

Appendix:

In[]:=

Supplementary Information (Mathematica v13.1.0 code of TraditionalForm)

In[]:=

Biased Stochastic Process of Randomly Moving Particles with Constant Average Velocity

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NOTE:

1. The "Euclid Math One" regular and bold fonts are needed to display the contents correctly in this Notebook.
2. If there is no special case, the Mathematica code starts with gray " $In[\bullet] :=$ " and is bold by default according to Mathematica's rules.

Part 1. Necessary Calculation Processes

$$In[\bullet] := \text{Simplify} \left[\frac{\text{Variance} \left[\text{RandomWalkProcess} \left[\frac{1 + \frac{6p-1}{5}}{2}, \frac{1 - \frac{6p-1}{5}}{2} \right] [t] \right]}{\text{Variance} \left[\text{RandomWalkProcess} \left[\frac{1}{2} \right] [t] \right]} \right]$$

$$Out[\bullet] = -\frac{12}{25} (3p^2 - p - 2)$$

$$In[\bullet] := \text{Simplify} \left[\left(\frac{\sqrt{c^2 - \left(\frac{6p-1}{5} c \right)^2}}{c} \right)^2, \text{Assumptions} \rightarrow c > 0 \right]$$

$$Out[\bullet] = -\frac{12}{25} (3p^2 - p - 2)$$

Part 2. Process of Obtaining the Lorentz Factor for Randomly Moving Particles with Speed Following Maxwell Distribution

$$In[\bullet] := \text{PDF}[\text{TransformedDistribution}[a r, \{r \approx \text{MaxwellDistribution}[\lambda]\}], x]$$

$$Out[\bullet] = \begin{cases} \frac{\sqrt{\frac{2}{\pi}} x^2 e^{-\frac{x^2}{2a^2\lambda^2}}}{a^3 \lambda^3} & x > 0 \\ 0 & \text{True} \end{cases}$$

$$In[\bullet] := \text{PDF}[\text{MaxwellDistribution}[a \lambda], x]$$

$$Out[\bullet] = \begin{cases} \frac{\sqrt{\frac{2}{\pi}} x^2 e^{-\frac{x^2}{2a^2\lambda^2}}}{a^3 \lambda^3} & x > 0 \\ 0 & \text{True} \end{cases}$$

$$In[\bullet] := \text{Mean}[\text{MaxwellDistribution}[\lambda]]$$

$$Out[\bullet] = 2 \sqrt{\frac{2}{\pi}} \lambda$$

For the case on the x - or y -axis, we can regard the particles in $\mathcal{A}_i$ as a particle swarm possessing a mixed distribution with weight $w = \frac{c+u}{2c}$ (this code takes approximately 36 seconds).

`In[*]:= D = TransformedDistribution[r Cos[θ] Sin[ArcCos[η]], {θ ≈ UniformDistribution[{-π, π}],`

`η ≈ UniformDistribution[{-1, 1}], r ≈ MaxwellDistribution[ $\frac{1}{2} \sqrt{\frac{\pi}{2}} c$ ]}];`

`D1 = TransformedDistribution[r Cos[θ] Sin[ArcCos[η]], {θ ≈ UniformDistribution[{-π, π}],`

`η ≈ UniformDistribution[ $\{\frac{u}{c}, 1\}$ ], r ≈ MaxwellDistribution[ $\frac{1}{2} \sqrt{\frac{\pi}{2}} c$ ]}];`

`D2 = TransformedDistribution[r Cos[θ] Sin[ArcCos[η]], {θ ≈ UniformDistribution[{-π, π}],`

`η ≈ UniformDistribution[ $\{-1, \frac{u}{c}\}$ ], r ≈ MaxwellDistribution[ $\frac{1}{2} \sqrt{\frac{\pi}{2}} c$ ]}];`

`w =  $\frac{c + u}{2c}$ ;`

`D12 = MixtureDistribution[{w, 1 - w}, {D1, D2}];`

`σu = Simplify[StandardDeviation[D12], Assumptions → 0 < u < c]`

`Out[*]=  $\frac{1}{2} \sqrt{\frac{\pi}{2}} \sqrt{c^2 - u^2}$`

`In[*]:= Simplify[σu/StandardDeviation[D], Assumptions → 0 < u < c]`

`Out[*]=  $\frac{\sqrt{c^2 - u^2}}{c}$`

When $c = 10$ and $u = 6$, the distribution of $\mathcal{D}12$ on the x- or y-axes is like this (this code takes approximately 18 seconds):

`In[*]:= c = 10;`
`u = 6;`
`data = RandomVariate[D12, 30 000 000];`
`D0 = SmoothKernelDistribution[data, {"Adaptive", Automatic, Automatic}];`
`s2 = Plot[PDF[D0, x], {x, -20, 20}, PlotRange → {{-21, 21}, {0, 0.091}},`
`PlotStyle → {Blue, Thickness → 0.004}, AxesLabel → {HoldForm[Speed], HoldForm[Probability Density]},`
`AxesStyle → Directive[Black, Thickness → 0.0018], TicksStyle → Directive[Black, Thickness → 0.0014],`
`LabelStyle → Directive[Black, FontFamily → "Arial", FontSize → 15]]`

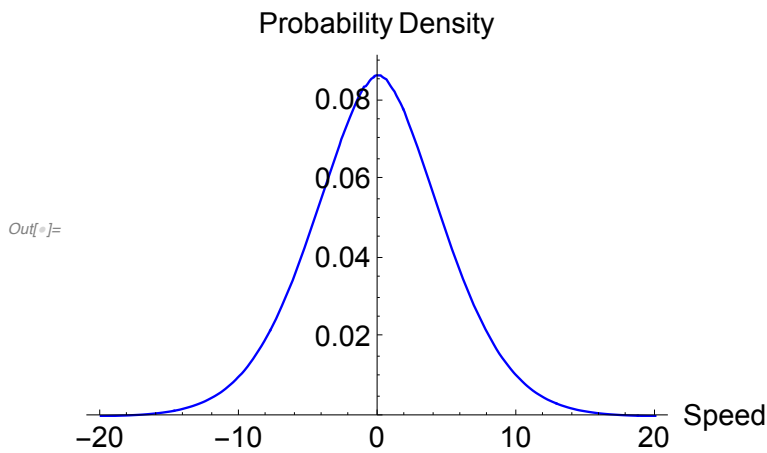


Figure S1 | Simulated probability density of the mixed distribution $\mathcal{D}12$ when $\bar{r} = c = 10$ and $u = 6$.

For the case on the z-axis:

```

In[ ]:= w =  $\frac{c + u}{2 c}$ ;

D3 = TruncatedDistribution[ $\left\{r \frac{u}{c}, r\right\}$ , UniformDistribution[ $\{-r, r\}$ ]];

D4 = TruncatedDistribution[ $\left\{-r, r \frac{u}{c}\right\}$ , UniformDistribution[ $\{-r, r\}$ ]];

D34 = MixtureDistribution[{w, 1 - w}, {D3, D4}];

Simplify[StandardDeviation[D34], Assumptions  $\rightarrow r > r \frac{u}{c} > 0$ ]

Out[ ]:=  $\frac{r \sqrt{1 - \frac{u^2}{c^2}}}{\sqrt{3}}$ 

In[ ]:= StandardDeviation[UniformDistribution[ $\{-r, r\}$ ]]

Out[ ]:=  $\frac{r}{\sqrt{3}}$ 

In[ ]:= Dmz = TransformedDistribution[ $\frac{r \sqrt{1 - \frac{u^2}{c^2}}}{\sqrt{3}}, r \approx \text{MaxwellDistribution}\left[\frac{1}{2} \sqrt{\frac{\pi}{2}} c\right]$ ];

Dz = TransformedDistribution[ $\frac{r}{\sqrt{3}}, r \approx \text{MaxwellDistribution}\left[\frac{1}{2} \sqrt{\frac{\pi}{2}} c\right]$ ];

Simplify[Mean[Dmz]/Mean[Dz], Assumptions  $\rightarrow c > 0$ ]

Out[ ]:=  $\sqrt{1 - \frac{u^2}{c^2}}$ 

```

Part 3. Calculation Process about Ito Equation

```

In[ ]:= Clear[v]

proc = ItoProcess[dx[t] == v dt +  $\sqrt{1 - v^2}$  σ dw[t], x[t], {x, 0}, t, w ≈ WienerProcess[]];

PDF[proc[t], x] // Simplify

PDF[WienerProcess[v,  $\sqrt{1 - v^2}$  σ][t], x] // Simplify

SliceDistribution[WienerProcess[v,  $\sqrt{1 - v^2}$  σ], t]

Out[ ]:=  $\frac{e^{\frac{(x-t)v^2}{2\sigma^2 t(v^2-1)}}}{\sqrt{2\pi} \sqrt{\sigma^2 (-t)(v^2-1)}}$ 

Out[ ]:=  $\frac{e^{\frac{(x-t)v^2}{2\sigma^2 t(v^2-1)}}}{\sqrt{2\pi} \sigma \sqrt{t} \sqrt{1-v^2}}$ 

Out[ ]:= NormalDistribution[t v, σ  $\sqrt{t} \sqrt{1-v^2}$ ]

```

```
In[ ]:= proc = ItoProcess[dx[t] == v dt + sqrt(1 - v^2) sigma dw[t], x[t], {x, 0}, t, w ~ WienerProcess[]];
proc["KolmogorovForwardEquation"] // TraditionalForm
```

Out[]//TraditionalForm=

$$p^{(0,1)}(x, t) = \frac{1}{2} \frac{\partial^2 ((1 - v^2) \sigma^2 p(x, t))}{\partial x \partial x} - \frac{\partial (v p(x, t))}{\partial x}$$

Part 4. Figures Used in the Main Text

NOTE: To run these codes correctly, the contents in "MyDirection = **" in the next cell should be modified. It is similar to MyDirection = "/Users/yourdirection/". Then, run it (Shift+Enter) beforehand.

```
In[ ]:= MyDirection = "/Users/gotall/Library/Mobile
Documents/com~apple~CloudDocs/SPaper/Normal Paper/conti/zhengshi/LaTeX/";
Protect[MyDirection];
Off[General::wrsym];

### ### ### ### ### ### ### ### ### ### Figure1 ### ### ### ### ### ### ### ### ### ###
```

```
In[ ]:= v = 0.6;
sigma = 1;

proc = WienerProcess[v, sqrt(1 - v^2) sigma];
SeedRandom[123];
sample = Table[RandomFunction[proc, {0, 50, 0.1}, 3][["ValueList"]^T, {500}];
figure1 = Graphics3D@Table[{ColorData["SolarColors"][RandomReal[]], Line@sample[[i]], {i, 500}}];
Export[MyDirection <> "figure1.png", figure1, Background -> None, ImageResolution -> 700];

### ### ### ### ### ### ### ### ### ### Figure1 ### ### ### ### ### ### ### ### ### ###
```