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Article

A Simple Explanation for the Flat Rotation Curves of Spiral Galaxies

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Abstract: This paper presents a simple model and explanation for the flat rotation curves of Spiral Galaxies without resorting to: (1) The introduction of Dark Matter, (2) a modification of Newton's 2nd Law (MOND) or (3) a modification of Einstein's theory of General Relativity. The model that is developed involves a rotating baryonic disk (a rotating frame of reference) with an "effective angular velocity (ω)" about its center surrounded by a rotating cloud of hydrogen (HI). The model also assumes that the measurements that are made on the rotating HI cloud are made from an earth based Inertial frame of reference. The predictions of the model rely on the relationship between velocities as measured by earth based radio telescope and the velocities as measured in the rotating frame. Calculations of the rotation curves of three galaxies are made using this model and compared with readily available experimental data.

Keywords: galaxy; galactic rotation curves; galactic kinematics

I. Introduction

Lamda CDM emerged in the late 1990's as a parametrization of Big Bang Cosmology. This model proposes that 68.3% of the Universe's Mass-Energy is due to Dark Energy with 26.8% due to Dark Matter and the remaining 4.9% due to the ordinary baryonic matter that we see all around us. Big Bang Cosmology and Lamda CDM have proved very successful in predicting the observed behavior of our universe. However, cracks have appeared over the decades since the model was introduced and some have pointed to these cracks as indicative of a crisis in Physics and a need for a paradigm change.

The existence of Dark Matter was originally suggested by Zwicky¹ in the 1930s to explain the motion of "Nebulae" in galactic clusters. In 1978 Rubin et al² found additional evidence in support of the Dark Matter hypothesis during their investigation of the rotation curves of the hydrogen surrounding spiral galaxies. Since then a great deal of effort, both experimental and theoretical, has been devoted to shoring up the Dark Matter hypothesis. Of particular importance are the earth based experiments designed to detect Dark Matter directly. However, to date, no Dark Matter has been detected by these experiments. This begs the question "does Dark Matter actually exist?" Perhaps we have a situation reminiscent of the 20th century search for the ether?

In 2010 the author, together with an undergraduate student, became interested in the Dark Matter problem and published a couple of papers^{3,4} in the Physics teaching literature focused on exercises in computational physics that involve galactic dynamics. In the conclusion to reference 3, the authors pose the question: "An intriguing possibility is that these experiments could prove to be the next (after Michelson-Morley) great null experiments and then What? The present paper attempts to proactively address the "then What?".

In section II we first set the stage for our model by considering the measurements that are made to determine galactic rotation curves. Following that we introduce a simple explanatory and predictive framework (Theory) intended to explain the flat rotation curves. Section III applies the

theory to examples of readily available rotation curve data. A summary and conclusions are given in section IV

II. Explanatory and Predictive Framework

The rotation curves of spiral galaxies are frequently measured by interpreting the Doppler shifted 21cm hyperfine spectral line emitted by a rotating HI cloud of gas surrounding the luminous rotating galactic disk consisting of ordinary baryonic matter. In this paper we consider the rotating disk to be a rotating frame of reference with an “effective angular velocity (ω)”

The basic heliocentric, line of sight measurements of the rotating HI cloud are typically made by earth based radio telescopes. For specificity we will limit our discussion to measurements made using radio telescopes although the radio telescope data are often supplemented by a variety of spectroscopic measurements. In this paper we consider the radio telescopes to be fixed Inertial frames of reference. A schematic of the measurement arrangement is shown in Figure 1.

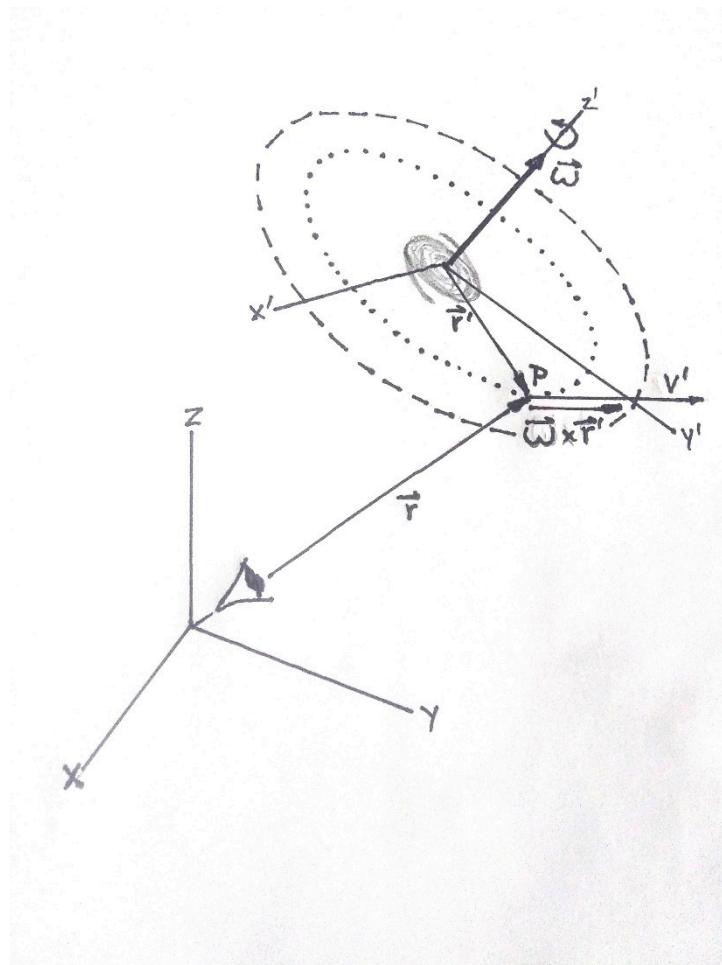


Figure 1. A typical galactic measurement set up. The dashed oval represents the extent of the rotating HI cloud surrounding a rotating disk shown in the center. The dotted oval is an assumed circular path of the HI. The primed coordinate system is fixed to a rotating disk. An earth based coordinate system is shown to the left.

In Figure 1 the primed coordinate system (x', y', z') is fixed to the rotating baryonic disk as shown. The disk is assumed to have an “effective angular velocity (ω)” about the z' axis. The dashed oval curve indicates the extent of the HI cloud rotating around the disk shown in the center. The dotted oval curve represents an assumed circular path followed by a sample of HI. The earth based coordinate system (x, y, z) is depicted to the left. The earth bound radio telescopes record line of sight data from a point P in the HI cloud. The velocity (v') of the HI sample is the velocity relative to the

primed coordinate system. This velocity results from the effect of the gravitational field of the disk and is tangent to the dashed oval at point P. The vector cross product $\omega \times \mathbf{r}'$ is also tangent to the circular path and will be introduced below.

The interpretation of the line of sight measurements by a earth based observer relies on a complex model of the rotating HI cloud called the "Tilted Ring Model". This model is described, for example, by Begeman⁵. As noted, the measurements are "line of sight" and as a result collect Doppler Shifted radiation from cylindrical samples through the tilted disk. This model assumes, *a priori*, that these cylindrical samples of HI material are part of a circular rotating mass of HI. From this raw data an algorithm is used to construct a contour map of the speeds. An example of such a contour map is shown in Figure 2⁵

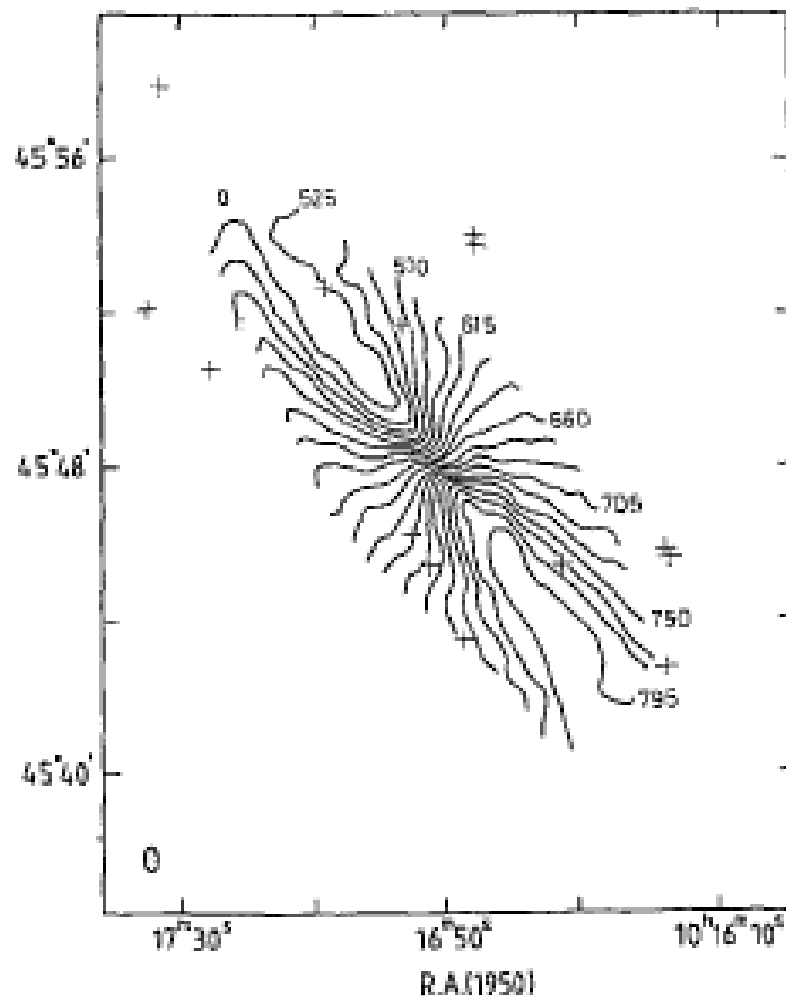


Figure 2. Figure 2 is taken from Begeman⁵(Fig 5, page 28) and shows a typical rotation speed contour map for a spiral galaxy. Such a map is obtained after significant processing of the line of site data due to the Doppler shifted 21 cm hyperfine line emitted by HI.

The experimental rotation curve is established by recording the speeds along the major axis of this plot. Two sets of data points are obtained. One from the side tilted towards the observer and one tilted away from the observer. The average is recorded as the rotation curve. Standard practice assumes that this is a best guess as to the rotation speed of the HI cloud as measured by the earth based radio telescopes.

From a kinematic perspective, any velocity (\mathbf{v}) measured by an earth based inertial observer is related to the velocity (\mathbf{v}') measured in the rotating primed frame by the straight forward vector equation (see for example reference 6 equation 5.2.13, page 177 and Figure 5.2.4)

$$\mathbf{v} = \mathbf{v'} + \boldsymbol{\omega} \times \mathbf{r'} + \mathbf{V} \tag{1}$$

In this expression \mathbf{V} is the velocity of translation of the primed system. In the present context \mathbf{V} would be the systemic velocity of a galaxy. \mathbf{V} is not relevant as far as the galactic rotation is concerned.

From figure1, the cross product $\boldsymbol{\omega} \times \mathbf{r'}$ is always aligned with the direction of the tangent to the assumed circular paths and is in the same direction as the circular velocity ($\mathbf{v'}$). Recall that ($\mathbf{v'}$) is a consequence of the gravitational field of the disk acting on the HI cloud. The magnitude of the cross product $\boldsymbol{\omega} \times \mathbf{r'}$ is $\omega r' \sin(\alpha)$ where α is the angle between the vector $\boldsymbol{\omega}$ and $\mathbf{r'}$. We will assume that the disk is flat and therefore α is 90 degrees and $\boldsymbol{\omega} \times \mathbf{r'}$ is therefore simply $\omega r'$.

Theoretically, the velocity $\mathbf{v'}$ due to the gravitational field of the disk is determined as follows: It is assumed that the mass distribution of the baryonic disk mimics the light distribution. This is quantified by the “Mass to Light” ratio. The light distribution is usually assumed to be a decreasing exponential function. With this assumed mass distribution the gravitational field due to the baryonic disk can be calculated. Because the HI paths are assumed to be circular this gravitational field is by Newton’s second law equal to (v'^2/r'). From this we get the well known expression for the circular speed (v') due to the gravitational force of the exponential disk (see for example reference 7).

$$v'^2 = u_0 R_d (y)^2 [(I_0(y))K_0(y) - I_1(y) K_1(y)] \tag{2}$$

R_d is a radial scale length dependent on the galaxy.

$$y = \frac{r'}{2R_d}$$

I_0 , K_0 are modified Bessel functions of the first kind and I_1 , K_1 are modified Bessel functions of the second kind.

From equation 1 the rotational speed measured by the earth based observer is then given by:

$$v = \sqrt{[u_0 \cdot R_d \cdot (y)^2] (I_0(y) K_0(y) - I_1(y) \cdot K_1(y)) + \omega \cdot r'} \tag{3}$$

Equation (3) is used to fit the theory to the experimental data. In the non linear fitting procedure u_0 and ω are treated as adjustable parameters.

IV. Results of Fitting to Selected Galaxies

To illustrate the application of equation (3) to the experimental data we have chosen the 3 galaxies listed in Table 1.

Table 1.

Galaxy.	$R_d(\text{kpc})$	$D(\text{Mpc})$	$u_0(\text{km/s})^2\text{kpc}^{-1}$	$\omega(\text{km/s})(1/\text{kpc})$
NGC 3198	2.63	9.4	3650	3.042
UGC 4499	1.5	13.0	6214	5.592
LSB F583-1	1.6	32.0	4561	5.497

Table 1 lists the galaxies studied in this paper. Column two gives the values of R_d used in the calculations. Column three gives the distance that the galaxies are from earth using a Hubble Constant $H_0 = 75 \text{ (km/s)/Mpc}$. Columns four and five give the values found for the fitting variables.

The first galaxy is a typical spiral galaxy (NGC3198) that has been studied by a number of authors. We have used the data given by Begaman⁵ for this galaxy. The second and third galaxies are a late type dwarf galaxy (UGC4499) and a low surface brightness galaxy (LSBC F583-1) respectively. These two galaxies are usually assumed to be dominated by Dark Matter. Here we have used data taken from figures published by Marcesini et al⁸

Figures 3–5 show the fits of equation 3 to the data.

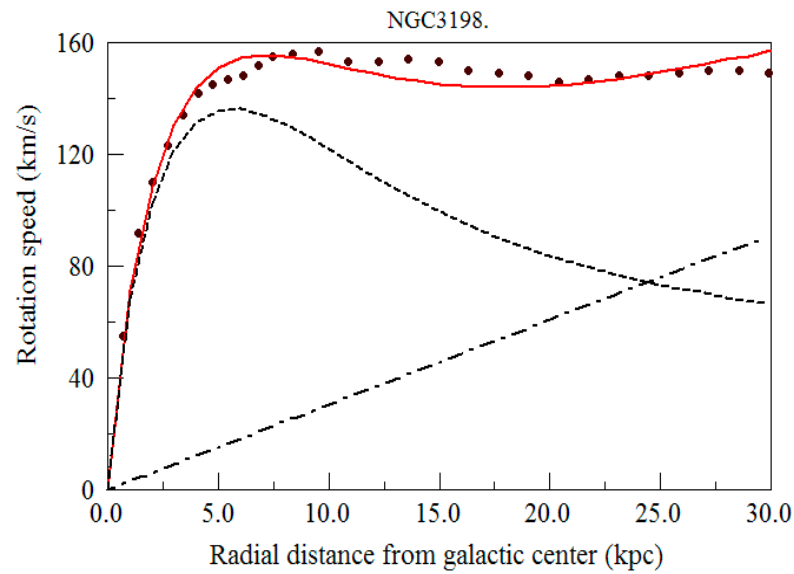


Figure 3. Dots are the experimental rotation curve data for NGC 3198 taken from Begeman⁵. Dashed curve is the rotational speed due to the galactic disk. Dot dashed line is the inertial contribution to the rotational speed ($\omega r'$). The solid curve is the sum of the rotational speeds.

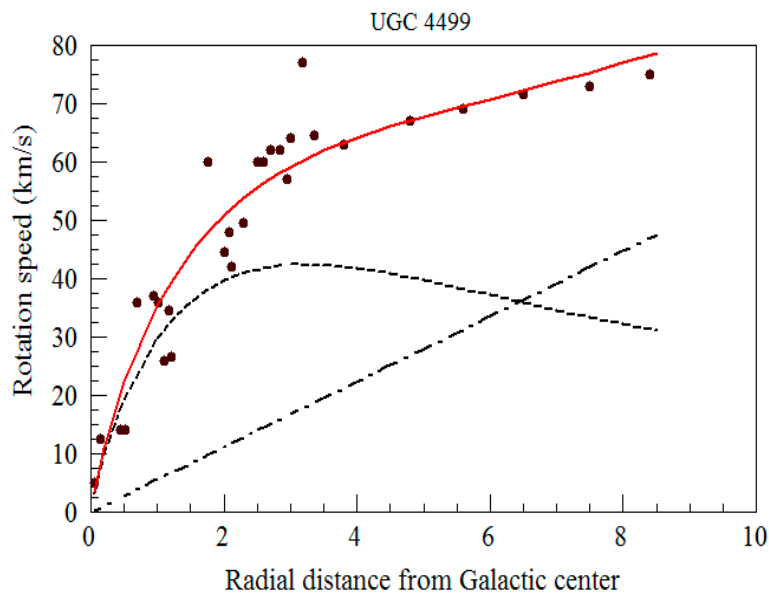


Figure 4. Dots are the experimental rotation curve data for a Dwarf galaxy UGC 4499 taken from Marchesine et al⁸. Dashed curve is the rotational speed due to the galactic disk. Dot dashed line is the inertial contribution to the rotational speed ($\omega r'$). The solid curve is the sum of the rotational speeds.

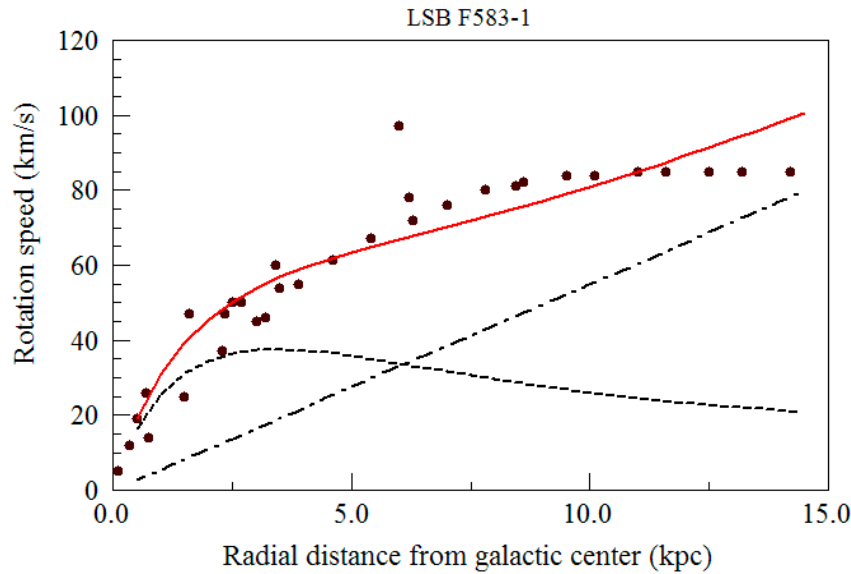


Figure 5. Dots are the experimental rotation curve data for a Low Brightness galaxy LSB F583-1 taken from Marchesine et al⁸. Dashed curve is the rotational speed due to the galactic disk. Dot dashed line is the inertial contribution to the rotational speed ($\omega r'$). The solid curve is the sum of the rotational speeds.

IV. Discussion and Conclusion

From Figures 3–5 we see that the agreement between the theoretical framework introduced in this paper and the experimental rotation curve is fairly good given the complexity of the experimental observations and the uncertainty associated with the concept of an “effective angular velocity”. It should also be noted that there is plenty of room for fine tuning the basic model. For example the mass distribution of the disk is only approximately a decreasing exponential. In addition the disk is often not thin and the angle between ω and r' will vary due to warping. In addition we have not included a bulge and the possibility of a Bar.

The essential aspect of the model presented in this paper is the prediction of “effective angular velocities (ω)” for the galaxies

It would be interesting to extend the calculations to all spiral galaxies for which data is currently available. Especially relevant would be those galaxies that are assumed to be dominated by Dark Matter such as those studied by Swaters et al in connection with their investigation of MOND (see reference 9). Also of particular interest would be the Milky Way and the Andromeda galaxies for which additional contributions to the disk gravitational field are generally included..

From such a collection of galactic studies it might be possible to establish systematic trends associated with the “effective angular velocity” of the disks. Perhaps the predicted “effective angular velocities” could be related to the morphology of Spiral galaxies (see references 7 and 10 in this regard).

The veracity of the model presented in this paper depends on what is meant by the concept of “effective angular velocity” and whether or not some independent measurements of this can be found. In this regard we note that the baryonic matter in the disk is rotating but the disk is not a merry-go-round. In essence what the present paper does is replace the mysterious Dark Matter with a somewhat less mysterious “effective angular velocity”. Acceptance of this would lead to a monumental shift in the Lamda CDM paradigm. In addition, the impact on Dark Matter searches and the path forward for particle physics would also be significant.

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