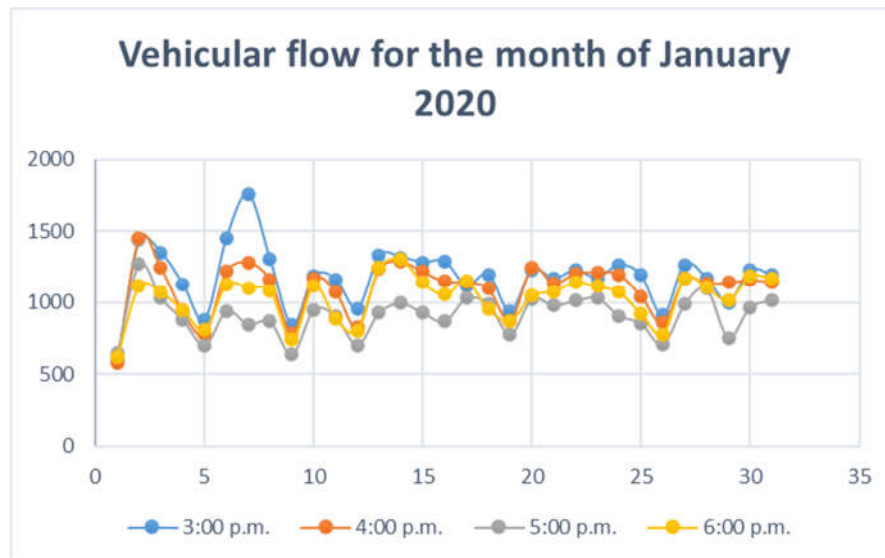


Figure 3. M/M/1 Queuing System

For the development of this model, we will use:

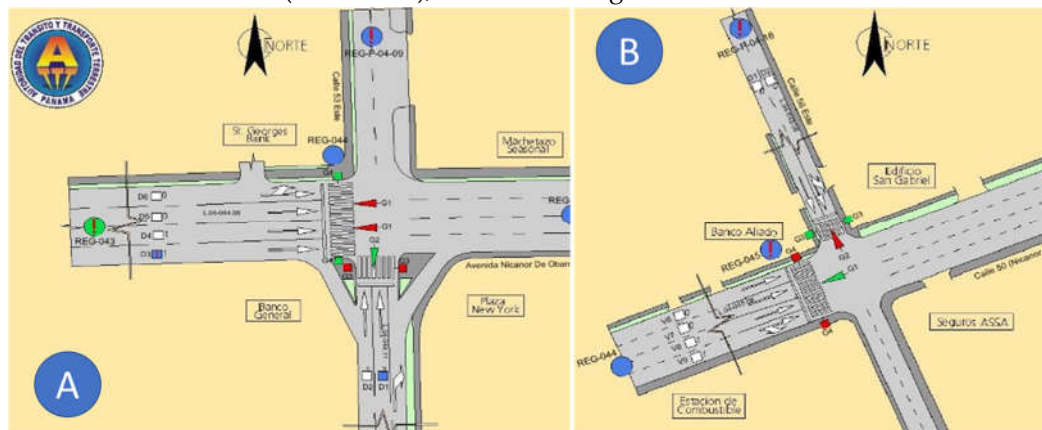
- Population: The population to be used in the model to be generated is all vehicles that travel on the main avenues of Panama City.
- Sample: It will correspond to the vehicles that travel along the stretch of roads that comprise Calle 50 - Aquilino de la Guardia to Calle 50 - Calle 56. January 2020 will be considered, with a cycle of 180 seconds, from 3:00 p.m. to 6:00 p.m. (see Figure 4).



**Figure 4.** Vehicular flow for January 2020. Schedule from 3:00 p.m. to 7:00 p.m.

- Variables: The independent variables we will use are:  $\lambda$  which is the number of arrivals and  $\mu$  number of departures or service rate. The dependent variables are:  
 $\rho$ : is the average system utilization.  
 $L$ : average number of vehicles in the service system.  
 $L_q$ : average number of vehicles in the waiting queue.  
 $W$ : average time elapsed in the system, including service.  
 $W_q$ : average waiting time in the queue.

The information will be provided by the Panama Transit and Land Transportation Authority (ATTT). Here we analyze the vehicular movement through two traffic lights 044 and 045 located on Nicanor De Obarrio Avenue (50th Street), as shown in Figure 5.



**Figure 5.** 50 Nicanor de Obarrio Street, A-53rd Street East; B-56th Street East.

From Figure 5 we can see the L04-044-08 and L04-045-08 counter, which provides the initial data for the queuing theory model, the number of customers or vehicles entering the system per hour. The queuing system study required a switch to a traffic network to understand the queuing processes at both server's input and output as shown in Figure 6.

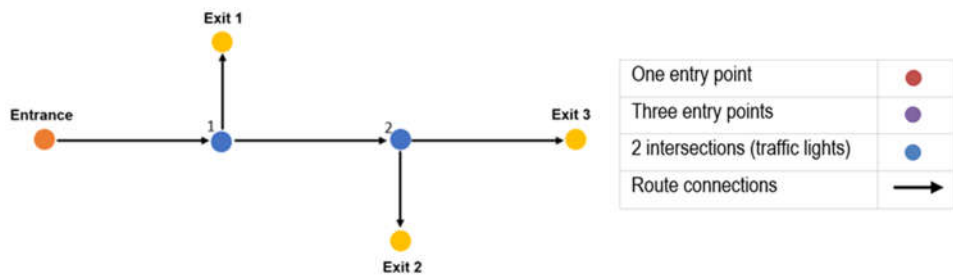


Figure 6. Vehicle traffic network

6. Results and discussions

For the stability analysis of the queuing system, we first adapted the traffic lights and the streets where the queues are formed to a serial M/M/1 queuing system; the traffic network was taken to a system where we have the queue vs. the servers (see Figure 7).

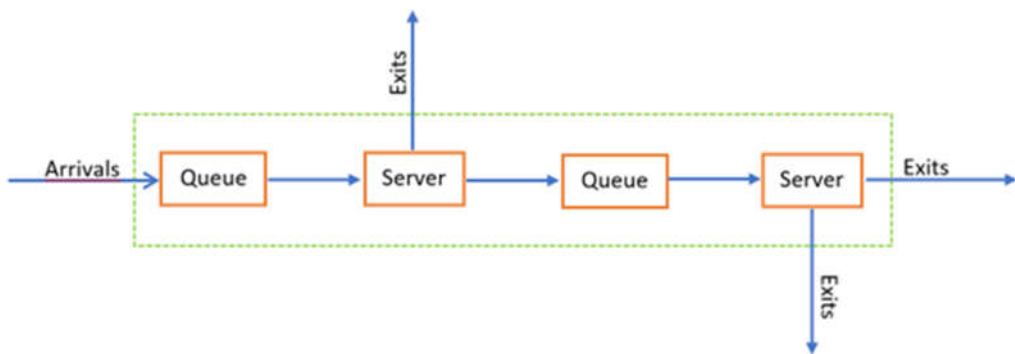


Figure 7. Queuing system at the 50th Street intersection (Nicanor de Obarrio Ave.).

Applying Kendall's notation to these data, this waiting system is characterized by the fact that both the inter-arrival times and the service times are exponentially distributed, and the number of servers is one after another (in series). We analyze the stability of the vehicles entering the first traffic light at 50th Street to determine the % of the days of the month where the system is stable.

Through MATLAB software, a function called `tcola50` was made to run through the vectors and generate the value of  $\rho$ , to see the days the system was stable. Moreover, thus calculate a percentage in January where the system was stable for the first traffic light (see Figure 8).

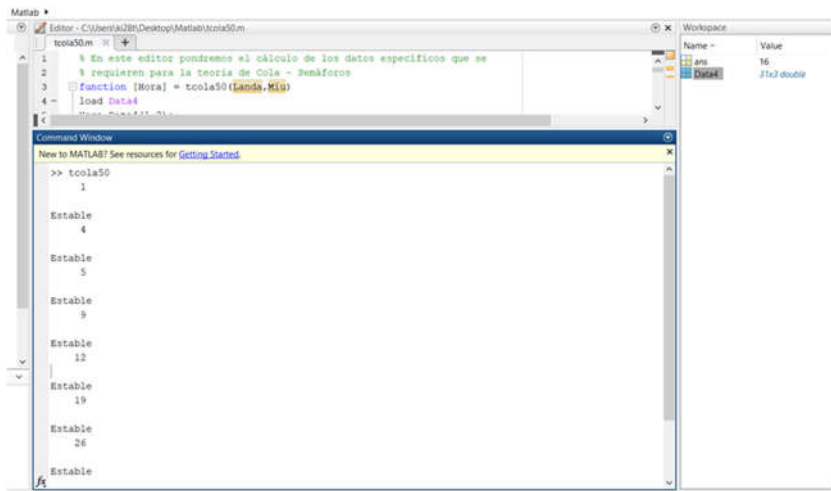


Figure 8. Results of the `tcola50` function.



6.1. Optimization of the queuing system

1. Analysis for the month of January for a traffic light capacity of 1000 vehicles/h.  
The analysis of the first server (traffic light 1) shows the results of stability of 22.58 % and instability of 77.42 %. The results of the other variables analyzed on the days selected for this server are shown in Table 1.

**Table 1.** Results of L, Lq W, Wq. For a capacity of one thousand vehicles. First Server. 3:00 p.m.

January Day 2020	L	Lq	W	Wq
1	1.4510	0.8590	0.0025	0.0015
5	7.1301	6.2531	0.0081	0.0071
9	5.2112	4.3722	0.0062	0.0052
12	20.7391	19.7851	0.0217	0.0207
19	13.9254	12.9924	0.0149	0.0139
26	9.7527	8.8457	0.0108	0.0098
29	499	498.002	0.5	0.499

In addition, the following information is obtained for this queuing system:

- The average number of vehicles in the queue is 18 vehicles per light change.
- The average time to be served is: 65 seconds, under the condition that it only occurs on holidays and weekends.
- For the capacity of 1000 vehicles per hour, the system was 77% of the time over saturated, with an average occupancy rate of 118%.
- Of the days that the Kendall notation results could be obtained are holidays and weekends, i.e., the value of  $\rho < 1$ , the system was at an average occupancy of 87%.

Using the program, it was found that the average number of cars leaving the system after being served by the server (traffic light 1) corresponds to 20% and 80% continue. The system has four parallel lanes that continue, and one leaves the main road because the left lane forks. Taking this information into account, the percentage of distribution that will be the restriction to advance to the next server (traffic light 2) is calculated and 100% of cars entering the system is divided by five to obtain this percentage.

From the results obtained in the first server, 80% of cars waiting to be served by the second server (Traffic Light 2) are considered. In this second server, stability improves, and 64.5% stability and 35.5% instability are obtained. Additionally, the following results are obtained:

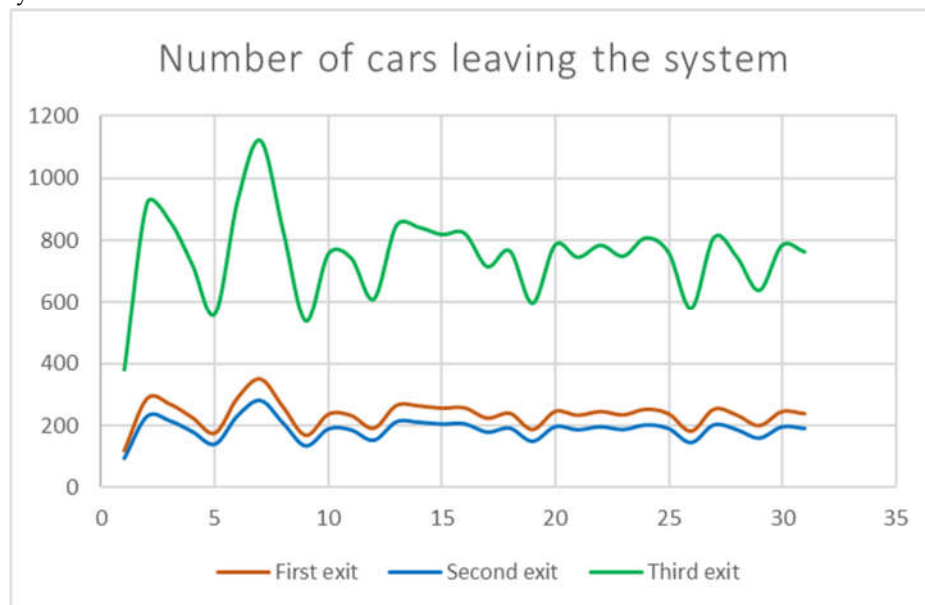
- The average number of vehicles in the queue is 11 vehicles per light change.
- The average time to be served is: 40 seconds, under the condition of stability on holidays and weekends.
- For the capacity of 1000 vehicles per hour, the system was 65% stable and 35% unstable.
- For the full month, the system had an occupancy level of 94%, which confirms the improvement in stability.
- The system was at an average occupancy of 85% on days where the value of  $\rho$  is less than 1, i.e.,  $\rho < 1$ .

Additionally, stability and instability values were obtained for the capacity of 1,000 vehicles at the 4:00 p.m., 5:00 p.m. and 6:00 p.m. schedules. The results obtained are shown in Table 2:

**Table 2.** Stability and instability values for the capacity of 1,000 vehicles during the hours of 4:00 p.m., 5:00 p.m. and 6:00 p.m.

Server	Hour	Stability	Instability
First traffic light	4:00 p.m.	23%	77%
Second traffic light		90%	10%
First traffic light	5:00 p.m.	74%	26%
Second traffic light		97%	3%
First traffic light	6:00 p.m.	32%	68%
Second traffic light		97%	3%

From this second traffic light we have two outputs. The second output corresponds to cars leaving the second server and leaving the system (see Figure 9) and the third output is not considered in this study.



**Figure 9.** Number of cars that leave the system according to the outputs.

## 2. Analysis for the month of January for a traffic light capacity of 1300 vehicles/h.

After several tests for the traffic light capacity of vehicles per hour, it was determined that the value of 1300 vehicles per hour for  $\mu$  is where stability of more than 70% is achieved. An improvement in stability was obtained with 77.4% and an instability of 22.6%. By having a stable queuing system, it was possible to calculate the average values by means of Kendall notation with a more accurate study of the system. For the first server (traffic light 1) the following results were obtained:

- The average number of vehicles in the queue is 13 vehicles per light change.
- The average time to be served is: 37 seconds, if and only if the vehicle arrives now when the light is green and there is no vehicle waiting.
- The system had an occupancy level of 91% for the entire month. This confirms the improvement in stability.

It is observed that 20% of the vehicles leave the system after being served by the first server. While 80% continue to be served by the second server (traffic light 2). This improves the stability in this second server by 96.8%, with an instability of 3.2% and the following results:

- The average number of vehicles in the queue is 3 vehicles per light change.
- The average time to be served is: 11 seconds, if and only if the vehicle arrives now when the light is green and there is no vehicle waiting.

- The system had an occupancy level of 72% for the entire month. This confirms the improvement in stability.

The values obtained for L, Lq, W, Wq at 4:00 p.m., 5:00 p.m. and 6:00 p.m. are shown in Table 3.

**Table 3.** Stability and instability values for the capacity of 1,000 vehicles during the hours of 4:00 p.m., 5:00 p.m. and 6:00 p.m.

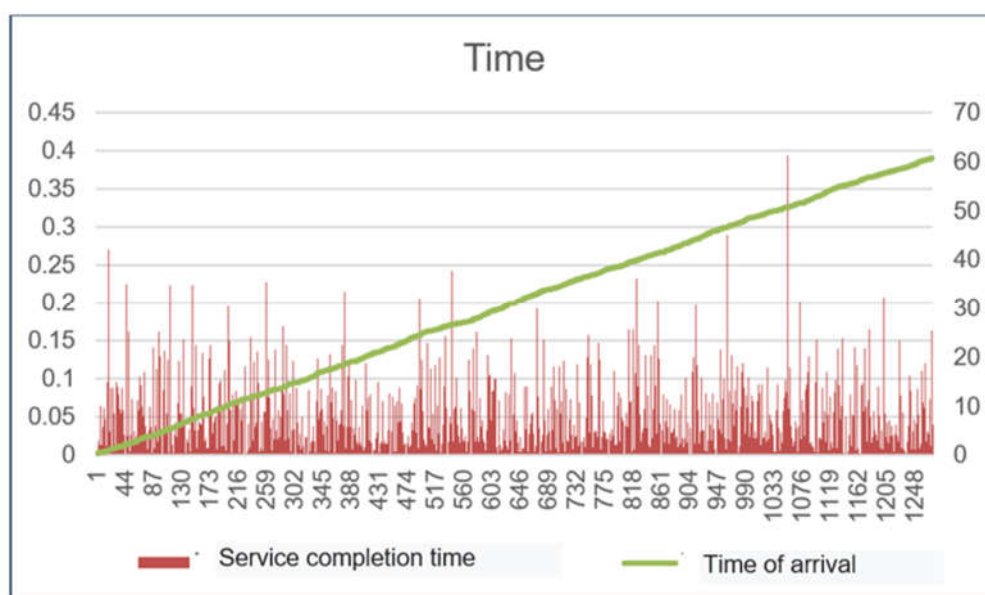
Server	Hour	Stability	Instability
First traffic light	4:00 p.m.	97%	3%
Second traffic light		100%	0%
First traffic light	5:00 p.m.	100%	0%
Second traffic light		100%	0%
First traffic light	6:00 p.m.	97%	3%
Second traffic light		100%	0%

## 6.2. Simulation

For the simulation, a specific day in January was taken, January 15, 2020, for a capacity of 1,300 vehicles. For this day there are 1280 vehicles per hour, which is equivalent to 98% of the system capacity. The Kendall notation has been used in theoretical analysis. For the first traffic light on this day, there are about 64 vehicles in the waiting queue at the first server of the system (traffic light 1); each vehicle takes about 180 seconds (3 minutes) to be served.

With this data, in Excel, we simulate the movement of these vehicles, remembering that we are on the schedule from 3:00 p.m. to 4:00 p.m. and that each cycle takes 180 seconds. Two green cycles of 60 seconds (1 minute) and one red cycle of 60 seconds are included. Each vehicle has their random arrival, and the time between arrivals is calculated, observing that their arrivals are very close to 3 vehicles every 3 seconds, which causes the queue to fill up with vehicles when in the red-light cycle with an average of 180 vehicles.

The time in the system is analyzed, where at the beginning of the system, each vehicle takes about 180 seconds to be attended; however as seconds advance, the queue increases, and the last vehicles must wait approximately 60 minutes to be attended to either to leave the server or to continue to the second traffic light.



**Figure 10.** Simulated arrival times compared to vehicle completion times at 3:00 p.m. on January 15, 2021.

### 6.3 AI Predictive Model

Statistical and AI techniques are increasingly used with numerical models to produce more accurate forecasts. Linear regression (LR) forecasting is one of the most widely used AI models in various applications, such as queueing theory [35,36].

LR is a data analysis technique that predicts the value of unknown data by using another related and known data value. It mathematically models the unknown or dependent variable and the known or independent variable as a linear equation [37]. The regression model consists of an approach to model the relationship between a dependent scalar variable "Y" and one or more explanatory variables named "X" and then plot a line that will indicate the trend of a set of continuous data, whose formula is:

$$Y = mX + b$$

Where Y is the result, X is the variable, m is the slope (or coefficient) of the line, and b is the constant or also known as the "point of intersection with the Y-axis" on the graph (when X = 0).

The predictive models are obtained using simple linear regression using the demand data of each traffic flow intensity from 3:00 p.m. to 6:00 p.m. provided by Autoridad de Tránsito y Transporte Terrestre of Panamá (ATTT) [38]. Table 4 shows the data used to train the model.

The demand of the model variables w, x, y and z is obtained by applying simple linear regression on the data set that constitutes the traffic flow intensity demand for each hour. The variables used for the intensity are:

I<sub>15H</sub> -- traffic flow intensity demand at 3:00 p.m. (15 hours)

I<sub>16H</sub> -- traffic flow intensity demand at 4:00 p.m. (16 hours)

I<sub>17H</sub> -- traffic flow intensity demand at 5:00 p.m. (17 hours)

I<sub>18H</sub> -- traffic flow intensity demand at 6:00 p.m. (18 hours)

The models for each intensity are as follows:

$$w = i_1 + m_1 I_{15H}$$

$$x = i_2 + m_2 I_{16H}$$

$$y = i_3 + m_3 I_{17H}$$

$$z = i_4 + m_4 I_{18H}$$

where  $i_1$ ,  $m_1$ ,  $i_2$ ,  $m_2$ ,  $i_3$ ,  $m_3$ ,  $i_4$ , and  $m_4$  are the coefficients of the linear regressions of the intensities.

Predictive models are obtained using simple linear regression and the demand values of each intensity.

**Table 4.** Maximum demand from traffic flow intensity by hours

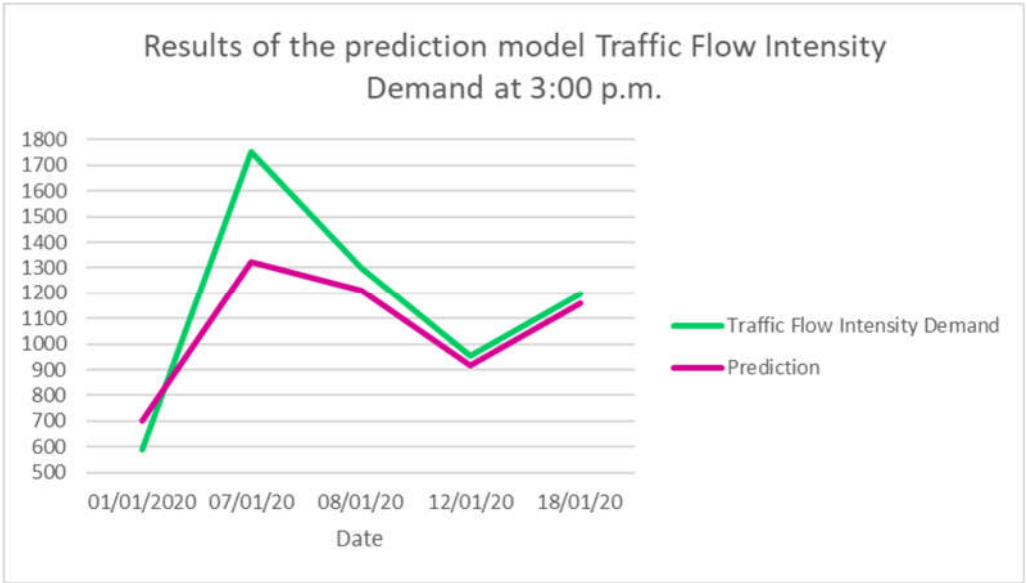
Date	3:00 p.m.	4:00 p.m.	5:00 p.m.	6:00 p.m.
01/01/20	592	577	650	620
02/01/20	1437	1453	1270	1117
03/01/20	1348	1240	1029	1071
04/01/20	1127	929	877	945
05/01/20	877	778	699	809
06/01/20	1452	1219	938	1127
07/01/20	1754	1281	844	1103
08/01/20	1300	1156	865	1083
09/01/20	839	787	634	739
10/01/20	1183	1167	947	1116
11/01/20	1162	1076	902	888
12/01/20	954	824	701	802
13/01/20	1326	1236	928	1247
14/01/20	1316	1286	993	1301
15/01/20	1280	1216	926	1138
16/01/20	1287	1147	866	1053
17/01/20	1118	1142	1027	1150
18/01/20	1197	1096	991	952
19/01/20	933	871	774	870
20/01/20	1228	1245	1024	1047
21/01/20	1165	1135	979	1077
22/01/20	1226	1203	1010	1143
23/01/20	1171	1214	1031	1113
24/01/20	1263	1196	905	1072
25/01/20	1189	1042	853	923
26/01/20	907	856	704	777
27/01/20	1265	1164	985	1170
28/01/20	1169	1134	1097	1108
29/01/20	998	1138	753	1015
30/01/20	1225	1159	965	1185
31/01/20	1192	1140	1015	1169

These data were stored in Pandas DataFrame and were programmed using Python [39,40]. During the training of the models, 80% of the data for training and 20% of the data for testing were used. Tables 5, 6, 7 and, 8 and Figures 11, 12, 13 and, 14; show the values resulting from the model training for each intensity.



**Table 5.** Results of the prediction model for traffic flow intensity demand at 3:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
01/01/2020	592	699.61208
07/01/20	1754	1323.617801
08/01/20	1300	1212.821331
12/01/20	954	918.545905
18/01/20	1197	1159.639025



**Figure 11.** Prediction model for traffic flow intensity demand at 3:00 p.m.

**Table 6.** Results of the prediction model for traffic flow intensity demand at 4:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
02/01/20	1453	1501.728419
09/01/20	787	804.933475
10/01/20	1167	1147.852999
12/01/20	824	878.337974
21/01/20	1135	1182.911864

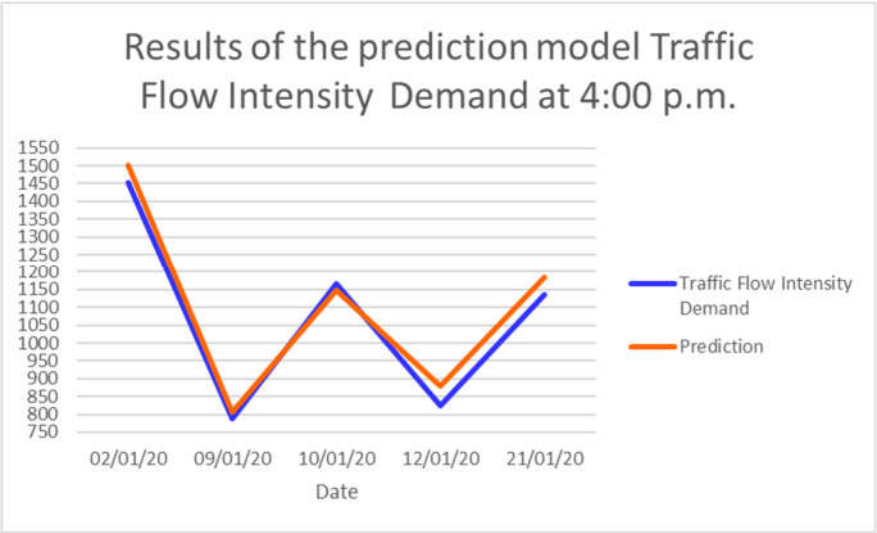


Figure 12. Prediction model for traffic flow intensity demand at 4:00 p.m.

Table 7. Results of the prediction model for traffic flow intensity demand at 5:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
04/01/20	877	845.417087
05/01/20	699	758.486939
12/01/20	701	754.012594
18/01/20	991	849.891432
23/01/20	1031	952.801386

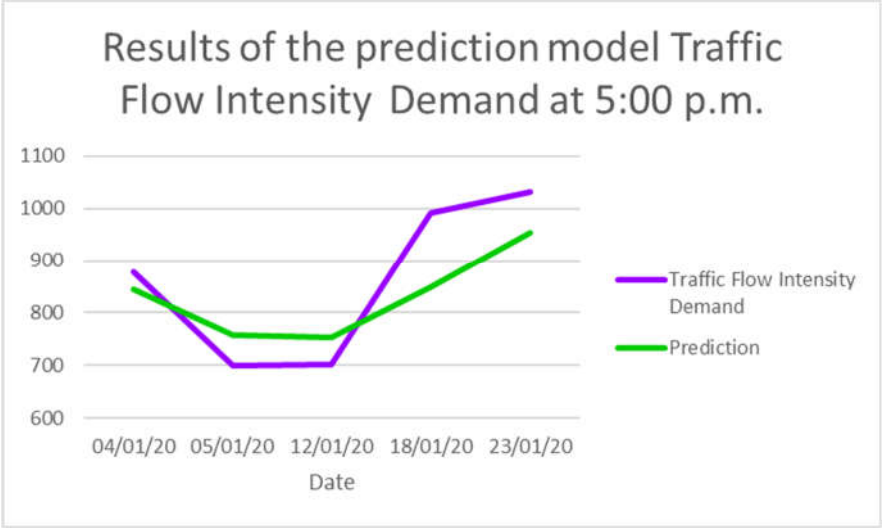
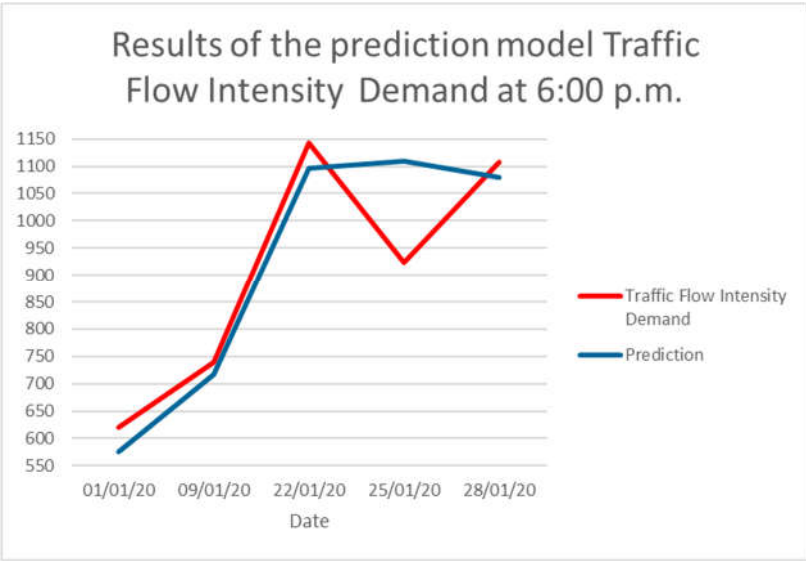


Figure 13. Prediction model for traffic flow intensity demand at 5:00 p.m.

Table 8. Results of the prediction model for traffic flow intensity demand at 6:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
01/01/20	620	575.04631
09/01/20	739	717.509395
22/01/20	1143	1096.252718
25/01/20	923	1110.151556
28/01/20	1108	1078.879171



**Figure 14.** Prediction model for traffic flow intensity demand at 6:00 p.m.

The coefficient of determination ( $r^2$ ) is used to evaluate how well the data of each model fit. In this, a value of 1 is equivalent to an optimal fit. The coefficients of determination ( $r^2$ ) for each traffic flow intensity demand for hours are shown in Table 9.

**Table 9.** Coefficients of determination ( $r^2$ ) by traffic flow intensity demand for hours

Traffic flow intensity demand at:	Coefficient $r^2$
3:00 p.m.	0.7148
4:00 p.m.	0.9713
5:00 p.m.	0.6423
6:00 p.m.	0.7262

7. Conclusions and future work

An M/M/1 queuing model based on Kendall notation was proposed to solve the problem that currently exists in the synchronization of traffic lights on 50th Street in Panama City. The mathematical model included a stability analysis of the system, performing the analysis with two capacities of the system until achieving the stability of the queuing system at 1300 vehicles per hour.

For the AI component, we measured the accuracy, as shown in Table 9. We are considering evaluating other models to make the AI component even more robust, which will be studied in future research. Evaluating other models can help identify the strengths and weaknesses of the current model and compare its performance with other models in different scenarios. In addition, it can also improve the ability of the current model to make more accurate and valuable predictions in more diverse situations.

The algorithm developed in MATLAB was based on a stability analysis, which shows the stability and instability for January 2020. The stability analysis found that the system is not saturated on holidays and weekends. On weekdays, we observed that the system is oversaturated with the capacity currently having the traffic light cycle of 50th Street. With this, we can analyze that the current scheduling system of 50th Street is an unstable queuing system over saturated, which would generate large queues with non-estimated departure times.

A simulation was carried out in Excel over a single day. The chosen day was at 98% of its capacity. The results obtained are that approximately the duration in the queuing system is 0.002 seconds in the best scenario with 0 elements in the queue and with waiting times of up to 60 minutes.

Regarding the service capacity of the system, several system stability analyses were carried out, where the current traffic light capacity is insufficient for the number of vehicles passing through the road at the 3:00 p.m. peak hour. The first analysis is of a capacity of one thousand vehicles per hour (current capacity), giving us a stability of 22.58% and an instability of 77.42%. With this capacity, the system was so saturated that it was not feasible to apply the model. Starting with a capacity of one thousand vehicles per hour, we continued analyzing one hundred at a time until we reached a capacity of one thousand three hundred vehicles per hour (suggested capacity). With this capacity, we obtained stability of over 70%. Compared with the previous capacities, this result is feasible, and the system analysis could be carried out using Kendall's notation.

For the waiting time in the system, two scenarios were analyzed:

- Scenario 1. Vehicle capacity of 1000 vehicles per hour, in this scenario only employing Kendall's notation model, it would be possible to estimate the average waiting values on holidays and weekends in January at 3:00 p.m., where a short queue with an average of 18 vehicles waiting for each cycle and a waiting time of 65 seconds on average was determined. In the rest of the days, the queuing system becomes unstable, and it would not be possible to estimate the waiting times in the system employing this model.
- Scenario 2, for a capacity of 1300 vehicles per hour, the service time for each vehicle is 37 seconds, having at least 13 vehicles in the queue in the best-case scenario where the light is green, and no vehicles are stacked in the queue. The capacity was analyzed from 3:00 p.m. to 4:00 p.m. every day of January 2020. It was observed that, at that time, there was more congestion. A vehicle could take between 2 to 60 minutes to be served.

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