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Article

# Specific Relativistic Uncertainty in Angular Incidence

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**Abstract:** The relativistic effects of the dynamical properties of light at angular incidence were analyzed from the perspectives of Bohr indeterminacy and Heisenberg uncertainties and statistical dispersion. It was found that these effects report minimal uncertainties that agree with one or the other according to the angular range of incidence and that decrease with increasing refringence of the medium, constituting a specific relativistic uncertainty at angular incidence. An anomaly is indicated for the uncertainty principle in the Quantum Theory (QT) setting for small angles of incidence, where the accuracy of the angular position does not imply an increase in the uncertainty of the linear momentum. The anomalies arise because QT does not predict the alternation between the classical and relativistic regimes of photon inertia at angular incidence. Specific relativistic uncertainty particularizes the uncertainty principle in the transmission of light between media pairs at angular incidence for the relativistic scenario, considering an observer that registers the relativistic effects of measurements that interfere with the observed system, in another inertial referential.

**Keywords:** relativity; uncertainty principle; light dynamics; photon inertia; relativistic angular constant; relativistic uncertainty; light transmission

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## 1. INTRODUCTION

Heisenberg's uncertainty principle stands out in the Quantum Theory (QT) scenario, in such a way as to delimit the confidence interval in the relationship between physical quantities and naturally shows itself as a pillar of QT, with notoriety as highlighted by Richard Feynman [13].

In the scenario of quantum computing and information, as well as those in which QT has wide employment, the uncertainty principle is always present adjusting the contour of the variables involved. In Physical Optics with emphasis on new technologies and concepts for computing and information, variables such as orbital angular momentum (OAM) and spin angular momentum (SAM) are treated with modulations [1], [7], [14], [16], [18], which in turn satisfy their quantizations and are bounded by the uncertainty principle while the minimum standard deviation is expected.

Although Relativity and Quantum theories are in different scenarios, they naturally interrelate through the quantization of light and relativistic energy if one considers the quasi-relativistic Shrödinger wave function, for cases where the momentum is well defined [15]. In turn, relativistic effects do not arise naturally in the context of quantum theory [5]. Most of the time, the triggers for transposition between theories arise through the relativistic constant, considering quantized particles with speeds close to that of light in a vacuum.

The uncertainty principle can be considered in three treatments: Heisenberg uncertainty, Bohr indeterminacy and dispersion uncertainty. In agreement with Chibeni [10], The Heisenberg uncertainty relation assumes that the observed object is a particle, where experimental measurements cannot have arbitrarily large precision, characterizing the term uncertainty properly. In Bohr's derivation of the Heisenberg uncertainty relation, quantum objects are assumed to be wave entities, and can be represented in wave packets, where the location of the wave packet is associated with the uncertainty of the wave number, establishing a relation of indeterminacy of quantities conjugated to the pairs [10], [2]. The third relation is the statistical version, which presents a minimum of dispersion between conjoined quantities in pairs [10], [2].

In recent studies, we have discussed the relativistic properties of photon dynamics at angular incidence, in the transition between two media [9], [4], [5], [8], demonstrating that the Abraham

moment appears as a relativistic ignition device of the Minkowski moment. In this scenario the photon is subjected to two torques in different scenarios, one classical and one relativistic, where the relativistic one is able to introduce its trajectory with the increase of the predominance of the relativistic inertia from the classical-relativistic synchronization points, an region of incidence in which the photon has the facility to change its directional properties [4].

In this analysis the uncertainty associated with relativistic properties is verified [9], [4], [5] of light for certain materials at angular incidence, in the face of Heisenberg, Bohr, and statistical dispersion uncertainties, considering deformations associated with linear momentum and angular position in transmission processes.

## 2. METHODOLOGY

From the perspective of Heisenberg gamma-ray microscopy, for small wavelengths of the order of  $\lambda \sim \Delta x$ , in accord with Boughn [3], in scattering measurement the impulse imparted to the electron is proportional to the photon's momentum, where the impulse can be treated as an unknown momentum disturbance:

$$\delta p \sim \frac{\hbar}{\lambda} \quad (1)$$

from which one can describe the laws of quantum mechanics as thought by Heisenberg [3]:

$$\delta p \delta x \approx \hbar. \quad (2)$$

In turn, the Bohr indeterminacy admits a wider range of values:

$$\delta p \delta x \geq \hbar. \quad (3)$$

The uncertainty associated with statistical dispersion presents a larger range of uncertainty in the same way as Bohr, although it introduces a smaller minimum uncertainty when compared to Heisenberg's and Bohr's, where:

$$\delta p \delta x \geq \hbar/2. \quad (4)$$

In the Abraham and Minkowisk moment analysis, the deformations of linear momentum and angular position were described [9], where:

$$\frac{\Delta p}{p} = \frac{\Delta \theta}{\theta} = \gamma(\theta, n_{12})(1 - n_{12}), \quad (5)$$

being the relativistic angular constant [9], [4], [5]:

$$\gamma(\theta, n_{12}) = \frac{\sin \theta_1}{\sqrt{1 - n_{12}^2}}. \quad (6)$$

In the treatment of quantized particles at the microscopic level the uncertainty of trajectories and dispersion, the interpretations are slightly associated with the results and expectations found and discussed for the position and momentum of the electron, considering the basis of Heisenberg's arguments [2]. The present analysis considers that the uncertainty principle can be extended to all quantized particles, such as the photon, even if subject to relativistic effects that disturb the Heisenberg scenario, in the perception of an observer. The Heisenberg uncertainty principle does not consider the effects associated with the observer's intervention in the simple act of extracting measurements and thus does not consider the measurements in the relativistic scenario.

In this work we assume the photon to be a quantized particle, under relativistic effect, of which the trajectories can be easily verified in transmission processes instantaneously before and after the photon-matter interaction that takes place on the separating surface between media pairs. Thus, in an analog to determining the standard deviation of energy when we know the finite interval of an event, considering that we can completely know one of the variables while the other is uncertain [20], we will obtain the standard deviation (uncertainty) of the photon's linear momentum assuming the determination of the photon's angular position variation in transmission.

### 3. DEVELOPMENT AND DISCUSSION

From eq. (5), we can establish the specific uncertainty relation for certain materials, considering the relativistic effects that do not arise naturally in the quantum theory setting:

$$\Delta p \Delta \theta = \gamma^2(\theta, n_{12}) [p(1 - n_{12})] [\theta(1 - n_{12})], \quad (7)$$

being  $p(1 - n_{12})$  and  $\theta(1 - n_{12})$  the variations of the classical linear momentum<sup>1</sup> and angular position for small angles, respectively. Thus, we can constitute the relation of relativistic uncertainty from the perspective of the referential of the source with respect to the referential of the observer in the laboratory:

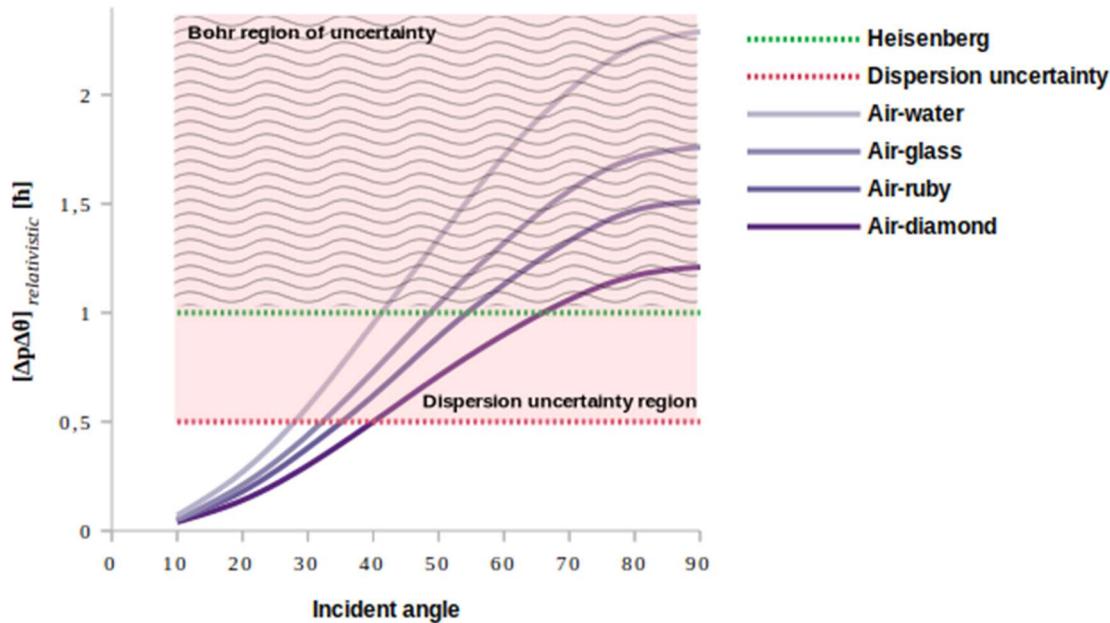
$$[\Delta p \Delta \theta]_{source} = \gamma^2(\theta, n_{12}) [\Delta p \Delta \theta]_{observer}. \quad (8)$$

Let us consider that the observer in the laboratory considers the uncertainty principle, for a single photon (Heisenberg uncertainty), so that the specific relativistic uncertainty for a single photon is given by:

$$[\Delta p \Delta \theta]_{relativistic} \geq \gamma^2(\theta, n_{12}) \hbar. \quad (9)$$

In Figure 1, we see that the uncertainty relation given by eq. (9) for large angles agrees with the expected uncertainty in statistical scattering and Bohr indeterminacy, although the uncertainty of scattering and Bohr indeterminacy agree for values larger than  $\hbar$ . It seems clear that we can separate these regions by the classical-relativistic synchronization angle ( $\theta_{\text{sync}}$ ) [9], [4], [5], exactly at the intersection of the curves with the straight line in green.

In these terms, for large angles before source-observer synchronizations, we find that the relativistic uncertainty agrees with the scattering uncertainty, while for incidence angles larger than  $\Theta_{\text{sync}}$  it agrees with Bohr. The synchronization points are found just when the relativistic uncertainty agrees with the Heisenberg uncertainty. For small angles<sup>2</sup>, the relativistic uncertainty is smaller compared to the minimum expected dispersion uncertainty.

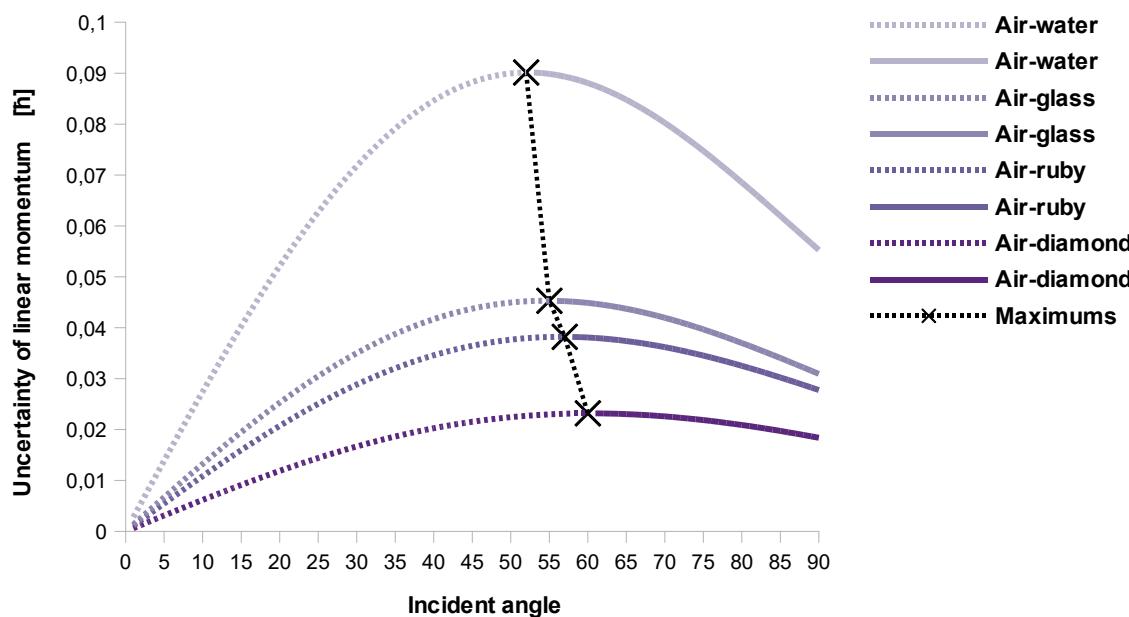


- 1 The variation of the classical linear momentum occurs under the action of Abraham 9.
- 2 We do not present the uncertainty in normal incidence due to the fact that the refracted angle agrees with the incident one, which may generate a wrong interpretation to the reader, of a zero uncertainty. However, in fact the curves maintain their tendency for angles smaller than  $10^\circ$ , with decreasing minimum uncertainty..

**Figure 1.** Minimum specific relativistic uncertainty in angular incidence (Source: author).

In the analysis of the momentum uncertainty of the relativistic energy wave (REW) [9], the accuracy of the angular position variation between the incident and refracted angles was considered. The refraction data known in the literature constituted the angular positions of the photon before and after the photon-matter interaction, in the scenario of constant frequency transmission, with conservation of the linear and angular moments for both particles involved, photon and electron interaction [9], [4], [5].

Note that the estimates treated in the Figures 1 and 2, report the relativistic effects considering the minimum Heisenberg uncertainty and naturally could be reduced by half if we choose the dispersion uncertainty in eq. (9). It is noteworthy that the model given by eq. (9) reports the relativistic effects for the QT scenario, without requiring changes in the Heisenberg algebra, because according to Faizal et al. [12] is subject to deformation whenever the uncertainty principle is deformed.



**Figure 2.** Specific relativistic minimum uncertainty of the photon's linear momentum at angular incidence (Source: author).

It can be seen in Figure 2 that the maxima of the minimum uncertainty are found in the vicinity of Brewster's angle. The classical-relativistic synchronization angles between source-observer [9], [4], which in the quantum-relativistic scenario characterize incidences in which the chances are equal of finding one or another quantized state of light [5], are pseudo Brewster angles [5], [19]. In this sense, the maxima of the minimum uncertainties of the photon linear momentum at angular incidence lie in the range of the reflectance minima.

The region of dashed curves characterizes the range of incidence of anomalies of the uncertainty principle in the unique scenario of quantum mechanics, because the uncertainty of the linear momentum decreases in proportion to the sharp drop in the uncertainty of the angular position. After the maxima of the linear momentum, the relativistic uncertainty agrees with what is expected by Quantum Mechanics (QM) where the certainty of one decreases with increasing uncertainty of the other physical quantity.

The causes of the anomalies are not in the of QM scenario, as they are due to the characteristic relativistic effects of the relativistic angular constant in determining the photon's trajectory. The switch from classical predominance of the photon mass to relativistic occurs at classical-relativistic synchronization angles, with increasing relativistic effects for large angles until  $\odot(\theta, n_{12}) = \odot(n_{12})$  [9], [4], [5].

In QM De Broglie's wavelength is consistent with the wave function considering Schrödinger's own interpretation that his mechanics was an analog to optics [17], and naturally in the description of microscopic particles with speeds close to that of light in vacuum, relativistic mass is assumed, although one expects the best relativistic representation of kinetic energy and momentum for a wave function that reports relativistic effects [15]. Therefore, it is precisely in this assumption that the anomaly arises, because MQ does not predict the classical predominance of the photon mass preceding the synchronizations, nor the alternation of predominances in angular incidence..

The minima of the momentum uncertainty for small angles do not indicate a region (incidence range) in the classical domain where the effects are deterministic, although the predominance of the classical photon inertia, because these limits are not predicted in the classical theory. The relativistic effects occur from angular incidence [9], [4], [5], although the relativistic trajectory appears with relativistic delay until inertial dominance switches to relativistic. In the present case, the uncertainty minima reside in the quantum scenario under the relativistic effects of the recent relativistic properties of photon dynamics at angular incidence [9], [4], [5].

#### 4. CONSIDERATIONS

In this analysis the consistency of the specific relativistic uncertainty of some materials was verified, with the uncertainty principle from the perspective of Heisenberg, Bohr and statistical dispersion. The transposition of the relativistic effects associated with the photon in angular incidence to the uncertainty principle was treated through the uncertainties of the linear momentum and angular position of the photon, before and after the photon-matter interaction.

The specific relativistic uncertainty before clock synchronizations in matter is consistent with the statistical scattering uncertainty, showing smaller specific uncertainty for small angles of incidence. At synchronizations it agrees with the Heisenberg uncertainty for a particle, while for larger angles it is mixed between the scattering uncertainty and the Bohr indeterminacy for an energy wave.

It was found that the relativistic specific uncertainty reports anomalies to the uncertainty principle that do not arise in the quantum theory setting, showing agreement in the uncertainty drops of the pair of variables considered, in the incidence range that precedes the maxima of the minimum specific uncertainty of the linear momentum. The anomalies in the uncertainty principle of quantum mechanics are due to the fact that the alternation between the classical and relativistic predominances of photon inertia arise in the relativistic setting and are not predicted in the quantum theory.

While the uncertainty principle of quantum mechanics is generalized, the specific relativistic uncertainty is a property that reports the particularities of photon-matter interaction, presenting smaller minimum uncertainties compared to those expected by quantum theory, which are distinguished among different materials and angles of incidence.

Stands out that the specific relativistic uncertainty is distinguished from Heisenberg, Bohr and statistical dispersion uncertainty in that it considers the records of an observer who performs relativistic measurements, relative to those that perturb the system in an inertial referential.

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