Article

Modeling the Area of Interest for a Mobile Sensory System

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1 **Abstract:** Autonomous electronic area, territory, or space protection is getting more
2 important. One of the areas of application of the system is to monitor the quarantine
3 zones created during epidemics. The idea to create a system of mobile sensory units
4 for defending quarantine zones was born during one of the phases of the Covid
5 epidemic in 2020. The paper contains a brief technical description of a mobile sensor
6 units and their network. This network surrounds an area of interest. The work
7 includes a mathematical and data model of the protected area. In addition, the paper
8 presents a method for distributing mobile sensor units in areas of interest.

9 **Keywords:** earth centered inertial; geodetic coordinate systems; digital model of
10 terrain; digital model of the protected area; network data model; network of mobile
11 sensory units

1. Introduction

Solutions for external protection of an object or space are very specific problems. Creating adequate protection is affected by various surrounding conditions such as weather, visibility, and the possibility of false detection or failure. Therefore, it is necessary to combine and integrate many various units and methods that would overlap and complement each other and thereby ensure comprehensive protection, which would be resistant to various external influences. Various modern technologies used nowadays allow trespassers to intrude into the area of their interest (increasing the probability of success of terrorist attacks, sabotage theft or another type of attacks). The aim of these attacks is destruction or damage to the protected objects or to persons in the objects. Regarding the spread of viral diseases caused by COVID-19, the high dynamics of the spread of the disease and the formation of outbreaks of infection were demonstrated, which may not be identified soon enough. The problem is not just the isolation of one person, but also the isolation of whole communities. Despite the high consciousness of the whole community being affected by the behavior of individuals, crowd psychology and bad affect the influence of the presence of a relatively large commitment to the safety components on the psyche of people.

The idea to create a system of mobile sensory units for electronic area protection to isolate possible infection outbreaks was based on these facts. This type of community isolation is a very complex question. In our article, we have contemplated the solution's technological and algorithmic elements. These are the basic thoughts of our project called Movir. The goal of the project is to create and examine a mobile monitoring system for defending an isolated group of people from spreading viral diseases. Of course,

this also applies the other way around, the system is meant to protect the population from an isolated identified group. The system should be independent from the local electrical grid and visually is not meant to harm any of the protected parties.

2. Electronic area protection

The main goal of electronic area protection is to catch the trespasser before or directly intruding into the protected area. Secondly to locate and monitor its movement with technical tools to eliminate it or prevent its illegal activity. If we detected the intruder before breaking, or just after breaking into the territory of the interest, the security service would have enough time to react and eliminate the given threat.

Such devices and systems are required problem-free functionality and reliability. The reliability of the system is affected, not just by the quality of components, but by their resistance to the wind and other environmental effects, especially if these devices are located within the reach of people or animals. These conditions could be the source of unforeseen complications. It is important to prevent these types of malfunctions by placing detectors outside the visible perimeter or using the appropriate masking to blend to the surrounding environment. It is important to use high-quality sensors and software, which ensure the correct detection of space violations without false alarms and malfunctions. The external electronic area protection has to be designed and constructed correctly to fulfill its purpose. Technological progress makes it possible to enter protected areas in new ways.

New systems and equipment could easily eliminate security. Staying up-to-date is necessary when researching and developing new systems and sensors for external use. Various intrusion detection systems are used to secure a defined guarded area of interest. According to the principle used to detect intrusion into the protected area, we can operate sensors with infrared detectors, microwave detectors, accelerometers, magnetometers, sound detectors, various hygrometers and thermometers detectors, microwave barriers, ultrasonic sensors and optical laser sensors, FMCW (Frequency Modulated Continuous Wave Radar), radar systems that measure the distance and speed of moving objects and Fence intrusion detection systems which use a various combination of detection sensors.

3. Network of mobile sensory units

3.1. Project MOVIR - Mobile sensory system

With the project MOVIR, our goal was to create and test a mobile monitoring system for the defense of an isolated and at-risk group of people. The system is meant to protect against the spreading of viral diseases or conversely, to protect the population from an infected group identified as a source of risk. The proposed solutions will be independent from the local electricity distribution. Also, it will not change visually the sight of the environment, so it will not cause additional psychological burdens for any of the protected groups.

The actuality of this problem is generally known and frequently discussed. It directly affects all inhabitants of our country and neighboring states too. The proposed solution of the monitoring system is to ensure interest areas are original, especially from the point of view of its mobility. The current studies show that localization and subsequent isolation of outbreaks of infection play a key role in the fight against viral diseases. Another aspect is the fact that the current pandemic is particularly dangerous and risky for certain selected groups of the population, especially for seniors, or for people with some serious or chronic diseases. Protection of the risk groups of the population is also one of the reasons proving the topicality of the issue being addressed.

Our project touches on many areas of science. To model the sensor network, we drew on the following theoretical sources: geographic [1], coordinate systems and projections [2], plane geometry [3], coordinates for geographical information system (GIS) [4], computational geography and astronomy [5], GIS algorithms [6], and finally software computations [7],[8].

The results of the project are multidisciplinary by combining knowledge from various fields such as engineering [9], transport systems [10], sensorics [11], modeling and simulations [12], electrical engineering, electronics, engineering, information technologies [13], cloud technologies [14] and communication technologies.

A database of technologies similar to ours can be found in the publication [15]. Individual records of the database contain a theoretical description of the application of sensory systems.

3.2. Mobile (Portable) sensory units

Basic units, and mobile sensory units will be implemented in the form of separate free-standing units and portable devices as needed. The devices will have separate hardware equipment, which distribution in space creates a sensory network. Our work aims to model the formation of such a network.

The mobile sensory unit is a modular element of a transportable monitoring system with clear diversity of used sensors. The electrical independence of the system is provided by suitable batteries. Depending on the solar activity the batteries will be charged up by photovoltaic panels, to extend the period of operation of the monitoring system. The mobile sensory units are equipped with a control module to provide collection and preliminary data processing from individual sensors to make ensure autonomous operations.

The mobile sensory unit is designed for continuous monitoring of the surrounding area to inform about possible unauthorized entry into the area of interest or about leaving the monitored area. The obtained data During the monitoring of the area of interest are processed, evaluated and transmitted by the communication module using the low-energy LoRa communication network. The data is sent from the mobile sensory unit to the central sensor station where it is recorded to a storage medium and data backup is realized by forwarding them from the mobile monitoring station to the data server.

Although the mobile sensory unit is fully autonomous during normal operations, its activity can be partially controlled by an operator from a mobile monitoring station. The unit can be placed single, in a group, or combined manner, which defines the perimeter of the area of interest. It is possible to create a sensory communication network with the help of several mobile sensory units. In this case, the task could be to monitor whether there has been a violation of the defined area. All sensory units communicate and are controlled by the central sensor station. This means that in the communication concept, individual sensory units are involved in a communication star topology with a central control unit. In case of malfunction of this central station, it is possible to replace it with another suitable unit. If it is impossible to find this central station, the units can communicate with their neighboring units in an emergency communication topology circle and thus forward important data further. The number and location of these free-standing units are directly dependent on the topography of the country. Depending on the sensor units used, it is necessary to evaluate the appropriate placement according to distance and direct visibility between these free-standing column-shaped units, with a separate power source and a separate set of sensors. Each portable sensor unit has a function of its localization using the global navigation satellite system (GNSS). Here arises the importance of how to deploy individual sensor units in the field so that they can cover the protected perimeter with their range and at the same time be within communication range with the central unit too. It is essential to find an algorithm or method for the automated design of the optimal placement of sensory units in the target area.

One of the other requirements for the features of the scheme was the possibility of displaying the location of the object on the virtual map of the terrain and its 3D display for the needs of the operator. It is ideal to use a digital terrain model (DTM) as background material for the target area, which comprehensively describes the shape of the land surface with elevations. It is also possible to directly insert a vector layer of objects into the given model to display the digital model of the relief. To display these objects in 3D form, it is necessary to list their heights above the ground in the attribute table.

3.3. Sustainability of the project

Within the implementation of the project, as well as within the sustainability period, the applicant will emphasize the extensive dissemination of research and development results on a non-exclusive and non-discriminatory basis through teaching, open-access databases and publicly accessible publications. Application of the knowledge gained during the implementation of the aforementioned scientific and research activities into the educational process will lead to steps to increase the quality of human resources. Also, it will cause reconciliation of the interests of individuals with the needs of society, especially in connection with the possible occurrence and spread of viral diseases.

4. Digital model of the area of interest

4.1. Area of interest - Estimation of the number of sensory units

Our project aimed to test mobile sensory units and the network of sensory elements in a small monitored area. A small monitored space means a rectangular area with edges of a few dozen meters. In this chapter, we define the mathematical and data model of the protected area.

The basic principle of determining the area of interest is the principle used by Free Route Airspace (FRA) see. [12] when defining the horizontal part of the airspace. It is needed to enter special significant points of space which belong to the given area. These points will be given by GPS coordinates. Significant points with coordinates $[u_l, v_l, h_l]$ will be divided into three sets. One of the sets will be the internal points (blue color in the picture (1), obstacles in the terrain can be entered this way). In the second set are possible boundary points of the area (marked with red color), and in the third set are necessary points that must be included into the sensory network units (yellow color). We assume that some boundary points will form a simple closed polygon determined by the system user. It will be the polygon of the area of interest.

$$P = \{[u_i, v_i, h_l]; i = 0, 1, \cdots, n\} \tag{1}$$

Unlike FRA, this polygon has three dimensions (FRA points only have GPS coordinates, heights are solved otherwise). We consider the height because of the terrain profile. If we denote the length of the polygon as $L(P)$, the range of the signaling sensor with s like $range(s)$ and the sensitivity of the sensor element s with τ, then we can set the maximum distances between signaling sensors as $\tau \times range(s)$. While parameter τ can take on values from the interval $(1, 2\rangle$. The value of $\tau = 2$ means that neighboring sensory units work at a distance of $2 * range(s)$. Using these parameters, we can estimate the number of sensory units needed to create the network of mobile sensory units.

$$p(P) = L(P)/(\tau \times range(s)) \tag{2}$$

This value is the minimum number of sensory network units. Here we assume that there are used the same length measurement units for the length of the polygon and the range of sensory units. We note that the length of the polygon is the circumference of the area of interest. The number of the nodes of the closed simple polygon P is marked with $|P|$. Polygon which does not contain any nodes is marked $P = \emptyset$.

4.2. Digital terrain model

We suppose that input significant points $S = \{[u_l, v_l, h_l]; l = 0, 1, \cdots, k\}$ to the area $[u_l, v_l]$ are given. The $[u_l, v_l]$ are GPS coordinates of individual significant points, h_l is their altitude for $l = 0, 1, \cdots, k$. By using these points, we will create a 3D model of the terrain in which signaling sensors will be placed. There will be created a network of sensory units. It is also necessary to consider the spatial resolution of the technology that will create the digital model of the terrain. This parameter is denoted as d - detail of terrain modeling. Dimensions of the sensory element (width and depth) are approximately $50 \times 50\,cm$

so in this case, we can set the detail to $d = 2$ by default. This parameter means that a rectangle of size 100×50 cm can be covered with two frames that have size 50×50 cm. For the value of d, it is true that $0 < d$. The set M_S contains points on which the sensory units must be placed. The set V_S will mean internal significant points. Therefore, $S - V_S$ is a set of boundary significant points.

To model the *mobile sensory units network* in the Euclidean space, we need to define a coordinate system, the center of the coordinate system and x, y, z - axis. The center of the coordinate system will be the center of the Earth, the x-axis will be vernal equinox in the equator plane, the y-axis will be perpendicular to the x-axis in the equator plane, and the z-axis will be perpendicular to x and y - axes. This coordinate system is called *Earth centered inertial*. The more general name of this system is the *Ecliptic coordinate system*. The coordinate system can be implemented in Cartesian and spherical coordinates, see [4] and [1].

For the sake of simplicity, we move the origin of the coordinate system to a point defined based on entry significant points. Calculation of the origin of the new coordinate system and the digital terrain model contains these steps

Algorithm for creating a digital model of the terrain around the area of interest
    Input: S, d, M_S, V_S
    Output: $S', mx, mz, my, O, M_{S'}, V_{S'}$

1. Declaration of coordinates of S in the Cartesian system:

$$S = \{[u_l, v_l, h_l]; l = 0, 1, \cdots, k\} = \{[x_l, y_l, z_l]; l = 0, 1, \cdots, k\} \tag{3}$$

2. Marking of $\min x = \min_{l=0,\cdots k} x_l$, $\min y = \min_{l=0,\cdots k} y_l$, $\min z = \min_{l=0,\cdots k} z_l$ a

$$S' = \{[d * (x_l - \min x), d * (y_l - \min y), d * (z_l - \min z)]; l = 0, 1, \cdots, k\} = \{[x'_l, y'_l, z'_l]; l = 0, 1, \cdots, k\}. \tag{4}$$

3. 3. Let's mark $mx = \max_{l=0,\cdots k} x'_l$, $my = \max_{l=0,\cdots k} y'_l$ a $mz = \max_{l=0,\cdots k} z'_l$, and point $O = [0, 0, 0]$.

4. In this way a Euclidean coordinate system with the origin O will be created, see [2]. Dimensions of the digital terrain model will be $mx \times my \times mz$. Points of the set $S', M_{S'}, V_{S'}$, are created based on relation (3).

5. The digital model of the terrain is going to be an $A = (a_{ij})$ matrix of type $mx \times my$. The significant points $[x'_l, y'_l, z'_l] \in S'$ are going to be elements $a_{x'_l y'_l}$ of the matrix A and their values are going to be the height of the terrain $a_{x'_l y'_l} = z'_l$.

Notes: Different heights can be distinguished by color during the modeling of the terrain (see.1). Height gradient of the terrain will be mz. For each point $U = [x, y, z] \in S'$ it holds that:

$$0 \leq x \leq mx, 0 \leq y \leq my, 0 \leq z \leq mz. \tag{5}$$

Our digital terrain model is raster (grid) and contains a regular distribution of points.

We can see in the picture (1) the shift of the origin of the coordinate system to the point $O = [0, 0, 0]$ (steps of the algorithm 1. 2. and 3.). This shift is necessary to calculate distances easier and locally with the help of the digital terrain model in applications. In the Digital terrain model section, a 90×130 matrix A is displayed. Significant points are marked as signs: +. Red points can be border and blue points internal significant points of the area ($V_{S'}$) and the significant points marked with yellow must be included in the network of sensory units ($M_{S'}$).

A digital image of the terrain (matrix A of type $mx \times my$ in units of d) can be created by using the point $O' = [0, 0, h_0]$, with dimensions $mx \times my \times mz$. It is necessary to find point O' in the field too (it is a point with GPS coordinates (min x, min y, h_0). It is necessary to digitally scan the terrain so the point

O' will be located in the lower-left corner of the digital model. The x -axis is defined by a straight line between point O and point O_x (this point has GPS coordinates (min x, y) and points O, O', O_x form one plane) and y - axis is placed between point O and point O_y (this point has GPS coordinates (x, min y) and the points O, O', O_y form one plane). The scanned area will be a rectangle of size $mx \times my$ in d units.
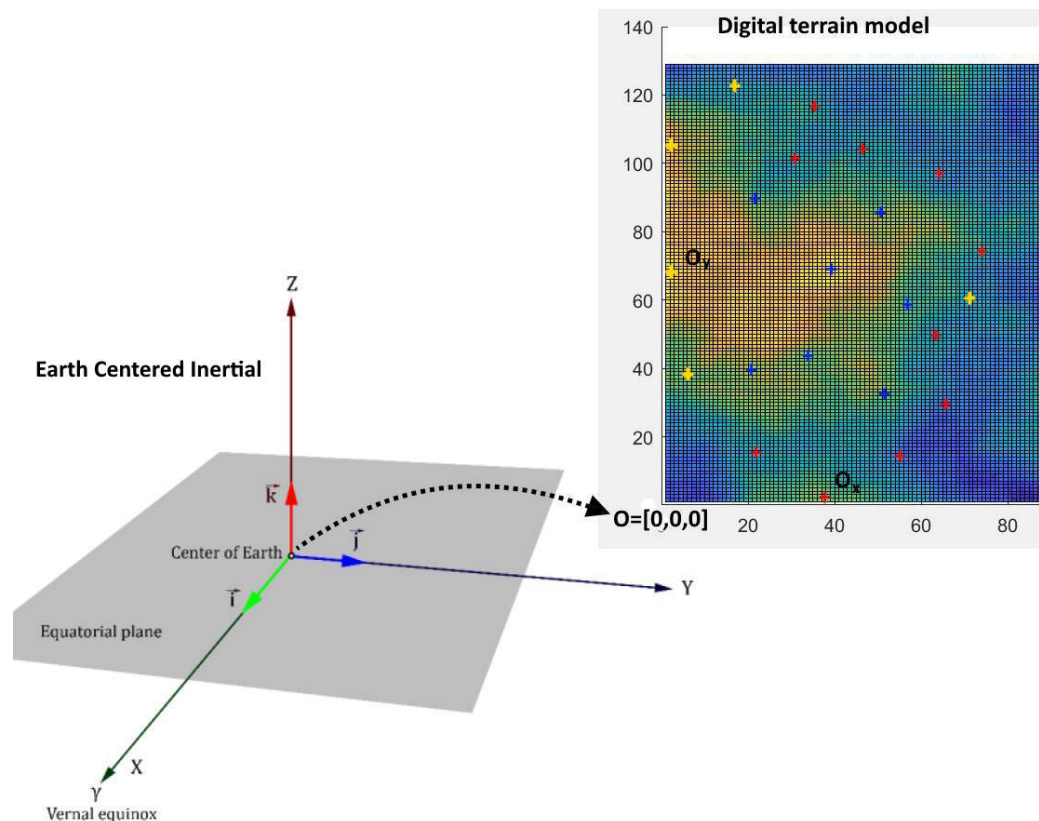

Figure 1. Digital terrain model and the origin O

A digital image of the terrain must be created to obtain a matrix A. In the matrix A it is necessary to mark significant points S' according to the assignment (4) and significant points that must be included in the area of interests $M_{S'}$ and internal significant points $V_{S'}$. Conversion of GPS coordinates to the Cartesian system (3) is possible, e.g. in MATLAB using the command:

$$[X, Y, Z] = geodetic2ecef(spheroid, lat, lon, h) \tag{6}$$

This command is used to convert geodetic coordinates to the Earth-centred, Earth-fixed coordinate system (ECEF). In this case, the height h is the ellipsoidal height of the point,(lat, lon) are the GPS coordinates of the point and the parameter *spheroid* is a parameter of the MATLAB system, which stands for the fact that we are examining in our considerations globe.

When we have available the ellipsoid heights of the points, we can apply the command (6). Then assignment

$$a_{xy} = Z \tag{7}$$

defines the elements of the matrix $A = (a_{xy})$ of type $mx \times my$ in d units. Matrix A represents the digital model of the terrain. This model using raster data structures.

When the altitude or orthometric height H is available, we need to calculate the ellipsoidal height h. The relation ships between heights are shown in Figure (2). Methods for calculating altitude (orthometric height) can be found in publications such as[4]. Another option is to apply newer versions of the program MATLAB (newer than R2022b), where theoretical calculations are already implemented and there are functions for this calculation [7].

Of course, other mathematical systems such as SageMath or Geogebra also enable geographic calculations too. Publication [1] contains the basic mathematical principles of geography. Detailed theoretical information about geodetic coordinate systems can be found in the publication [5], and about plane geometry of the type we use in the design [3].
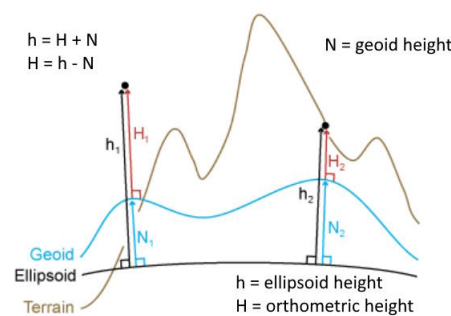

Figure 2. Above mean sea level (orthometric) height of the point

4.3. *Computation of X, Y, and Z coordinates in the ECI system*

The next MATLAB code (version R2022b or higher) calculates coordinates in the Earth-Centred Inertial (ECI) coordinate system (1). The input data are the GPS coordinates and the altitude (above mean see level height) of the peak of Mount Everest. The outputs are the coordinates of this point in the Euclidean coordinate system with the center at the center of the Earth (3 point of the algorithm). The calculation requires the Mapping Toolbox of the MATLAB system. This toolbox contains the digital models of the globe (geoid). The results of the computations are given in meters.

```
% Computation of X, Y, Z coordinates in the ECI system
% Input ( lat , lon ) GPS coordinates of the point , H − altitude
lat = 27.988056;
lon = 86.925278;
H = 8848;

% Definition of the digital model of the globe
% Computation of ellipsoidal height
% The Mapping Toolbox − contains command
N = egm96geoid(lat,lon);
h = H + N

% Determination the ellipsoid where the point [X,Y,Z] is located
spheroid = referenceEllipsoid('GRS_80');
[X,Y,Z]=geodetic2ecef(spheroid,lat,lon,h)

% The outputs are given in meters
h = 8.8193e+03
X = 3.0274e+05
```

Y = 5.6360 e+06
Z = 2.9795 e+06

"Projections for large areas usually use a simple sphere; for smaller areas, where accuracy gains in importance, the projection uses an ellipsoid which locally corresponds to a *geoid,* the name for the shape with the most accurate correspondence to the actual oblate and irregular shape of the earth at a given time. " [1]. The description of the *GRS 80* reference model can be found in the publication [8] and [7].

4.4. Polygon of the area of interest

As stated in the work [12], it is not possible to algorithmically determine a polygon based on significant points of the limited area. Therefore, it is necessary to enter which significant points and in what order will define the polygon that bounds the area of interest. In the mentioned work, it is also stated that a simple polygon can be divided into an upper and a lower part, as we can see in Figure 3. This is the way to obtain the points $r_0, r_1, \cdots, r_n = r_0$ which define the polygon of the bounded area.

Now we need to determine the length of the polygon to be able to estimate the number of sensory units of the network according to relation (2). To calculate the length of the polygon, we need to define the term *spatial curve.*

Definition 1: Let $\langle a, b \rangle \subset \mathbb{R}$ a closed interval. Representation $r : \langle a, b \rangle \to \mathbb{R}^3$ is a *spatial curve,* $r(a)$ is the *starting point,* $r(b)$ is the *endpoint* of the curve. If $r(a) = r(b)$ then the curve is *closed.* The curve r is *simple,* when $r : (a, b) \to \mathbb{R}^3$ is a *one-to-one* correspondence.

The correspondence is one-to-one if each element of the first set is made to correspond with exactly one element of the second set, and vice versa.

Definition 2: Let $r : \langle a, b \rangle \to \mathbb{R}^3$ be a *spatial curve* and $F = \{t_0 = a < t_1 < \cdots < t_n = b\}$ is the partition of the interval $\langle a, b \rangle$. The spatial polygon P, which has vertices $r_i = r(t_i), i = 0, 1, \cdots, n$. is called an *inscribed polygon* belonging to partition F. The length of the polygon is the sum of the lengths of the edges:

$$L(P) = \sum_{i=1}^{n} |r_i - r_{i-1}| \tag{8}$$

A *polygon of a area of interest* is a simple, closed spatial curve, which is given by the division of the interval $\langle a, b \rangle$. In three-dimensional space, a polygon is given by these points

$$r_i = [x_i, y_i, z_i] \tag{9}$$

for $i = 0, 1, \cdots, n$. Therefore, we can calculate the distances between the points r_i and r_{i-1} as (see [4,5])

$$|r_i - r_{i-1}| = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \tag{10}$$

The points $r_i = [x(t_i), y(t_i), z(t_i)], i = 0, 1, \cdots, n$ from relation (9) define three functions $x(t), y(t), z(t)$. These functions for our purposes at the moment are not needed, only their functional values in points t_i, so we do not examine the properties of the functions.

4.5. Digital model of the area of interest

We now summarize the procedure for defining a digital model of the area of interest.
The method of defining the area of interest in the terrain model

A method of defining the area of interest in the terrain model
Input: A - digital terrain model,
S' - set of significant points (+) , M_S- set of significant points which are necessary (+, yellow color),
$V_{S'}$ set of internal significant points (+, blue color)

(+, red color in figure (3) alternative boundary significant points, it is a set $(S' - V_{S'}) - M_{S'}$)
Output:
P - polygon of the area of interest,$p(P)$ - minimum number of sensory units.

1. Determination of the sequence of significant points $r_0, r_1, \cdots, r_n = r_0$ in matrix A so that all necessary boundary significant points have to be included (+, yellow color). These points are completed with boundary points (+ red color) to create a simple polygon P of the area of interest.
2. For a given polygon P, we calculate the value $p(P)$ - it is an estimate of the minimum number of sensory units which are needed.

We note that this method cannot be algorithmized. This is the reason why the application is needed, which with the data inputs and outputs will help solve this problem. When the polygon P is defined, the length of the polygon is calculated based on (8) and according to (2) we can estimate the minimum number of sensory units in the network. The next task will be to create a method to deploy the sensory units on the polygon P.
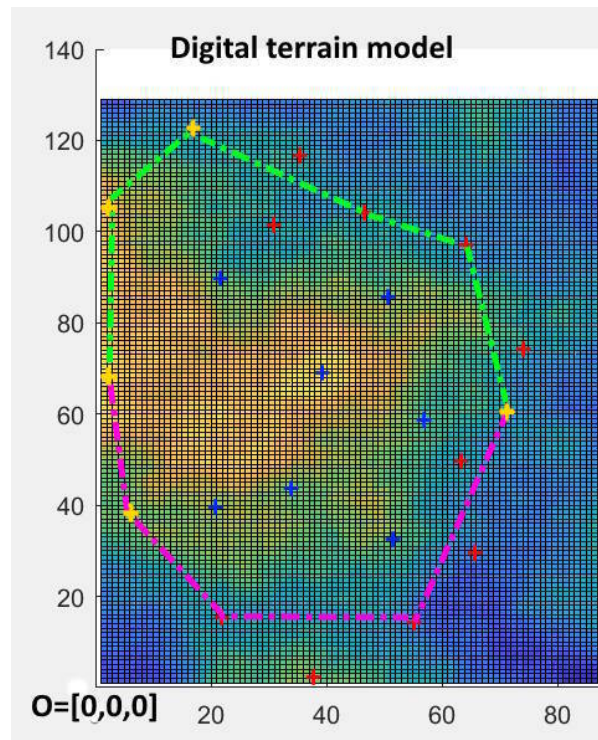

Figure 3. Polygon of the area of interest

4.6. Algorithm for creating a network of mobile sensory units

Suppose that the polygon P of the area of interest is given. Polygon P contains significant points $r_0, r_1, \cdots, r_n$ a $r_0 = r_n$. All the necessary significant points that belong to the set M_S are vertices of the polygon P. In a digital model of the terrain is given point $O = [0,0,0]$. According to this point, the coordinates of the points are given $r_i = [x_i, y_i, z_i]$..

The algorithm for creating a network of sensory units

Input: P - polygon of the area of interest is given with significant points $r_0, r_1, \cdots, r_n$ where $r_0 = r_n$. The set of significant points S', the set of necessary significant points $M_{S'}$ and the set of internal significant

points $V_{S'}$. Digital model of terrain A, $range(s)$ – the reach of the sensory element, τ - sensitivity of the sensory element.

Output: Q - polygon of the area of interest, which contains points $q_0, q_1, \cdots, q_k$ and $q_0 = q_k$ which form the network of sensory units.

1. for $i = 1, \cdots, n$
2. $j = i - 1$, point selection $r_j \in P$, $Q_j = \varnothing$
3. $l=1, k = 1, q_k = r_j$.
4. $k = k + 1$
5. if $\left| r_j - r_{j-1} \right| \leqq \tau * range(s)$ then $q_k = r_j, j = j + 1, l = l + 1$ (see, point I. on the Figure (4))
6. if $\left| r_i - r_{i-1} \right| > \tau * range(s)$ then

      if $r_i \in M_{S'}$ then $q_k = r_j, j = j + 1, l = l + 1$ (see, point II. on the Figure (4))

      if $r_i \notin M_{S'}$ then it is possible (there is no obstacle between the points r_j and D)

            $q_k = D, j = j + 1, l = l + 1$

         if there is an obstacle between the points r_j and D, then

            $q_k = r_j, j = j + 1, l = l + 1$ (see, point III. on the Figure (4))
7. if $i < n$ then go to 2 (in this step the polygon Q_j with vertices $q_1, q_2, \cdots, q_l$ was created)
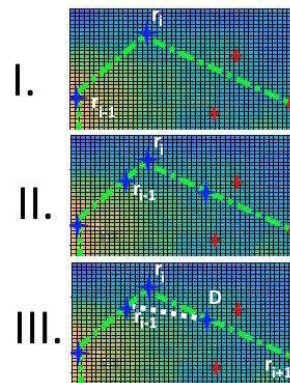8. if $i = n$ then $Q = Q_k$, $|Q_k| = \min\limits_{i=1,\cdots,n} |Q_i|$.

Figure 4. Selecting the next point

Note: Point D lies on the line r_i and r_{i+1} at a distance of $2 * range(s)$. Point D is not a significant point.

Brief description of the algorithm: We have a cycle considering the variable j in which we select the vertices of the polygon P. According to points 5-7. we create a polygon Q_j. with vertex r_{j-1}. Polygons Q_j contain a set of necessary significant points and other points where sensory units are located. In point 8, we determine the polygon, which contains the smallest number vertices from that set of polygons $Q_1, \cdots, Q_n$.

This method finds a polygon Q such that the length of the polygon is $p(Q) \leq p(P)$ and the number of vertices of Q is less than or equal to the number of vertices of P. The polygon Q is a digital model of the boundary of the area of interest, the vertices of the polygon define the location of the sensor units in the area of interest.

4.7. Data model of the network

The coordinate system (coordinate system origin, coordinate system axes), the set of significant points, the polygon of the area of interest and the polygon of the network belong to the mathematical model of the network. We also have other inputs, such as a digital terrain model. This terrain model

depends on the detailedness of the terrain d. All these objects need to be defined in the data model of the network.

The *raster data model* contains the following data structures:

Origo - a table that contains one record in which there are the following items: lat, lon, H, d. The origins of the coordinate system (lat, lon) are the actual GPS coordinates, H is its altitude and d is the detailedness of the terrain, which was applied in the creation of a digital terrain model. This information determines the place where the terrain is located in which the network will be set. Values (lat, lon) need to be calculated according to relations (3, 4). Parameters H and d are going to be input parameters. These data define the origin of the coordinate system, it is the *reference point* of the system and the *detailedness* of the terrain model, in which resolution we will work.

T_{Origo}- table (matrix $A = (a_{xy})$) of size $mx \times my$ - digital terrain model. Digital terrain model can be obtained from existing software such as MATLAB (for detail $d = 1$) or it is possible to apply special scanners such as LiDAR, where we can achieve a detail of $d = 10$, which means we can divide a 100 cm section into 10 parts, or in other words, the resolution of one table point will be10×10 cm.

S_{Origo}- a table that contains rows of $x, y, type$ - set of significant points$[x, y]$ coordinates in the table T_{Origo}, $type$ - significant point coding, 1- internal (blue color), 2- necessary boundary (yellow color), 3 - borderline (red color in Figures 1 and 3). We note that a suitable choice of significant points helps to define different types of obstacles in the field.

With the help of significant internal points, we can define obstacles as things, events or constructions. Different observation layers can also be created. In that case, matrix A will be three-dimensional. The third dimension can contain layers; for example, the first layer includes the elevation, the second layer structural obstacles and the third layer natural obstacles. In the case of layers, the data complexity of the system grows.

P_{Origo}- table (polygon of the area of interest): contains the lines $[x, y]$ - coordinates of points from the table S_{Origo}, which create the polygon of the area of interest.

Q_{Origo} - table (polygon of the grid): include lines $[x, y]$ - some coordinates of P_{Origo} points and some position data of T_{Origo} points.

We can match this basic raster data model to a mathematical model. For completeness, let us note that there may also be a *vector data model* (see [1]) of the monitored area; but we do not deal with this type of model in this work.

The data model is needed for the next step, for the software implementation of the network modeling application, which can be considered a small geographic information system (GIS). Algorithms applied for GIS systems can be found in the publication [6].

4.8. Steps for software implementation

Steps for implementation of the application:

1. step of the software implementation is the selection of significant points in the monitored area.

2. step is determining the coordinate system (determining the points O, O', O_xO_y and *Origo*) (with this step, we locate the task).

3. step is obtaining a digital terrain model of the monitored area. Determination of input parameters ($Origo$, T_{Origo}, S_{Origo}).

This input information is necessary for the application's software implementation to design the network of significant elements.

4. step - the output of the computations are tables of polygons P_{Origo}, Q_{Origo}.

Several solutions are available for the software implementation of a GIS network of sensory elements. One solution is to apply MATLAB v. R2022b and above with the mapping toolbox. This system houses digital models of the globe with functions that enable geographic calculations. Another option is to

use the Python programming language. There are extensions for this language, such as ArcGIS, for geographic applications, see [8].

5. Conclusion

The main goal of this work was to create a mathematical and data model of a mobile sensor unit network. In the article, a mathematical model was proposed, which contains the definition of the coordinate system, sets of significant points, digital terrain model, terrain detail, area of interest polygon and polygon of the network. Based on the mathematical model, a data model was also designed. This model should serve as an input for the program implementation of a geographic information system for modeling the creation of a network of mobile sensory units.

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References

1.   Harvey, F. *A Primer of GIS: Fundamental Geographic and Cartographic Concepts*; GUILFORD PUBN, 2015.
2.   Maling, D.H. *Coordinate Systems and Maps Projections*; Pergamon Press, 1992.
3.   Chiossi, S.G. Elementary Plane Geometry. In *Essential Mathematics for Undergraduates*; Springer International Publishing, 2021; pp. 401–437. doi:10.1007/978-3-030-87174-1_18.
4.   Sickle, J.V. *Basic GIS Coordinates*; CRC Press. Taylor & Francis Group, 2017.
5.   Lawrence, J.L. *Celestial Calculations - a Gentle Introduction to Computational Astronomy*; The MIT Press, Cambridge, Massachusetts, London England, 2018.
6.   Xiao, N. *GIS Algorithms*; SAGE Publications Ltd, 2015.
7.   Mathworks. *Find Ellipsoidal Height from Orthometric Height*, 2023. https://www.mathworks.com/help/map/ellipsoid-geoid-and-orthometric-height.html.
8.   Conley, J. *A Geographer's Guide to Computing Fundamentals*; Springer International Publishing, 2022. doi:10.1007/978-3-031-08498-0.
9.   Kurdel, P.; Češkovič, M.; Gecejová, N.; Labun, J.; Gamec, J. The Method of Evaluation of Radio Altimeter Methodological Error in Laboratory Environment. *Sensors* **2022**, *22*, 5394. doi:10.3390/s22145394.
10.   Novotňák, J.; Fiľko, M.; Lipovský, P.; Šmelko, M. Design of the System for Measuring UAV Parameters. *Drones* **2022**, *6*, 213. doi:10.3390/drones6080213.
11.   Fiľko, M.; Kessler, J.; Semrád, K.; Novotňák, J. Manufacturing of the Positioning Fixtures for the Security Sensors Using 3D Printing. *Transportation Research Procedia* **2022**, *65*, 98–105. doi:10.1016/j.trpro.2022.11.012.
12.   Szabó, P.; Ferencová, M.; Blišťanová, M. A Spatially Bounded Airspace Axiom. *Axioms* **2022**, *11*, 244. doi:10.3390/axioms11060244.
13.   Kasper, P.; Lipovsky, P.; Smelko, M. Application Software of Modular System for Magnetic Characteristics Measurement. In Proceedings of the 2022 New Trends in Aviation Development (NTAD). IEEE, 2022. doi:10.1109/ntad57912.2022.10013625.
14.   Szabó, P.; Ferencová, M.; Železník, V. Cloud Computing in Free Route Airspace Research. *Algorithms* **2022**, *14*, 123. doi:10.3390/a15040123.
15.   Nayak, S.; Zlatanova, S., Eds. *Remote Sensing and GIS Technologies for Monitoring and Prediction of Disasters*; Springer Berlin Heidelberg, 2008.