Research on Wavelet Transform Modulus Maxima and OTSU in Edge Detection

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Abstract: In the course of routine bridge maintenance, edge detection allows the partial condition of the bridge to be viewed. However, many edge detection methods often have unsatisfactory performances when dealing with images with complex backgrounds. Moreover, the processing often involves the manual selection of thresholds, which can result in repeated testing and comparisons. To addresses these problems in this paper, the wavelet transform modulus maxima method is used to detect the target image, and then the threshold value of the image can be determined automatically according to the OTSU method to remove the pseudo-edges. Thus, the real image edges can be detected. The results show that the information entropy and SSIM of the detection results are the highest when compared with the commonly used Canny and Laplace algorithms, which means that the detection quality is optimal. To more fully illustrate the advantages of the algorithms, images with more complex backgrounds was detected, and the processing results of the algorithms in this paper are still optimal. In addition, the automatic selection of thresholds saves the operator's effort and improves the detection efficiency. Thanks to the combined use of the above two methods, detection quality and efficiency are significantly improved, which has a good application in engineering practice.

Keywords: edge detection; wavelet transform modulus maxima; OTSU; complex background; threshold

1. Introduction

Edge detection is an important process in techniques, such as image analysis and processing, computer vision and pattern recognition, with the aim of detecting information about the shape of objects in an image [1–4]. In the real world, there are four situations in which edges can be formed in an image: (1) discontinuities in depth, (2) discontinuities in surface orientation, (3) different materials of objects in an image, and (4) different lighting in a scene. Edges often locate in an image where there are significant changes in grey scale, colour and texture [5], so the edges of an object can be determined by detecting gradients. Commonly used algorithms for edge detection include Roberts, Prewitt, Sobel, Laplace and Canny. These detection methods are easy to implement, demonstrate many advantages when detecting images with relatively simple backgrounds, and are widely used in many industries [6,7]. However, when dealing with images with complex backgrounds [8], they are not sufficient to recover much high-level semantic information, so they have some shortcomings.

The wavelet transform is a time-frequency domain analysis method that has the ability to characterize the local features of a signal. The singularities of the wavelet-transformed signal are closely related to the location where the signal changes apparently. A. Tjirkallis [9] applied the wavelet transform modulus maxima method to image processing which determined the damage of the object caused by external forces or environmental conditions in the image. Xiangxing Kong[10] used this method to successfully denoise tunnel monitoring, providing a basis for tunnel health

diagnosis; Yunting Gu [11] used an improved wavelet transform mode maxima method to subdivide and fuse two edges obtaining the clearer and better connected edge information; Mohammad Barr [12] embedded a 1-bit watermark in wavelet transform mode maxima to achieve rotation, scaling and panning of the image, improving the robustness of the watermarking technique. Wensi Ding [13] used wavelet transform modulus maxima based on Bayesian threshold estimation to effectively remove noise in MEMS.

In the process of applying wavelet transform modulus maxima, thresholds need to be set to remove pseudo-edges in gradient images, but manual setting requires repeated debugging and is not efficient. OTSU method can use the maximum inter-class variance of the image to automatically calculate the threshold and improve the debugging efficiency. Using the OTSU, Ta Yang Goh [14] carried out threshold segmentation on the real image data and predicted the segmentation results based on the Monte Carlo statistical method; Kishore Dutta [15] used OTSU to segment plant images to discriminate between healthy leaves, unhealthy leaves and other backgrounds, and could identify the infected parts of leaves; Nidhi Gupta [16] used OTSU to pre-process and segment brain MR images and extracted many important features from the segmented images; Assma Azeroual [17] used OTSU to work out the FSW extrema coefficients and obtain the edges of the image by connecting the edge points using the predicted edges; Noorhan M Sobhy [18] used OTSU and watershed marker-controlled technique to distinguish between single cells and groups of cells.

In the process of applying the wavelet transform modulus maxima method for edge detection, this paper convolves a convolution kernel generated by a two-dimensional Gaussian function firstly with the target image, which is also the CWT (continuous wavelet transform). Based on the convolution result, the modulus value and magnitude angle of the image pixel have been calculated, and then the modulus maxima in the image pixel are determined according to the magnitude angle. Then the threshold value is calculated automatically using OTUS. Comparing the threshold value and the modulus maxima, we can remove the pseudo-edges of the image to obtain the final edge image. The edge detection results of Canny algorithm and Laplace algorithm are shown, and detection results of the three were evaluated with information entropy and SSIM as the evaluation indexes. The results demonstrate that the detection quality and detection efficiency of the algorithm in this paper have been improved. To fully illustrate this, image with more complex backgrounds was processed by the three algorithms. The results still show that the information entropy and SSIM of the wavelet transform modulus maxima method are the highest, which shows that this algorithm has obvious advantages, when processing images with more complex backgrounds.

2. Study Area and Data

2.1. Study Area

This paper selects the Jialing River Bridge in Chongqing, China, as the test area. The Jialing River Bridge is located between Shangqing Temple and Huaxin Street in Chongqing, China. The Jialing River Bridge is approximately 600-metre long, with 5 main bridge holes, 384-metre long, and 7 approach holes, 216.56-metre long, with a deck width of 21.5-metre, including 14-metre for the carriageway and 3.75-metres for the footpaths on each side. The bridge structure is a riveted steel truss double cantilever bridge, and the approach bridge is a reinforced concrete T-girder. The bridge was opened to traffic in the 1960s, putting an end to the history that residents on both sides of the river could only cross by ferry. 2017 saw a new round of renovations, bringing the bridge back to life, and it is still an important travel route for local residents. Figure 1 shows a partial image of the bridge taken from a side angle.

Figure 1. Partial image of the bridge.

2.2. Data

Figure 2 shows an overhead image of the Jialing River Bridge in Chongqing. Figure 2 was chosen as the data for this study. The image was taken by an unmanned aerial vehicle (UAV) and shows well the characteristics of the environment around the Jialing River Bridge. The content of the image is relatively feature-focused, with clear contours and a relatively homogeneous background in the middle part, which facilitates the removal of interference and is beneficial for subsequent edge detection and other related processing.
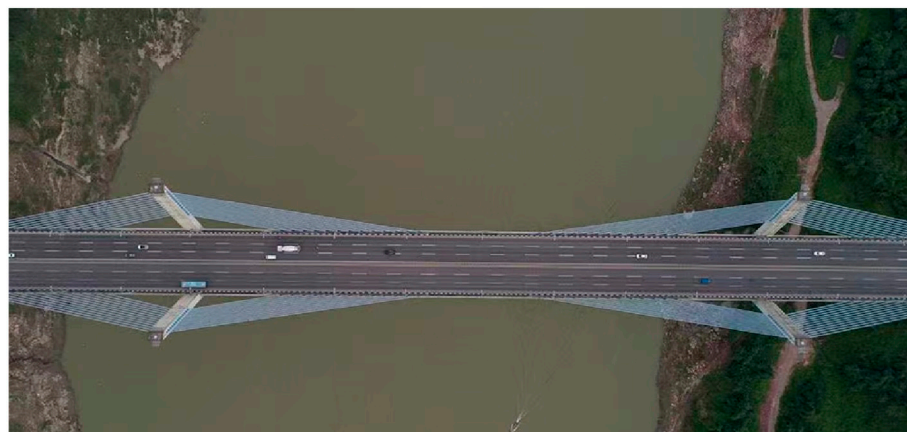
Figure 2. Overhead image of the bridge.

3. Materials and Methods

3.1. Theory

3.1.1. Wavelet Transform Modulus Maxima

The wavelet transform inherits and develops the idea of localisation of the Fourier transform, while overcoming the disadvantages of the window size not varying with frequency. Its main feature is to highlight the characteristics of certain aspects of the signal after the transform, and to gradually refine the signal on multiple scales through the telescopic translation operation, eventually achieving time subdivision at high frequencies and frequency subdivision at low frequencies, which can automatically adapt to the requirements of time-frequency signal analysis and thus focus on arbitrary details of the signal [19–21]. Therefore, wavelet transform has been successfully applied to many fields, especially the discrete numerical algorithm of wavelet transform is widely used in the study of many theoretical problems.

The modulus maximum is defined as follow：$W_{2^l}[f(t)]$ is the coefficient of the scale I of the wavelet transform of the signal f(t), and its modulus maximum is written as $Max[W_{2^l}[f(t_{k_l})]], l = 1, 2, \cdots, L,$,l is the scale, $k_l = 1_l, 2_l, \cdots, M_l,$ is the serial number of the modulus maximum on the scale l. The modulus maximums are located at the sudden changed points [22]. The modulus maxima satisfies the 2 conditions in Equation 1.

$$\begin{cases} |W_{2^l}[f(t)]| > |W_{2^l}[f(t-1)]| \\ |W_{2^l}[f(t)]| > |W_{2^l}[f(t+1)]| \end{cases} \tag{1}$$

If there is $W_{2^l}[f(t)] = |W_{2^l}[f(t+1)]|$ the equation $W_{2^l}[f(t+1)] = |W_{2^l}[f(t+2)]|$ must be satisfied as well ; and if there is $W_{2^l}[f(t+1)] = |W_{2^l}[f(t+2)]|$, the equation $|W_{2^l}[f(t-1)]| = |W_{2^l}[f(t-2)]|$ must be satisfied as well.

The general steps of the wavelet transform mode maxima method applied in edge detection [23] are as follow: (1) use a fast algorithm, find the multi-scale wavelet coefficients of the image; (2) calculate the modulus and magnitude angle of each pixel respectively; (3) calculate the mode maxima point after wavelet transform; (4) set the threshold, and where the modulus value is greater than the threshold is the edge point; (5) output the multi-scale edge of the image. The specific workflow is shown in Figure 3.
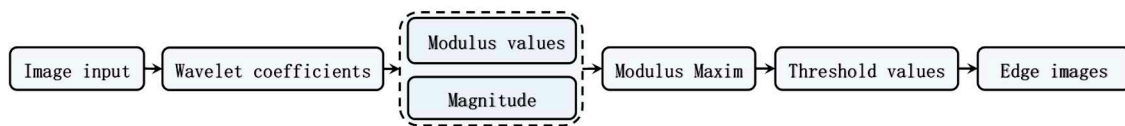
Figure 3. Wavelet transform modulus maxima flow.

3.1.2. OTSU

The OTSU algorithm, also known as the Nobuyuki OTSU method, is considered the best algorithm for threshold selection in image segmentation. Because OTSU is simple to compute and is independent to image brightness and contrast, it has been widely used in digital image processing [24–26]. It divides the image into two parts, background and foreground, according to the grey-scale characteristics of the image. The larger the interclass variance between the background and foreground, the greater the difference between the two parts that make up the image. When part of the foreground is divided into the background or part of the background is divided into the foreground, both result in a smaller difference between the two parts [27]. Therefore, the segmentation that maximizes the interclass variance means the smallest probability of misclassification [28]. Let the grey scale image grey level be L, then the grey scale range is [0,L-1], and the optimal threshold for calculating the image using the OTSU algorithm is shown in Equation 2.

$$t = Max[w_0(t) * (u_0(t) - u)^2 + w_1(t) * (u_1(t) - u)^2] \tag{2}$$

The variables are stated as follow: when the threshold for segmentation is t, w0 is the background scale, u0 is the background mean, w1 is the foreground scale, u1 is the foreground mean and u is the mean of the whole image. The t that maximizes the value of the above equation is the optimal threshold for segmenting the image.

3.1.3. Information Entropy and SSIM

Information entropy is a concept introduced by Claude Shannon, a famous American mathematician and founder of information theory. Information entropy is used as a measure of uncertainty, that is, the probability of discrete random events occurring. Briefly , the more chaotic things are, the higher the information entropy, and conversely, the lower the information entropy [29–34].

For an image, random variable described by the information entropy is the pixel value of the image. The information entropy of an image is a statistical form of a feature and reflects the size of the average amount of information in the image. Typically, the greater the information entropy of an image, the more informative and detailed it is, and the better the image quality. The one-dimensional information entropy of an image can show the concentration characteristics of the grey-scale distribution in the image, but it cannot reflect the spatial characteristics of the grey-scale distribution. On the basis of the one-dimensional entropy, characteristic quantities which can reflect the spatial characteristics of the grey-scale distribution can be introduced to compose the two-dimensional entropy of an image [35]. The neighbourhood grey mean of the image is chosen as the spatial characteristic quantity of the grey scale distribution, and combated with the pixel grey scale of the image to form into the feature binary (i,j). Equation 3 and Equation 4 are the calculation methods of image information entropy [36].   f(i,j) is the frequency of occurrence of (i,j) , N is the scale of the image.

$$H = -\sum_i \sum_j P_{i,j} \log P_{i,j} \tag{3}$$

$$P_{i,j} = f(i,j) / N^2 \tag{4}$$

SSIM (Structural Similarity Index Measure) is a measure of the degree of similarity between two images, and the structural similarity between the two can be seen as a quality measure of distorted images. Compared to traditional image quality measures, SSIM is more in line with the human eye's judgement of images. Given two signals x and y, the structural similarity between the two is defined [37–41] as shown in Equation 5 and Equation 6.

$$SSIM = [l(x,y)]^{\partial} [c(x,y)]^{\beta} [s(x,y)]^{\gamma} \tag{5}$$

$$l(x,y) = \frac{2\mu_x \mu_y + C_1}{\mu_X^2 + \mu_Y^2 + C_1}, c(x,y) = \frac{2\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2 + C_2}, s(x,y) = \frac{\sigma_{xy} + C_3}{\sigma_x \sigma_y + C_3} \tag{6}$$

where l(x,y) is used to compare the brightness of x and y, c(x,y) to compare the contrast of x and y, and s(x,y) to compare the structure of x and y. α, β, γ are parameters to adjust the relative stability of l(x,y), c(x,y), and s(x,y). μ_x and μ_y are the mean values of x and y, σ_x and σ_y are the standard deviations of x and y respectively, and σ_{xy} is the covariance of xy. C1, C2 and C3 are all constants used to maintain the stability of l(x,y), c(x,y) and s(x,y). In practice, the default will generally be, C3= 0.5* C2, so Equation 5 can be simplified to Equation 7.

$$SSIM = \frac{(2\mu_x \mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{7}$$

3.2. Data Processing

3.2.1. Pre-treatment

In order to facilitate the edge detection on the target image, appropriate pre-treatment is necessary. The pre- treatment used in this paper is image enhancement. Image enhancement can improve the visual effects, highlight the useful information of an image and compress invalid information facilitating computer analysis. Image enhancement methods can be broadly classified into 2 categories: spatial domain methods and frequency domain methods. The spatial domain method is to directly perform linear or non-linear operations on the image to process on the pixel grey values of the image [42]. The frequency domain method, on the other hand, considers an image as a two-dimensional signal and performs signal enhancement on it based on Fourier transform [43].

In this paper an image frequency domain method is adopted based on the wavelet transform. With efficient scaling and translation capabilities, the wavelet transform can analyze signals at

multiple scales [44] and resolve some problems that cannot be solved by traditional frequency domain transforms. After the two-dimensional wavelet transform is applied to the image, some kinds of two-dimensional signal, the approximate signal and the detailed signal (horizontal, vertical and diagonal directions) can be obtained. The contour information of the image is mainly concentrated in the approximate signal [43], and some details and noise are concentrated in the detailed signal. In order to promote the contrast of the target image and suppress the background, non-linear image enhancement can be applied to the approximate signal [45]. Eventually, based on the processed approximate and detail signals, wavelet reconstruction is performed to obtain the enhanced image. The specific processing process [46] is shown in Figure 4, where wavelet decomposition, non-linear enhancement, denoising and wavelet reconstruction are the main operational steps.
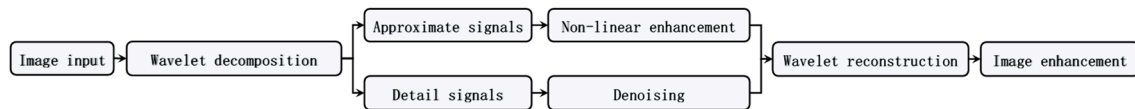
Figure 4. Image enhancement process.

When performing non-linear enhancement, histogram equalization can generally be used. Histogram equalization causes pixels with a high probability of grey values to be stretched, and in the process of stretching, the noise will be locally enhanced. After the wavelet transform, the noise information is concentrated in the detail signal and not in the approximate signal, so the histogram equalization for the approximate signal avoids the problem of amplifying the noise, which is the advantage of the wavelet transform.

Now the image enhancement is carried out on the target image. Firstly, db4 (Daubechies) is selected as the wavelet function to decompose the target image in 3 levels to obtain the approximate signal and the detail signal, then the histogram equalization is carried out on the approximate signal, and finally the approximate signal and the detail signal are reconstructed again to obtain the enhanced image. The enhancement result is shown in Figure 5(b). From the comparison of (a) and (b) in Figure 5, it can be seen that the image becomes brighter overall, and the overall contour of the bridge is more prominent, and the contrast becomes greater and the features become more obvious, providing favourable conditions for edge detection in the next step.
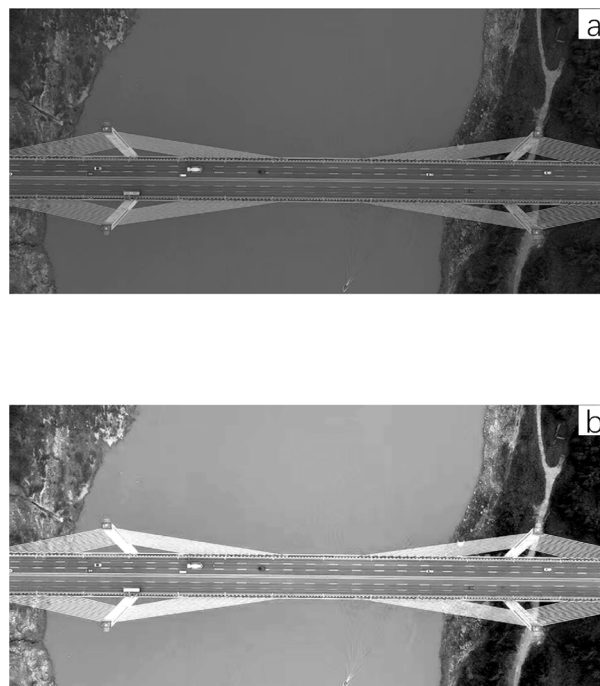
Figure 5. (**a**) Original image (**b**) Enhanced greyscale image.

3.2.2. Wavelet Transform Modulus Maxima Image

After pre-processing, the image is enhanced. The next step will be edge detection using the wavelet transform modulo maxima method. In the process, a convolution kernel, i.e., a wavelet function, is firstly generated from the image and the constructed 2D Gaussian function. Then the generated convolution kernel is used to convolve with the image to obtain the detail information, during which the continuous wavelet transform (CWT) is carried out, as shown in Figure 6. Based on the wavelet transform results, the modulus and amplitude angles of the image are calculated. The resulting magnitude angle is divided into four directions: horizontal, vertical, positive diagonal and negative diagonal. Each pixel point is examined in turn to determine whether it is an extreme value in the direction to which it belongs, and if so, the gradient value is recorded, if not, the gradient value is set to zero. The maximum value of the gradient is found and the value is normalized. Compare the normalized gradient value for each pixel and consider it a true edge when the gradient value is greater than the threshold, otherwise it is considered a pseudo-edge.
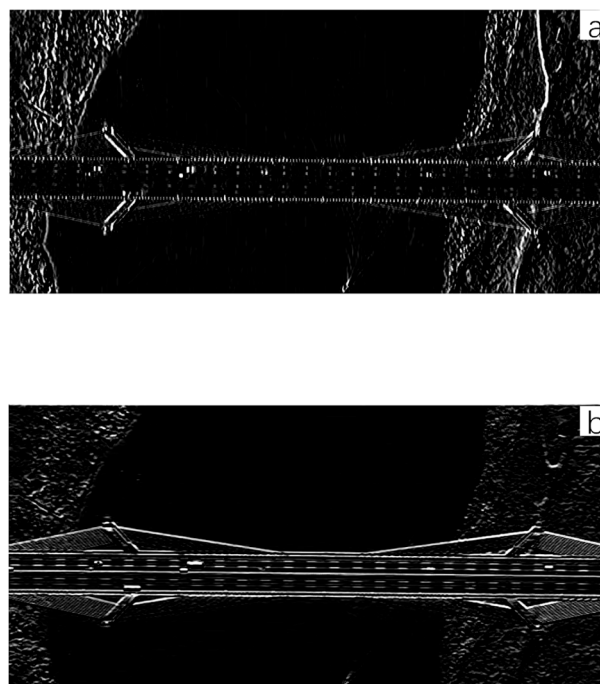
Figure 6. (**a**) Horizontal (**b**) Vertical.

The results of the transformation of the image in both horizontal and vertical directions after the continuous wavelet transform have been shown in Figure 6. Taking the pixel values of the corresponding pixels of the two images, finding the sum of the squares of the two and performing a square root calculation, the overall modulus image can be obtained as shown in Figure 7. The modulus image is further processed, i.e. the extreme value of the pixel in the current image is found and retained, and the result is shown in Figure 8.
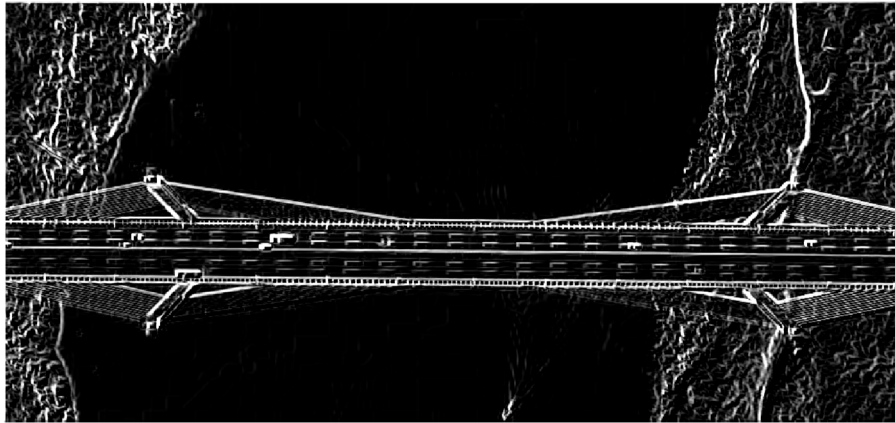
Figure 7. Modulus image.
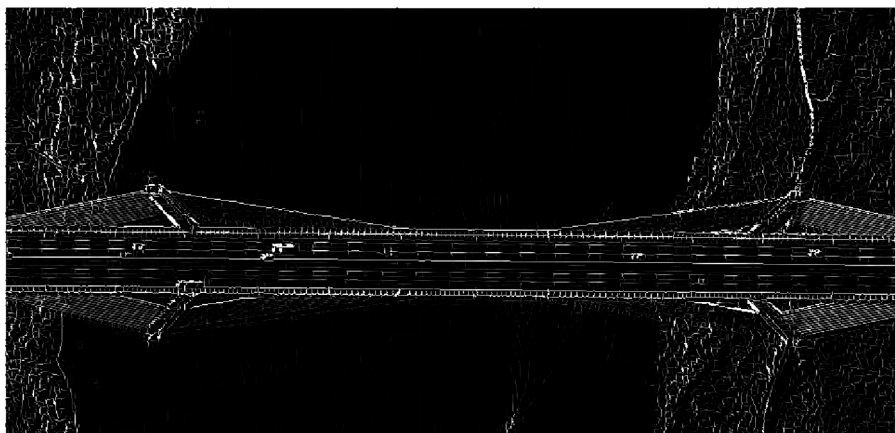
Figure 8. Modulus maximum image.

The advantages of OTSU in calculating global thresholds are simple, fast, and unaffected by brightness or contrast. The disadvantage is that it is sensitive to noise. It is not effective when the difference between the target and background areas is too large. With equal proportion of the target and background, Modulus maxima images are suitable for the OTSU method to calculate thresholds. The result is 0.06.

4. Results and Verification

The modulus image is processed according to the threshold value calculated by the OTSU above. Pixel points that are larger than the threshold are retained, and pixel points that are smaller than the threshold are deleted by setting their pixel values to zero. Then the final edge image is shown in Figure 9. As can be seen in the figure, the overall shape of the bridge is basically detected. The steel cables on one side of the bridge are also clear, and even the white lane lines on the bridge deck can be shown regularly. The ground on both sides of the image is clearly divided from the water in the middle, and areas with and without coverage of vegetation are basically distinguished.
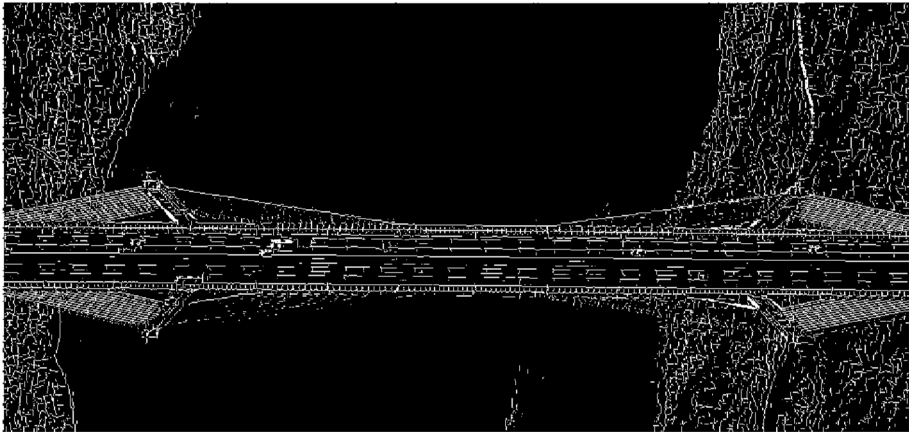
Figure 9. Edge detection results.

To further illustrate the advantages of the combined use of the wavelet mode maxima method and the OTSU, two other detection algorithms, the Canny algorithm and the Laplace algorithm, are used to detect the edge of the same image. Detection results are shown in Figure 10. The information entropy and SSIM are also used as indicators to compare these three detection results.
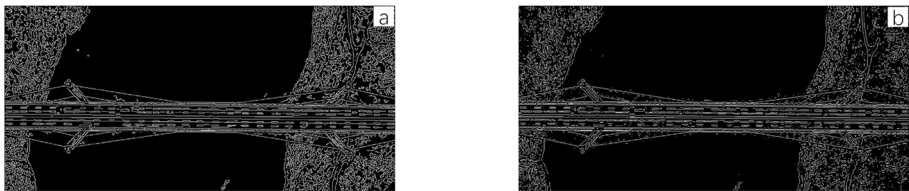
Figure 10. Detection results (**a**) Canny's algorithm (**b**) Laplace's algorithm.

Based on the test results, the information entropy and SSIM of the three were calculated respectively for a more intuitive comparison, as shown in Table 1.

Table 1. Information entropy and SSIM for the three algorithms.

Algorithms	Information entropy	SSIM
Wavelet transform modulus maxima method	2.5691	0.0023
Canny	2.1792	0.0022
Laplace	2.1792	0.0017

As can be seen from Table 1, the edge images detected by the wavelet transform modulus maxima method have greater information entropy and SSIM compared to the other two algorithms, indicating that the images are richer in detail information and providing more information and better detection results. the results demonstrate that the algorithm in this paper has relative advantages.

5. Analysis and Discussion

In the comparison above, the test area image was chosen with a relatively clear outline and homogeneous background in order to make the detection more effective. It is then worthwhile to illustrate whether the advantages of the wavelet transform modulo maxima method still exist in the case of a more complex background. Therefore, a bridge image with a more complex background is selected, as shown in Figure 11. Edge detection using wavelet transform modulo maxima is performed, and the results are shown in Figure 12.
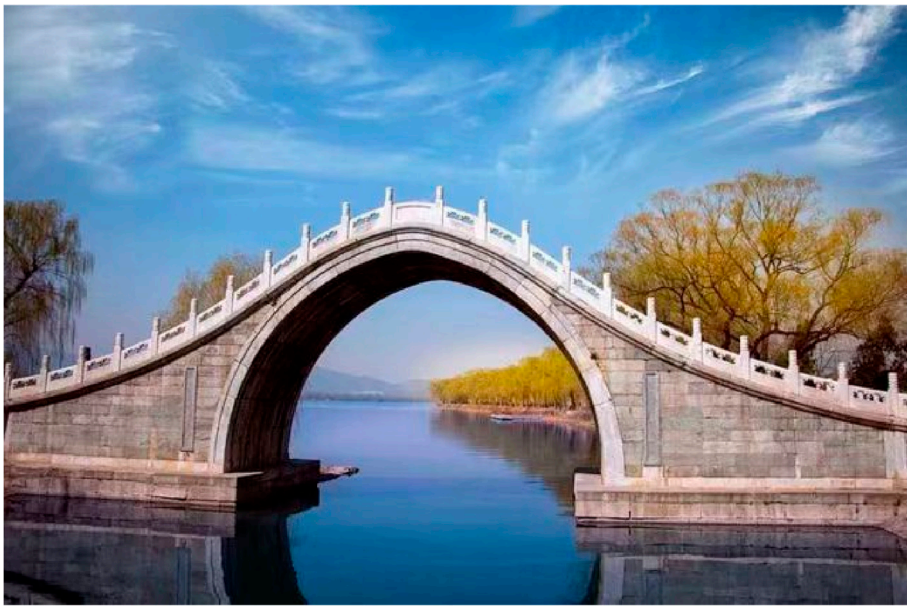
Figure 11. Original drawing of the bridge.
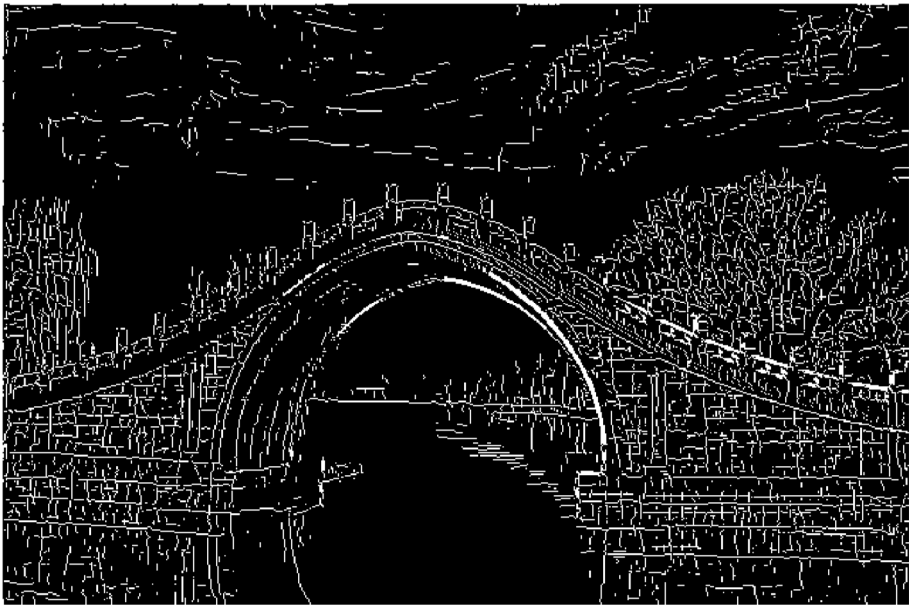
Figure 12. Wavelet transform modulus maxima detection results.

To better illustrate the detection effect, the Canny algorithm and Laplace algorithm are still used again for edge detection, and the results are shown in Figure 13. The information entropy and SSIM of each of the three are calculated for comparison, as shown in Table 2.
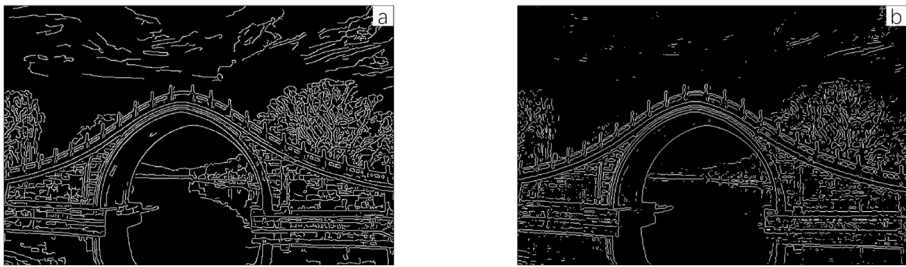
Figure 13. Detection results (**a**) Canny's algorithm (**b**) Laplace's algorithm.

Table 2. Information entropy and SSIM for the three algorithms.

Algorithms	Information entropy	SSIM
Wavelet transform modulus maxima method	2.5796	0.003
Canny	2.4989	0.003
Laplace	2.1128	0.0027

As can be seen from Table 2, the edge image detected by the wavelet transform modulus maxima method has the highest information entropy and SSIM, which means that its edge image quality is optimal. In other words, the advantage of the wavelet transform modulus maxima method is still present, and even more obvious, when edge detection is performed on images with more complex backgrounds.

Actually, the process of edge detection is to find out grey-scale changes in an image, separate high-frequency signals from the image and accurately locate the edge position. The modulus maxima of the wavelet transform mirrors the sharply changing signals, i.e., the edges of the image. With the flexible scale, the wavelet transform has excellent local qualities in both the time and frequency domains. A small scale can distinguish between real edges and noisy signals, while a large scale can remove the influence of noise. A reasonable choice of the scale can optimize the combined performance of both and improve the overall detection effect of the method in this paper.

Although this paper demonstrates some of the advantages of the combined application of wavelet transform modulus maxima and OTSU, some shortcomings were found during the processing. After edge detection, the overall outline of the image is very clear, but the detail is not yet fine enough. For example, the cars on the bridge are still not fully detected, the connectivity of the edges still needs to be improved and the visual effect is not yet friendly enough. All need to be further optimized to meet the needs of engineering applications. Moreover, the situation reflected by information entropy sometimes does not exactly match human intuition, which requires more fully comprehensive metrics to evaluate image quality, and this is where the author of this paper continue to work.

6. Conclusions

Edge detection of images can be used for routine maintenance for bridges. Especially when combined with drone photography technology, it not only saves a lot of manpower, but also allows people focus on the areas that are not easily observed. However, there are many images with complex background in the inspection process. Conventional inspection algorithms do not perform satisfactorily.

In this paper, the wavelet mode maxima method is used for edge detection of the test area images, and the OTSU method is used to determine the threshold value in the detection process, instead of the manual debugging of the threshold value, which improves the overall edge detection effect and the efficiency of edge detection at the same time. In addition to this, information entropy and SSIM is used as evaluation metrics when judging the detection effect, to evaluate detection results of different detection algorithms.

In summary, compared with the traditional method, the combination of wavelet transform mode maxima and OTSU has advantages in both efficiency and effectiveness in the process of image edge detection, and good prospects for engineering applications.

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