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Article

Novel Quick Value Assessment Procedure of the Bond Work Index

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Abstract: The bond index is an indicator of the grindability of the material and is widely used in the preparation of mineral raw materials and the cement industry. Paper offers new, abbreviated and simplified procedure to determine Bond work index that relies on first-order kinetics law and can be performed with any number of grinding cycles, depending on the desired accuracy of the required data. The parameters G and P_{80} of each grinding cycle are multiplied by the newly founded coefficients g_{IDT} and p_{IDT} to obtain values approximately equal to these parameters of the last grinding cycle when the equilibrium state is reached in the standard test. The paper presents comparative results obtained by standard Bond and new shortened procedure on individual samples of andesite, limestone, copper-ore and smelter slag and on composite samples of andesite from limestone and copper-ore with smelter slag in different mass ratios. As the number of grinding cycles increases, the precision of the shortened procedure increases, and the mean square error decrease 3.59%, 2.61% and 1.74% for two, three and four grinding cycles.

Keywords: grinding; kinetics; Bond index; simplified procedure; composite samples

1. Introduction

The resistance of the material to the crushing and grinding in a ball mill is represented by the comminution parameter called Bond work index. The determination of the energy required for comminution by means of a closed-cycle grinding test in a ball mill until a stable circular batch is established was devised by Bond et al. in Alice-Chalmers [1].

The Bond grinding test determines the work index and is performed by simulating dry grinding in a closed cycle in a Bond standardized laboratory mill with balls until a circulating batch of 250% is achieved [2]. Numerically, the work index is the energy (kWh) per one short ton of raw material required to reduced from theoretically infinite feed size to 80% of the raw material passing through a 100 μm square sieve [3–5]. To perform the test, a sample weight of approximately 10 kg is required, which is crushed to a size of 100% -3,327 mm. The Bond test involves a series of successive grinding cycles. The grinding is repeated until the circular batch in the last three grinding cycles is 250%. This is usually achieved with 7–10 grinding cycles [6]. The Bond working index is calculated using the parameters of the initial sample and the parameters from the last three grinding cycles and the formula:

$$W_i = 1,1 \cdot \frac{44,5}{P_c^{0,23} \cdot G^{0,82} \cdot \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}} \right)} \quad (1)$$

Wi – Bond work index (kWh/t);

Pc – the size of the opening of the control sieve (μm);

G – the mass of the newly created screen of the control sieve per revolution of the mill (g/rev);

F_{80} – the size of the opening of the sieve through which 80% of the feed passes before grinding (μm);
 P_{80} – the size of the sieve opening through which 80% of the comparative sieve from the last grinding cycle passes (μm).

The description of the procedure shows that its execution is complicated, that it takes a lot of time and that it is prone to errors due to its complexity. Because of all this, a large number of scientists have tried with more or less success to shorten and simplify this procedure [7]. Crushing and grinding is a very important aspect of mineral processing, cement production and other branches of industry where material grinding is applied, therefore comminution process has been subjected to many investigations through decades and still continues to be challenging.

In order to facilitate the process of determining the comminution parameter some of the scientists provided a procedure that enable the usage of any mill, not only Bond mill [8–10]. Some other scientist proposed method that is using specially designed mills smaller than the Bond mill and, therefore, the required amount of the sample is much smaller [11–13]. The other scientists focus their investigations on mathematical algorithms [14–16] and mathematical correlations[17] that helped them to calculate Bond work index upon the data collected after the first or second grinding cycle as an input parameter. Grinding kinetic model was basis for many investigations and some resulted in promising models [18–22].

This paper provides novel fast procedure for determining the Bond work index and is based on many years of experience and observations of the authors. Procedure relies on the first-order kinetics present in the Bond mill with balls when crushing raw materials [23,24]. The presented results obtained by the standard Bond procedure and the fast procedure aim to compare the data obtained by these two procedures and show their differences.

The advantage of this novel procedure in relation to the others is that it is made on a heterogeneous mixture of raw materials with large differences in the grindability of the components, and the results obtained are of high satisfactory precision.

2. Objectives of the Investigation

Due to the complexity and length of performing this procedure for determining the Bond work index, process errors and incorrect results are possible. Because of all the above, attempts were made to facilitate procedure and make it shorter.

Also, there is a problem of determining the grindability of inhomogeneous composite materials. In the mineral industry, it is important to understand how milling will affect mixtures with different grindability of the ores that originate from different parts of the deposit or from two deposits.

The aim of the research presented in this paper is to give a theoretical and practical contribution to the knowledge of the comminution process in Bond's laboratory ball mill.

The subject of this research is to determine the grindability of inhomogeneous composite materials, test the abbreviated procedure for determining Bond's work index using comparative sieves of $105\mu\text{m}$ and $150\mu\text{m}$, and finally determine and demonstrate its reliability.

Research includes:

- Monitoring of changes in the ratio of G_e/G_{ie} parameters during different grinding cycles while performing standard Bond procedure on inhomogeneous composite materials at different mass fractions.

- Monitoring of changes in the ratio of P_{80}/P_{i80} parameters during different grinding cycles while performing standard Bond procedure on inhomogeneous composite materials with different mass fractions.

- Testing the abbreviated procedure for determining Bond's work index on inhomogeneous composite materials at different mass fractions and determining its reliability.

3. Theoretical Basis of Novel Procedure

The kinetics of grinding the raw material in the Bond mill with balls takes place according to the law of first order kinetics [25]:

$$R = R_0 \cdot e^{-kt}, \quad (2)$$

Where are:

R – reflection of the comparative sieve at the moment (t),
 R₀ – screening of the comparative sieve at the beginning of grinding (t=0),
 k – grinding rate constant,
 t – grinding time.

For each grinding cycle of the Bond process, the grinding rate constant (k) can be determined according to the formula:

$$k = \frac{\ln R_0 - \ln R}{t} = \frac{n(\ln R_0 - \ln R)}{N}, \quad (3)$$

Where are:

N – the total number of revolutions of the mill,
 n – number of mill revolutions per minute.

As the ordinal number of grinding cycles of the standard Bond procedure for determining the grindability of the raw material increases, so does the grinding rate constant (k). The reason for this is that the size of the circular batch decreases with each subsequent grinding cycle and it is easier to get the grinding product of the desired size, so k₂ < k₃ < k₄ < k₅ < ... (speed constants for the second, third, fourth, fifth... grinding cycle).

The grinding time required to obtain a circular batch of 250% (ie the number of revolutions of the mill) can be calculated using the known value of k.

When the equilibrium state is reached, ie a circular batch of 250%:

$$R_0 = \frac{2,5}{3,5} \cdot M + \frac{M}{3,5} \cdot X \quad \text{and} \quad R = \frac{2,5}{3,5} \cdot M \quad (4)$$

Where are:

M – the mass of the starting sample (700cm³),
 X – class content +P_c in the initial sample (in parts of the unit).

$$t = \frac{1}{k} \left[\ln \left(\frac{2,5}{3,5} \cdot 100 + \frac{X}{3,5} \cdot 100 \right) - \ln \left(\frac{2,5}{3,5} \cdot 100 \right) \right], \quad (5)$$

$$N = \frac{n}{k} \left[\ln \left(\frac{2,5}{3,5} \cdot 100 + \frac{X}{3,5} \cdot 100 \right) - \ln \left(\frac{2,5}{3,5} \cdot 100 \right) \right] \quad (6)$$

In this way, values can be obtained tie and N_{ie} for each grinding cycle (t_{2e} and N_{2e} for the second cycle, t_{3e} and N_{3e} for the third cycle, t_{4e} and N_{4e} for the fourth cycle...).

When the equilibrium is reached, the circular batch of 250%, the newly created screen of the comparative sieve is $Z = \frac{M}{3,5} - \frac{M}{3,5} \cdot (1 - X) = \frac{M}{3,5} \cdot X$.

The value of G (g / rev) can be calculated using the value of N:

$$G = \frac{Z}{N} = \frac{\frac{M}{3,5} \cdot X}{N} \quad (7)$$

This is how values can be calculated G_{ie} for each grinding cycle (G_{2e}, G_{3e}, G_{4e},... for the second, third, fourth ... grinding cycle).

By performing a vast number of tests on different raw materials by the standard Bond procedure, it was observed that the ratios of the obtained calculated values G_{ie} and the values for the last grinding cycle G (250% circular batch) are approximately the same Figure 1 presents the relative mean values of the parameter G_{ie} in relation to the parameter G_e of more than 30 performed Bond tests.

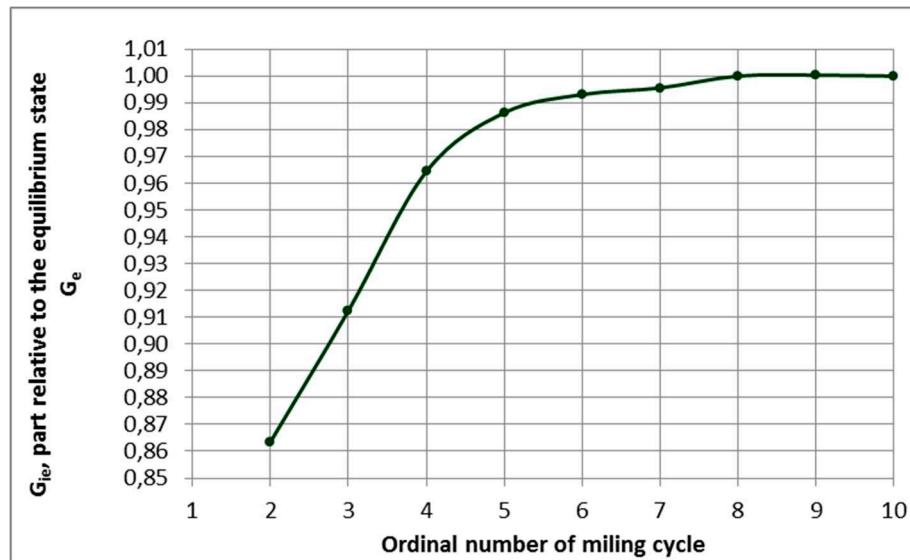


Figure 1. Ratios of the obtained calculated values G_{ie} and the values for the last grinding cycle G_e (250% circular batch)

For two, three and four grinding cycles, the mean values of these ratios are

$$g_{2DT} = \frac{G}{G_{2e}} \approx \text{const.} = 1,158 \quad g_{3DT} = \frac{G}{G_{3e}} \approx \text{const.} = 1,096 \quad g_{4DT} = \frac{G}{G_{4e}} \approx \text{const.} = 1,037 \quad (8)$$

It follows from the above that it is possible to calculate the approximate value of parameter G of the last grinding cycle of the standard Bond procedure as a product of the parameter G_{ie} and the corresponding constants for a certain grinding cycle g_{iDT} [26].

Simultaneously, by performing a large number of tests on different raw materials by the standard Bond procedure, it was noticed that the ratios of the parameters P_{i80} of a certain grinding cycle and the parameters P_{80} for the last grinding cycle (250% circular batch) are approximately the same. Figure 2 presents the relative mean values of the parameter P_{i80} in relation to the parameter P_{80} of more than 30 performed Bond tests.

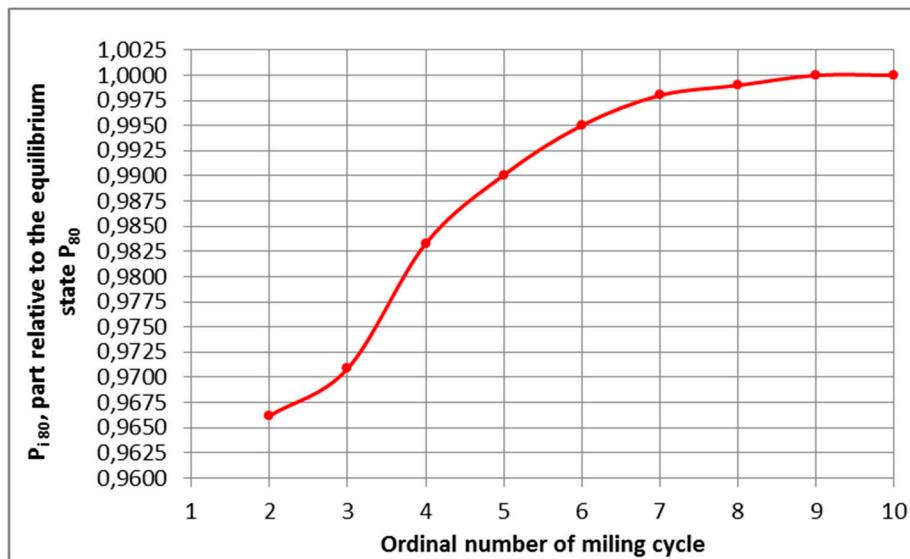


Figure 2. Ratios of the P_{i80} obtained calculated values and the P_{80} values for the last grinding cycle (250% circular batch)

For two, three and four grinding cycles, the mean values of these ratios are:

$$p_{2DT} = \frac{P_{80}}{P_{280}} \approx \text{const.} = 1,035 \quad p_{3DT} = \frac{P_{80}}{P_{380}} \approx \text{const.} = 1,030 \quad p_{4DT} = \frac{P_{80}}{P_{480}} \approx \text{const.} = 1,017 \quad (9)$$

It follows from the above that it is possible to calculate the approximate value of parameter P_{80} of the last grinding cycle of the standard Bond procedure as a product of parameter P_{i80} and the corresponding constants for a certain grinding cycle p_{iDT} ([26]).

4. Experimental

4.1. Materials

The experiments were performed on four raw different materials: andesite, limestone ore, copper ore and smelter slag.

Andesite – Density $\rho = 2,77 \text{ g/cm}^3$. The chemical composition of the andesite sample is given in Table 1.

Table 1. Chemical composition of andesite

Chemical element	Cu	S	Cu _{ox}	SiO ₂	Al ₂ O ₃	CaO
Content, %	0.184	3.51	0.040	64.130	17.430	0.420
Chemical element	MgO	Fe ₂ O ₃	K ₂ O	Na ₂ O	TiO ₂	Gž
Content, %	1.250	6.000	3.990	0.849	0.50	5.15
Chemical element	Zn	Cr	Pb	Cd	Mn	Ni
Content, ppm	30.49	42.49	20.00	0.10	93.50	44.90
Chemical element	Mo	/	/	/	/	/
Content, ppm	47.99	/	/	/	/	/

Limestone ore – Density $\rho = 2.72 \text{ g/cm}^3$. The insoluble part of the limestone sample in HCl was 0.43%, CaCO₃ content was 99.57%.

Copper ore – Density $\rho = 2.95 \text{ g/cm}^3$. The chemical composition of the copper ore sample is shown in Table 2.

Table 2. Chemical composition of andesite

Chemical element	Cu _{uk.}	Cu _{ox.}	Cu _{sulf.}	S	Al ₂ O ₃	SiO ₂
Content, %	0.440	0.017	0.423	4.88	17.10	57.52
Chemical element	Fe	Fe ₂ O ₃	CaO	/	/	/
Content, %	3.21	<0.03	7.22	/	/	/
Chemical element	Au	Ag	Mo	/	/	/
Content, g/t	30.49	42.49	20.00	/	/	/

Smelter slag - Density $\rho = 3.52 \text{ g/cm}^3$. The chemical composition of the smelter slag sample is given in Table 3.

Table 3. Chemical composition of smelter slag

Chemical element	Cu	Cu _{sulf}	Cu _{ox}	S
Content, %	0.881	0.761	0.120	0.720

Experiments were performed with andesite and limestone ore on pure samples and on their mixtures in different ratios. Andesite and limestone ore have large differences in grinding, ie differences in the size of the Bond work index. Experiments were performed on a mixture of such raw materials in order to better confirm the validity and accuracy of the fast procedure for estimating the size of the Bond work index. When performing the Bond process on mixtures of raw materials with different grindings, the content of harder raw materials in a circular batch increase with each subsequent grinding cycle until equilibrium is reached. So, such systems can best show the precision of a shortened procedure, because they do not perform grinding cycles until an equilibrium state is reached when the composition of the circular batch no longer changes.

Experiments were also performed with copper ore and smelted slag on pure samples and their mixtures in different ratios. In these samples, we also have large differences in the grindability of the components of the mixture, where the accuracy of the fast procedure on a very heterogeneous raw material can be confirmed. These raw materials were chosen also because it is a real example that we find in plant when grinding copper ore and smelting slag for copper flotation.

Andesite and limestone samples were grinded to a size of 100% -3.327 mm. Composite samples of andesite and limestone in mass ratios were made from such crushed samples: limestone:andesite=25:75, limestone:andesite=50:50, limestone:andesite=75:25. Experiments were performed on composite samples and on samples of pure limestone and pure andesite.

Copper ore and slag samples were grinded to a size of 100% -3.327 mm. Composite samples of copper ore and slag in mass ratios were made from such crushed samples: slag:copper ore=25:75, slag:copper ore=50:50, slag:copper ore=75:25. Experiments were performed on composite samples and on samples of pure slag and pure copper ore.

The characteristics of the Bond mill and the experimental conditions for performing the Bond test are given in Table 4.

Table 4. Bond mill characteristics and experimental grinding conditions

Mill diameter, D _m	30.48 cm
The length of the mill, L _m	30.48 cm
Mill lining geometry	Smooth
The number of revolutions of the mill per minute, n	70 min ⁻¹
Mass of balls, M _b	21.125 kg
V _{sample}	700 cm ³
Type of grinding	Dry

On all the above samples, the determination of the Bond working index is according to the standard Bond test with comparative sieves of 105 μ m and 150 μ m. Granulometric analysis of comparative sieve screening was performed after each grinding cycle and the parameter P₁₈₀ was determined.

4.1. Method - Procedure for Performing a Quick Procedure for Assessing the Value of a Bond Work Index

The fast procedure for estimating the value of the Bond work index [26] with two grinding cycles is done in the same way as the first two grindings of the standard Bond test and consists of the following operations:

- Grinding of the sample to a size of 100 % -3.327 mm;
- Determination of the granulometric composition of the initial sample and the parameter F₈₀ (μ m) and the participation of a class larger than the opening of the comparative sieve X (in parts of the unit);

- A volume sample is taken 700 cm³, its mass M (g) is determined, it is loaded into a Bond mill with balls and ground for an arbitrary number of revolutions of the mill (N₁ = 50, 100 or 150 revolutions);
- After grinding, the sample is sieved on a comparative sieve and the mass of sieves D_u (g) and sieves R_n (g) is determined. Sieve D consists of the mass of sieve D_u introduced with the inlet and the mass of newly created sieve in mill D_n:

$$D = D_u + D_n, \text{ g} \quad (10)$$

- The mass of the newly created sieve is calculated D_n:

$$D_n = D - D_u, \text{ g} \quad (11)$$

- In the first experiment is:

$$D_u = M \cdot (1 - X) D_n = D - D_u, \text{ g} \quad (12)$$

- In the following experiments is:

$$D_u = D_{(n-1)} \cdot (1 - X), \text{ g} \quad (13)$$

Where is:

D_(n-1) – the sieve mass from the previous experiment, g.

- The mass of the newly created sieve is calculated per one revolution of the mill:

$$G = \frac{D_n}{N}, \text{ g/rev.} \quad (14)$$

Where is:

N – the number of revolutions in the experiment in question.

- The mill speed is calculated for the following grinding experiment:

$$N_n = \frac{\frac{M}{3.5} D_{(n-1)} (1 - X)}{G}, \text{ rev.} \quad (15)$$

- A fresh sample is added to the screening of the comparative sieve, the mass of which is equal to the mass of the sieve from the previous experiment D_(n-1). The entrance thus formed is inserted into the mill and grinds N_n rev.;
- After grinding, the sample is sown on a comparative sieve and the reflection is measured R (g);
- The constant k is calculated using formula (3):

$$k = \frac{n \cdot (ln R_0 - ln R)}{N} = \frac{n \cdot \left[ln \left(\frac{R_{(n-1)}}{M} \cdot 100 + \frac{D_{(n-1)}}{M} \cdot X \cdot 100 \right) - ln \left(\frac{R}{M} \cdot 100 \right) \right]}{N}. \quad (16)$$

- Using the constant k and formula (6), the required number of revolutions N is calculated in case the quantity of raw material is ground at the same speed as in the second grinding as when the circular batch is 250 %;
- The parameter G_{2e} (g / rev.) is calculated using formula (7) and the value of N. The value of G_{2e} is multiplied by the constant g_{2DT} = 1.158 and the value of G_e is obtained, which is close to the value of G in the last grinding cycle of the standard Bond test;
- On the sieving of the second grinding cycle, granulometric analysis is performed and parameter P_{2 80} is read from the graph. The read value is multiplied by the constant p_{2DT} = 1.035. The obtained P_{e80} result is close to the value of the P₈₀ sieving parameter of the last grinding cycle of the standard Bond test;
- Using the formula (1) and the calculated values of the parameters G_e and P_{e80}, an approximate value of W_i (kWh / t) is obtained.

In the fast process with three or four grinding cycles, the appropriate number of grinding cycles of the standard Bond process is performed. The parameters G_e and P_{e80} are calculated as when two fast mills are made and multiplied by the corresponding constants g_{iDT} and p_{iDT} for a given number of grinding cycles. The Bond index is also calculated using formula (1).

5. Results and Discussion

Tables 5, 6 and 7 show the comparative results, obtained by the standard Bond procedure and the fast procedure with two, three and four grinding cycles, parameters G and P_{80} and the Bond work index.

Table 5. Comparison of parameter G obtained by standard Bond procedure and fast procedure with two, three and four grinding cycles

Sample	Sieve, μm	G, g/rev	2 Grinding		3 Grinding		4 Grinding	
			G_{e2} , g/rev	Differ., %	G_{e3} , g/rev	Differ., %	G_{e4} , g/rev	Differ., %
Limestone : andesite	105	1.190	1.223	+2.77	1.261	+3.28	1.234	+1.15
	150	1.540	1.645	+7.14	1.580	+2.60	1.556	+1.01
Limestone : andesite	105	1.270	1.259	-0.79	1.289	+1.57	1.276	+0.43
	150	1.660	1.697	+2.14	1.678	+1.20	1.649	-0.67
Limestone : andesite	105	1.480	1.436	-2.70	1.442	-2.70	1.441	-2.61
	150	1.810	1.893	+4.42	1.864	+2.76	1.804	-0.31
Limestone : andesite	105	1.600	1.538	-3.75	1.529	-4.38	1.556	-2.78
	150	1.960	1.908	-2.55	1.958	0.00	1.898	-3.18
Limestone : andesite	105	1.810	1.785	-1.66	1.733	-4.42	1.742	-3.75
	150	2.160	2.029	-6.02	2.112	-2.31	2.126	-1.58
Slag : Cu ore 100 : 0	150	1.85	2.00	+8.11	1.92	+3.78	1.89	+2.16
	150	1.97	2.11	+7.11	2.03	+3.05	1.97	0.00
Slag : Cu ore 75 : 25	150	2.00	2.17	+8.50	2.13	+6.50	2.00	0.00
	150	2.11	2.13	+0.95	2.16	+2.37	2.19	+3.79
Slag : Cu ore 100 : 0	150	2.13	2.19	+2.82	2.08	-2.35	2.10	-1.41
Maximum error	/		8.11		6.50		3.79	
Mean error	/		4.09		2.88		1.65	

When performing the fast procedure with two, three and four grinds and the standard Bond procedure for parameter G, it is noticed that the largest error decreases from 8.11 % to 3.79 %, and the average error value decreases from 4.09 % to 1.65 %. From this it can be seen that as the number of grinding cycles increases, the accuracy of the estimated parameter G also increases (Table 5).

Table 6. Comparison of parameter P_{80} obtained by standard Bond method and fast process with two, three and four grinding cycles

Sample	Sieve, μm	P_{80} , μm	2 Grinding		3 Grinding		4 Grinding	
			$P_{80\text{ e2}}$, μm	Differ., %	$P_{80\text{ e3}}$, μm	Differ., %	$P_{80\text{ e4}}$, μm	Differ., %
Limestone : andesite	105	86.00	87.98	+2.30	87.55	+1.80	87.46	+1.70
	0 : 100	150	121.00	+0.08	121.54	+0.45	121.02	+0.02
Limestone : andesite	105	86.00	84.87	-1.31	87.55	+1.80	86.45	+0.52
	25 : 75	150	121.00	+0.08	120.51	-0.40	122.04	+0.86
Limestone : andesite	105	87.00	87.98	+1.13	90.64	+4.18	88.48	+1.70
	50 : 50	150	122.00	+0.96	125.66	+3.00	124.07	+1.70
Limestone : andesite	105	89.00	87.98	-1.15	90.64	+1.84	90.51	+1.70
	75 : 25	150	124.00	-1.51	123.60	-0.32	124.07	+0.06
Limestone : andesite	105	91.00	94.19	+3.51	90.64	-0.40	88.48	-2.77
	100 : 0	150	124.00	-5.68	120.51	-2.81	123.06	-0.76
Slag : Cu ore 100 : 0	150	127.00	126.27	-0.57	126.69	0.00	129.16	+1.70
	Slag : Cu ore 75 : 25	150	122.00	125.00	+2.65	126.00	+3.00	124.00
Slag : Cu ore 50 : 50	150	122.00	123.00	+0.95	124.00	+1.31	123.00	+0.87
	Slag : Cu ore 25 : 75	150	120.00	116.00	-3.40	118.00	-1.29	121.00
Slag : Cu ore 100 : 0	150	118.00	115.00	-2.60	115.00	-2.60	115.00	-2.60
Maximum error		/		5.68		4.18		2.77
Mean error		/		1.86		1.68		1.30

When performing the fast procedure with two, three and four grindings and the standard Bond procedure for the P_{80} parameter, the largest errors ranged from 5.68 % to 2.77 %, and the mean error value ranged from 1.86 % to 1.30 % (Table 6).

Table 7. Comparison of Bond work index W_i obtained by standard Bond procedure and fast procedure with two, three and four grinding cycles

Sample	Sieve, μm	W_i , kWh/t	2 Grinding		3 Grinding		4 Grinding	
			W_{ie2} , kWh/t	Differ., %	W_{ie3} , kWh/t	Differ., %	W_{ie4} , kWh/t	Differ., %
Limestone : andesite	105	16.93	17.15	+1.26	16.68	-1.53	16.96	+0.14
	0 : 100	150	16.01	-5.24	15.72	-1.76	15.88	-0.81

Limestone : andesite	105	16.41	16.38	-0.15	16.39	-0.09	16.40	-0.02
25 : 75	150	15.13	14.87	-1.74	14.96	-1.14	15.30	+1.14
Limestone : andesite	105	14.60	15.07	+3.23	15.31	+4.88	15.08	+3.30
50 : 50	150	14.26	13.83	-3.00	14.20	-0.39	14.46	+1.41
Limestone : andesite	105	13.91	14.27	+2.55	14.61	+5.00	14.40	+3.46
75 : 25	150	13.59	13.75	+1.18	13.57	-0.15	13.96	+2.73
Limestone : andesite	105	12.77	13.21	+3.45	13.20	+3.37	12.94	+1.32
100 : 0	150	12.63	12.77	+1.12	12.62	-0.13	12.73	+0.78
Slag : Cu ore	150	15.02	14.01	-6.73	14.55	-3.14	14.95	-0.50
100 : 0								
Slag : Cu ore	150	13.71	13.17	-3.93	13.63	-0.58	13.89	+1.34
75 : 25								
Slag : Cu ore	150	13.40	12.62	-5.81	12.83	-4.27	13.47	+0.50
50 : 50								
Slag : Cu ore	150	12.58	12.19	-3.15	12.25	-2.63	12.27	-2.51
25 : 75								
Slag : Cu ore	150	12.22	11.73	-3.99	12.26	+0.34	12.15	-0.56
100 : 0								
Maximum error		/		6.73		5.00		3.46
Mean error		/		3.10		1.96		1.37
Root mean square error		/		3.59		2.61		1.74

When performing the fast procedure with two, three and four grinds and the standard Bond procedure at the value of the Bond work index, the largest errors ranged from 6.73 % to 3.10 %, and the mean error value ranged from 3.10 % to 1.37 % (Table 7).

The reliability of the fast procedure for estimating the value of the Bond working index can best be assessed by comparing the values of G and Ge shown in Table 5.

In previous years, since Bond's procedure for determining the ore grindability has been in use, there have been many attempts and suggestions on how to simplify and shorten this long and demanding procedure. Figure 1 compares some of the shortened/simplified procedures for obtaining the Bond index and the mean errors obtained by their application in relation to the actual value of W_i . The mean square errors in these procedures range from 0.41 % to 24.1 %. When performing the standard Bond procedure, due to the complexity of the procedure, an error of up to 5 % is considered frequent and tolerable (when performing two standard Bond's test on the same sample). For this reason, shortened and simplified procedures that give the mean square error of less than 5 % compared to the original Bond procedure can be considered absolutely acceptable. Procedures with an error above 5 % will be excluded from further discussion. All the presented procedures can be divided into two groups: the first group includes procedures that simulate the standard Bond test with a simplified procedure, and the second group includes procedures that determine the grindability of raw materials using a reference sample of known grindability and a certain mathematical simulation. The procedure proposed by Horst and Baassarear (1977) [9] relies on the grindability of the referent material. The advantage of this procedure is that can be performed on any laboratory mill with balls and demand a small amount of sample (1kg). However, the time required to perform this procedure is quite long and is similar to the time required to perform the standard

Bond procedure. Karra (1981) [15] proposed a mathematical algorithm that can estimate the W_i based on data from two grinding cycles using the standard Bond method. This procedure taking into account that the grindability of a circular batch is smaller than that of the initial sample. Mular and Jergensen (1982) [10] proposed the so-called Anaconda method, which can be performed on any laboratory mill. This method requires a reference raw material whose value of W_i is known, by means of which the calibration factor for a given mill is determined by a series of grinding. This method is performed quickly and gives good results, but extensive work is required to determine the calibration factor of the mill. Nematollahi (1994) [11] and Menéndez-Aguado et al. (2005) [12] proposed procedures that are performed in mills smaller than Bond's. A much smaller amount of ore is required to perform both of these processes, and the processes are performed identically to the Bond process. Saeidi et al. (2013) [13] proposed a procedure using a mill constructed by Nematollahi (1994). Due to the large deviations of the parameters P_{80} and G , he defined correction formulas for them. The relative error of the Bond working index was 0.41 %. Although Saeidi et al. obtained results almost identical results with the original Bond procedure, the lack of this procedure as well as the Nematollahi (1994) and Menéndez-Aguado et al. (2005) is the special construction of the mill that is used. Lewis et al. (2000) [16] proposed a complex mathematical algorithm that simulates the standard Bond test. The data obtained from the first grinding cycle of the standard Bond procedure are used for the input parameters of the algorithm. Aksani and Sönmez (2000) [18] developed a computer simulation of the standard Bond test based on the cumulative kinetic model. The model is based only on the correlation of grinding speed and grain size, for easier interpolation. The sample is ground in a Bond mill for 0.5, 1, 2 and 4 minutes, after each cycle G and P_{80} is determined. Based on these data, a simulation model is formed. Since they developed their own program for the simulation it's not available for general use. Ford and Sithole (2015) [19] have developed two methods for estimating the Bond working index that are performed in the Bond mill. The first method is performed with only one grinding cycle for 0.5, 1, 2 and 4 minutes. Parameter G and P_{80} is determined for each time interval. Further, with the results obtained in this way, a mathematical simulation is performed, which is used to obtain W_i . However, one-grinding cycle method provide poor results with mean square error of 11 %. The second method is performed with three grinding cycles identical to the standard Bond procedure. After the third cycle, the parameters G and P_{80} are determined and W_i is calculated using the mathematical formula they proposed. The three-grind procedure gives more precise results. Magdalinović (1989) [22] proposed a procedure that is performed in a Bond mill with balls on a standard sample and which is based on the law of first order kinetics with only two grinding cycles. In the first grinding cycle, the sample is ground for any number of revolutions (50, 100 or 150) and after grinding, the amount of sieving of the comparative sieve is determined. Based on the initial data and data after grinding, the grinding speed constant (k) is determined. The second grinding cycle is performed for the number of mill revolutions (N) which is calculated using the constant k to obtain a circular batch of 250%. After the second grinding cycle, the parameters G and P_{80} are determined, which are used to determine the value of the Bond working index using a standard formula. Magdalinović (2003) [25] proposed a method with three grinding cycles. The method is identical to the two-cycle grinding method with the addition of one cycle. The procedure with three grinding cycles gives more precise results. Gharehgheshlagh (2015) [20] developed a method based on the kinetics of grinding in a Bond mill. This method grinds a standard sample of raw material for 0.33, 1, 2, 4 and 8 minutes. After each time interval, the parameters G and P_{80} are determined, and on the basis of these data, the grinding kinetics and the value of the Bond working index are determined. Since this procedure demands five grinding cycles, the time frame for this procedure is not much shortened compared to the original one.

The results presented in this paper obtained by the proposed fast and simplified procedure in comparison with the results of other procedures have a satisfactory accuracy. The fast process with four grinding cycles is one of the four most accurate results shown in the graph. However, it's shorter compared to the procedures proposed by Horst and Bassarear (1977) and Gharehgheshlagh (2015), and does not require specially designed mill like Saeidi et al. This implies that results presented in

this paper shows highest accuracy and has been significantly simplified compared to the original Bond's procedure.

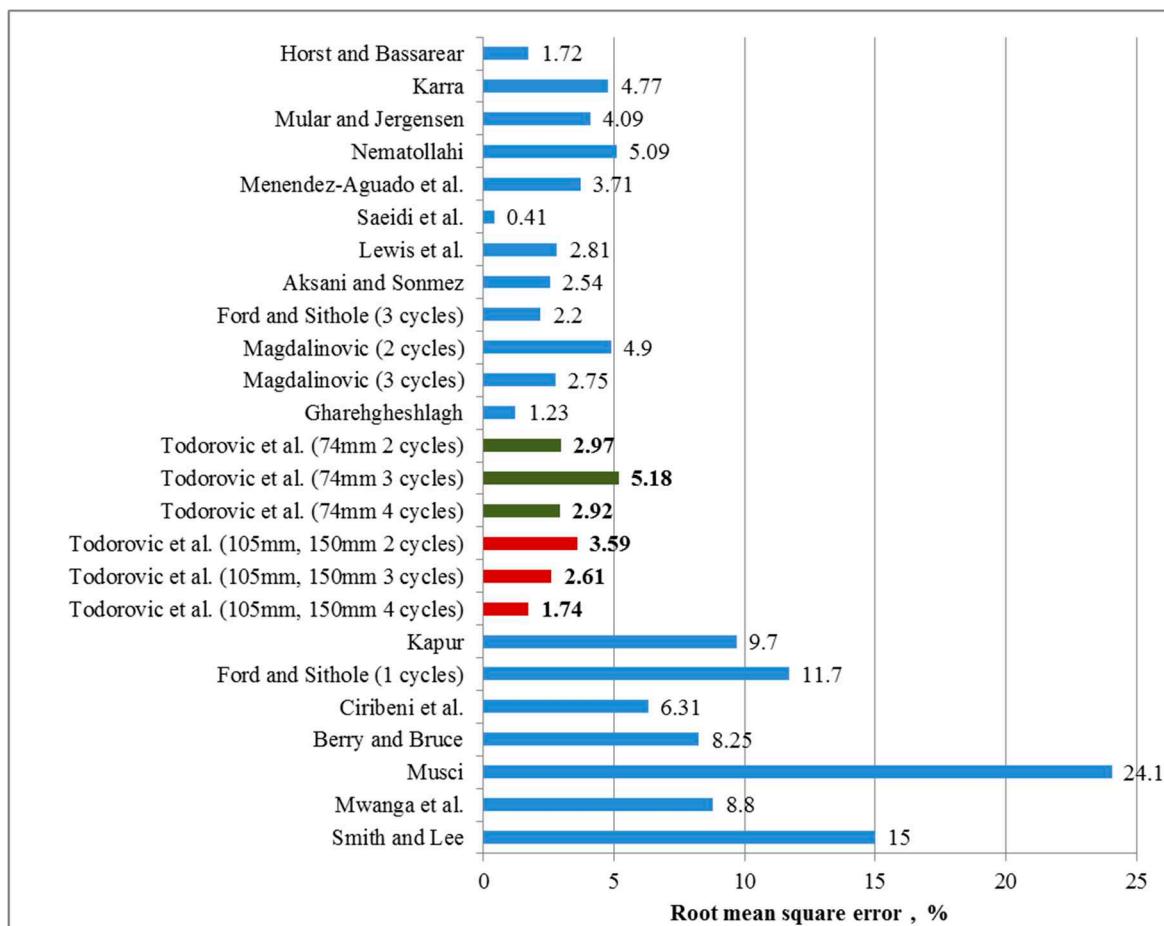


Figure 3. Summary of relative errors of alternative procedures [27].

5. Conclusion

Crushing and grinding is a very important aspect of mineral processing, cement production and other branches of industry where material grinding is applied, therefore comminution process has been subjected to many investigations through decades and still continues to be challenging. The raw material is ground in a laboratory mill with balls according to the law of first order kinetics. Knowing this, it is possible to calculate the approximate values of the parameter G of the last grinding cycle on the basis of the data obtained by any grinding cycle of the standard Bond test and the corresponding g_{iDT} constants. Using the P_{80} screening parameter of the comparative sieve of any standard test grinding cycle and the corresponding p_{iDT} constants, the approximate P_{80} screening values of the last standard test grinding cycle can be calculated. Using the obtained approximate values of parameters G and P_{80} , the approximate value of the Bond working index W_i can be calculated.

The largest errors of the obtained values of the Bond working index W_i by the fast procedure for two, three and four grinding cycles were 6.73 %, 5.00 % and 3.46 %, respectively. The mean square errors of the obtained values of the Bond working index W_i by the fast procedure for two, three and four grinding cycles were 3.59 %, 2.61 % and 1.74%, respectively. The accuracy of the obtained data increases with the number of performed grinding cycles.

The values of the Bond working index obtained by the fast and simplified procedure described in this paper on composite samples of limestone and andesite at their different mass fractions and on composite samples of smelting slag and copper ore, with comparative sieves of 100 and 150 μm , gave very good results. The verification of this procedure should be performed on more different raw materials and if the accuracy of the results obtained so far is confirmed, this procedure could be

practically applied in cases when there is a small amount of test sample or when there is limited time to determine the grindability of raw materials.

The reliability of the parameter G , which is obtained by this method and which significantly affects the value of W_i , can be checked using data from already performed experiments by the standard Bond procedure on different raw materials and appropriate mathematical formulas.

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