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## Article

# A Quantum-Classical Model of Brain Dynamics

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**Abstract:** The study of the human psyche has elucidated a bipartite structure of cognition reflecting the quantum-classical nature of any process that generates knowledge and learning governed by brain activity. Acknowledging the importance of such a finding for modelization, we posit an approach to study brain by means of the quantum-classical dynamics of a Mixed Weyl symbol. The Mixed Weyl symbol is used to describe brain processes at the microscopic level and, when averaged over an appropriate ensemble, provides a link to the results of measurements made at the mesoscopic scale. Within this approach, quantum variables (such as, for example, nuclear and electron spins, dipole momenta of particles or molecules, tunneling degrees of freedom, etc) may be represented by spinors while the electromagnetic fields and phonon modes involved in the processes are treated either classically or semi-classically, by also considering quantum zero-point fluctuations. Zero-point quantum effects can be incorporated into numerical simulations by controlling the temperature of each field mode *via* coupling to a dedicated Nosé-Hoover chain thermostat. The temperature of each thermostat is chosen in order to reproduce quantum statistics in the canonical ensemble. In this first paper, we introduce a quantum-classical model of brain dynamics, clarifying its mathematical structure and focusing the discussion on its predictive value. Analytical consequences of the model are not reported in this paper, since they are left for future work. Our treatment incorporates compatible features of three well-known quantum approaches to brain dynamics - namely the electromagnetic field theory approach, the orchestrated objective reduction theory, and the dissipative quantum model of the brain - and hints at convincing arguments that sustain the existence of quantum-classical processes in the brain activity. All three models are reviewed.

**Keywords:** quantum-classical dynamics; quantum brain; open quantum systems; neuroscience; electromagnetic brain stimulation; clinical psychology

## 1. Introduction

The human brain is perhaps the most complicated condensed matter system known. It contains about  $10^{12}$  billions of neurons and at least as many glia cells [1]. The brain is composed of 77 to 78 % water, 10 to 12 % lipids, 8 % proteins, 2 % soluble organic substances, and 1 % carbohydrates and inorganic salts [2]. It is also extremely fascinating that higher brain functions precisely define what it means to be human. Brain states and their dynamics have so far eluded a well-founded physical understanding. This means that one cannot describe brain dynamics by brute force, *i.e.*, starting from the behavior of all atoms and reconstruct macroscopic time evolution. Some of the most complex brain functions are delocalized over long distances and require synchronization processes that do not seem easy to explain through classical mechanics alone. In particular, the wholeness of perception requires

to integrate the activity of an enormous number of brain cells. With respect to this, quantum models [3–35] may hold the key to a possible microscopic understanding of some brain functions.

However, the study of the human psyche and its applications in clinical psychology [36–48] seem to strongly support a quantum-classical modelization of brain dynamics going beyond the one adopted in Refs. [3–35]. We will discuss how psychological processes can be structured on two different kinds of logic [36–48]. One form of logic is called Aristotelian and it is basically the logic one applies to understand classical reality. The other logic is termed non-Aristotelian and it can be likened to the logic followed by quantum physical events. Hence, quantum-classical theories seem good candidates to reflect such a bipartite structure of human psyche. This observation is reinforced by the fact that, from a practical point of view, one wants to effectively ‘regulate’ the brain to devise effective therapies for the cure of its pathologies [49–51]. This can only come from a deeper understanding of the basic microscopic mechanisms governing the brain activity and on our capability to interpret and fully exploit the achieved results in practical terms. With respect to this, the central theorem of cybernetics, due to Conant and Ashby, states that “every good regulator of a system must be a model of that system” [52]. Therefore, given that logic, in accordance with the current theories of the human psyche [45–48], has a bipartite ‘quantum-classical’ structure, a clear quantum-classical theory adopted as “regulator” model of the brain activity complies with this theorem. We wish to remark, in passing, that any advancement of such an approach, would produce a significant consolidation of the quantum-like models of cognition developed in the past years [53–61].

In recent years, three main quantum models of the brain have been introduced in the literature. These are the electromagnetic field (EMF) approach [3–11], the orchestrated objective reduction (Orch OR) theory [12–26], and the dissipative quantum model of brain (DQMB) [27–35]. Even if there are several key differences between the EMF, the Orch OR approaches, and DQMB, these three theories study the brain from the perspective of condensed matter physics and matter-EMF interactions. While DQMB [27–35] is mainly concerned with the explanation of memory storage and retrieval, long-range correlations between brain clusters of cells and brain correlates of perception, both the EMF [3–11] and Orch OR [12–26] approaches claim to describe aspects of consciousness. We do not consider EMF [3–11] and Orch OR [12–26] approaches as models for explaining consciousness. Instead, we consider them just as microscopic theories of brain dynamics.

Motivated by the germinal considerations in Refs. [3–35] and by the existence of a bipartite structure of logic, in this paper we introduce an explicit quantum-classical model of brain dynamics. Such a model is based on the hybrid quantum-classical (QC) formalism of Refs. [62–83]. In many QC theories, the nature of the interaction between the classical and quantum subsystems is somewhat unclear and the quantum variables are not treated on the same footing of the classical DOF. The formulation of Refs. [62–83] is based on Mixed Weyl symbols and it is conceptually free from this drawback. In fact, such an approach is founded upon a statistical operator depending parametrically on phase space points. This implies that the dynamics must be considered at each phase space point without the possibility of separating quantum dynamics from the classical-like dynamics of the phase space.

We are not concerned here with the problem of describing a single gigantic wave function for the whole brain. We are interested in study those brain processes that can be described in terms of a few variables embedded in a classical environment. As an example, one can consider that single quantum particles, such as electrons and protons, retain a quantum properties [77–83] at every temperature [84]. For such a reason, our QC approach can cope ‘almost by design’ with the controversial issue of decoherence [20,85–87] in warm and wet environments, such as those found in biological systems. Small numbers of quantum particles are naturally found in small biological structures [20,21], in molecular structure, down to atomic nuclei [22–24]. Even a small number of quantum variables can have a significant effect on the dynamics of a large classical systems by means of four mechanisms. One is given by non-adiabatic transitions between energy states [77–83]. The second one is caused by the stochastic collapse of the wave function [88,89]. The third one is generated by the motion of

quantum sources of the electromagnetic fields in the brain. The fourth one is the famous ‘order from order’ mechanism elaborated by Schrödinger [90], which led to the discovery of DNA [91,92]. All these mechanisms follow Pascual Jordan’s idea [93–95] about the necessary role of amplification of quantum processes in order to steer classical dynamics.

QC spin-boson models [96–99], and their non-linear extension [83], appear to be apt to describe a finite number of quantum variables coupled to classical DOF. Non-Hamiltonian deterministic thermostats [74–76,100–102] can be used to generate the dissipative dynamics Mixed Weyl symbols. The QC formalism simplifies numerical calculations of averages and response functions. In turn, response functions can be compared to electromagnetic signals that could be provided by macroscopic experiments on the brain [103–120].

The paper is structured as follows. We present the historic evolution of logic’s bipartite structure by discussing General Semantics (GS) in Subsec. 2.1, Synchronicity in Subsec. 2.2, and Blanco’s Bi-logic in Subsec. 2.3. We review the EMF approach in Sec. 3, Orch OR in Sec. 4, and DQMB in Sec. 5. Our QC approach is presented in Sec. 6. Finally, our conclusions are given in Sec. 7.

## 2. The Bipartite structure of psychology as the root for quantum-classical models of brain

In the field of clinical psychology, Korzybski was one of the pioneers making use of non-Aristotelian logic [36–39] for theurapetical application. A more abstract and somewhat implicit approach to such a bi-partite structure of logic can be found in the work of Jung and Pauli [40–44]. Instead, the most complete formulation and application in clinical psychology (until now) is found in the work of Blanco [45–48], where it is called Bi-logic. Since in QM the law of the excluded middle is not valid, so that a cat can be both alive and dead [121,122], at a fundamental level, quantum logic is non-Aristotelian and we propose to identify it with Blanco’s Bi-logic.

Hence, the quantum world does not follow Aristotelian logic. QM cannot indeed be separated from the classical world because of the processes of ‘measurement’ and the stochastic collapse of the wave function [88,89]. From this perspective, physical processes have a hybrid structure, where both Aristotelian and non-Aristotelian logic must be employed. Thus, we sustain that quantum physical processes must be described by hybrid quantum-classical models.

Since Bi-logic [45–48] can be likened to a quantum-classical worldview and there are already models supporting the quantum-classical nature of the brain [3–35], the idea of developing models by means of an explicit quantum-classical theory naturally arises. Our parallelism between Bi-logic [45–48] and quantum-classical phenomena in the brain can also be considered as the motivation for extending the current quantum-like models of cognition [53–61] to take into account quantum-classical processes.

In the remaining part of this section, we discuss the historical development of non-Aristotelian logic in psychology and clinical psychology.

### 2.1. General Semantics

Roughly speaking, GS is a specific instance of Process Philosophy [123]. However, it has the specific goal of improving mental health and adaptation to the world [36,37]. One key aspect of this approach is that a non-Aristotelian logic is more conform to reality. Once non-Aristotelian logic is accepted as the correct way of thinking, our language must be adjusted accordingly. GS’s link with QM has been briefly mentioned in the famous book “The Tao of Physics” [124]. An interesting connection of GS to quantum models of decision making, which is to be fully explored yet, may be founded on the Free Energy Principle [125,126]. However, a first application of this principle to quantum decision making can be found in Ref. [53].

Although it constitutes the historical roots of many modern philosophical point of view and many areas of man’s knowledge, GS is rarely acknowledged in contemporary theories. The premises of GS are “A map is not the territory”, “A map does not represent all of a territory”, and “A map is self-reflexive”, meaning that an ‘ideal’ map would include a map of the map, etc., indefinitely” [38].



These assumptions can be translated to daily life in order to improve the mental sanity of human beings [37]. In this case, GS premises become “A word is not what it represents”, “A word does not represent all of the facts”, and “Language is self-reflexive” in the sense that in language we can speak about language. Alas, human being reactions to verbal communication are largely based on unconscious beliefs, violating the first two assumptions and disregarding the third. Mathematics and GS are the only languages that rigorously take into account the above non-Aristotelian premises at all times. For such a reason, Korbizski strongly suggested to psychologists to study mathematical structures. At page 280 of his “Science and Sanity” [36], we find a discussion of the importance of the Theory of Aggregates and the Theory of Groups in Psychology, something that it will be further examined in Blanco’s Bi-logic [45–48].

It is very common to find that the influence of Korzybski’s GS on various approaches is not properly acknowledged [39]. Luckily, there are exceptions. For example, Ellis acknowledges Korzybski’s influence on his Rational Emotive Behavior Therapy [127]. Almost similarly, Wysong pays the dues of Gestalt Therapy to GS by writing a commentary in *The Gestalt Journal* [128]. How much Gestalt Therapy owes to GS is also discussed in the thesis of Allen Richard Barlow [129], which is downloadable from The University of Wollongong Thesis Collection on-line. One of many counter-examples [39] is given by Family Therapy [130,131], where it is stressed that one must be aware of abstractions leading to disregarding the wholeness of processes [130] (non-elementalism [36,37]) and it is also underlined the difference between the verbal and the non-verbal [131], but without citing GS. Hence, GS may be considered (either directly or indirectly) as the hidden root of various therapeutic practices.

Given the above discussion, it is not difficult to see the logical connections between GS and QM. If we consider that scientific theories are “maps” of reality, with classical theories providing a first level of abstraction, then QM is clearly characterized by a second level of abstraction. QM does not provide laws for the dynamics of models of phenomena. QM gives laws for the probability amplitudes that models of phenomena have a certain dynamics [132,133], *i.e.*, QM provides laws for models of models. GS classifies this as self-reflexiveness of the language. The influence of the quantum mechanical worldview on the formulation of GS was explicitly acknowledged by Korzybski [36]. From this perspective, we can consider GS an application of certain QM concepts to clinical psychology.

## 2.2. *Pauli and Jung’s Synchronicity*

The goal of the collaboration between Jung and Pauli was to find a unified view of reality in terms of both the psychological and the physical point of view. Jung’s approach to the psyche was based on certain in-forming (in the sense of having the power of giving “form”) structures that he called archetypes [134]. As universal regulators of the psyche, archetypes transcended the individual and belonged to a collective unconscious, common to all humankind.

Pauli was one of the founders of QM. He interpreted QM in term of the concept of statistical causality. This facilitated the collaboration with Jung. He explained to Jung that QM is about ‘forms’, *e.g.*, wave amplitudes, and it is also intrinsically probabilistic. While the causality of the classical world requires the exchange of physical quantities (such as energy, momentum, angular momentum and so on), statistical causality describes correlations between systems that exchange energy (and other physical quantities) through synchronic events, even if they may potentially interact in a classical way.

At the same time, Jung considered random coincidences in the classical world as the analogue of the statistical causality in the quantum world. Moreover, the origin of subjective meaning in the psyche was assigned to random coincidences themselves. According to Jung, the organizing principle of reality, which he called Synchronicity, is found in meaningful coincidences. Afterwards, the concept of Synchronicity was further generalized to include acausal correlations without any psychological component. We can conclude that Jung’s Synchronicity unsurprisingly reflects the quantum-classical nature of the world.

### 2.3. The Bi-Logical Structure of Psychology

Korzybiski's GS [36] proposes a new psychology founded on mathematical structures and, to this end, briefly dealt with both set and group theory. However, it is only in the work of Blanco [45–48] that these ideas are fully exploited in order to generalize Freud's formulation of the unconscious. While Freud defined the unconscious in a qualitative way, *i.e.* what is hidden and repressed in the psyche, Blanco describes it as a bipartite structure. Such a bipartite structure has one side that is asymmetric (which we may call Aristotelian by following GS language), pertaining man's common-day experience, and another side that is symmetric (which we may call non-Aristotelian), where space and time do not exist and the logical principle of non-contradiction is no longer valid. Blanco stated that both logics are at work in the human psyche [45–48] and that clinical practice must accurately take into account this point.

Blanco's and GS's conceptual structures share concepts taken from QM. However, while GS is fully non-Aristotelian (without any form of classical-like logic attached to it), Blanco's Bi-logic has an Aristotelian component (congruent with a classical worldview) and another non-Aristotelian component (in agreement with the logic of QM). Taking both aspects into account, we conclude that Blanco's Bilogic [45–48] formulates a QC conceptual perspective of the psyche, reflecting the QC nature of the phenomenological world. A full acknowledgement of this parallelism and its possible consequences on clinical practice are a matter of novel researches.

### 3. Electromagnetic Fields in the Brain

The role of EMFs in bridging space and time scales is very important [3–11]. Brain states are routinely studied *via* computer simulation [135] and various noninvasive stimulation techniques such as alternating current stimulation (ACS) [103–107] and transcranial direct-current stimulation (tDCS) [108,109]. In particular, tDCS is one of the most investigated methods in the field of non-invasive brain stimulation. It modulates the excitability of the cerebral cortex with direct electrical currents ( $1 \approx 2$  mA [110]) delivered *via* two or more electrodes of opposite polarities (*i.e.*, anode and cathode) placed on the scalp. tDCS modulates resting neuronal membrane potentials at sub-threshold levels [108], with anodal and cathodal stimulation increasing and decreasing cortical excitability, respectively [109]. Although their tDCS-induced physiological mechanisms are not yet fully understood, it is assumed that effects are based on long-term potentiation (LTP) and long-term depression-like (LTD) mechanisms [109,111].

In the history of brain research, it was assumed that higher brain functions, such as learning and memory, arise from electrical impulses passing through neurons. The physical explanation of permanent information storing was assigned to multiple reflections of impulses through neuronal circuits [136,137]. This idea is basically exemplified by the Hodgkin-Huxley model [138]. This model runs into problems since also glia cells take part in brain functions [139,140]. Of course, more complicated models, based on intricate networks including glia and other molecules, have been proposed (see Ref. [141], for example). However, even assuming that, by means of Darwinian evolution, the most efficient hyper-network can emerge, given enough time, there still would be the problem of explaining the origin of *software* running on hyper-networks. One has also to consider that the classical theories of self-assembling of hyper-networks are based on statistical fluctuations, *e.g.*, on the mechanism that Schrödinger called "order from disorder" [90]. What Schrödinger actually wanted to express with the expression "order from disorder" is that there are some ordered macroscopic structures, such as hyper-networks, that can arise from the statistical disorder at the microscopic level. In truth 'microscopic statistical disorder' is a misnomer that stands for the great number of microscopic states that correspond to the same macroscopic state [142,143]. Von Neumann entropy (and its quantum-classical generalization defined in terms of the Mixed Weyl of the statistical operator) is a property of the macrostates given in terms of the probability of microstates [144,145]. The belief that the passage to macroscopic 'order' is associated with entropy decrease is mistaken. The first reason is that macroscopic 'order' is somewhat an anthropomorphic concept that can only be defined

once some macroscopic variables are chosen. On the contrary, microscopic order is physical since it is defined in terms of the number of microstates that are compatible with the macroscopic constraints. A system must be considered microscopically ordered if there is a small number of states associated to the macroscopic constraints. In agreement with the Third Law of thermodynamics, for example, this takes place at  $T = 0$  where there is only one accessible microstate and the system is maximally ordered on the microscopic level. Another example is given by the phenomenon of reentrant phase transitions [146–149], where the macroscopic ‘ordered’ phase has higher entropy than the microscopic ‘disordered’ one because of the unfreezing of certain DOF. Irreversible microscopic dynamics, such as diffusive motion, does not conserve the number of accessible microstates of the system conditioned by the macroscopic constraints and, thus, lead to an increase of entropy [144,145]. This is the essence of Schrödinger’s “order from disorder” mechanism [90]: in our macrocosm we are surrounded by structures that we classify as ordered but that are based on microscopic disorder in agreement with the Second Law of Thermodynamics.

As discussed by Schrödinger, an “order from disorder” mechanism cannot explain the synchronization of molecular processes, which is required by brain functions and living matter in general. To explain living matter, Schrödinger proposed a second mechanism he named “order from order”. The mechanism of “order from order” is basically a quantum-mechanically zero-temperature clockwork, in agreement with the Third Law of Thermodynamics [90]. Only solid forms of matter allows for quantum clockworks to exist in high-temperature disordered biological environment. This is caused by the existence of energy gaps protecting, for example, long wavelength electronic wavefunction in a solid. This idea led Schrödinger to predict that an aperiodic solid (ultimately identified with DNA [91,92]) would contain in a stable manner the information needed by living organism to survive entropic decay. Nowadays, the idea has become more general and it is not limited to solid structures as shields from molecular disorder. One example is found in the Orch OR theory according to which quantum effects are protected inside hydrophobic regions of biological microstructures [20,21]. Another mechanism to protect quantum clockwork is provided by rigid boundaries enclosing quantum variables [150–152].

The idea that other physical agents, rather than the sole dynamics of neural networks, must be invoked to describe highly coordinated brain activity is not new [112–115]. Electric charges (*e.g.*, electrons, protons, ions), together with their associated currents, are the sources of EMFs [3–10]. In turn, these EMFs interact with water dipoles and also influence van der Waals and Casimir interactions among brain macromolecules. ACS has shown the importance of EMFs in the brain [103–107], tDCS of human subjects [108,109] has shown the importance of both cognition processes and psychological state changes can be modulated. For instance, Anodal (excitatory) tDCS of the prefrontal cortex boosts affective memory such as fear extinction learning [116–118]. Moreover, the cathodal (*i.e.*, inhibitory) stimulation of the tongue motor neurons of the primary motor cortex reduces appetite [119].

The working of tDCS might be understood through a mechanical analogy. The complex dynamics of brain EMFs can be reduced to the time evolution of their sources. Such a dynamics can be mapped onto that of a harmonic spring mattress. Within this pictorial description, tDCS can be equated to the nonlinear effect generated by the application of a constant pressure to specific extended regions of the spring mattress. The applied pressure changes the harmonic dynamics of the mattress so that oscillations with principal frequencies (phonons) scatter with each other. This mechanical model might be useful to perform computer simulations of certain processes which are observed in tDCS. We note that the same model has been used to give a pictorial representation of quantum fields [153]. Both ACS and tDCS provide evidence that brain EMFs are not ephemeral; they are correlated to the dynamics of their sources, but they also react back and influence both cognitive functions and emotions.

When studying brain dynamics on the mesoscopic scale of EMFs, it may seem that there is no necessity to invoke any quantum effect. The original EMF approach was formulated only in terms of classical physics [3–11]. Nevertheless, our analysis below can elucidate the fundamental quantum coherent properties of the microscopic EM fields invoked by such an approach. [150–156]. Observable

coherent EMFs have by definition a well-defined phase. Quantum mechanically, phase  $\Phi$  and photon number  $N$  are conjugate variables. This implies that they obey the indeterminacy relation

$$\Delta\Phi\Delta N \geq 1. \quad (1)$$

According to Eq. (1) when the number of quanta of the photon field  $N$  is not fixed and  $\Delta N$  can be large, it follows that the phase  $\Phi$  is well determined and the quantum photon field is coherent.

The only way for the number of photon  $N$  to fluctuate is that photons are continuously absorbed and re-emitted. In other words, coherent EMFs are ‘composed’ of virtual photons [154], e.g., packets of energy in momentum space whose existence is ephemeral. Interestingly, experimental evidence shows that dendrimers can act as trap for photons [150,151]. According to quantum electrodynamics, [155], a trapped photon can be represented in terms of virtual photons continuously emitted and re-absorbed between fermions. This picture can be developed considering that, in terms of Feynman diagrams, a photon line connecting two fermion lines is a virtual photon describing Møller scattering [155]. Thus, an exchange of virtual photons along the time direction between the two fermion lines, generating a so-called “ladder” diagram [157,158], may very well be considered the microscopic picture of a trapped photon.

#### 4. Penrose and Hameroff’s Orch OR

Orch OR theory [12–26] provides a detailed molecular mechanism for the time evolution of brain states. According to Orch OR theory [12–26], quantum effects in tubulin proteins (which are organized in arrays of microtubules inside the cytoplasm of brain cells) play an important role in brain function. Quantum dynamics of the electronic orbitals of carbon rings inside tubulins, time evolution of the nuclear spins, quantum energy transport among microtubules, and spontaneous collapse of microtubules’ wave function are the main ingredients of this theory. Upon collapse of the wave function, classical brain dynamics ensues. Thus, even if its originators presented it as a theory of consciousness, here we only consider Orch OR as a theory of brain processes.

One peculiar characteristic of Orch OR is that neurons are not considered the fundamental units of information processing [11]. Instead, in Orch OR it is proposed that information processing takes place in ordered arrays of microtubules inside the cell. This idea slowly took form during the 1980s and the first part of the 1990s when Hameroff noticed the effects of anesthetics on networks of microtubules inside the cell. In a series of papers, Hameroff *et al.* [159–164] proposed that some kind of digital computation was taking place in arrays of microtubules. Such a computation was based on nonlinear electrodynamic effects [159–164]. However, the question of how the results of local digital calculations could be efficiently transferred between distant brain regions by classical diffusive mechanisms remained. Hence, Hameroff started his search for different mechanisms. On a different path, looking for a fundamental explanation of wave function collapse in QM, Penrose elaborated the theory of Objective Reduction (OR) [12–16].

In the standard interpretation of QM, the collapse of the wave function, i.e., the transition from the worlds of possibilities to that of classical events [165,166], is explained only through the stochastic interaction of quantum systems with a classical ones. The collapse of the wave function is called the ‘measurement’ process because of the interaction with a classical system [167]. It is not explained within the theory but it is assumed as a postulate. OR proposes that the superposition of different stationary mass distributions becomes unstable because of quantum gravitational effects, and, beyond a certain threshold time interval, it naturally collapses according to the standard probabilistic rules of QM, but without any external intervention of a “measuring instrument”. A simple way to discuss this process is to consider

$$\omega_{\text{Bohr}} = \Delta E / \hbar, \quad (2)$$



as the Bohr frequency of the energy eigenvalues of two eigenstates involved in a certain superposition. Penrose gives a number of reasons why the superposition must become unstable in the presence of quantum gravitational effects. The lifetime of the superposition is given by

$$\tau \approx \frac{h}{\Delta E} . \quad (3)$$

Looking at Eqs. (2) and (3), one might say that, in a certain sense, the deterministic time evolution of the gravitational field acts as the instrument measuring the superposition. However, according to Penrose [12–16], there is an important difference between the measurement of the superposition by a classical instrument and by a quantum gravitational field. A measurement performed by a quantum gravitational field, is still a fully quantum mechanical process and as such it is intrinsically random and absolutely non-computable. Penrose considered that brain dynamics is interspersed with discrete events (see Ref. [7] for experimental support of this idea). On a phenomenological basis, such events parallel the discontinuity of wakefulness and awareness [7] and other rhythmic phenomena in the brain. Penrose identified discrete events in the brain with series of wave function collapses. Between one collapse and the other, the brain can evolve coherently so that new superpositions are formed. We note that such a coherent evolution of the wave function, interrupted by quantum gravitational collapses, is reminiscent of both piecewise deterministic processes in open quantum systems [168] and nonadiabatic dynamics of QC system in the adiabatic basis [80–83].

While Penrose put forth the idea that OR could have an important role in brain dynamics, Hameroff fleshed out the detailed biomolecular mechanisms. Inside each tubulin protein making up a given microtubule, Hameroff hypothesized the existence of quantum matter systems able to support stable quantum dynamics in between OR events. One example is given by carbon rings and their delocalized molecular orbitals, which can evolve coherently in a superposition of states. The carbon rings are pushed by hydrophobic forces into the tubulin's interior, shielding them from the decoherence [85,86] caused by the polar environment outside the protein. The carbon rings form helical structures inside each microtubule. They also create oriented arrangements that can act as quantum channels [20,21] through which quantum signals travel among the lattice of microtubules inside the cell's cytoskeleton.

Various types of quantum oscillators are therefore found in microtubules' ordered structures, *e.g.*, time-dependent electric fields arising from the dynamic polarization of molecular charges (which produce van der Waal and Casimir-Polder forces), magnetic fields originating from electron spin dynamics, etc. Notably, it has also been suggested [22–24] that nuclear spins can play an important role in Orch OR theory since they are shielded from decoherence for longer time intervals than other quantum systems in the brain. Recently, this theory [22–24] has gained experimental support [25]. The frequencies of all such quantum oscillators range from kilohertz to terahertz. Orch OR theory requires the feedback [26] between the quantum coherent evolution of microtubules and, for example, the classical dynamics of microtubule-associated proteins (MAPs) [169,170]. Such a classical dynamics concerns the classical dynamics of MAPs [169,170] and CAMKII [171–176], *viz.*, the direction of motion, the place where MAPs and CAMKII halt their motion, the case in which they interact or non-interact with the tubulins, and the precise time when they interact. According to the Orch OR theory [12–26], the coherent evolution of the microtubule's wave function and its OR determine all detailed molecular events. However, we must note that the possibility that extended brain regions, may be free from decoherence [20,85–87] is rather controversial.

Lately, there has been a convergence of ideas between the approach to brain dynamics *via* quantum EMFs [150,151,154] and Orch OR [177]. The physical process underlying quantum signaling in Orch OR has been assumed to be photon emission. Due to the work of Alexander Gurwitsch, it has been known since the beginning of the 20th century, that tissues inside the body emit biophotons [178–180]. Such biophotons may be supported by the hydrophobic interior region of tubulins, where tryptophanes, with their indole rings of  $\pi$  electron orbital forming optically active molecular orbitals, are found. The

packing of indole rings may give rise to resonant energy transfer between molecular orbitals [177] much in the same way Förster resonant energy transfer takes place between close chromophores. Kurian *et al* [181] represented the microtubule as a chain of two-level systems and calculated the coupling constants in the Hamiltonian by means of Molecular Dynamics simulations and quantum chemical calculations. Exciton propagation was performed by means of the Haken and Strobl method [182]. Their main result is that energy transfer occurs on a length scale of microns, at least. What is even more interesting from the Quantum Optical perspective is that Kurian *et al*'s simulation [181] does not consider the geometric structure of the left-handed helices of microtubule in mammals. There are reasons to believe that superradiance can be important in such complicated geometric arrangements [183–185]. Very recently, the experimental study of Kalra *et al* [186] found that photonic energy transfer in microtubules occurs over 6.6 nm, it cannot be explained in terms of Förster theory, and it is damped by anesthetics. The idea that electromagnetic resonance is the fundamental mechanism of communications among molecules was first proposed by Veljkovic *et al.*, who also suggested that such a mechanism could provide a long-range effective communication [187]. At this stage, we believe that a unification of the EMF and Orch OR theories of brain dynamics is conceptually very probable [156].

Nevertheless, Orch OR model remains very controversial. It is based on quantum gravitational effects to objectively induce the wave function collapse by using only a provisional theory of quantum gravity [17–19].

## 5. The Dissipative Quantum Model of Brain

The precursor of of DQMB [27] was the seminal paper [188] of Ricciardi and Umezawa, where the quantum field theory model (QFTMB) of brain was introduced [189,190]. An interacting QFT can naturally describe the creations of dynamical correlations. Whenever a quantum field has an average value different from zero in the vacuum, the vacuum state will be no longer unique. Instead, there will be different vacua and each of them will spontaneously break the symmetry of the Hamiltonian density [195–197]. In order to compensate for the SSB, one observes the proliferation of bosonic modes establishing long range correlations with the local configurations of the field. The symmetry-breaking mechanism in the QFTMB [188] can qualitatively describe both long-term memory storage in the ground states with broken symmetry and long range correlations between distant clusters of neurons by means of the Nambu-Goldstone bosons. Nambu-Goldstone bosons also act as the agents for memory retrieval [188] while excited energy states of the field describe short-term memory.

In DQMB the dissipation is ascribed to excited thermal states, which are represented through a doubling the number of fields according to Thermo Field Dynamics [189,190]. DQMB also predicts that long-range correlations between distant excited areas of the brains do not occur *via* chemical transport but by means of Nambu-Goldstone bosons [195–197]. One example of such long-distance correlations is observed when brain is locally stimulated. In this case, there is experimental evidence [103–109] that the response is given by simultaneous excitations in several regions [198,199], which are far from one another.

In DQMB, quantum coherent fields interacts with classical neurons and glia cells. DQMB presents us with a hybrid description where memory storage finds a quantum explanation and biochemical reactions a classical one. Such a hybrid description requires to coarse-grain the classical degrees of freedom (DOF) and to describe them in terms of some kind of waves. Only at this level of description it is possible to formulate the interaction between the Nambu-Goldstone bosons [195–197] the condensed quantum field predicted by the model and the classical waves, much in the same way phonons in an ordered solid interact with acoustic waves [27–35].

Due to its mesoscopic nature, DQMB does not aim to describe the behavior of the molecular constituents of the brain with atomistic detail, *e.g.*, neurons, glia cells, membranes, neurotransmitters or other macromolecules. Today, we know that all these structures form brain clusters [198,199] that, once stimulated [103–109] can influence human behavior [106,107]. Since normal mesoscopic brain dynamics is not chaotic, brain response to stimuli cannot be expected to depend on the number  $N$  of

the fundamental constituents of the clusters. If  $N$  is not fixed, Eq. (1) is valid, the phase  $\Phi$  of the matter field is well defined and the matter field will be coherent.

DQMB does not specify the physical nature of the bosonic fields of the brain. The bosonic fields in Fourier space may be identified with the modes of the quantum oscillators considered in Orch OR theory [20,21] and discussed in Sec. 4. However, another proposal suggested to interpret the bosonic fields in term of the the dipoles of water molecules [200–204]. According to the theory in Refs. [200–202], when water molecules have a high density, the approximation of weak coupling to the electromagnetic vacuum field [205] may not hold. It has been suggested that water in the cytoplasm is found in a structured state [206], so that the considerations of Refs. [200–202] are definitely relevant for brain dynamics. Since a water molecule is dipolar, a coherent superposition of the dipoles of many water molecules can be described by a coherent quantum dipolar field. Hence, in this model it is the condensation of the quantum dipolar field to produce a ground state with broken symmetry, *i.e.*, and many unitarily inequivalent subspaces [207]. Consequently, Nambu-Goldstone modes arise for restoring symmetry at long range.

In the following we use the Hamiltonian of the noninteracting dipolar wave quanta of Ref. [35] in order to elucidate the theoretical description of dissipation by means of doubling the DOF as described by Umezawa's Thermo Field Dynamics [189,190]. The dynamical variables of DQMB are doubled upon introducing creation and annihilation operators of physical dipolar wave quanta,  $\hat{a}^\dagger, \hat{a}$ , respectively, and dual creation and annihilation operators of fictitious dipolar wave quanta,  $\hat{v}^\dagger, \hat{v}$ , respectively. For example, the Hamiltonian of the noninteracting dipolar wave quanta might be defined as [35]:

$$\hat{H}_0 = \sum_k \hbar \omega_k \left( \hat{a}_k^\dagger \hat{a}_k - \hat{v}^\dagger \hat{v}_k \right), \quad (4)$$

where  $\omega_k$  is the oscillation frequency of each mode. The interaction between the physical modes and their doubles can be taken as

$$\hat{H}_I = i \sum_k \hbar \gamma_k \left( \hat{a}_k^\dagger \hat{v}^\dagger - \hat{a}_k \hat{v}_k \right), \quad (5)$$

where  $\gamma_k$  is the damping constant of each mode. Finally, the total many-body Hamiltonian of the thermal system is

$$\hat{H} = \hat{H}_0 + \hat{H}_I. \quad (6)$$

A thorough study of the Hamiltonian in Eq. (6), and its associated equations of motion has led to finding a number of interesting results over the years [30].

DQMB has been applied by Vitiello and collaborators to study various brain processes [29]. Some applications include nonlinear dynamics [31], cortical patterns in perception [32], the relation between fractal properties and the coherent states in the brain [33], rhythmic generators in the cortex [34], and correlations of brain regions that are realized through entanglement [35]. DQMB dynamics has also been adopted by Nishiyama *et al.* in a number of works [208–211]. As reported in Ref. [208], one notes that the phenomenon of superradiance, which is expected to occur in complicated geometric arrangements of microtubules, also occurs in DQMB.

## 6. The Quantum-Classical Model of Brain

Our aim is to model multi-scale brain dynamics, explicitly treating classical DOF and quantum variables on the same footing. To this end, our approach considers Mixed Weyl symbols of dynamical variables (represented by operators in the standard formulation of quantum mechanics) and a Mixed Weyl symbol of the statistical operator (corresponding to the density matrix of the systems in the standard representation of QM) [62–83]. We imagine that the brain is described by quantum operators  $(\hat{r}, \hat{p}, \hat{R}, \hat{P})$ , where  $(\hat{r}, \hat{R})$  are position operators while  $(\hat{p}, \hat{P})$  are the respective conjugated momenta operators. Now,  $(\hat{r}, \hat{p}) = \hat{x}$  corresponds to the brain variables with a long de Broglie wavelength that, for this reason, must be treated quantum mechanically, while  $(\hat{R}, \hat{P}) = \hat{X}$  can be treated semi-classically

because of their much shorter de Broglie wavelength. A partial Wigner transform over the  $(\hat{X})$  operators [72] introduces the Mixed Weyl symbols  $\tilde{\mathcal{O}}(X)$  and  $\tilde{\mathcal{W}}(X)$  arising from  $\hat{\mathcal{O}}(\hat{x}, \hat{X})$  and  $\rho(\hat{x}, \hat{X})$ , respectively. Please, note that the following notation is adopted: when a quantum operator depends both on quantum variables and classical DOF, a  $\sim$  is written on it, while if the quantum operator does not depend on  $X$  a  $\hat{\cdot}$  is used. No hat is used in the case of a dynamical variable depending only on  $X$ . A practical example of a possible application of this mixed QC representation can be given when considering molecular orbitals, electron and nuclear spins, light ions, neurons, glia cells, and electromagnetic interactions. Conformational dynamics of cells may be represented through phonons, *i.e.*, harmonic DOF. Other harmonic DOF can be used to describe coherent EMFs. The inclusion of non-Harmonic perturbation terms provides a description of non-trivial interactions among all the DOF of the model. Zero-point effects on the motion of classical-like DOF can be described by means of advanced algorithms that will be explained in the following. As in the case of DQMB, the goal is to set up a mesoscale approach to brain dynamics, noting however that in our case the QC dynamical variables are explicitly represented.

If we now introduce the coordinates of the EMF modes  $(Q, \Pi) = Y$ , a possible model Mixed Weyl symbol of the Hamiltonian  $\tilde{\mathcal{H}}(X, Y)$  can be written as

$$\tilde{\mathcal{H}}(X, Y) = \hat{\mathcal{H}}_S + \mathcal{H}_B(X) + \mathcal{H}_F(Y) + \tilde{\mathcal{V}}_{SB}(R) + \tilde{\mathcal{V}}_{SEM}(Q) \quad (7)$$

In Eq. (7),  $\hat{\mathcal{H}}_S(t)$  is the Hamiltonian operator of the quantum subsystem with quantum variables  $\hat{x}$ . The phononic Hamiltonian is

$$\mathcal{H}_B(X) = \sum_{J=1}^{N_{PH}} \left( \frac{P_J^2}{2} + \frac{(\omega_J^{PH})^2}{2} R_J^2 \right), \quad (8)$$

where  $\omega_J^{PH}$ ,  $J = 1, \dots, N_{PH}$ , is the frequency of each phonon. Similarly, The EMF Hamiltonian is

$$\mathcal{H}_F(Y) = \sum_{K=1}^{N_{EM}} \left( \frac{\Pi_K^2}{2} + \frac{(\omega_K^{EM})^2}{2} Q_K^2 \right), \quad (9)$$

where  $\omega_K^{EM}$ ,  $K = 1, \dots, N_{EM}$ , is the frequency of the EMF mode. The interaction operators  $\tilde{\mathcal{V}}_{SB}(R)$  and  $\tilde{\mathcal{V}}_{SEM}(Q)$  describe the coupling of the phonons and of the EMF to the quantum subsystem, respectively. Assuming for simplicity a bilinear approximation, these can be written as

$$\tilde{\mathcal{V}}_{SB}(R) = - \sum_{J=1}^{N_{PH}} C_J R_J \hat{\chi} \quad (10)$$

$$\tilde{\mathcal{V}}_{SEM}(Q) = - \sum_{K=1}^{N_{EM}} F_K Q_K \hat{\zeta}, \quad (11)$$

where the  $C_J$  and the  $F_K$  are the coupling constants of the quantum operators  $\hat{\chi}$  and  $\hat{\zeta}$ , respectively. The operators  $\hat{\chi}$  and  $\hat{\zeta}$  acts on the same space of  $\hat{x}$ .

The dynamics of the Mixed Weyl symbol  $\tilde{\mathcal{O}}(X, Y, t)$  of an arbitrary operator  $\hat{\mathcal{O}}$  is given by a QC bracket [62–83]. The QC bracket is a quasi-Lie bracket [73–76] that breaks the time-translation invariance of Lie algebras because it does not satisfy the Jacobi relation. In the case of a system with both phononic and EMF modes, it can be written by introducing two antisymmetric matrices,  $\Omega = -\Omega^{-1}$  and  $\Lambda = -\Lambda^{-1}$ :

$$\Omega = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad (12)$$



and

$$\Lambda = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}. \quad (13)$$

The QC equation of motion in the Heisenberg picture reads

$$\begin{aligned} \partial_t \tilde{\mathcal{O}}(t) &= \frac{i}{\hbar} \left[ \tilde{\mathcal{H}} \quad \tilde{\mathcal{O}}(t) \right] \Omega \begin{bmatrix} \tilde{\mathcal{H}} \\ \tilde{\mathcal{O}}(t) \end{bmatrix} - \frac{1}{2} \tilde{\mathcal{H}} \overleftarrow{\nabla}^{X,Y} \Lambda \overrightarrow{\nabla}^{X,Y} \tilde{\mathcal{O}}(t) \\ &+ \frac{1}{2} \tilde{\mathcal{O}}(t) \overleftarrow{\nabla}^{X,Y} \Lambda \overrightarrow{\nabla}^{X,Y} \tilde{\mathcal{H}}, \end{aligned} \quad (14)$$

where  $\nabla^{X,Y} = ((\partial/\partial R), (\partial/\partial Q), (\partial/\partial P), (\partial/\partial \Pi))$  is the phase space gradient operator.

The lhs of Eq. (14) defines the quantum-classical bracket of  $\tilde{\mathcal{O}}(t)$  with  $\tilde{\mathcal{H}}$ . The first term in the lhs of Eq. (14) is the quantum commutator while the other two terms are Poisson brackets. All terms are written in matrix form [74–76]. The super propagator associated to the QC bracket is

$$\begin{aligned} \tilde{\mathcal{U}}(t) &= \exp \left\{ (it/\hbar) \left[ \tilde{\mathcal{H}} \quad \dots \right] \Omega \begin{bmatrix} \tilde{\mathcal{H}} \\ \dots \end{bmatrix} - (t/2) \left( \tilde{\mathcal{H}} \overleftarrow{\nabla}^{X,Y} \Lambda \overrightarrow{\nabla}^{X,Y} \dots \right) \right. \\ &+ \left. (t/2) \left( \dots \overleftarrow{\nabla}^{X,Y} \Lambda \overrightarrow{\nabla}^{X,Y} \tilde{\mathcal{H}} \right) \right\} \end{aligned} \quad (15)$$

The super-operator  $\tilde{\mathcal{U}}(t)$  defines the dynamics of Mixed Weyl symbols of standard operators as

$$\tilde{\mathcal{O}}(t) = \tilde{\mathcal{U}}(t) \tilde{\mathcal{O}}, \quad (16)$$

where  $\tilde{\mathcal{O}} = \tilde{\mathcal{O}}(t=0)$ . QC averages are calculated using the formula

$$\langle \tilde{\mathcal{O}}(t) \rangle = \text{Tr}' \int dX dY \tilde{\mathcal{W}}(X, Y; t) \tilde{\mathcal{O}}(X, Y, t). \quad (17)$$

In Eq. (17) the parametric time-dependence of the Mixed Weyl symbol of the statistical operator of the system,  $\tilde{\mathcal{W}}(X, Y; t)$  describes possible non-equilibrium initial conditions. The formalism here presented can be easily adapted to more general non-equilibrium situations, arising from an explicit time dependence of the Mixed Weyl symbol of the Hamiltonian in Eq. (7). In such a case, it would be more convenient to adopt the Schrödinger scheme of motion and propagate the Mixed Weyl symbol of the statistical operator. One would also have to take into account the time-ordering of the propagator, something that can be implemented by the algorithm [81]. Non-equilibrium dynamics is important if one considers the Free Energy Principle proposed by Karl Friston [125,126]. Recently, such a direction of research has witnessed interesting developments [53]. As for QC correlation functions, they are defined in the following way

$$\langle \tilde{\mathcal{O}}_1(t) \tilde{\mathcal{O}}_2 \rangle = \text{Tr}' \int dX dY \tilde{\mathcal{W}}(X, Y; t) \tilde{\mathcal{O}}_1(X, Y, t) \tilde{\mathcal{O}}_2(X, Y). \quad (18)$$

The operator  $\text{Tr}'$  found in Eqs. (17) and (18) takes the trace over the quantum operators  $\hat{x}$ , while  $\tilde{\mathcal{O}}_1$  and  $\tilde{\mathcal{O}}_2$  are two arbitrary Mixed Weyl symbols.

### 6.1. Constant Temperature Quantum-Classical Dynamics

In order to illustrate the advanced techniques for controlling the temperature of the harmonic modes, we consider a simple systems with just two phononic modes with coordinates  $(X_1, X_2)$  and two NHC chains of length one (which is usually enough to generate ergodic dynamics for stiff harmonic degrees of freedom) [100–102]. Thus, the extended phase space point can be written as  $X^e = (R_1, \eta_1^{(1)},$

$\eta_2^{(1)}, R_2, \eta_2^{(1)}, \eta_2^{(2)}, P_1, P_{\eta_1}^{(1)}, P_{\eta_1}^{(2)}, P_2, P_{\eta_2}^{(1)}, P_{\eta_2}^{(2)}$ , consequently, the extended phase space gradient is  $\nabla^e = ((\partial/\partial R_1), (\partial/\partial \eta_1^{(1)}), (\partial/\partial \eta_2^{(1)}), (\partial/\partial R_2), (\partial/\partial \eta_1^{(2)}), (\partial/\partial \eta_2^{(2)}), (\partial/\partial P_1), (\partial/\partial P_{\eta_1}^{(1)}), (\partial/\partial P_{\eta_2}^{(1)}), (\partial/\partial P_2), (\partial/\partial P_{\eta_1}^{(2)}), (\partial/\partial P_{\eta_2}^{(2)}))$ . If we now define the antisymmetric matrix  $\mathcal{R} = -\mathcal{R}^{-1}$  as

$$\mathcal{R} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & P_1 & 0 & -P_{M_{\eta_2}}^{(1)} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & P_{M_{\eta_2}}^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -P_2 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & P_2 & 0 & -P_{\eta_1}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & P_{\eta_1}^{(2)} & 0 \end{bmatrix} \quad (19)$$

together with the Mixed Weyl symbol of the extended Hamiltonian

$$\tilde{\mathcal{H}}^e(X^e) = \hat{\mathcal{H}}_S + \mathcal{H}_B(X) + \tilde{\mathcal{V}}_{SB}(R) + \sum_{I=1}^2 \sum_{L=1}^2 \frac{P_{\eta_L}^{(I)}}{2M_{\eta_L}} + \sum_{I=1}^2 \sum_{L=1}^2 k_B T^{(I)} \eta_L^{(I)}. \quad (20)$$

the QC equation of motion at constant temperature can be written in compact form [74–76] as

$$\begin{aligned} \partial_t \tilde{\mathcal{O}}^e(t) &= \frac{i}{\hbar} \left[ \tilde{\mathcal{H}}^e \quad \tilde{\mathcal{O}}^e(t) \right] \Omega \begin{bmatrix} \tilde{\mathcal{H}}^e \\ \tilde{\mathcal{O}}^e(t) \end{bmatrix} - \frac{1}{2} \tilde{\mathcal{H}}^e \overleftarrow{\nabla}^e \mathcal{R} \overrightarrow{\nabla}^e \tilde{\mathcal{O}}^e(t) \\ &+ \frac{1}{2} \tilde{\mathcal{O}}^e(t) \overleftarrow{\nabla}^e \mathcal{R} \overrightarrow{\nabla}^e \tilde{\mathcal{H}}^e, \end{aligned} \quad (21)$$

where  $\tilde{\mathcal{O}}^e(t) = \tilde{\mathcal{O}}^e(X^e, t)$ , the NHC variables are  $(\eta_L^{(I)}, P_{\eta_L}^{(I)})$ , with  $I$  and  $L$  running over the phonons and the coordinates of the chain, respectively;  $k_B$  is the Boltzmann constant,  $T^{(I)}$  is the temperature of each mode, and  $M_{\eta_L}^{(I)}$  are the inertial parameters of the NHC variables.

Constant temperature averages and correlation functions can be calculated choosing the Mixed Weyl symbol  $\tilde{\mathcal{W}}^e(X^e)$  of the statistical operator in extended space as

$$\tilde{\mathcal{W}}^e(X^e) = \hat{w}_S \mathcal{W}^\beta(X) \prod_{I=1}^2 \prod_{L=1}^2 \delta(\eta_L^{(I)}) \delta(P_{\eta_L}^{(I)}) \quad (22)$$

where  $\hat{w}_S$  is the Mixed Weyl symbol of the statistical operator of the quantum subsystem while the thermal Mixed Weyl symbol of the statistical operators of the phonons is

$$\mathcal{W}^\beta(X) = \prod_{I=1}^2 \frac{\tanh(\beta\omega_I/2)}{2} \exp \left[ -\frac{2 \tanh(\beta\omega_I/2)}{\omega_I} \left( \frac{P_I^2}{2} + \frac{\omega_I^2}{2} R_I^2 \right) \right], \quad (23)$$

where  $\beta = 1/k_B T$  and  $\omega_I$  is the frequency of phonon  $I$ . If in the Mixed Weyl symbol of the Hamiltonian in Eq. (20) one defines  $T^{(I)} = T \quad \forall I$ , then the dynamics defined by Eq. (21) defines constant-temperature evolution. Instead, the choice of  $T^{(I)} = 1/k_B \beta^{(I)}$ , with

$$\beta^{(I)} = \frac{2 \tanh(\beta\omega_I/2)}{\omega_I}, \quad \forall I. \quad (24)$$

describes a time evolution of the phonons where zero-point effects are taken into account. The structure of the extended QC super-propagator  $\tilde{U}^e$  is similar to that displayed in Eq. (15):

$$\begin{aligned} \tilde{U}^e(t) = & \exp \left\{ (it/\hbar) \begin{bmatrix} \tilde{H}^e & \dots \end{bmatrix} \Omega \begin{bmatrix} \tilde{H}^e \\ \dots \end{bmatrix} - (t/2) \left( \tilde{H}^e \overleftarrow{\nabla}^e \mathcal{R} \overrightarrow{\nabla}^e \dots \right) \right. \\ & \left. + (t/2) \left( \dots \overleftarrow{\nabla}^e \mathcal{R} \overrightarrow{\nabla}^e \tilde{H}^e \right) \right\}. \end{aligned} \quad (25)$$

Since we are interested in thermal and zero-point QC averages and correlation functions of non-fictitious dynamical variables, we must consider Mixed Weyl symbols  $\tilde{O}(X)$  that at  $t = 0$  do not depend on the extended phase space point  $X^e$  but they depend on the non-fictitious phase space point  $X$ . However, the key of temperature control is that the phase space variable dependence found at  $t = 0$  is not preserved at  $t \neq 0$ . We have  $\tilde{U}^e(t)\tilde{O}(X) = \tilde{O}(X^e, t)$ . Finally, we can write the expression for thermal (or zero-point) QC averages as

$$\langle \tilde{O}(X, t) \rangle_e = \text{Tr}' \int dX^e \mathcal{W}^e(X^e) \tilde{O}(X, t), \quad (26)$$

$$\langle \tilde{O}_1(X, t) \tilde{O}_2(X) \rangle_e = \text{Tr}' \int dX^e \mathcal{W}^e(X^e) \tilde{O}_1(X, t) \tilde{O}_2(X). \quad (27)$$

## 7. Conclusions

In this work, we have brought to light a parallelism between the bipartite structure of the psyche and the quantum-classical view of physical phenomena. The conscious part of the psyche follows the Aristotelian logic of the classical world while the unconscious follows the non-Aristotelian logic of quantum mechanics. Following the suggestions, already presented in the literature, about the quantum-classical character of brain dynamics and the central theory of cybernetics, saying that a 'regulator' must contain a model of the system, we have been motivated to build an explicitly quantum-classical model suitable for studying brain dynamics. As a by-product of this line of reasoning we have realized that quantum-like models of cognition should be revised in term of the bi-partite structure of logic and the quantum-classical worldview.

Motivated by the above concepts, we have proposed a quantum-classical model for studying brain processes. One idea behind this proposal, *i.e.*, the need to mix a quantum and a classical level of description together, had already been supported in a somewhat less explicit form by three models, which we have reviewed in the first part of this work. The very formulation of our model is given in terms of quantum and classical variables that are treated on the same level. It does not need to invoke quantum gravitational effects in the brain. Instead, the crux is that the quantum variables play the fundamental role of providing a quantum guiding mechanism for the classical variables they are coupled with. With respect to this, the existence of a quantum coherent wave function of the whole brain at high temperature is not needed. There are some important quantum properties of few-body systems that are not lost at high temperature. These have been discussed in the text. What is needed for the quantum biology of the brain has been suggested long ago by Pascual Jordan: The collapse of the wave function works as amplification mechanism acting as a bridge between the quantum and the classical world.

We have shown that the quantum-classical model provides a statistical mechanic formulation of averages and correlation functions. In turn, as it is well-known, correlation functions lead to the definition of response functions. Non-invasive brain stimulation techniques can provide the numerical data to which our theory can be compared. Moreover, electromagnetic DOF are also taken into account by our model. At least the model capable to produce results to be compared to numerical data coming from electroencephalograms.

We have taken the risk to discuss many complex ideas using only logic and our scientific knowledge. We have presented a synthesis of subtle concepts, we have introduced our

quantum-classical model, with which we plan to take on big scientific challenges, and we have declared the direction that our future work will take. We have done so with the belief that science is not only made by numbers. It is also made by understanding and sharing concepts with the community. Subsequently, such concepts can be discussed and refined, possibly leading to new advancements. Our future work will be devoted to interpret biochemical processes in the brain in terms of quantum-classical dynamics. This will also require to perform quantum-classical calculations of neural response functions. The implications of the interplay between the bipartite structures of both the world and the psyche will be investigated through the formulation of quantum-classical models of decision-making.

This section is not mandatory, but can be added to the manuscript if the discussion is unusually long or complex.

## References

1. von Bartheld, C.S.; Bahney, J.; and Herculano-Houzel, S. The Search for True Numbers of Neurons and Glial Cells in the Human Brain: A Review of 150 Years of Cell Counting. *Journal of Comparative Neurology* **2016**, *524*, 3865.
2. McIlwain, H.; Bachelard, H. S. *Biochemistry and the Central Nervous System*, Churchill Livingstone: Edinburgh, Scotland, 1985.
3. McFadden, J. Integrating information in the brain's EM field: the cemi field theory of consciousness. *Neuroscience of Consciousness* **2020**, *6*, niaa016.
4. McFadden, J. Synchronous Firing and Its Influence on the Brain's Electromagnetic Field. *Journal of Consciousness Studies* **2002**, *9*, 23.
5. McFadden, J. The CEMI Field Theory: Closing the loop. *Journal of Consciousness Studies* **2013**, *20*, 153.
6. Hales, C. G.; Pockett, S. The relationship between local field potentials (LFPs) and the electromagnetic fields that give rise to them. *Frontiers in Neuroscience* **2014**, *8*, 1-4.
7. Pockett, S.; Brennan, B. J.; Bold, G. E. J.; Holmes, M. D. A possible physiological basis for the discontinuity of consciousness. *Frontiers in Psychology* **2011**, *2*, 377.
8. Pockett, S.; Holmes, M. D. Intracranial EEG power spectra and phase synchrony during consciousness and unconsciousness. *Consciousness and Cognition* **2009**, *18*, 1049.
9. Liboff, A. R. Magnetic correlates in electromagnetic consciousness. *Electromagnetic Biology and Medicine* **2016**, *35*, 228.
10. Liboff, A. R. A human source for ELF magnetic perturbations. *Electromagnetic Biology and Medicine* **2016**, *35*, 337.
11. Fröhlich, F.; McCormick, D. A. Endogenous Electric Fields May Guide Neocortical Network Activity. *Neuron* **2010**, *67*, 129.
12. Penrose, R. On Gravity's Role in Quantum State Reduction. *General Relativity and Gravitation* **1996**, *8*, 581.
13. Penrose, R. On the Gravitization of Quantum Mechanics 1: Quantum State Reduction. *Found. Phys.* **2014**, *44*, 557.
14. Penrose, R. On the Gravitization of Quantum Mechanics 2: Conformal Cyclic Cosmology. *Found. Phys.* **2014**, *44*, 873.
15. Penrose, R. *The Emperor's New Mind*, Oxford University Press: Oxford, UK, 1989.
16. Penrose, R. *Shadows of the Mind*, Oxford University Press: Oxford, UK, 1994.
17. Hameroff, S.; Penrose, R. Consciousness events as orchestrated space-time selections. *Journal of Consciousness Studies* **1996**, *2*, 36.
18. Hameroff, S.; Penrose, R. Orchestrated reduction of quantum coherence in brain microtubules: A model for consciousness. *Mathematics and Computers in Simulation* **1996**, *40*, 453.
19. Hameroff, S.; Penrose, R. Consciousness in the universe. A review of the 'Orch OR' theory. *Physics of Life Reviews* **2014**, *11*, 39.
20. Hameroff, S.; Nip, A.; Porter, M.; Tuszynski, J. Conduction pathways in microtubules, biological quantum computation, and consciousness. *Biosystems* **2002**, *64*, 149.



21. Craddock, T. J. A.; Hameroff, S. R.; Ayoub, A. T.; Klobukowski, M.; Tuszynski, J. A. Anesthetics Act in Quantum Channels in Brain Microtubules to Prevent Consciousness. *Current Topics in Medicinal Chemistry* **2015**, *15*, 523.
22. Fisher, M. P. A. Quantum cognition: The possibility of processing with nuclear spins in the brain. *Annals of Physics* **2015**, *362*, 593.
23. Weingarten, C. P.; Doraiswamy, P. M.; Fisher, M. P. A. A new spin on neural processing: Quantum cognition. *Frontiers in Human Neuroscience* **2016**, *10*, 541.
24. Ettenberg, A.; Ayala, K.; Krug, J. T.; Collins, L.; Mayes, M. S.; Fisher, M. P. A. Differential effects of lithium isotopes in a ketamine-induced hyperactivity model of mania. *Journal of Pharmacology, Biochemistry and Behavior* **2020**, *190*, 172875.
25. Kerskens, C. M.; Pérez, D. L. Experimental indications of non-classical brain functions. *J. Phys. Communications* **2022**, *6*, 105001.
26. Hameroff, S. R. The Brain is Both Neurocomputer and Quantum Computer. *Cognitive Science* **2007**, *31*, 1035.
27. Vitiello, G. Dissipation and memory capacity in the quantum brain model. *International Journal of Modern Physics B* **1995**, *9*, 973.
28. Pessa, E.; Vitiello, G. Quantum dissipation and Neural Net Dynamics. *Bioelectrochemistry and Bioenergetics* **1999**, *48*, 339–342.
29. E.; Alfinito, Vitiello, G. The dissipative quantum model of brain: how does memory localize in correlated neuronal domain. *Information Sciences* **2000**, *128*, 217-229.
30. W. J. Freeman and G. Vitiello, The Dissipative Quantum Model of Brain and Laboratory Observations, in *Physics of Emergence and Organization*, 233-251 (World Scientific, Singapore, 2008).
31. Freeman, W. J.; Vitiello, G. Nonlinear brain dynamics as macroscopic manifestation of underlying many-body field dynamics. *Physics of Life Reviews* **2006**, *3*, 93.
32. Freeman, W. J.; Vitiello, G. Dissipative neurodynamics in perception forms cortical patterns that are stabilized by vortices. *J. Phys.: Conference Series* **2009**, *174*, 012011.
33. Vitiello, G. Fractals as macroscopic manifestation of squeezed coherent states and brain dynamics. *J. Phys.: Conference Series* **2012**, *380*, 012021.
34. Vitiello, G. The use of many-body physics and thermodynamics to describe the dynamics of rhythmic generators in sensory cortices engaged in memory and learning. *Current Opinion in Neurobiology* **2014**, *31*, 7.
35. Sabbadini, S. A.; Vitiello, G. Entanglement and Phase-Mediated Correlations in Quantum Field Theory. Application to Brain-Mind States. *Applied Sciences* **2019**, *9*, 3203.
36. Korzybski, A. *Science and Sanity. An Introduction to Non-Aristotelian Systems and General Semantics*, Institute of General Semantics: Fort Worth, US, 2005.
37. Kodish, S. B.; Kodish, B. I. *Drive Yourself Sane. Using the Uncommon Sense of General Semantics* Extensional Publishing: Pasadena, US, 2011.
38. Korzybski, A. *Alfred Korzybski: Collected Writings 1920-1950*, Institute of General Semantics: Englewood, US, 1990.
39. Christopher, P. They're Stealing Our General Semantics. *ETC* **1998**, *55*, 217.
40. Meier, C. A., Ed., *Atom and the Archetype: The Pauli/Jung Letters 1932-1958*, Princeton University Press: Princeton, US, 2014.
41. Atmanspacher, H.; Fuchs C., Eds., *The Pauli-Jung Conjecture*, Imprint Academics: Exter, UK, 2014.
42. Lindorss, D. *Pauli and Jung*, Quest Books: Wheaton Illinois, US, 2009.
43. Jung, C. G. *Synchronicity: An Acausal Connecting Principle*, Bollingen Foundation: Bollingen, Switzerland, 1993.
44. C. G.; Jung Pauli, W. *The Interpretation of Nature and Psyche*, Pantheon Books: New York, US, 1955.
45. Blanco, I. M. *The Unconscious as Infinite Sets. An Essay in Bi-logic*, Karnac Books: London, UK, 1980.
46. Blanco, I. M. *Thinking, Feeling, and Being. Clinical Reflections on the Fundamental Antinomy of Human Beings and World*, Routledge: London, UK, 1988.
47. E. Rayner, *Unconscious Logic. An Introduction to Matte Blanco's Bi-Logic and Its Uses*, Routledge: London, UK, 1995.
48. Lombardi, R. *Formless Infinity. Clinical Explorations of Matte Blanco and Bion*, Routledge: London, UK, 2015.
49. Goh, B. H.; Tong, E. S.; Pusparajah, P. Quantum Biology: Does quantum physics hold the key to revolutionizing medicine? *Prog. Drug. Discov. Biomed. Sci.* **2020**, *3*, a0000130.

50. Pessa, E.; Penna, M. P.; Bandinelli, P. L. Is quantum brain dynamics involved in some neuropsychiatric disorders? *Medical Hypotheses* **2000**, *54*, 767.
51. Schwartz, J. M.; Stapp, H. P.; Beauregard, M. Quantum physics in neuroscience and psychology: a neurophysical model of mind brain interaction. *Philosophical Transactions of the Royal Society B* **2005**, *360*, 1309.
52. Conant, R. C.; Ashby, W. R. Every good regulator of a system must be a model of that system. *Int. J. Systems Sci.* **1970**, *1*, 89.
53. Tanaka, S.; Umegaki, T.; Nishiyama, A.; Kitoh-Nishioka, H. Dynamical Free Energy Based Model for Quantum Decision Making. *Physica A* **2022**, *605*, 127979.
54. Khrennikov, A. Quantum-like modeling of cognition. *Frontiers in Physics* **2015**, *3*, 77.
55. Khrennikov, A. Quantum-like modeling: cognition, decision making, and rationality. *Mind & Society* **2020**, *19*, 307.
56. Busemeyer, J. R.; Bruza, P. D. *Quantum Models of Cognition and Decision*; Cambridge University Press: Cambridge, UK, 2012.
57. Khrennikov, A. *Ubiquitous Quantum Structure: from Psychology to Finances*; Springer: Berlin, Germany, 2010.
58. Bond, R. L.; He, Y.-H.; Ormerod, Thomas C. A quantum framework for likelihood ratios. *International Journal of Quantum Information* **2018**, *16*, 1850002.
59. Basieva, I.; Pandey, V.; Khrennikova, P. More Causes Less Effect: Destructive Interference in Decision Making. *Entropy* **2022**, *24*, 725.
60. Busemeyer, J. R.; Pothos, E.; Franco, R.; Trueblood, J. S. A quantum theoretical explanation for probability judgment 'errors'. *Psychological Review* **2011**, *118*, 193.
61. Van den Noort, M.; Lim, S.; Bosch, P. On the need to unify neuroscience and physics. *Neuroimmunology and Neuroinflammation* **2016**, *3*, 271.
62. Silin, V. P. The Kinetics of Paramagnetic Phenomena. *Zh. Teor. Eksp. Fiz.* **1956**, *30*, 421.
63. Rukhazade, A. A.; Silin, V. P. On the magnetic susceptibility of a relativistic electron gas. *Soviet Phys. JETP* **1960**, *11*, 463.
64. Balescu, R. A. Covariant Formulation of Relativistic Quantum Statistical Mechanics, I. Phase Space Description of a Relativistic Quantum Plasma. *Acta Phys. Aust.* **1968**, *28*, 336.
65. Zhang, W. Y.; Balescu, R. Statistical Mechanics of a spin-polarized plasma. *J. Plasma Phys.* **1988**, *40*, 199.
66. Balescu, R.; Zhang, W. Y. Kinetic equation, spin hydrodynamics and collisional depolarization rate in a spin polarized plasma. *J. Plasma Phys.* **1988**, *40*, 215.
67. Aleksandrov, I. V.; The Statistical Dynamics of a System Consisting of a Classical and a Quantum Subsystem. *Z. Naturforsch. A* **1981**, *36*, 902.
68. Gerasimenko, V. I. Dynamical equations of quantum-classical systems. *Theor. Math. Phys.* **1982**, *50*, 49.
69. Boucher, W.; Traschen, J. Semiclassical physics and quantum fluctuations. *Phys. Rev. D* **1988**, *37*, 3522.
70. Petrina, D. Y.; Gerasimenko, V. I.; Enolskii, V. Z. Equations of motion of one class of quantum-classical systems. *Sov. Phys. Dokl.* **1990**, *35*, 925.
71. Prezhdo, O. V.; Kisil, V. V. Mixing quantum and classical mechanics. *Phys. Rev. A* **1997**, *56*, 162.
72. Kapral, R.; Ciccotti, G. Mixed quantum-classical dynamics. *J. Chem. Phys.* **1999**, *110*, 8919.
73. Nielsen, S.; Kapral, R.; Ciccotti, G. Statistical mechanics of quantum-classical systems. *J. Chem. Phys.* **2001**, *115*, 5805.
74. Sergi, A. Non-Hamiltonian Commutators in Quantum Mechanics. *Phys. Rev. E* **2005**, *72*, 066125.
75. Sergi, A. Deterministic constant-temperature dynamics for dissipative quantum systems. *J. Phys. A* **2007**, *40*, F347.
76. Sergi, A.; Hanna, G.; Grimaudo, R.; Messina, A. Quasi-Lie Brackets and the Breaking of Time-Translation Symmetry for Quantum Systems Embedded in Classical Baths. *Symmetry* **2018**, *10*, 518.
77. Osborn, T. A.; Kondratëva, M. F.; Tabisz, G. C.; McQuarrie, B. R. Mixed Weyl symbol calculus and spectral line shape theory. *J. Phys. A Math. Gen.* **1999**, *32*, 4149.
78. Martens, C. C.; Fang, J. Y. Semiclassical-Limit Molecular Dynamics on Multiple Electronic Surfaces. *J. Chem. Phys.* **1996**, *106*, 4918.
79. Donoso, A.; Martens, C. C. Simulation of Coherent Nonadiabatic Dynamics Using Classical Trajectories. *J. Phys. Chem. A* **1998**, *102*, 4291.

80. Sergi, A.; Kapral, R. Quantum-Classical Limit of Quantum Correlation Functions. *J. Chem. Phys.* **2004**, *121*, 7565.
81. Uken, D. A.; Sergi, A. Quantum dynamics of a plasmonic metamolecule with a time-dependent driving. *Theor. Chem. Acc.* **2015**, *134*, 141.
82. Sergi, A.; Sinayskiy, I.; Petruccione, F. Numerical and Analytical Approach to the Quantum Dynamics of Two Coupled Spins in Bosonic Baths. *Phys. Rev. A* **2009**, *80*, 012108.
83. Sergi, A.; Kapral, R. Quantum-Classical Dynamics of Nonadiabatic Chemical Reactions. *J. Chem. Phys.* **2003**, *118*, 8566.
84. McFadden, J. *Quantum Evolution*; Norton: New York, USA, 2002.
85. Joos, E.; Zeh, H. D.; Kiefer, C.; Giulini, D.; Kupsch, J.; Stamatescu, I.-O. *Decoherence and the Appearance of a Classical World in Quantum Theory*, Springer: Berlin, Germany, 2003.
86. Zurek, W. H. Decoherence, einselection, and the quantum origins of the classical. *Rev. Mod. Phys.* **2003**, *75*, 715.
87. Tegmark, M. Importance of quantum decoherence in brain processes. *Phys. Rev. E* **2000**, *61*, 4194.
88. Stapp, H. P. The Copenhagen Interpretation. *Am. J. Phys.* **1972**, *40*, 1098.
89. von Neumann, J. *Mathematical Foundations of Quantum Mechanics*; Princeton University Press: Princeton, UK, 1983.
90. Schrödinger, E. *What is life? with Mind And Matter, and Autobiographical Sketches*, Cambridge University Press: Cambridge, UK, 2013.
91. Watson, J. D.; Crick, F. H. C. A structure for deoxyribose nucleic acid. *Nature* **1953**, *171*, 737.
92. Pray, L. Discovery of DNA structure and function: Watson and Crick. *Nature Education* **2008**, *1*, 100.
93. Beyler, R. From Positivism to Organicism: Pascual Jordan's Interpretations of Modern Physics in Cultural Context. Ph.D diss., Harvard University, Harvard, 1994.
94. Beyler, R. Targeting the Organism. The Scientific and Cultural Context of Pascual Jordan's Quantum Biology, 1932-1947. *Isis* **1996**, *87*, 248.
95. Al-Khalili, J.; McFadden, J. *Life on the Edge. The Coming of Age of Quantum Biology*; Bantam Press: London, UK, 2014.
96. Leggett, J. A.; Chakravarty, S.; Dorsey, A. T.; Fisher, M. P. A.; Garg, A.; Zwerger, W. Dynamics of the dissipative two state system. *Rev. Mod. Phys.* **1987**, *59*, 1.
97. Bakemeier, L.; Alvermann, A.; Fehske, H. Quantum phase transition in the Dicke model with critical and noncritical entanglement. *Phys. Rev. A* **2012**, *85*, 043821.
98. Hwang, M.-J.; Puebla, R.; Plenio, M. B. Quantum Phase Transition and Universal Dynamics in the Rabi Model. *Phys. Rev. Lett.* **2015**, *115*, 180404.
99. Finney, G. A.; Gea-Banacloche, J. Quasiclassical approximation for the spin-boson Hamiltonian with counterrotating terms. *Phys. Rev. A* **1994**, *50*, 2040.
100. Martyna, G. J.; Klein, M. L.; Tuckerman, M. Nosé-Hoover chains: The canonical ensemble via continuous dynamics. *J. Chem. Phys.* **1992**, *92*, 2635.
101. Sergi, A.; Ferrario, M. Non-Hamiltonian Equations of Motion with a Conserved Energy. *Phys. Rev. E* **2001**, *64*, 056125.
102. Sergi, A. Non-Hamiltonian Equilibrium Statistical Mechanics. *Phys. Rev. E* **2003**, *67*, 021101.
103. Riddle, J.; McFerren, A.; Frohlich, F. Causal role of cross-frequency coupling in distinct components of cognitive control. *Progress in Neurobiology* **2021**, *202*, 102033.
104. Riddle, J.; Scimeca, J. M.; Cellier, D.; Dhanani, S.; D'Esposito, M. Causal Evidence for a Role of Theta and Alpha Oscillations in the Control of Working Memory. *Current Biology* **2020**, *30*, 1748.
105. Abubaker, M.; Al Qasem, W.; Kvašňák, E. Working Memory and Cross-Frequency Coupling of Neuronal Oscillations. *Frontiers in Psychology* **2021**, *12*, 756661.
106. Croce, P.; Zappasodi, F.; Capotosto, P. Offline stimulation of human parietal cortex differently affects resting EEG microstates. *Scientific Reports* **2018**, *8*, 1287.
107. Caruana, F.; Gerbella, M.; Avanzini, P.; Gozzo, F.; Pelliccia, V.; Mai, R.; R. O.; Abdollahi, Cardinale, F.; Sartori, I.; Lo Russo, G.; Rizzolatti, G. Motor and emotional behaviours elicited by electrical stimulation of the human cingulate cortex, *Brain* **2018**, *141*, 3035.
108. Nitsche, M. A.; Paulus, W. Excitability changes induced in the human motor cortex by weak transcranial direct current stimulation. *Journal of Physiology* **2000**, *527*, 633.

109. Stagg, C. J.; Nitsche, M. A. Physiological Basis of Transcranial Direct Current Stimulation. *Neuroscientist* **2011**, *17*, 37.
110. Papazova, I.; Strube, W.; Wienert, A.; Henning, B.; Schwippel, T.; Fallgatter, A. J.; Padberg, F.; Falkai, P.; Plewnia, C.; Hasan, A. Effects of 1 mA and 2 mA transcranial direct current stimulation on working memory performance in healthy participants. *Consciousness and Cognition* **2020**, *83*, 102959.
111. Yavari, F.; Jamil, A.; Samani, M. M.; Vidor, L. P.; Nitsche, M. A. Basic and functional effects of transcranial Electrical Stimulation (tES)—an introduction. *Neurosci. Biobehav. Rev.* **2018**, *85*, 81–92.
112. Frölich, F.; McCormick, D. A. Endogenous Electric Fields May Guide Neocortical Network Activity. **2010**, *67*, 129.
113. Anastassiou, C. A.; Perin, R.; Markram, H.; Koch, C. Ephaptic coupling of cortical neurons. *Nature Neuroscience* **2011**, *14*, 217.
114. Martinez-Banaclocha, M. Ephaptic Coupling of Cortical Neurons: Possible Contribution of Astroglial Magnetic Fields? *Neuroscience* **2018**, *370*, 37.
115. Pinotsis, D. A.; Miller, E. K. Beyond dimension reduction: Stable electric fields emerge from and allow representational drift. *NeuroImage* **2022**, *253*, 119058.
116. Vicario, C. M.; Nitsche, M. A.; Hoysted, I.; Yavari, F.; Avenanti, A.; Salehinejad, M. A.; Felmingham, K. L. Anodal transcranial direct current stimulation over the ventromedial prefrontal cortex enhances fear extinction in healthy humans: A single blind sham-controlled study. *Brain Stimul.* **2020**, *13*, 489–491.
117. Ney, L. J.; Vicario, C. M.; Nitsche, M. A.; Felmingham, K. L. Timing matters: Transcranial direct current stimulation after extinction learning impairs subsequent fear extinction retention. *Neurobiol Learn Mem.* **2021**, *177*, 107356.
118. Markoviř, V.; Vicario, C. M.; Yavari, F.; Salehinejad, M. A.; Nitsche, M. A. A Systematic Review on the Effect of Transcranial Direct Current and Magnetic Stimulation on Fear Memory and Extinction. *Front Hum Neurosci.* **2021**, *22*, 655947.
119. Vicario, C. M.; Salehinejad, M. A.; Mosayebi-Samani, M.; Maezawa, H.; Avenanti, A.; Nitsche, M. A. Transcranial direct current stimulation over the tongue motor cortex reduces appetite in healthy humans. *Brain Stimul.* **2020**, *13*, 1121–1123.
120. Nunez, P. L.; Srinivasan, R. *The Neurophysics of EEG*, Oxford University Press: Oxford, UK, 2006.
121. Schrödinger, E. Die gegenwärtige Situation in der Quantenmechanik. *Naturwissenschaften* **1935**, *23*, 807.
122. Jaeger, G. *Entanglement, Information, and the Interpretation of Quantum Mechanics*, Springer: Berlin, Germany, 2009.
123. Rescher, N. *Process Metaphysics: An Introduction to Process Philosophy*; SUNY Press: New York, USA, 1996.
124. Capra, F. *The Tao of Physics*; Shambhala: Boston, USA, 2013.
125. Friston, K. A free energy principle for biological systems. *Entropy* **2012**, *14*, 2100.
126. J. Sánchez-Cañizares, The free energy principle: Good science and questionable philosophy in a grand unifying theory, *Entropy* **2021**, *23* 238.
127. Ellis, A.; Harper, R. A. *A New Guide to Rational Living*, Wilshire Books: North Hollywood, US, 1977.
128. Alfred Korzybski and Gestalt Therapy. Wysong, J. *The Gestalt Journal* **1998**, Available online: [www.gestalt.org/alfred.htm](http://www.gestalt.org/alfred.htm) (accessed on 9 January 2023).
129. Barlow, A. R. *The Derivation of a Psychological Theory: Gestalt Therapy*. PhD Thesis, University of Wollongong, Wollongong, Australia, 1983.
130. Minuchin, S. *Families and Family Therapy*, Harvard University Press: Cambridge Massachusetts, US, 1974.
131. Bowen, M. *Family Therapy in Clinical Practice*, Jason Aronson: New York, US, 1978.
132. Ballentine, L. E. *Quantum Mechanics*, World Scientific: Singapore, Republic of Singapore, 2001.
133. Weinberg, S. *Lectures on Quantum Mechanics*, Cambridge University Press: Cambridge, UK, 2013.
134. Jung, C. G. *The Archetypes and the Collective Unconscious*, Routledge: New York, US, 1991.
135. Deco, G.; Cruzata, J.; Cabral, J.; Tagliazucchi, E.; Laufs, H. Logothetis, N. K.; Kringelbach, M. L. Awakening: Predicting external stimulation to force transitions between different brain states. *PNAS* **2019**, *116*, 18088.
136. McCulloch, W. S.; Pitts, W. A logical calculus of the ideas immanent in nervous activity. *Bulletin of Mathematical Biophysics* **1943**, *5*, 115.
137. Caianiello, E. R. J. *Theor. Biol.* **1961**, *1*, 204.
138. Hodgikin, A. L.; Huxley, A. F. A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiology* **1952**, *117*, 500.



139. Schmitt, F. O.; *New Scientist* **1966**, 23, 643.
140. Arbib, M. *Brain Machines and Mathematics*, McGraw-Hill: London, UK, 1964.
141. Agnati, L. F.; Marcoli, M.; Maura, G.; Woods, A.; Guidolin, D. The brain as a “hyper-network”: the key role of neural networks as main producers of the integrated brain actions especially via the “broadcasted” neuroconnectomics. *J. Neural. Transm.* **2018**, 125, 883.
142. Callen, H. B. *Thermodynamics and an Introduction to Thermostatistics*; John Wiley & Sons: New York, USA, 1985.
143. Blundell, S. J.; Blundell, K. M. *Concepts in Thermal Physics*, Oxford University Press: Oxford, UK, 2006.
144. Ohya, M.; Petz, M. *Quantum Entropy and its Use*; Springer: Berlin, Germany, 1993.
145. Heusler, S.; Dür, W.; Ubben, M. S.; Hartmann, A. Aspects of entropy in classical and in quantum physics. *J. Phys. A: Math. Theor.* **2022**, 55, 404006.
146. O. Portmann, A. Glzer, N. Saratz, O. V. Billoni, D. Pescia, and A. Vindign, Scaling hypothesis for modulated systems. *Phys. Rev. B* **2010**, 82, 184409.
147. Borycki, D.; Marćkowiak, J. Reentrant behavior of superconducting alloys. *Supercond. Sci. Technol.* **2011**, 24, 035007.
148. Avraham, N.; Khaykovich, B.; Myasoedov, Y.; Rappaport, M.; Shtrikman, H.; Feldman, D. E.; Tamegai, T.; Kes, P. H.; Li, M.; Konczykowski, M.; van der Beek, K.; Zeldov, E. ‘Inverse’ melting of a vortex lattice. *Nature* **2001**, 411, 451. doi: 10.1038/35078021.
149. Wu, W. J.; He, Y. W.; Zhao, Z. G.; Liu, M.; Yang, Y. H. Inverse Melting of Vortex Lattice in Layered Superconductors. *International Journal of Modern Physics B* **2005**, 19, 451. doi: 10.1142/S0217979205028797
150. Mukamel, S. Trees to trap photons. *Nature* **1997**, 388, 425-427.
151. Jiang, D.-L.; Aida, T. Photoisomerization in dendrimers by harvesting of low-energy photons. *Nature* **1997**, 388, 454-456.
152. Sergi, A.; Grüning, M.; Ferrario, M.; Buda, F. A Density Functional Study of the PYP Chromophore. *Journal of Physical Chemistry B* **2001**, 105, 4386.
153. Zee, A. *Quantum Field Theory in a Nutshell*, Princeton University Press: Princeton, US, 2003.
154. Romijn, H. Are virtual photon the elementary carriers of consciousness? *J. Consciousness Study* **2002**, 9, 61-81.
155. Mandl, F.; Shaw, G. *Quantum Field Theory*, John Wiley & Sons: New York, US, 1990.
156. Różyk-Myrta, A.; Brodziak, A.; Muc-Wierzoń, M. Neural Circuits, Microtubule Processing, Brain’s Electromagnetic Field—Components of Self-Awareness. *Brain Sci.* **2021**, 11, 984.
157. Mattuck, R. D. *A Guide to Feynman Diagrams in the Many-Body Problem*; Dover: New York, USA, 1992.
158. Mahan, G. D. *Many-Particle Physics*, Kluwer: Dordrecht, The Netherlands, 2000.
159. Hameroff, S. R.; Watt, R. C. Information Processing in Microtubules. *J. theor. Biol.* **1982**, 98, 549.
160. Smith, S. A.; Watt, R. C.; Hameroff, S. R.; Cellular Automata In Cytoskeletal Lattices. *Physica* **1984**, 10D, 168.
161. Hameroff, S. R.; Smith, S. A.; Watt, R. C. Automaton Model of Dynamic Organization in Microtubules. *Annals of the New York Academy of Science* **1986**, 446, 949.
162. Rasmussen, S.; Karampurwala, H.; Vaidyanath, R.; Jensen, K. S.; Hameroff, S. Computational Connectionism Within Neurons: A Model Of Cytoskeletal Automata Subserving Neural Networks. *Physica D* **1990**, 42, 428.
163. Lahoz-Beltra, R.; Hameroff, S. R.; Dayhoff, J. E. Cytoskeletal logic: a model for molecular computation via Boolean operations in microtubules and microtubule-associated proteins. *BioSystems* **1993**, 29, 1.
164. Dayhoff, J.; Hameroff, S.; Lahoz-Beltra, R.; Swenberg, C. E.; Cytoskeletal involvement in neuronal learning: a review. *Eur. Biophys. J.* **1994**, 23, 79.
165. Kastner, R. E.; *The Transactional Interpretation of Quantum Mechanics*, Cambridge University Press: Cambridge, UK, 2013.
166. Kastner, R. E. *Understanding our unseen Reality. Solving Quantum Riddles*, Imperial College Press: London, UK, 2015.
167. Wick, D. *The Infamous Boundary. Seven Decades of Controversy in Quantum Physics*, Springer: Berlin, Germany, 1995.
168. Breuer, H.-P.; Petruccione, F. *The Theory of Open Quantum Systems* Oxford University Press: Oxford, UK, 2007.
169. Goodson, H. V.; Jonasson, E. M. Microtubules and Microtubule-Associated Proteins. *Cold Spring Harb. Perspect. Biol.* **2018**, 10, a022608.
170. Steiner, B.; Mandelkow, E.-M.; Biernat, J.; Gustke, N.; Meyer, H. E.; Schmidt, B.; Mieskes, G.; Soling, H. D.; Drechsel, D.; Kirschner, M. W.; Goedert, M.; Mandelkow, E. Phosphorylation of microtubule-associated

- protein tau: identification of the site for Ca<sup>2+</sup>-calmodulin dependent kinase and relationship with tau phosphorylation in Alzheimer tangles. *EMBO Journal* **1990**, *9*, 3539.
171. Waxham, M. N. Calcium-Calmodulin Kinase II (CaMKII) in Learning and Memory, In *Encyclopedia of Neuroscience* **2009**, 581-588.
  172. Baratier, J.; Peris, L.; Brocard, J.; Gory-Fauré, S.; Dufour, F.; Bosc, C.; Fourest-Lieuvin, A.; Blanchoin, L.; Salin, P.; Job, D.; Andrieux, A. Phosphorylation of Microtubule-associated Protein STOP by Calmodulin Kinase II. *J. Biol. Chem.* **2006**, *281*, 19561.
  173. Craddock, T. J. A.; Tuszynski, J. A.; Hameroff, S. Cytoskeletal Signaling: Is Memory Encoded in Microtubule Lattices by CaMKII Phosphorylation? *Comput. Biol.* **2012**, *8* e1002421.
  174. Vallano, M. L.; Goldenring, J. R.; Buckholz, T. M.; Larson, R. E.; Delorenzo, R. J. Separation of endogenous calmodulin- and cAMP-dependent kinases from microtubule preparations. *Proc. Natl. Acad. Sci.* **1985**, *82*, 3202.
  175. Gradin, H. M.; Marklund, U.; Larsson, N.; Chatila, T. A.; Gullberg, M. Regulation of Microtubule Dynamics by Ca<sup>2+</sup>/Calmodulin-Dependent Kinase IV/Gr-Dependent Phosphorylation of Oncoprotein 18. *Molecular and Cellular Biology* **1997**, *17*, 3459.
  176. Schulman, H.; Kuret, J.; Jefferson, A. B.; Nose, P. S.; Spitzer, K. H. Ca<sup>2+</sup>/Calmodulin-Dependent Microtubule-Associated Protein 2 Kinase: Broad Substrate Specificity and Multifunctional Potential in Diverse Tissues. *Biochemistry* **1985**, *24*, 5320.
  177. Craddock, T. J. A.; Kurian, P.; Tuszynski, J. A.; Hameroff, S. R. Quantum Processes in Neurophotons and the Origin of Brain's Spatiotemporal Hierarchy. In *Neurophotons and Biomedical Spectroscopy* Elsevier, Amsterdam, Holland, 2019; p. 189.
  178. Chang, J.-J.; Fisch, J.; Popp F.-A., Eds.; *Biophotons*, Springer: Dordrecht, Germany, 1998.
  179. Popp, F.-A.; Belousov L., Eds.; *Integrative Biophysics. Biophotons*, Springer: Dordrecht, Germany, 2003.
  180. Fels, D.; Cifra, M.; Scholkmann, F., Eds.; *Fields of the Cell*, Research Signpost: Kerala, India, 2015.
  181. Kurian, P.; Obisesan, T. O.; Craddock, T. J. A. Oxidative species-induced excitonic transport in tubulin aromatic networks: Potential implications for neurodegenerative disease. *J. Photochem. Photobiol. B Biol.* **2017**, *175*, 109.
  182. Haken, H.; Strobl, G. An exactly solvable model for coherent and incoherent exciton motion. *Z. Phys.* **1973**, *262*, 135.
  183. Abasto, D. F.; Mohseni, M.; Lloyd, S.; Zanardi, P. Exciton diffusion length in complex quantum systems: the effect of disorder and environmental fluctuations on symmetry-enhanced supertransfer. *Phil. Trans. R. Soc. A* **2012**, *172*, 3750.
  184. Celardo, C. L.; Giusteri, G. G.; Borgonovi, F. Cooperative robustness to static disorder: superradiance and localization in a nanoscale ring to model light-harvesting systems found in nature. *Phys. Rev. B* **2014**, *90*, 075113.
  185. Celardo, C. L.; Poli, P.; Lussardi, L.; Borgonovi, F. Cooperative robustness to dephasing: single-exciton superradiance in a nanoscale ring to model light-harvesting systems. *Phys. Rev. B* **2014**, *90*, 085142.
  186. Kalra, A. P.; Benny, A.; Travis, S. M.; Zizzi, E. A.; Morales-Sanchez, A.; Oblinski, D. G.; Craddock, T. J. A.; Hameroff, S. R.; Maclever, M. B.; Tuszynski, J. A.; Petry, S.; Penrose, R.; Scholes, G. D. Electronic Energy Migration in Microtubules. arXiv: 2208.10628 **2022**, available online: <https://arxiv.org/abs/2208.10628> (accessed on 9 January 2023).
  187. Veljkovic, V.; Veljkovic, N.; Esté, J. A.; Dietrich, U. Application of the EIIP/ISM Bionformatics in Development of New Drugs. *Current Medical Chemistry* **2007**, *14*, 133.
  188. L. M.; Ricciardi, Umezawa, H. Brain and Physics of Many-Body Problems. *Kybernetik* **1967**, *4*, 44-48.
  189. Umezawa, H.; Matsumoto, H.; Tachiki, M. *Thermo Field Dynamics and Condensed States*, North-Holland: Amsterdam, Holland, 1982.
  190. Umezawa, H. *Advanced Field Theory. Micro Macro Thermal Physics*, AIP: New York, US, 1995.
  191. Stone, M. H.; On One-Parameter Unitary Groups in Hilbert Space. *Annals of Mathematics* **1932**, *33*, 643.
  192. Stone, M. H. Linear Transformations in Hilbert Space: III. Operational Methods and Group Theory. *PNAS* **1930**, *16*, 172.
  193. Neumann, J. v. Über Einen Satz Von Herrn M. H. Stone. *Annals of Mathematics* **1932**, *33*, 567.
  194. Neumann, J. v. Die Eindeutigkeit der Schrödingerschen Operatoren. *Mathematische Annalen* **1931**, *104*, 570.

195. Nambu, Y. Quasiparticles and Gauge Invariance in the Theory of Superconductivity. *Phys. Rev.* **1960**, *117*, 648–663.
196. Goldstone, J. Field Theories with Superconductor Solutions. *Nuovo Cimento* **1961**, *19*, 154–164.
197. Goldstone, J.; Salam, A.; Weinberg, S. Broken Symmetries, *Phys. Rev.* **1962**, *27*, 965–970.
198. Flannery, J. S.; Riedel, M. C.; Bottenhorn, K. L.; Poudel, R.; Salo, T.; Hill-Bowen, L. D.; Laird, A. R.; Sutherland, M. T. Meta-analytic clustering dissociates brain activity and behavior profiles across reward processing paradigms. *Cognitive, Affective, and Behavioral Neuroscience* **2020**, *20*, 215.
199. Bhaduri, A.; Sandoval-Espinosa, C.; Otero-Garcia, M.; Oh, I.; Yin, R.; Eze, U. C.; Nowakowski, T. J.; Kriegstein, A. R. An atlas of cortical arealization identifies dynamic molecular signatures. *Nature* **2021**, *598*, 200.
200. Del Giudice, E.; Doglia, S.; Milani, M.; Vitiello, G. A Quantum Field Theoretical Approach to the Collective Behaviour of Biological Systems. *Nucl. Phys.* **1985**, *B251*, 375–400.
201. Del Giudice, E.; Doglia, S.; Milani, M.; Vitiello, G. Electromagnetic field and spontaneous symmetry breakdown in biological matter. *Nucl. Phys.* **1986**, *B275*, 185–199.
202. Del Giudice, E.; Vitiello, G.; Preparata, G. Water as a free electron laser. *Phys. Rev. Lett.* **1988**, *61*, 1085–1088.
203. Jibu, M.; Yasue, K. *Quantum brain dynamics and consciousness*; John Benjamins, Amsterdam, The Netherlands, 1995.
204. Jibu, M.; Yasue, K. What Is Mind? Quantum Field Theory of Evanescent Photons in Brain as Quantum Theory of Consciousness. *Informatica* **1997**, *21*, 471.
205. Preparata, G. *QED Coherence in Matter* World Scientific: Singapore, Republic of Singapore, 1995.
206. Ling, G. N. *Life at the Cell and Below-Cell Level*, Pacific Press: New York, US, 2001.
207. Blasone, M.; Vitiello, G.; Jizba, P. *Quantum Field Theory and its Macroscopic Manifestations. Boson Condensation, Ordered Patterns, and Topological Defects*; Imperial College Press: London, UK, 2011.
208. Nishiyama, A.; Tanaka, S.; Tuszynski, J. A. Nonequilibrium quantum brain dynamics: Super-Radiance and Equilibration in 2+1 Dimensions. *Entropy* **2019**, *21*, 1066.
209. Nishiyama, A.; Tuszynski, J. A. Non-Equilibrium  $\Phi^4$  theory for networks: toward memory formations with quantum brain dynamics. *J. Phys. Communications* **2019**, *3*, 055020.
210. Nishiyama, A.; Tanaka, S.; Tuszynski, J. A. Nonequilibrium quantum brain dynamics, Chap. 5 in *Advances in Quantum Chemistry* **82**, 159 **2020**,
211. Nishiyama, A.; Tanaka, S.; Tuszynski, J. A. Non-Equilibrium Quantum Brain Dynamics II: Formulation in 3+1 Dimensions. *Physica A* **2021**, *567*, 125706.

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