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The mass of our observable Universe

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ABSTRACT
The standard Cosmological model (LCDM) assumes that the expanding spacetime around us is infinite, which is inconsistent with the observed cosmic acceleration unless we include Dark Energy (DE) or a Cosmological Constant (Λ). But the observed cosmic expansion can also be explained with a finite mass M, inside a uniform expanding sphere, with empty space outside. An object with mass M has a gravitation radius $r_S = 2GM$. When M is all contained within r_S, this is a Black Hole (BH). Nothing can escape from r_S, which becomes a boundary for the inside dynamics. In the limit where there is nothing outside, the inside corresponds to a local isolated Universe. The r_S boundary condition (or surface term) corresponds to an effective force which is mathematically equivalent to $\Lambda = 3/r_S^2$. We can therefore interpret cosmic acceleration as a measurement of the gravitational boundary of our Universe, with a mass $M = \frac{c^2}{2G}\sqrt{3/\Lambda} \simeq 6 \times 10^{22} M_\odot$. Such BH Universe (BHU) is observationally very similar to the LCDM, except for the very large scale perturbations, which are bounded by r_S.

1 INTRODUCTION

The standard cosmological model (Dodelson 2003; Weinberg 2008), also called LCDM, assumes that our Universe correspond to a global space-time that began in a hot Big Bang (BB) expansion at the very beginning time. Such initial conditions seem to violate the classical concept of energy conservation and are very unlikely (Tolman 1931; Dyson et al. 2002; Penrose 2006; Brandenberger 2017). According to this singular start BB model, the full observable Universe came out of (macroscopic) nothing, resulting from some Quantum Gravity vacuum fluctuations that we can only speculate about. We will never be able to test experimentally these ideas because of the enormous energies involved ($10^{19} GeV$) and here is no direct evidence that this ever occurred. The BB model also requires three more exotic patches: Cosmic Inflation, Dark Matter and Dark Energy (DE), for which we have no direct evidence or understanding at any fundamental level.

Despite these shortfalls, the LCDM model seems very successful in explaining most observations by fitting just a handful of free cosmological parameters. Here we discuss the Black Hole Universe (BHU) as an alternative paradigm to the LCDM and elaborate that the main difference between these two approaches reside in whether the mass-energy of our Universe is finite or not.

The fact that the universe might be generated from the inside of a BH has been studied extensively in the literature. Pathria (1972) and Good (1972) proposed that the FLRW metric could be the interior of a BH. But these early proposals were not proper GR solutions, but just incomplete analogies (see Knutsen 2009). Stuckey (1994) presented a rigorous demonstration within classical GR that that the FLRW metric could be inside a BH, something that was also clear from the works of Oppenheimer & Snyder (1939). But many of the previous approaches (Smolin 1992; Easson & Brandenberger 2001; Daghigh et al. 2000; Firouzjahi 2016; Popławski 2016; Oshita & Yokoyama 2018; Dymnikova 2019) involve modifications to Classical GR. There are also some simple scalar field $\varphi(x)$ examples (e.g. Daghigh et al. 2000) which presented models within the scope of a classical GR and classical field theory with a false vacuum interior. A particular case are Bubble or Baby Universe solutions where the BH interior is de-Sitter metric (Gonzalez-Diaz 1981; Grøn & Soleng 1989; Blau et al. 1987; Frolov et al. 1989; Aguirre & Johnson 2005; Mazur & Mottola 2015; Garriga et al. 2016; Kusenko 2020). In the BHU solution (Gaztañaga 2022a,b,c,d) no Bubble is needed and the inside is not filled with a false vacuum, but with regular matter and radiation. The gravitational radius r_S of such regular mass-energy inside plays the role of a surface term. The difference between the recent BHU and the previously cited works of Oppenheimer & Snyder (1939) and Stuckey (1994), is the role of the gravitational radius r_S (or effective Λ term) as a boundary condition in the Einstein-Hilbert action (Gaztañaga 2022c). The model by Zhang (2018) has the same name and similar features as the BHU, but the model was proposed as a new postulate to GR and not as a classical GR solution.

Here, we will start by reviewing in §2 the classical GR solutions to the problem of a uniform spherical ball with fixed mass. In §3 we will interpret what we mean by a fixed mass and we will then focus in §4 on the corresponding BH solutions to show that the observed cosmic expansion is consistent with the BHU idea. We end with some conclusion and discussion of how the BHU form in §5.

2 A UNIFORM SPHERICAL BALL OF FIXED MASS

A metric $g_{\mu\nu}$ with spherical symmetry in spherical coordinates and proper time $dx^\mu = (d\tau, d\chi, d\theta, d\phi)$ can be expressed as (Tolman 1934; Oppenheimer & Snyder 1939; Misner & Sharp 1964):

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + e^{\lambda(\tau,\chi)} d\chi^2 + r^2(\tau,\chi) d\Omega^2, \quad (1)$$

where we use units of speed of light $c = 1$ and the radial coordinate χ will be taken here to be comoving with the matter content. This metric is sometimes called the Lemaitre-Tolman metric (Lemaître 1927; Tolman 1934) or the Lemaitre-Tolman-Bondi (LTB) metric. This is a metric localized around a reference central point in space which we have set to be the origin ($\vec{r} = 0$) for simplicity.

For an observer moving with a perfect fluid the energy-momentum tensor is diagonal: $T_\mu^\nu = diag[-\rho, p, p, p]$, where $\rho = \rho(\tau, \chi)$ is the energy density and $p = p(\tau, \chi)$ is the pressure. The off-diagonal

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terms of the field equation are zero, e.g.: $8\pi G T^1_0 = G^1_0 = 0$, which translates into:

$$(\partial_\tau \lambda)(\partial_\chi r) = 2\partial_\tau(\partial_\chi r) \tag{2}$$

This equation can be solved as: $e^\lambda = C(\partial_\chi r)^2$, where $\partial_\tau C = 0$. The case $C = 1$ corresponds a flat geometry:

$$ds^2 = -d\tau^2 + [\partial_\chi r]^2 d\chi^2 + r^2 d\Omega^2, \tag{3}$$

so that there is only one function we need to solve: $r = r(\tau, \chi)$ which corresponds to the radial proper distance. The field equations for r (with $\Lambda = 0$) are:

$$H^2 \equiv r_H^{-2} \equiv \left(\frac{\dot r}{r}\right)^2 = \frac{2GM}{r^3} \tag{4}$$

$$M \equiv \int_0^\chi \rho\, 4\pi r^2 (\partial_\chi r) d\chi = M(\tau, \chi) \tag{5}$$

where the dot here is $\dot r \equiv \partial_\tau r$. When $\rho = \rho(\tau)$ is uniform, we have $M = \frac{4\pi}{3} r^3 \rho$ and $r = a(\tau)\chi$, so that Eq.3 reproduces the flat (global) FLRW metric and Eq.4 is the corresponding solution: $3H^2(\tau) = 8\pi G \rho(\tau)$, which corresponds to a global solution (any point can be chosen as to be the center of the metric $\vec r = 0$). The non flat case can also be reproduced if we consider the more general case $e^\lambda = (\partial_\chi r)^2/[1 + K(r)]$. For matter dominated universe with $\rho = \rho_0 a^{-3}$ we have a constant mass inside χ: $M = \frac{4\pi}{3}\chi^3 \rho_0$. More generally M could be a function of time. But at any given time, the total mass M_T is always infinite: $M_T \equiv M(\chi < \infty) = \infty$.

The solution in Eq.4-5 can also be used to solve non-homogeneous cases. The simplest is the case of an expanding (or collapsing) uniform spherical ball inside a comoving coordinate χ_*:

$$\rho(\tau, \chi) = \begin{cases} \rho(\tau) & \text{for } \chi \le \chi_* \\ 0 & \text{for } \chi > \chi_* \end{cases} \tag{6}$$

which reproduces the exact same FLRW metric and solution $r = a\chi$ with the same mass inside $\chi < \chi_*$ as in the infinite FLRW case. The Hubble-Lemaitre expansion law: $\dot r = H(\tau) r$ of Eq.4 with $3H^2(\tau) = 8\pi G \rho(\tau)$ is the same inside χ_*. The only difference is that this is now a local FLRW solution with empty space outside χ_*, so that the total mass is finite:

$$M_T \equiv M(\chi < \infty) = M(\chi < \chi_*) = \frac{4}{3}\pi R^3 \rho(\tau) < \infty \tag{7}$$

where $R \equiv a\chi_*$. This is a consequence of *Birkhoff's theorem* (see Johansen & Ravndal 2006; Faraoni & Atieh 2020), since a sphere cut out of an infinite uniform distribution has the same spherical symmetry. Thus, the FLRW metric is both a solution to a global homogeneous (i.e. infinite M_T) uniform background and also to the inside of a local (finite mass) uniform sphere centered around one particular point. The local solution is called the FLRW cloud (Gaztañaga 2022c).

3 RELATIVISTIC MASS

Misner & Sharp 1964) mass-energy M_{MS} inside a spatial hypersurface Σ of Eq.1, given by $r < R$ or $\chi < \chi_*$, is:

$$M_{MS} = \int_0^R \rho 4\pi r^2 dr = \int_0^{\chi_*} \rho \left(1 + \frac{\dot r^2}{c^2} - \frac{2GM_{MS}}{c^2 r}\right)^{1/2} dV_3 \tag{8}$$

where $dV_3 = d^3y\sqrt{-h} = 4\pi r^2 e^{\lambda/2} d\chi$ is the 3D spatial volume element of the metric in Σ (we recover here units if $c \neq 1$ to check the non relativistic limit). The first term is the material or passive mass (which we call here M):

$$M = \int_\Sigma \rho\, dV_3 = \int_0^{\chi_*} \rho\, 4\pi r^2 (\partial_\chi r) d\chi \tag{9}$$

and corresponds to Eq.5. We can then interpret the next two terms in Eq.8 as the contribution to mass energy from the kinetic and potential energy (see also Hayward (1996) for a more general description). In the non relativistic limit ($c = \infty$) these two terms are negligible and $M_{MS} = M$. But in general, as indicated by Eq.8, M_{MS} can not be expressed as a sum of individual energies as M_{MS} also appears inside the integral, reflecting the non-linear nature of gravity. But in the case of Eq.4, the kinetic and potential energy terms cancel for $M = M_{MS}$ and we can interpret M as the total mass-energy of the system.

For the case in Eq.4-5, the mass inside χ is constant for matter dominated fluid when $\rho \sim a^{-3}$. But in the early stages of the expansion, when the Energy-density is dominated by radiation or a fluid with a different equation of state the mass inside χ is a function τ. If we want M_T in Eq.7 to be constant throughout the evolution we need the junction χ_* in Eq.6 to be a function of time τ:

$$R^3(\tau) \equiv a^3(\tau)\chi_*^3(\tau) = \frac{3M_T}{4\pi\rho(\tau)} \tag{10}$$

Note that both R and χ_* here are just radial coordinates and not proper distances between events. A different approach to the problem of having a finite fixed mass M_T is the use of junction conditions to match the FLRW solution inside R to a Schwarzschild solution outside, see Gaztañaga (2022c).

4 BLACK HOLE SOLUTION

For the local top-hat distribution of Eq.6, if all the mass M_T is contained within $R < r_S = 2GM$ the solution correspond to a Black Hole (BH, see Firouzjaee & Mansouri 2010). In the limit where the exterior is empty, the gravitational radius r_S should be interpreted as a boundary that separates the interior ($r < r_S$) from the exterior manifold. This creates and isolated Universe inside with a boundary condition at r_S. Having a boundary condition or surface term changes the Einstein-Hilbert action and therefore the field equations. Without surface terms, but including a Λ term:

$$S = \int_{V_4} dV_4 \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}\right], \tag{11}$$

where $dV_4 = \sqrt{-g}\, d^4x$ is the invariant volume element, V_4 is the volume of the 4D spacetime manifold, $R = R^\mu_\mu = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar curvature and $\mathcal{L}$ the Lagrangian of the energy-matter content. We can obtain Einstein's field equations for the metric field $g_{\mu\nu}$ by requiring S to be stationary $\delta S = 0$ under arbitrary variations of the metric $\delta g^{\mu\nu}$. The solution is (e.g. Padmanabhan 2010):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}, \tag{12}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ and $\mathcal{L}$ is the matter Lagrangian. The field equations in Eq.12 require that boundary or surface terms vanish (e.g. see Padmanabhan 2010). Otherwise, we need to add a Gibbons-Hawking-York (GHY) boundary term (York 1972; Gibbons & Hawking 1977; Hawking & Horowitz 1996) to the action:

$$S = \int_{V_4} dV_4 \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}\right] + \frac{1}{8\pi G} \oint_{\partial V_4} d^3y \sqrt{-h}\, K. \tag{13}$$

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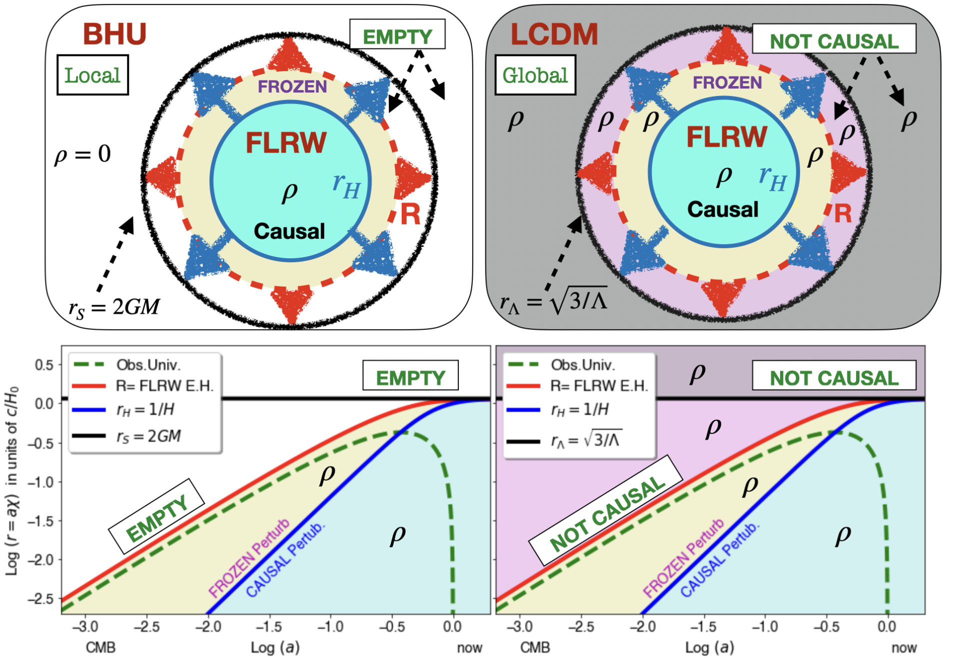

Figure 1. TOP PANELS: Comparison of the proper radius $r = a\chi$ of the BHU (left) and the LCDM (right) models. The blue circle represents a sphere of radius $r_H = 1/H$. The red one corresponds to the future event horizon R of the FLRW metric inside. Both r_H and R expand towards the fixed black sphere: the gravitational radius r_S in the BHU and $r_\Lambda = \sqrt{\Lambda/3}$ in LCDM. Scales $r > R$ are not causally connected, despite this, the LCDM has the same density everywhere. BOTTOM PANELS: evolution of the the different radius as a function of time (given by the scale factor a) for $\Omega_\Lambda = 0.7$. The dashed green line is the past light-cone or observable Universe: $a \int_a^1 \frac{da}{Ha^2}$. We can only see photons emitted in the past along this green line radial trajectory (in all directions).

where K is the trace of the extrinsic curvature at the boundary ∂V_4 and h is the induced metric. Gaztañaga (2022c) showed how when the FLRW evolution happens inside r_S, we have that $K = -2/r_S$ (independently of Λ), and such GHY boundary generates an effective Λ_e term, $\Lambda_e = 3/r_S^2$ (even if we started from $\Lambda = 0$), so that:

$$S_{GHY} = \frac{1}{8\pi G}\int d\tau\, 4\pi r_S^2\, K = \int_{V_4} dV_4 \left[\frac{-2\Lambda_e}{16\pi G}\right] = -\frac{r_S^3 \Lambda_e}{3G}\tau \quad (14)$$

So we have that the new action, including the surface term, is the same as the original action but it has, in principle, two degenerate contributions to the observed Λ value, a possible raw value Λ_{raw} (that we added a priori by hand) and the surface term $\Lambda_e = 3/r_S^2$:

$$\Lambda = \Lambda_{raw} + \Lambda_e = \Lambda_{raw} + \frac{3}{r_S^2} \quad (15)$$

The fact that a GHY surface term from r_S mimics a Λ term was originally proposed in Gaztañaga (2021) and also developed in Gaztañaga (2022b). This changes the field equations Eq.4 into:

$$H^2 = \frac{2GM}{r^3} + \frac{\Lambda}{3} = \frac{8\pi G}{3}\sum_i \rho_i^0 a^{-3(1+\omega_i)} + \frac{\Lambda}{3} \quad (16)$$

where in the last step we have used $r = a\chi$ and $\rho = \sum_i \rho_i^0 a^{-3(1+\omega_i)}$, with $\omega_i \equiv p_i/\rho_i$ is the equation of state of each fluid component. Note that Eq.5 does not change when we modify the field equations with a Λ term because it results from energy conservation: $\nabla_\mu T^\mu_\nu = 0$ (while Eq.4 comes from the G_0^0 component of Einstein's field equations Eq.12). Also note that we still have $M = M_{MS}$, because the Λ term adds another contribution to M_{MS} in Eq.12:

$$M_{MS} = \int_\Sigma \rho \left(1 + \frac{\dot{r}^2}{c^2} - \frac{2GM_{MS}}{c^2 r} - \frac{\Lambda}{3}r^2\right)^{1/2} dV_3 \quad (17)$$

which cancels out with the new Hubble law in Eq.16. In terms of $\Omega_i = \rho_i/\rho_c$, where $\rho_c = 3H_0^2/(8\pi G)$:

$$H^2 = H_0^2\left[\sum_i \Omega_i a^{-3(1+\omega_i)}\right] + \frac{\Lambda}{3} \quad (18)$$

where typically we have $i = \{1, 2\}$ with $\omega_1 = 0$ for matter and $\omega_2 = 1/3$ for radiation. This is exactly what we measure in cosmological surveys. It shows how we can interpret the observed cosmic expansion as being a local BH solution, the BHU (with $\Lambda_e = 1/r_S^2$ instead of a global LCDM model with Λ_{raw} or DE). If there is also dark energy (DE) and/or Λ_{raw} we have to add all contributions:

$$H^2 = H_0^2\left[\sum_i \Omega_i a^{-3+\beta_i} + \Omega_{DE} a^{-3(1+\omega_{DE})}\right] + \frac{\Lambda_{raw}}{3} + \frac{1}{r_S^2}, \quad (19)$$

where $\omega_{DE} = p_{DE}/\rho_{DE}$ is the DE equation of state. Given that the last 3 terms are approximately constant (for $\omega_{DE} = -1$) there is no way to distinguish them using measurements of the Hubble-Lemaitre law. Current observations tell us that indeed $\omega_{DE} \simeq -1$ (DES Collaboration 2019) and the sum of the 3 terms is a new effective Λ term such that $\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2} \simeq 0.7$ or, in other words:

$$\Lambda \equiv 3/r_\Lambda^2 \equiv 3\Omega_\Lambda H_0^2 \equiv 8\pi G\rho_{DE} + \Lambda_{raw} + 3/r_S^2 \simeq 2.1 H_0^2, \quad (20)$$

If $\rho_{DE} = \Lambda_{raw} = 0$, we have that $r_S = r_\Lambda$ and the mass of our Universe is:

$$M = \frac{c^2}{2G}\sqrt{3/\Lambda} = \frac{c^2}{2G}\Omega_\Lambda^{-1/2} H_0^{-1} \simeq 6 \times 10^{22} M_\odot \quad (21)$$

where we have returned to units of $c \neq 1$ here to be more explicit. This corresponds to the BHU. In the other limit, if $r_S = \infty$ we recover the infinite LCDM model with $M_T = \infty$ and $\Omega_\Lambda = \Omega_{DE} \simeq 0.7$. Both

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models have a future Event Horizon (E.H.) R::

$$R(a) = a \int_a^\infty \frac{da}{Ha^2} < r_\Lambda \tag{22}$$

which is bounded by $R < r_\Lambda$ (Gaztañaga 2022c): anything that is at $r > R(a)$ is outside causal reach. Here $R(a)$ is the maximum proper distance that a photon can travel. Fig.1 shows a comparison of the two possible interpretations.

We could also have an intermediate situation, but it would be quite unnatural that r_S (from M_T) and r_Λ (from DE or Λ_{raw}) are fine tuned to both contribute to the observed cosmic acceleration. We therefore have to choose one of the two interpretations. Given that $M_T = \infty$ is a non physical (and not causally possible) solution and that we do not know what DE or Λ_{raw} is, it seems more plausible and simpler to interpret cosmic acceleration as a measurement for the mass M_T of our Universe. In this case $\rho_{DE} \simeq \Lambda_{raw} \simeq 0$ and our Universe is inside a local BHU.

5 CONCLUSION

We have interpreted the observed Λ to be an effective term that corresponds to the gravitational radius $r_S = \sqrt{3/\Lambda} = 2GM$ of our local Universe. This has several implications:

- The mass of our Universe can be estimated to be: $M \simeq 6 \times 10^{22} M_\odot$ (see Eq.21), which agrees well with what we have observed in the largest Galaxy Surveys, such as DES Collaboration (2019).
- Our Universe is a local solution inside its gravitational radius r_S. It is therefore a Black Hole Universe (BHU).
- The dynamical time associated to M in our Universe is $\tau \sim GM \sim 14 Gyr$, which is close to the measured age of the oldest galaxies and stars that we observe.
- An observer placed anywhere within the local BHU measures the same background as one within the LCDM. This becomes obvious when we note that the future event horizon R in Eq.22 is a null geodesic and therefore no signal can reach us from outside R, no matter how close the observer is to the boundary (see Gaztañaga 2022c for more details).
- The smoking gun of the BHU is a cut-off in the scale of the largest perturbations which has already been measured in CMB maps (Fosalba & Gaztañaga 2021; Gaztañaga & Camacho-Quevedo 2022).
- Our BHU might not be unique: there could also exist other Universes, like ours, elsewhere. This is part of the The Copernican Revolution: our place is not special.

How did such a BHU form? Here we enter the more speculative grounds of the Big Bang theories. The BHU could have formed in a similar way as the standard LCDM universe: out of one of the many existing models of Cosmic Inflation (Starobinskiĭ 1979; Guth 1981; Linde 1982; Albrecht & Steinhardt 1982; Liddle 1999) or from some of the Quantum Gravity alternatives (e.g. Easson & Brandenberger 2001; Novello & Bergliaffa 2008; Popławski 2016; Brandenberger 2017; Ijjas & Steinhardt 2018).

The BHU could have also formed in a much simpler way, just like the first stars: collapsing and exploding in a Supernova following the known laws of Physics (Gaztañaga 2022d). Left panel of Figure 2 shows the Crab Nebula, a remnant of a Supernova, which can be thought as a small version of our Universe today. In such case, Cosmic Inflation or Quantum Gravity explanations are not needed to understand the origin of our cosmic expansion. Such beginning would provide us with an anthropic explanation for the observed value of r_S and the coincidence problem. It also yields new candidates for Dark Matter in the form of compact remnants of the collapse and bouncing phases, such as primordial BHs and primordial Neutron Stars (Gaztañaga 2022d).

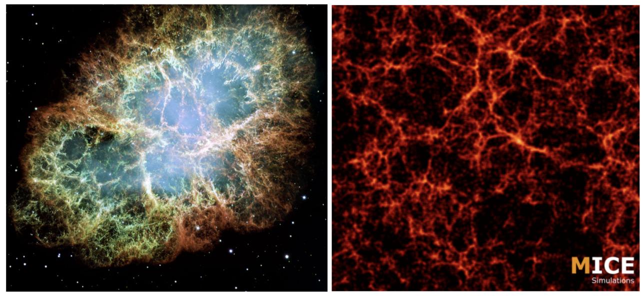

Figure 2. LEFT: The expanding Crab Nebula Supernova remnant, as observed in 1999 from the Hubble Space Telescope, 945 years after the explosion. The observed pulsar remnant could be part of the Dark Matter, in the form of primordial Neutron Stars and primordial BHs. RIGHT: the MICE simulation (Carretero et al. 2015) of large scale structures in our expanding Universe.

The BHU exists within a larger background that may or may not be totally empty outside. In the later case, r_S will increase if there is accretion from outside. This case is more speculative and needs to be studied in more detail, but it would result in an effective Λ term that decreases with time ($\omega_{DE} > -1$). The alternative interpretation is that M (and therefore r_S) are infinite so that the measured Λ can only be attributed to DE. This is the standard (LCDM) interpretation. In our view, this alternative is less appealing because it involves non physical infinite, non causal structure (Gaztañaga 2020; Gaztañaga 2021) and new components, DE or Λ, which are not really needed.

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DATA AVAILABILITY STATEMENT

No new data is presented in this paper.

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