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Developed Method. Interactions and Their Quantum Picture

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Developed Method: Interactions and Their Quantum Picture

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Abstract: By developing the previously proposed method of combining continuum mechanics with Einstein Field Equations, explicit equations of interactions operating in a physical system were obtained. The explicit form of the electromagnetic four-potential was also derived, and the Lagrangian and Hamiltonian densities describing the physical system containing all the fields. A quantum picture of the system was also proposed, which, interestingly, allows for a simple transition to a curvilinear description based on the metric tensor in curved spacetime.

Keywords: field theory; lagrangian mechanics; quantum mechanics; general relativity

1. Introduction

Over the past decades, great strides have been made in attempts to combine quantum description of interactions with General Relativity [1]. There are currently many promising approaches to connecting the quantum mechanics and General Relativity, including perhaps the most promising ones: String Theory [2], [3], [4], Loop quantum gravity [5], [6], [7] and Noncommutative Spacetime Theory [8], [9].

A lot of work has also been done to clear up some General Relativity challenges. An explanation for the problem of dark energy [10] and dark matter [11] is still being sought, and efforts are still being made to explain the origin of the cosmological constant [12], [13], [14].

The author also tries to bring his own contribution to the explanation of the above physics challenges, based on a recently discovered method, described in [15]. As this article will show, this method seems very promising and can help clarify at least some of the issues mentioned above.

According to conclusions from [15], the description of motion in curved spacetime and its description in flat Minkowski spacetime with fields are equivalent, and the transformation between curved spacetime and Minkowski spacetime is known. This allows for a significant simplification of research, because the results obtained in flat Minkowski spacetime can be easily transformed into curved spacetime.

In this article, the author will focus on developing the method proposed in [15]. In the first chapter, the fields will be separated and the Lagrangian density for the system will be derived, allowing to obtain the tensor described in [15]. These conclusions will be used later in the article to propose quantum description of the system.

The author uses the Einstein summation convention, metric signature $(+,-,-,-)$ and commonly used notations. In order to facilitate the analysis of the article, the key conclusions from [15] are quoted in the subsection below.

1.1. Short summary of the method

Denoting rest mass density in the system as ϱ_o and

$$\varrho \equiv \varrho_o \gamma \tag{1}$$

thanks to the proposed [15] amendment to the continuum mechanics, in flat Minkowski spacetime occurs

$$\partial_\alpha U^\alpha = -\frac{d\gamma}{dt} \quad \rightarrow \quad \partial_\alpha \, \varrho U^\alpha = 0 \tag{2}$$

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Stress-energy tensor $T^{\alpha\beta}$ for a system in a given spacetime described by some metric tensor $g^{\alpha\beta}$ is equal to

$$T^{\alpha\beta} = \varrho\, U^{\alpha} U^{\beta} - \left(c^2 \varrho + \Lambda_\rho\right)\left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) \tag{3}$$

where

$$\frac{1}{\xi} = \frac{1}{4}\, g_{\mu\nu}\, h^{\mu\nu} \tag{4}$$

$$\Lambda_\rho \equiv \frac{1}{4\mu_o} \mathbb{F}^{\alpha\mu}\, g_{\mu\gamma}\, \mathbb{F}^{\beta\gamma}\, g_{\alpha\beta} \tag{5}$$

$$h^{\alpha\beta} = 2\, \frac{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}}{\sqrt{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}\, g_{\mu\beta}\, \mathbb{F}_{\alpha\eta}\, g^{\eta\xi}\, \mathbb{F}^{\mu}{}_{\xi}}} \tag{6}$$

where $\mathbb{F}^{\alpha\beta}$ represents electromagnetic field tensor, and where the stress–energy tensor for electromagnetic filed, denoted as $\Upsilon^{\alpha\beta}$ may be presented as follows

$$\Upsilon^{\alpha\beta} \equiv \Lambda_\rho \left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) = \Lambda_\rho g^{\alpha\beta} - \frac{1}{\mu_o} \mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma} \tag{7}$$

The pressure p in the system is equal to

$$p \equiv c^2 \varrho + \Lambda_\rho \tag{8}$$

Denoting four-momentum density as $\varrho U^{\mu} = \varrho_o \gamma\, U^{\mu}$ one may notice that total four-force density f^{μ} acting in the system is

$$f^{\mu} \equiv \varrho A^{\mu} = \partial_\alpha \varrho U^{\mu} U^{\alpha} \tag{9}$$

Denoting rest charge density in the system as ρ_o and

$$\rho \equiv \rho_o \gamma \tag{10}$$

electromagnetic four-current J^{α} is equal to

$$J^{\alpha} \equiv \rho\, U^{\alpha} = \rho_o \gamma\, U^{\alpha} \tag{11}$$

Total four-force density f^{α} acting in the system calculated from $\partial_\beta\, T^{\alpha\beta} = 0$ is the sum of electromagnetic (f^{α}_{EM}), gravitational (f^{α}_{gr}) and other (f^{α}_{oth}) four-force densities

$$f^{\alpha} = \begin{cases} f^{\alpha}_{EM} \equiv -\Lambda_\rho\, \partial_\beta \xi\, h^{\alpha\beta} \quad (electromagnetic) \\ + \\ f^{\alpha}_{gr} \equiv c^2 \left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) \partial_\beta \varrho \quad (gravitational) \\ + \\ f^{\alpha}_{oth} \equiv \frac{\varrho c^2}{\Lambda_\rho} f^{\alpha}_{EM} \quad (other) \end{cases} \tag{12}$$

As was shown in [15], in curved spacetime ($g_{\alpha\beta} = h_{\alpha\beta}$) presented method reproduces Einstein Field Equations with an accuracy of $\frac{4\pi G}{c^4}$ constant and with cosmological constant Λ dependent on invariant of electromagnetic field tensor $\mathbb{F}^{\alpha\gamma}$

$$\Lambda = -\frac{\pi G}{c^4 \mu_o} \mathbb{F}^{\alpha\mu}\, h_{\mu\gamma}\, \mathbb{F}^{\beta\gamma} h_{\alpha\beta} = -\frac{4\pi G}{c^4} \Lambda_\rho \tag{13}$$

where $h_{\alpha\beta}$ appears to be metric tensor of the spacetime in which all motion occurs along geodesics and where Λ_ρ describes vacuum energy density.

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2. Separation of fields and Lagrangian density

Since for the considered method the transition to curved spacetime is known (based on electromagnetic field tensor), the rest of the article will focus on the calculations in the Minkowski spacetime with presence of fields, where $\eta^{\alpha\beta}$ represents Minkowski metric tensor. The first step will be separation of interactions described in the section 1.1.

Referring to definitions from section 1.1 one may define following four-potential $\mathbb{A}^\mu$

$$\mathbb{A}^\mu \equiv -\frac{\Lambda_\rho}{p}\frac{\varrho_o}{\rho_o}U^\mu \tag{14}$$

Next, based on it, one may define following four-force density f^α_A as

$$f^\alpha_A \equiv J_\beta\left(\partial^\alpha\mathbb{A}^\beta - \partial^\beta\mathbb{A}^\alpha\right) = \frac{\Lambda_\rho}{p}\left(f^\alpha - \frac{\varrho}{p}\frac{dp}{d\tau}U^\alpha + \frac{\varrho c^2}{p}\partial^\alpha p\right) \tag{15}$$

where J_β is electromagnetic four-current and where Minkowski metric property was utilized

$$U_\beta U^\beta = c^2 \quad \rightarrow \quad U_\beta\partial^\alpha U^\beta = \frac{1}{2}\,\partial^\alpha\left(U_\beta U^\beta\right) = 0 \tag{16}$$

Let f^α_A be equal to electromagnetic four-force density f^α_{EM} with respect to some unknown f^α_Δ

$$f^\alpha_A = f^\alpha_{EM} + f^\alpha_\Delta \tag{17}$$

Next, one may calculate, that

$$\varrho U_\beta\left(\partial^\beta\frac{\varrho c^2}{p}U^\alpha - \partial^\alpha\frac{\varrho c^2}{p}U^\beta\right) = \frac{\varrho c^2}{\Lambda_\rho}f^\alpha_A + \frac{1}{p}\left(\varrho U^\alpha U^\beta - \varrho c^2\eta^{\alpha\beta}\right)\partial_\beta\varrho c^2 = \tag{18}$$

what, thanks to definitions (3) and (12) after easy transformations, may be rewritten as

$$= f^\alpha_{oth} + \frac{\varrho c^2}{\Lambda_\rho}f^\alpha_\Delta + \left(\frac{1}{p}\left[T^{\alpha\beta} - \varrho c^2\,\eta^{\alpha\beta}\right] + \eta^{\alpha\beta} - \xi\,h^{\alpha\beta}\right)\partial_\beta\varrho c^2 = \tag{19}$$

$$= f^\alpha_{oth} + f^\alpha_{gr} + \frac{\varrho c^2}{\Lambda_\rho}f^\alpha_\Delta + \frac{1}{p}\left(T^{\alpha\beta} - \varrho c^2\,\eta^{\alpha\beta}\right)\partial_\beta\varrho c^2 \tag{20}$$

Thanks to definition of p in (8) following equality occurs

$$J_\beta\left(\partial^\alpha\mathbb{A}^\beta - \partial^\beta\mathbb{A}^\alpha\right) + \varrho U_\beta\left(\partial^\beta\frac{\varrho c^2}{p}U^\alpha - \partial^\alpha\frac{\varrho c^2}{p}U^\beta\right) = \varrho U_\beta\left(\partial^\beta U^\alpha - \partial^\alpha U^\beta\right) = f^\alpha \tag{21}$$

thus

$$f^\alpha_{EM} + f^\alpha_{oth} + f^\alpha_{gr} + \frac{p}{\Lambda_\rho}f^\alpha_\Delta + \frac{1}{p}\left(T^{\alpha\beta} - \varrho c^2\,\eta^{\alpha\beta}\right)\partial_\beta\varrho c^2 = f^\alpha \tag{22}$$

Simplifying

$$f^\alpha_\Delta = \left(T^{\alpha\beta} - \varrho c^2\,\eta^{\alpha\beta}\right)\partial_\beta\frac{\Lambda_\rho}{p} \tag{23}$$

One may now extend the conclusions from [15] by adopting the following hypothesis

$$\left(T^{\alpha\beta} - \varrho c^2\,\eta^{\alpha\beta}\right)\partial_\beta\frac{\Lambda_\rho}{p} = 0 \tag{24}$$

what yields, that $\mathbb{A}^\mu$ is some gauge of electromagnetic four-potential. It will be shown later in the article that this leads to the correct solutions.

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Using a simplified notation

$$\frac{d\ln(p)}{d\tau} = U_\mu \partial^\mu \ln(p) = U_\mu \frac{\partial^\mu p}{p} \tag{25}$$

after expanding and calculating forces, one now obtains the following picture of the forces acting in the system

$$f^\mu_{gr} = \varrho \left(\frac{d\ln(p)}{d\tau} U^\mu - c^2 \partial^\mu \ln(p) \right) \tag{26}$$

$$f^\mu_{EM} = \frac{\Lambda_\rho}{p} \left(f^\mu - f^\mu_{gr} \right) \tag{27}$$

$$f^\mu_{oth} = \frac{\varrho c^2}{p} \left(f^\mu - f^\mu_{gr} \right) \tag{28}$$

From a simple qualitative analysis of the behavior of the above forces, it can be concluded that they behave as it is presently observed:

- the gravitational force will dominate at high energy density gradients, and, interestingly, at significant changes in pressure,
- the electromagnetic force is related to the charges and depends on their density and the electromagnetic four potential,
- the last force has a chance to dominate only at high energy density but its small gradient, in the presence of a strong electromagnetic field, which probably occurs on a micro-scale.

Now, one may recall the classical Lagrangian density [16] for electromagnetism and explain why, in the light of the conclusions from [15] and above, it does not allow to create a symmetric stress-energy tensor [17]. The classical value of the Lagrangian density for electromagnetic field, written with the notation (4), (14) used here, is

$$-\mathcal{L}_{EMclassic} = \Lambda_\rho + \mathbb{A}^\mu J_\mu \tag{29}$$

which, in the light of the conclusions from this chapter, is equal to

$$-\mathcal{L}_{EMclassic} = \Lambda_\rho - \frac{\Lambda_\rho}{p}\frac{\varrho_o}{\rho_o} U^\mu\, U_\mu \rho_o \gamma = \Lambda_\rho - \frac{\Lambda_\rho \varrho c^2}{p} = \frac{\Lambda_\rho^2}{p} \tag{30}$$

As it is seen, above Lagrangian density is not invariant under gradient over four-position and $\mathbb{A}^\mu$ and J_μ are dependent, what is not taken into account in classical calculation

$$\frac{\mathbb{A}^\alpha}{\mathbb{A}^\mu \mathbb{A}_\mu} = \frac{J^\alpha}{\mathbb{A}^\mu J_\mu} \tag{31}$$

Above yields

$$\frac{\partial \ln\left(\frac{1}{\sqrt{\mathbb{A}^\mu \mathbb{A}_\mu}}\right)}{\partial \mathbb{A}_\alpha} = -\frac{J^\alpha}{\mathbb{A}^\mu J_\mu} = \frac{p}{\varrho c^2}\frac{J^\alpha}{\Lambda_\rho} \tag{32}$$

One may decompose

$$\ln\left(\frac{1}{\sqrt{\mathbb{A}^\mu \mathbb{A}_\mu}}\right) = \ln\left(\frac{p\,\rho_o}{\Lambda_\rho \varrho_o c}\right) = \ln(p) - \ln(\Lambda_\rho) - \ln\left(\frac{\varrho_o c}{\rho_o}\right) \tag{33}$$

and since $\frac{\varrho_o c}{\rho_o}$ are constants, one may simplify (32) to

$$\frac{\partial \ln(p)}{\partial \mathbb{A}_\alpha} - \frac{\partial \ln(\Lambda_\rho)}{\partial \mathbb{A}_\alpha} = \frac{p}{\varrho c^2}\frac{J^\alpha}{\Lambda_\rho} = \frac{J^\alpha}{\varrho c^2} + \frac{J^\alpha}{\Lambda_\rho} \tag{34}$$

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It is now appropriate to propose a solution to the Lagrangian density problem, allowing the stress-energy tensor under consideration to be derived, noting that in (34) occurs

$$\frac{\partial \ln\left(\Lambda_\rho\right)}{\partial \mathbb{A}_\alpha} = -\frac{J^\alpha}{\Lambda_\rho} \tag{35}$$

which leads to the conclusion that Λ_ρ acts as the Lagrangian density of the system

$$\frac{\partial \Lambda_\rho}{\partial \mathbb{A}_\alpha} = \partial_\nu\left(\frac{\partial \Lambda_\rho}{\partial(\partial_\nu \mathbb{A}_\alpha)}\right) = -J^\alpha \tag{36}$$

thus stress-energy tensor for the system is equal to

$$T^{\alpha\beta} = \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma - \Lambda_\rho \eta^{\alpha\beta} \tag{37}$$

In fact, the proof of correctness of the electromagnetic field tensor (noted as $\Upsilon^{\alpha\beta}$) allows to see this solution

$$f^\beta_{EM} = \partial_\alpha \Upsilon^{\alpha\beta} = J^\gamma \mathbb{F}^\beta{}_\gamma - \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\alpha \mathbb{F}^\beta{}_\gamma \tag{38}$$

what yields following property of electromagnetic field tensor

$$\mathbb{F}^{\alpha\gamma}\partial_\alpha\partial_\gamma \mathbb{A}^\beta = \mathbb{F}^{\alpha\gamma}\partial^\beta\partial_\alpha \mathbb{A}_\gamma \tag{39}$$

Using the above substitution, one may note

$$\partial_\alpha \Upsilon^{\alpha\beta} = \partial_\alpha \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\gamma \mathbb{A}^\beta - \partial_\alpha \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma = \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta\partial_\alpha \mathbb{A}_\gamma - J^\gamma\partial_\gamma \mathbb{A}^\beta - \partial_\alpha \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma \tag{40}$$

Thus the invariance of Λ_ρ with respect to $\mathbb{A}_\alpha$ and $\partial_\nu \mathbb{A}_\alpha$ is both a condition on the correctness of the electromagnetic stress-energy tensor and on Λ_ρ in the role of Lagrangian density

$$0 = \frac{\partial \Lambda_\rho}{\partial(\partial_\alpha \mathbb{A}_\gamma)}\partial^\beta(\partial_\alpha \mathbb{A}_\gamma) + \frac{\partial \Lambda_\rho}{\partial \mathbb{A}_\gamma}\partial^\beta \mathbb{A}_\gamma = \frac{1}{\mu}\mathbb{F}^{\alpha\gamma}\partial^\beta\partial_\alpha \mathbb{A}_\gamma - J^\gamma\partial^\beta \mathbb{A}_\gamma = \partial_\alpha \frac{1}{\mu}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma \tag{41}$$

what yields for (40) that

$$\partial_\alpha \Upsilon^{\alpha\beta} = J^\gamma\partial^\beta \mathbb{A}_\gamma - J^\gamma\partial_\gamma \mathbb{A}^\beta = f^\beta_{EM} \tag{42}$$

Equations (3), (7) and (37) yield

$$\frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\gamma \mathbb{A}^\beta = \varrho\, U^\alpha U^\beta - \frac{c^2\varrho}{\Lambda_\rho}\Upsilon^{\alpha\beta} \tag{43}$$

what yields second representation of the stress-energy tensor

$$T^{\alpha\beta} = \frac{p}{\varrho c^2}\cdot\frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\gamma \mathbb{A}^\beta - \frac{\Lambda_\rho}{c^2}U^\alpha U^\beta \tag{44}$$

After four-divergence, it gives additional expression for relation between forces and gives useful clues about the behavior of the system when transitioning to the description in curved spacetime.

3. Hamiltonian density and quantum picture

Since

$$\frac{\partial \Lambda_\rho}{\partial(\partial \mathbb{A}_\beta/\partial x_\alpha)} = \frac{1}{\mu_o}\mathbb{F}^{\alpha\beta} \quad \to \quad \frac{\partial \Lambda_\rho}{\partial(\partial \mathbb{A}_\beta/\partial x_0)} = \frac{1}{\mu_o}\mathbb{F}^{0\beta} \tag{45}$$

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thus noting Hamiltonian density in terms of the Lagrangian density [18], one gets

$$\mathcal{H} = \left(\frac{\partial \Lambda_\rho}{\partial(\partial \mathbb{A}_\beta/\partial x_0)}\right)\frac{\partial \mathbb{A}_\beta}{\partial x_0} - \Lambda_\rho = \frac{1}{\mu_o}\mathbb{F}^{0\beta}\frac{\partial \mathbb{A}_\beta}{\partial t} - \Lambda_\rho \tag{46}$$

where conjugate momentum field Π^β is

$$\Pi^\beta \equiv \frac{1}{\mu_o}\mathbb{F}^{0\beta} \tag{47}$$

As it is seen, above Hamiltonian density agrees with the classical Hamiltonian density for electromagnetic field [18] except negative sign and except that this Hamiltonian density was currently only considered for sourceless regions and only to consider the system with electromagnetic field. According to the result above, this Hamiltonian density is always correct and describes the entire physical system, taking into account all known interactions. Above therefore significantly simplifies quantum field theory equations [19], [20], [21].

Introducing ε_ρ as energy density of electromagnetic field

$$\varepsilon_\rho \equiv \frac{1}{2\mu}\left(\frac{E^2}{c^2} + B^2\right) \tag{48}$$

where E and B are for the electric and magnetic fields, respectively, by transforming (46) and using (44), Hamiltonian density may also be noted as follows

$$\mathcal{H} = \frac{p}{\varrho c^2}\frac{1}{\mu_o}\mathbb{F}^{0\gamma}\partial_\gamma \mathbb{A}^0 - \Lambda_\rho \gamma^2 = \varrho c^2\left(\gamma^2 - \frac{\varepsilon_\rho}{\Lambda_\rho}\right) - \varepsilon_\rho \tag{49}$$

Lagrangian, according to (8), should be considered as description of thermodynamical process related to first law of thermodynamics

$$L = \int p\ dV - mc^2\gamma \tag{50}$$

where V is for considered volume d^3x. The above seems to be a missing link between the first law of thermodynamics and the principle of minimum action. Since $\int p\ dV$ is path-dependent Pressure–Volume Work [22], thus it should be actually related to action. Denoting $\mathbb{P}^\beta$ as four-momentum, one may therefore notice, that from property

$$\mathbb{P}^\beta U_\beta = mc^2 \quad \rightarrow \quad F^\beta U_\beta = 0 \quad \rightarrow \quad mc^2\frac{d\gamma}{dt} = \frac{1}{\gamma}\vec{F}\frac{d\vec{r}}{dt} \tag{51}$$

one also obtains

$$mc^2\gamma = \int \frac{d\vec{\mathbb{P}}}{dt}\,d\vec{r} = \int \frac{d\vec{\mathbb{P}}}{dV}\frac{d\vec{r}}{dt}\,dV \tag{52}$$

what yields

$$L = \int p\ dV - \int \frac{d\vec{\mathbb{P}}}{dV}\,\vec{u}\ dV \tag{53}$$

Since the forces acting in the system are known (26) and all of them meet the above condition ($f^\mu U_\mu = 0$), this can help in the analysis of the contribution of individual forces to thermodynamic processes.

One may now summarize the above and propose a method for quantizing the system.

At first, it should be noted, that the above reasoning changes the interpretation of what the relativistic principle of least action means. As described above, there is no inertial system in which no fields act, and in the absence of fields, the Lagrangian and the Hamiltonian should vanish. Since the metric tensor (6) for description in curved spacetime depends on the electromagnetic field tensor, it

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must also be assumed that the absence of the electromagnetic field means actually the disappearance of spacetime and the absence of any action.

Introducing generalized, canonical four-momentum H^μ as four-gradient on Hamilton principal function S, where relation with wave four-vector K^μ occurs

$$H^\mu \equiv -\partial^\mu S \equiv \hbar K^\mu \tag{54}$$

and where K^μ does not depend on four-position, one may also conclude from (21) that it should be in form of

$$H^\mu = \mathbb{P}^\mu - V^\mu \tag{55}$$

where V^μ represents generalized four-potential (sum of all four-potentials containing all considered interactions acting on the test body). Four-momentum $\mathbb{P}^\mu$ may be now considered as just other gauge of this four-potential acting on the body

$$\partial^\alpha \mathbb{P}^\mu = \partial^\alpha V^\mu \tag{56}$$

where equation (21) represents the above for a system analyzed as a continuum (densities instead of point-like particles).

Now, since the metric, as shown earlier, depends on the distribution of the field, one may require, that

$$V^\mu X_\mu \equiv mc^2\tau \tag{57}$$

and propose action as follows

$$-S \equiv H^\mu X_\mu = \mathbb{P}^\mu X_\mu - V^\mu X_\mu \tag{58}$$

what yields Lagrangian independent on four-position in form of

$$-L = \frac{1}{\gamma} F^\mu X_\mu \tag{59}$$

and also

$$H^\mu H_\mu - m^2c^2 = 2m X_\mu F^\mu - V^\mu V_\mu \tag{60}$$

Using the above definitions, the action, the Hamiltonian and Lagrangian vanish for an inertial system (when $\mathbb{P}^\mu = V^\mu$). To relate the Lagrangian to the generalized field, it is enough to assume the following

$$-\gamma L = X_\mu F^\mu = \frac{V^\mu V_\mu}{2m} \tag{61}$$

and by introducing quantum wave function Ψ in form of

$$\Psi \equiv e^{i\frac{H^\mu X_\mu}{\hbar}} \tag{62}$$

from (60) one obtains Klein-Gordon equation

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right)\Psi = 0 \tag{63}$$

which allows for further analysis of the system in the quantum approach, eliminating the problem of negative energy appearing in solutions [23].

The above representation allows the analysis of the system in the quantum approach, classical approach based on (26) and the introduction of a field-dependent metric in (6) for curved spacetime, which connects previously divergent descriptions of physical systems.

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4. Conclusions and Discussion

As shown above, the proposed method of physical systems analysis seems to be a promising area of research.

It seems that it allows to combine previously divergent methods of curvilinear description with the quantum description, allows to derive an explicit form of forces and simplifies further research on the quantum picture of individual fields, significantly simplifying the equations of the quantum field theory.

Further analysis using the variational method should also allow for an explicit derivation of the generalized potential V^{μ}, which may lead to further discoveries regarding both the description of quantum fields and elementary particles associated with them, and the possibility of experimental verification of the obtained solutions.

5. Statements

Data sharing is not applicable to this article, as no datasets were generated or analyzed during the current study.

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