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Article

Wobbling Motion in Nuclei

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Abstract: Wobbling motion as an exotic collective mode in nuclei without axial symmetry, was intensively discussed during the last few years. The observation of the newly proposed transverse wobbling, first reported in ^{135}Pr and soon after in nuclei from other mass regions, was considered as a significant discovery in low-spin nuclear structure. However, both the reported experimental results and the proposed theoretical models were lively questioned in a series of works devoted to the investigation of the low-spin wobbling mode in the same nucleus. The electromagnetic properties of the $\Delta I = 1$ transitions connecting the one- to zero-phonon and the two- to one-phonon wobbling bands in ^{135}Pr were recently remeasured, demonstrating their predominant $M1$ magnetic character which is in contradiction with the wobbling motion interpretation. These new experimental observables being well reproduced by both quasiparticle-plus-triaxial-rotor model and interacting boson-fermion model calculations, are against the previously proposed wobbling characteristics of the low-spin bands in ^{135}Pr. On the other hand, we obtained conclusive experimental evidence for the theoretically suggested transverse wobbling bands at medium spin in ^{136}Nd. The comparison of the experimental data with calculations utilizing the triaxial projected shell model and a new developed particle-rotor model with frozen orthogonal geometry of the active nucleons, supports the description in terms of transverse wobbling of medium-spin bands in triaxial even-even nuclei.

Keywords: γ-ray spectroscopy; wobbling; collective models

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1. Introduction

One of the exotic rotational motions, known as nuclear wobbling motion, in which any of the inertia axes of a deformed nuclear body do not coincide with the rotational axis. It was first discussed by Bohr and Mottelson for even-even nuclei in the high spin region of nuclei without axial symmetry, and without active particles resulting from broken pairs [1]. The precession of a free asymmetric top in classical mechanics is analogous to this kind of collective motion in quantum physics. The triaxially deformed nucleus always rotates around the axis with the largest moment of inertia (MoI). The rotation axis executes harmonic oscillations about the space-fixed angular momentum vector in the limit of very high angular momentum. Wobbling motion is considered as an unambiguous fingerprint of stable triaxiality of a deformed nucleus which rotates simultaneously around all of its three axes with associated unequal moments of inertia.

Wobbling motion was identified via the observation of a series of rotational bands composed of $E2$ transitions and excitation energy increasing as the number of wobbling oscillation quanta increases. The signature quantum number ($\alpha = 0$ or 1 for even-even nuclei, and $\alpha = \pm 1/2$ for odd-even nuclei) of two rotational bands is distinct. The yrast and yrare bands correspond to zero- and one-phonon wobbling bands, respectively. The connection of the excited and yrast bands is realized by ΔI =1 dipole transitions; unlike the signature partners of a given configuration corresponding to spin up and down $s = \pm 1/2$ of the particle, the multipole $E2/M1$ mixing ratios δ are very large because the entire nuclear charge is involved in the wobbling motion, resulting in a collectively enhanced $E2$ component.

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The experimental wobbling evidence for nuclear wobbling motion was firstly reported in the triaxial superdeformed $^{161-167}$Lu and ^{167}Ta odd-even nuclei two decades ago [2–7]. The wobbling mode is recognized in the triaxial superdeformed bands and assigned to the $\pi i_{13/2}$ intruder configuration with a significant deformation of $\varepsilon_2 \approx$0.4 in all these nuclei. It should be emphasized that in all of these nuclei, the spin of the odd nucleon is assumed parallel to that of the core. Frauendorf and Dönau have recently refined the investigation of the wobbling motion in odd-mass nuclei [8]. Two types of wobbling modes, transverse wobbling (TW) and longitudinal wobbling (LW), were suggested based on whether the odd nucleon aligns its angular momentum perpendicular or along the axis of collective rotation.

The wobbling mode generates a series of rotational bands that are built on the same intrinsic structure. However, there bands have varying tilt angles of the collective angular momentum vector with respect to the principal axis of the nucleus. The yrast band with the quantum number of wobbling-phonon n = 0 corresponds to the rotation about the axis of with the largest moment of inertia (n is the number of excited wobbling phonons). The bands with n = 1, 2, ..., have increasing tilt angles with respect to that axis, and are referred to as one-phonon band, two-phonon band, and so on. The angular momentum of the odd nucleon in transverse wobbling is perpendicular to the angular momentum of the core, and the wobbling energy E_{wob} defined as

$$E_{\rm wob}(I) = E(n=1,I) - \frac{E(n=0,I+1) + E(n=0,I-1)}{2}, \tag{1}$$

Under the assumption of frozen orthogonal geometry, the transverse wobbling mode was firstly investigated using the triaxial particle-rotor model for the low-spin bands in ^{135}Pr, as well as for the triaxial superdeformed bands of the odd-mass Lu isotopes [8]. Recently, transverse wobbling bands at low spin were identified in ^{135}Pr [9,10]. Soon after these works, transverse wobbling motion was also documented in other odd-A nuclei, e.g., ^{105}Pd [11], ^{183}Au [12]. For the longitudinal wobbling the angular momentum of the odd particle is parallel to the axis with the largest moment of inertia and the wobbling energy E_{wob} increases with increasing angular momentum. Very recently, the longitudinal wobbling has been claimed in ^{133}La, ^{187}Au, and ^{127}Xe, where it was discovered that the wobbling energy increased with increasing spin [13–15]. The odd nucleon occupies a high-j orbital in all these nuclei (neutron only for the ^{105}Pd, and proton for all the other nuclei). However, the transverse wobbling description of the low-spin bands in all these nuclei are seriously questioned according to new re-measured experimental results and new theoretical calculations with geometric and algebraic models, see Section 2.

Wobbling motion has been widely observed in odd-A nuclei of various mass regions. However, the experimental evidence of wobbling motion in even-even nuclei is extremely rare over the nuclear chart. The only suggested wobbler candidates are ^{112}Ru and ^{104}Pd [16,17] in which the "γ-bands" were suggested as one- and two-phonon wobbling bands based on the observed odd-even staggering pattern of the level energies which is characteristic of rigid triaxiality. In fact, these nuclei are γ-soft, and the experimental evidence do not fully support the wobbling interpretation of these bands. At medium spin, under the effect of rotation and of the polarization induced by the unpaired particles resulting from the breaking of a nucleon pair, a more stable triaxial shape could be acquired by the nucleus. Possible one- and two-phonon transverse wobbling bands above I = 10 were initially suggested in two even-even nuclei, ^{134}Ce and ^{136}Nd [18]. However, because the mixing ratios and transition probabilities of the linking ΔI = 1 transitions between the hypothesized wobbling bands were not observed, that conjecture was not supported experimentally. Up to very recently, the only observation of wobbling bands above spin I =10 built on the $\pi h^2_{11/2}$ configuration was reported in the even-even nucleus ^{130}Ba [19]. The experimental findings supported the existence of transverse wobbling motion in two-quasiparticle bands developing at medium spin in ^{130}Ba, and were in good agreement with calculations employing the constrained triaxial covariant density-functional theory and quantum particle rotor model [19]. Two bands based on the $\pi h^2_{11/2}$ configuration in the even-even

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nucleus ^{136}Nd [20] were theoretically investigated in detail using the triaxial projected shell model, with a focus on the possibility of transverse wobbling interpretation. Very recently, we experimentally investigated the wobbling interpretation of the two-quasiparticle $\pi h^2_{11/2}$ bands in ^{136}Nd by measuring the mixing ratios character of the transitions linking the yrast and yrare bands [21], see Section 3.

In this article, the discussion of the wobbling motion at low spins of odd-A nuclei, particularly in ^{135}Pr, is presented in Section 2. The wobbling motion based on two-quasiproton configuration at medium spin of even-even nuclei is given in Section 3, and a summary is provided in Section 4.

2. Wobbling motion in odd-A triaxial nuclei

The negative-parity states on ^{135}Pr were firstly reported in Ref. [22]. New states were identified in Refs. [9,10], which were combined with previously known negative-parity states forming new bands. To investigate the nature of the bands, the $\Delta I = 1$ character of the interband transitions were extracted via angular distribution and polarization measurements. The large mixing ratios correspond to high E2 admixtures for the connection transitions were obtained, and a decrease in the wobbling energy give rise to the interpretation of one- and two-phonon transverse wobbling bands. This is the first time a wobbler has been discovered in a mass region other than A ≈160. New results on negative-parity states in ^{135}Pr were also recently published in Ref. [23], in which the electromagnetic characters of some key $\Delta I = 1$transitions used in the wobbling interpretation of the low-spin non-yrast bands differed from those described in Ref. [9], being predominantly $M1$ instead of $E2$ as expected for wobbling bands. These results raised numerous questions regarding the proposed transverse wobbling interpretation of the low-spin bands in ^{135}Pr [24–30].

Experimentally, the major argument is centered on the precise extraction of the mixing ratios, δ, of the ΔI = 1 transitions between yrast and yrare wobbling bands. In general, the mixing ratios of ΔI = 1 transitions obtained from the χ^2 fit to the experimental angular distributions always yields two solutions, $|\delta_1|$ >1 and $|\delta_2|$ <1, and therefore, excepting measurements with very high statistics, is not conclusive. In fact, the transitions linking the yrast and excited wobbling bands are weak, and the experimental statistics not sufficient to definitely conclude on their electromagnetic character based on only angular distributions, but this approach was exactly that used in Refs. [9,10]. By measuring additional observables, i.e., the linear polarization, the ambiguity on the electromagnetic character of ΔI = 1 transitions can be resolved. Only two of the six connecting transitions in ^{135}Pr have been studied using linear polarization measurements in Ref. [9]. However, only the sign of the detected linear polarization values were employed to draw conclusions about their predominant $E2$ character, not the magnitude of the measured linear polarization. For the two-phonon wobbling band in ^{135}Pr reported in Ref. [10], the wobbling interpretation was based on only angular distributions, without linear polarization measurements. Furthermore, the polarization data for the two linking transitions presented in Ref. [9] are inconsistent with the results reported in Ref. [23] which were published shortly after Ref. [9] and obtained using the same reaction and experiment setup. Following these works, an erratum paper was published [31] in which similar polarization results as in Ref. [9] were reported. However, the questions about the analysis remained unanswered, see Ref. [28]. Theoretically, there has been an intense debate on whether transverse wobbling motion in odd-mass nuclei is actually valid since the time when the mode was proposed. In particular, it appears that the frozen approximation employed in Ref. [8] does not adequately account for the effect of the Coriolis force on the odd nucleon, whose angular momentum is connected orthogonally to that of the core. Recently, a thorough investigation of the suggested wobbling bands in odd-A nuclei was conducted in Refs. [30,32], and a new collective mode called tilted precession (TiP) was proposed in Ref. [30]. It was demonstrated that the three-dimensional rotation of a triaxial odd-A nucleus is a precession of the total angular momentum around a specific tilted axis rather than wobbling motion. In Ref. [32], in addition to the known $\pi h_{11/2}$ band, two new excited bands built on the $\pi h_{11/2}$ configuration have been discovered in ^{135}Nd. The experimental available electromagnetic transition probabilities and the energy spectra of the excited bands are in general good agreement with the theoretical findings from calculations using the

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quasiparticle-plus-triaxial-rotor model. The features of the bands distinguish them as tilted precession bands rather than wobbling bands. That work sheds new light on the interpretation of low-lying bands in odd-A mass nuclei and may inspire further researches focused on the nuclear triaxiality.

In this context, a new experiment was conducted at the University of Jyväskylä, Finland, utilizing ^{100}Mo(^{40}Ar, 1p4n)^{135}Pr reaction at 152 MeV beam energy. The properties of the negative-parity bands at low spin in ^{135}Pr were investigated in detail. The JUROGAM II spectrometer, which included 24 clovers [33] and 15 phase-one [34] Germanium detectors with Compton suppression, was used to detect the prompt γ-rays. To overcome the aforementioned drawbacks in Refs. [9,10], a new method combining both angular correlation and linear polarization analyses was used to accurately extract the mixing ratios values of the linking transitions of the low spin bands in ^{135}Pr. Mixing ratio values $|\delta| < 1$ have been obtained for the 747-, 813-and 450-keV transitions in Ref. [35], see Figure 1. These results are in contradiction with the $|\delta|$ >1 values reported in Refs. [9,10], and claims in favor of a non-wobbling nature of the low-spin bands in ^{135}Pr. In addition, the 827-, 764-, and 1009-keV transitions were combined in one band in Ref. [10], and interpreted as two-phonon wobbling band in ^{135}Pr. However, these three transitions do not have increasing energy as expected for a rotational band. As a result, the second excited $19/2^-$ state is proposed as the band-head of the new band in ^{135}Pr that consists of the transitions of 688-and 871-keV with intensities significantly larger than the transition of 827-keV, and in agreement with the expected approximate $I(I+1)$ dependence of a rotational band Ref. [35]. A possible new rotational band built on the third excited $23/2^-$ state, with the 764-keV transition as the initial transition was also observed, while the 1009-keV transition reported in Ref. [10] was not observed. Thus, a total of five bands were established in Ref. [35].

In order to deeply understand the properties of the low spin bands in ^{135}Pr, the quasiparticle-plus-triaxial-rotor (QTR) and the interacting boson-fermion (IBMF) models were employed [35,36]. In those works the frozen approximation of the particle angular momentum was not adopted, nor was the modification of the relative magnitude of the irrotational-flow moments of inertia, and in particular the increase the moment of inertia along the short axis, as was the case in the QTR calculations of Refs. [9,10]. The present QTR model calculations predict rotational bands as result of a combination of collective and single-particle excitations. The single-particle angular momentum rapidly re-aligns from the short to the intermediate axis in all bands. Simultaneously, the orientation of the total angular momentum changes toward the intermediate axis. Three types of excitations that are associated with the predicted rotational bands have varying relative components, but the single particle and the total angular momenta undergo a re-alignment towards the intermediate axis for all the bands. This is in contrast to the prior QTR calculations which ignored this mode of excitation in the calculations and correlated the bands with total angular momentum precession along the short axis for the measured spin range [9,10,35]. In Ref. [36], an alternative interpretation of the identified low-spin yrast and excited bands in ^{135}Pr was also discussed. The calculated mixing ratios of the $\Delta I = 1$ linking transitions between one-phonon band to zero-phonon band, and two-phonon band to one-phonon band are consistently smaller in magnitude than the experimental values. These computed mixing ratios support the recent new experimental data and show a predominance of magnetic character. There are serious doubts the earlier wobbling assignments.

Figure 1 shows the experimental excitation energy versus spin $E(I)$ of bands 1, 3, and 4, as well as the mixing ratios values of the $\Delta I = 1$ transitions linking that claimed wobbling bands 3 to 1 in Ref. [35] of ^{135}Pr, which are in excellent agreement with the calculations using these two models, without invoking the wobbling mode. Figure 2 gives the comparisons of the computed $B(E2; I \to I-1)_{out}/B(E2; I \to I-2)_{in}$ and $B(M1; I \to I-1)_{out}/B(E2; I \to I-2)_{in}$ ratios with the new experimental results reported in Ref. [35], showing a good agreement, but being well different from the results of Ref. [9]. Instead of the fixed orthogonal geometry of the particle and core angular momenta as described as stated in previous works [8,9], it is discovered that the bands emerge from a fast re-alignment of the total angular momentum from the short to the intermediate nuclear axis. Based on these new experimental results and theoretical analysis, we conclude that the nucleus

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^{135}Pr is not a low-spin wobbler. It's also worth to mention that the results of IBMF calculations also strongly challenged the previously proposed wobbling interpretation of low-spin bands in ^{133}La, ^{127}Xe, and ^{105}Pd.

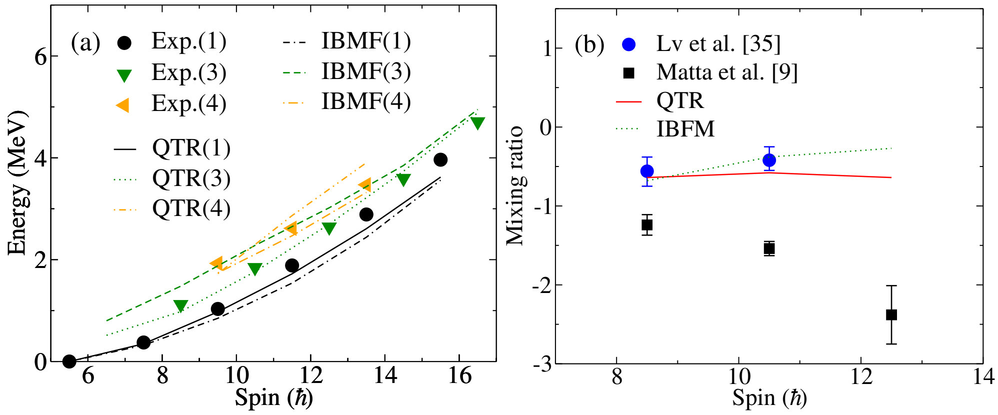

Figure 1. (**a**) Comparisons of theoretical QTR and IBMF model calculations with the experimental excitation energy spectra of bands 1, 3, 4 of ^{135}Pr. (**b**) Experimental mixing ratios compared with QTR and IBMF calculated values for the 747- and 813-keV transitions connecting the band 3 and the band 1. In addition, the values of Matta et al. [9] for $|\delta|$ >1 are shown for the comparison. The notations "1", "3", and "4" for the experimental bands corresponds those used in Ref. [35].

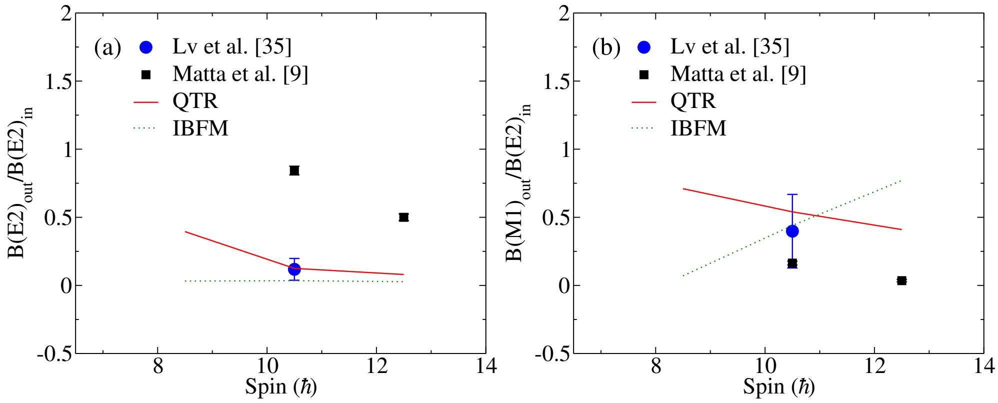

Figure 2. Ratios of transition probabilities obtained from this work (filled-blue circle) for the 813-keV ($21/2^- \rightarrow 19/2^-$) out-of-band and 726-keV ($21/2^- \rightarrow 17/2^-$) in-band transitions compared with the present QTR and IBFM calculations (solid and dashed lines), and to the experimental results of Ref. [9] (black squares).

In Ref. [37], the triaxial particle rotor model has been used to investigate the transverse wobbling bands in odd-A nuclei. In particular, various parameter sets for the moments of inertia were employed for ^{105}Pd, all showing good agreement with the experimental results. Analyzing the azimuthal geometry of the angular momenta of the core and valence nucleon, distinct modes of rotational excitation were revealed. These modes are confirmed to be sensitive to the ratio of the moment of inertia at the intermediate (m) and short (s) axes. When the ratio agrees with the rigid-body MoI, i.e., the total angular momentum wobbles around the s axis, the TW mode occurs. A wobbler around the m axis occurs when the ratio agrees with the hydrodynamical MoI. The planar tilted rotation occurs for both of the yrast and excited bands when the ratio is between the aforementioned two. The conclusion was drawn that tunneling and precession are two components of the quantum wobbling mode. In the yrare states of ^{105}Pd, the tunneling feature predominates and demonstrates the complexity of the transverse wobbling motion.

Out of the mass regions of A ≈ 110, 130, and 160, two bands were found in ^{183}Au of the A ≈ 190 mass region [12]. Both bands were suggested as the TW bands, however their wobbling excitation energies as a function of spin have different behaviors: the positive parity band increases with spin, while the negative parity band decreases with spin. Calculations using a particle rotor model with triaxial deformation successfully reproduce the experimental results. For the measured TW bands, a critical value of spin, I_c, was found: the wobbling energy decreases with spin above I_c and increases with spin below I_c. Thus, it is declared that the nucleus 183Au is the only nucleus in which both the increasing and decreasing components of the wobbling energy are observed, providing experimental proof of the full wobbling phenomena. A pair of longitudinal wobbling bands has been reported in ^{187}Au [13]. The longitudinal nature of the bands is supported by the $\Delta I = 1$ transitions between these bands with mainly $E2$ character and an increasing trend of the wobbling energy. It is found that theoretical calculations within the framework of the particle rotor model are in good agreement with the experimental findings. It proves that this exotic collective mode is a widespread phenomenon over the nuclear chart and provides the first experimental evidence for longitudinal wobbling bands where the expected signature partner band has also been observed [13]. However, the extraction of mixing ratio values of the linking transitions between these bands only used the angular distribution method, which is known to give rise to more than one solution from the fit of experimental data with low statistics. In order to clarify the nature of the low-spin bands in ^{187}Au, a new experiment was conducted at the Heavy Ion Research Facility in Lanzhou (HIRFL), China. Using combined measurements of linear polarization and of a two-point angular correlation ratio, the mixing ratios, δ, of the transitions linking the longitudinal bands were determined in the work [38]. They found that the new measured mixing ratio values are significantly different than the values reported in Ref. [13], and are in good agreement with the results previously obtained from internal conversion coefficient measurements. Thus, the interpretation of the low-lying bands in ^{187}Au as longitudinal wobbling motion was questioned. Also, after analyzing the experimental proofs reported in all wobbling bands at low spin of odd-A nuclei, it was pointed out that the previous flawed study paradigm leads to erroneous identification of low-spin wobbling bands.

3. Wobbling motion in even-even triaxial nuclei

Two bands built on the $\pi h^2_{11/2}$ configuration in the ^{130}Ba $(Z = 56, N = 74)$ nucleus were examined using constrained triaxial covariant density functional theory combined with quantum particle rotor model calculations in Ref. [19]. The experimental measured excitation energy spectra, wobbling energy, and the available electromagnetic transition probabilities are all well reproduced. It was shown that the band properties satisfy the character of transverse wobbling motion, establishing thus the first case of two-quasiparticle wobbling bands in even-even nuclei. The detailed analysis of the probability density distributions for the orientation of the angular momenta with respect to the body-fixed frame and of the total angular momentum geometry, further supports the bands wobbling nature. In addition, the calculations results reveals that the transverse wobbling regime is far more stable than the known situations with one odd-quasiparticle [19].

Very recently, two medium-spin bands (L1 and L3) built on two-quasiproton configuration in ^{136}Nd [21,40] were examined with the triaxial projected shell model (PSM) in Ref. [20], concentrating on the possible interpretation as wobbling bands. By analyzing the configuration components extracted from the microscopic wave functions, the angular momentum geometry, the probability density distribution profiles for the tilted angles of the angular momentum vector (θ, ϕ) with respect to the three principal axes, and the distribution of the angular momentum component on the short axis, the nature of these bands was predicted [20]. It revealed that near the bandhead of the bands L1 and L3 in ^{136}Nd, the transverse wobbling scenario is approximately valid. The wobbling pattern gradually vanishes with increasing spin. Its erosion may be caused by two factors. Due to its large moment of inertia, the rotational angular momentum converts from the s axis to the i axis. This results in the down-sloping wobbling energy against spin, which has been always identified as the hallmark of the

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transverse wobbling. The second cause of wobblling erosion is that as rotational angular momentum converts, the quasiparticle angular momentum prefers to align along the i axis rather than the s axis. Such an effect in the PSM model calculations is induced by configuration mixing. In addition, the effect of the rotational alignment of the quasiparticles on the picture of transverse wobbling is also explored. It shows that the zero-phonon wobbling candidate band is more affected than the one-phonon wobbling candidate band, which seems to go against expected decreasing trend of the wobbling energy in the transverse wobbling mode. The transverse wobbling scenario is affected by the rotational alignment of the quasiparticles and suggests the collapse of transverse wobbling with increasing spin, which is a more complex prediction than the frozen alignment approximation used in Ref. [8].

We recently studied the nature of bands L1 and L3 in ^{136}Nd experimentally. High spin and low spin states were populated using the ^{100}Mo (^{40}Ar, 4n) fusion-evaporation reaction. The ^{40}Ar beam with an energy of 152 MeV was provided by the K130 Cyclotron at the University of Jyväskylä, Finland. Refs. [41–43] provide detailed experimental information on the setup and data processing. By combining the angular correlation and linear polarization methods, the mixing ratio value of the $\Delta I = 1$, 751-keV transition liking the one-phonon and zero-phonon wobbling bands is determined. A mixing ratio of $|\delta| < 1$ is obtained, corresponding to a 19%, $E2$ component. This value is comparable to that of the $E2$ component ($\approx 25\%$) for the linking transitions between the one- and zero-phonon wobbling bands in ^{130}Ba. This mixing ratio value was then used to experimentally extract the transition probabilities ratio $B(E2)_{out}/B(E2)_{in}$. It is crucial to note that, in contrast to one-quasiparticle wobbling bands in which the linking $\Delta I = 1$ transitions have a predominant electric character, in the case of two-quasiparticle wobbling bands the connecting transitions have a predominant magnetic character due to the total gyromagnetic factor of two nucleons which is significantly larger than that of a single nucleon.

In order to further explore the nature of the transverse motion resulting in bands L1 and L3 in ^{136}Nd, a newly developed particle-rotor model, in which the total angular momentum of two quasiparticles is rigidly coupled to a triaxial core in an perpendicular geometry was performed by Budaca et al. [44]. The limits of various wobbling regimes are examined regarding to the dynamic evolution of the wobbling excitations and their associated electromagnetic properties. Additionally, this new model was successfully used to describe the wobbling modes suggested in the two-quasiparticle bands of ^{130}Ba, ^{134}Ce, and ^{138}Nd. Moreover, the application of the extension of this model calculations to the two-quasineutron hole bands in ^{138}Nd confirms their prior wobbling interpretation.

Figure 3 shows the comparison between the calculated energy spectra and wobbling energy of the proposed wobbling bands in ^{136}Nd using the PRM [44] and PSM [20] with the experimental data. Both models well reproduce the experimental energy spectra. For the wobbling energy, the results of PRM calculations globally reproduce all experimental data points, while the PSM results only reproduce the low-spin region, overestimating the high-spin part of the band. More important is that the calculated $B(E2)_{out}/B(E2)_{in}$ and $B(M1)_{out}/B(E2)_{in}$ ratios of the transitions between bands L3 and L1 of ^{136}Nd using both the PRM and PSM shown in Figure 4 are in good agreement with the experimental data. It is also worthwhile to mention that the possible wobbling motion based on two-quasiparticle configurations in ^{130}Ba, ^{134}Ce, and ^{138}Nd have been also investigated using a triaxial rotor with a rigidly aligned pair of quasiparticles in Ref. [44]. A good agreement between the calculated wobbling bands and the experimental data has been obtained for all studied nuclei. However, the confirmation of these predictions for ^{134}Ce and ^{138}Nd are yet be provided by measurements of the mixing ratios of the linking transitions between the claimed wobbling bands.

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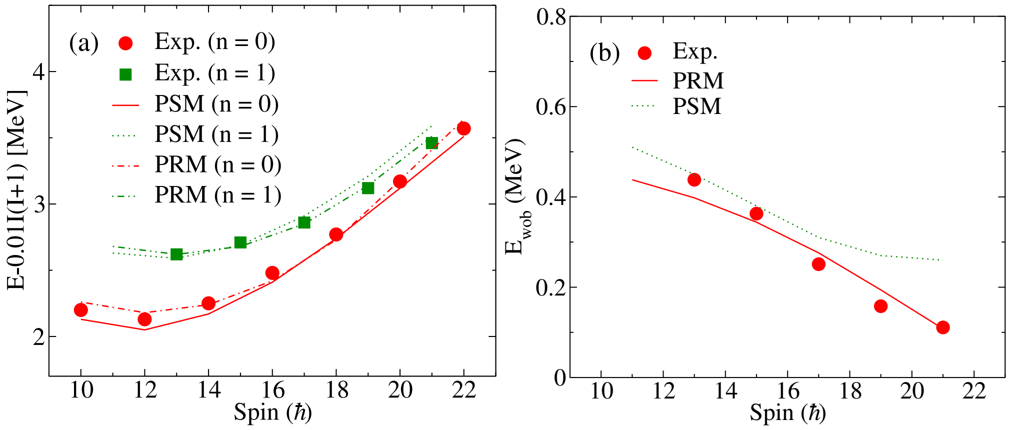

Figure 3. (**a**) Comparison of the experimental data with PSM and PRM models calculated excitation energies as a reference of common rigid-rotor for the L1 and L3 bands in ^{136}Nd [21,40]. (**b**) Comparison of the wobbling energies determined by PSM and PRM calculations with the experimental data from bands L1 (n = 0) and L3 (n = 1).

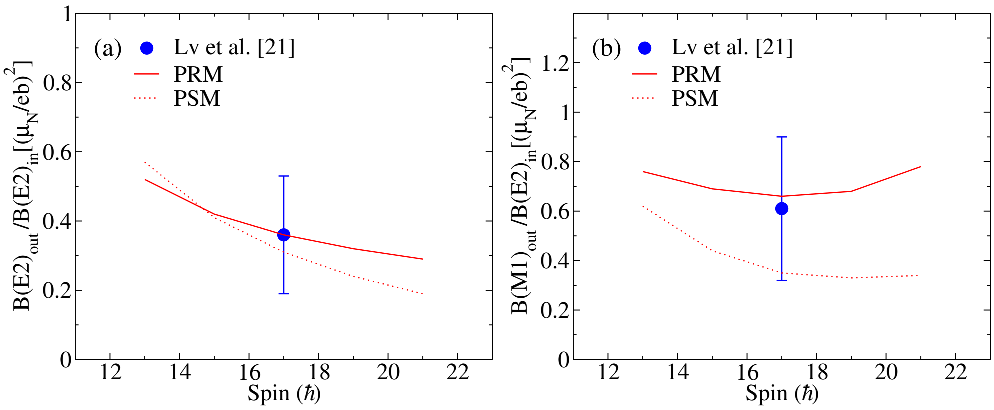

Figure 4. Comparison of the experimental data (symbols) and the $B(E2)_{out}/B(E2)_{in}$ as well as $B(M1)_out/B(E2)_in$ ratios computed by the PSM and PRM models for the $Delta I = 1$ transitions between the bands L3 and L1 of ^{136}Nd.

4. Conclusions

In summary, transverse wobbling and longitudinal wobbling modes have been theoretically suggested for low-spin bands in a series of odd-A nuclei. However, the validity of the reported experimental results supporting these predictions is seriously questioned by recent results. New experiments were performed to re-examine the low-spin bands in ^{135}Pr and ^{187}Au, and results in contradiction with the wobbling interpretation were obtained. These new experimental results are in excellent agreement with several theoretical models which do not involve the wobbling motion, giving a strong support against the interpretation of the bands in ^{135}Pr and ^{187}Au as wobbling bands, and shedding doubts on the reality of the low-spin wobbling modes. On the other hand, the mixing ratios of the $\Delta I = 1$ transition linking the suggested wobbling bands L1 and L3 in ^{136}Nd have been measured, and thus the ratios of reduced transition probabilities were deduced. The results of the PSM and a new developed PRM well reproduce the experimental data, supporting the transverse wobbling interpretation of the two-quasiparticle bands in even-even nuclei. These works shed new light on the nature of low-spin bands in odd-A and high-spin bands in even-even nuclei and put the phenomenon of wobbling on a solid experimental basis, providing reliable new experimental data to test different theoretical interpretations.

Author Contributions: C.M.P. and B.F.L. contributed to this article. All authors have read and agreed to the published version of the manuscript.

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Abbreviations

The following abbreviations are used in this manuscript:

TW	Transverse wobbling
LW	Longitudinal wobbling
E_{wob}	Wobbling energy

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