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Derivation of Zeldovich's Formula and Weinberg's Rlation of the Cosmological Constant Based on a Liquid Droplet Model of Electron

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Abstract: Inspired by Dirac's large number hypothesis (LNH), Zeldovich proposed a relation between the cosmological constant (CC), the mass of a proton, the Planck constant and Newton's gravitational constant. Zeldovich's formula suggests that the substance which is responsible for CC may interact with elementary particles. Since Newton's gravitational constant and the Planck constant appear in Zeldovich's formula, these interactions may be related to gravitational phenomena and quantum phenomena. Following this clue, we propose a liquid droplet model of electron. Applying the Laplace's equation, Zeldovich's formula of CC is derived. Weinberg proposes a relation between the mass of a pion, the Planck constant, the Hubble constant, Newton's gravitational constant and the velocity of light in vacuum when discussing cosmological models with a varying constant of gravitation. Weinberg pointed out that this relation may not merely be regarded as a meaningless random combinations of these fundamental constants in physics. The Weinberg's relation is derived based on the critical mass density formula in the Friedmann model of the universe and Zeldovich's formula.

Keywords: Zeldovich's formula; Weinberg's relation; cosmological constant; dark energy; droplet; surface tension

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1. Introduction

In 1967, inspired by the remarks of Eddington, Dirac and other authors concerning the curious numerical relations in cosmology, Y. B. Zeldovich proposes the following relation [1,2]

$$\Lambda = k_0 \frac{G^2 m_{\mathrm{p}}^6}{\hbar^4}, \tag{1}$$

where Λ is CC, G is Newton's gravitational constant, m_{p} is the mass of a proton, $\hbar \equiv h/2\pi$, h is the Planck constant, k_0 is a dimensionless parameter.

In 2008, C. G. Boehmer and T. Harko propose the following formula of CC Λ [3]

$$\Lambda = \frac{\hbar^2 G^2 m_{\mathrm{e}}^6 c^6}{e^{12}}, \tag{2}$$

where m_{e} is the mass of an electron, e is the charge of an electron, c is the velocity of light in vacuum.

Eq. (2) can be written as [4]

$$\Lambda = \frac{G^2}{\hbar^4} \left(\frac{m_{\mathrm{e}}}{\alpha_0} \right)^6, \tag{3}$$

where

$$\alpha_0 = \frac{e^2}{\hbar c}. \tag{4}$$

The Boehmer-Harko formula Eq. (3) can also be written as the same form of Zeldovich's formula Eq. (1).

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In 2009, C. Beck derive the following formula of CC Λ based on four axioms [5]

$$\Lambda = \frac{G^2}{\hbar^4}\left(\frac{m_e}{\alpha_{el}}\right)^6, \tag{5}$$

where m_e is the mass of an electron, α_{el} is the low-energy limit of the fine structure constant.

The Beck's formula Eq. (5) can also be written as the same form of Zeldovich's formula Eq. (1). The predicted cosmological constant $\Lambda_B = 1.36284 \pm 0.00028 \times 10^{-52}\mathrm{m}^{-2}$ according to the Beck's formula Eq. (5) is about 25.26% higher than the observed value ([6], p. 138).

The third independent method to construct similar formula of CC is based on the Dirac's LNH [4]. Eq. (3) and Eq. (5) are used to study some varying cosmological constant models and warm dark matter [4].

Zeldovich's formula Eq. (1) is a conjecture inspired by Dirac's LNH [1,2]. Zeldovich's formula Eq. (1) suggests that the substance responsible for CC Λ may interact with the substances which constitute elementary particles. Since Newton's gravitational constant G and the Planck constant h appear in Eq. (1), Zeldovich's formula may be related to gravitational phenomena and quantum phenomena. However, the physical foundations of Zeldovich's formula Eq. (1) are still unknown. In this manuscript, we propose a theoretical derivation of Zeldovich's formula Eq. (1) based on the theory of vacuum mechanics (VM) [7].

In 1972, S. Weinberg proposes the following relation when discussing cosmological models with a varying constant of gravitation ([8], p. 619)

$$m_\pi^3 = w_0 \frac{\hbar^2 H_0}{Gc}, \tag{6}$$

where m_π is the mass of a pion, H_0 is the Hubble constant, w_0 is a dimensionless parameter.

S. Weinberg pointed out that Eq. (6) may not merely be regarded as a meaningless random combinations of $\hbar$, H_0, G and c ([8], p. 620). He thought that Eq. (6) may be a possible clue, which suggests that Dirac's large numbers may be determined partly by the influence of the whole universe ([8], p. 619).

In 2008, A. Alfonso-Faus proposed a derivation of Weinberg's relation Eq. (6) based on Newton's universal gravitation and momentum conservation laws [9]. He proposed that Zeldovich's formula Eq. (1) is equivalent to Weinberg's relation Eq. (6) [10]. Therefore, he thought that the speed of light must be proportional to the Hubble parameter and thus decrease with time [10]. Using Weinberg's relation Eq. (6), he proposed that the self gravitational potential energy of any fundamental particle is independent of the mass of the particle [11].

According to Hubble's law, redshift of galaxies proportional to the distance of those galaxies, indicating a recession velocity of those galaxies proportional to the distance ([12], p. 395). In 1990-1999 two groups observed that the expansion of universe is accelerating ([13], p. 112). The concept 'dark energy' is commonly regarded as the origin of the observed accelerated expansion of the universe ([6], p. 490). Since the Hubble constant H_0 appears in Weinberg's relation Eq. (6), it is a natural idea that Eq. (6) may be related to dark energy. Following this clue, in this manuscript we propose a derivation of the Weinberg's relation Eq. (6).

2. A liquid droplet model of electron

Because P. A. M. Dirac was not satisfied with quantum electrodynamics, he suggested an ether hypothesis ([14], p. 201). P. A. M. Dirac's ether hypothesis may also be enlightening for the cosmological constant problem (CCP).

In 2022, we propose a mechanical model of vacuum and derive CC [7]. Vacuum is supposed to be filled with two kinds of continuums which may be called the $\Omega(1)$ and $\Omega(2)$ substrata [7].

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On August 17th 2017, the gravitational-wave (GW) event GW170817 and the gamma-ray burst (GRB) event GRB 170817A were observed independently [15]. The observed time delay of $+1.74 \pm 0.05s$ between GRB 170817A and GW170817 shows that the difference between the speed c_{gw} of GW and the speed of light is limited between $-3 \times 10^{-15}c$ and $+7 \times 10^{-16}c$, where c is the speed of light in vacuum [15].

According to VM [16], GW is the propagations of tensorial potential of gravitational fields in vacuum. The speed c of light in vacuum is the speed of transverse elastic waves in the $\Omega(1)$ substratum [17]. The $\Omega(1)$ substratum may be regarded as a new version of the the electromagnetic ether in the history [17]. Since the speed c_{gw} of GW coincides with the speed c of transverse elastic wave in the $\Omega(1)$ substratum, then the $\Omega(1)$ substratum is the medium which propagates the tensorial potential of gravitational fields. Therefore, the hypothetical $\Omega(2)$ substratum in Ref. [7] seems to be unnecessary.

The particles that constitute the $\Omega(1)$ substratum may be called the $\Omega(1)$ particles [7]. The $\Omega(1)$ particles are sinks in the $\Omega(0)$ substratum [7]. For a macroscopic observer with whose time scale is very large comparing to the Maxwellian relaxation time of the $\Omega(1)$ substratum, vacuum behaves like a Newtonian-fluid. Based on some assumptions, we have the following equation of state of vacuum [7]

$$p_{\text{vac}} = -\phi_{\text{vac}}, \tag{7}$$

where p_{vac} is the pressure of vacuum, ϕ_{vac} is the energy density of vacuum.

Liquid drop models have been proposed to describe the the nuclear binding energies since 1935 [18]. Weizsäcker and Bethe's concept of nuclear drop was used to explain the nuclear fission phenomenon by Meitner, Frisch, Bohr and Wheeler [18]. The behaviors of nuclear matter is similar to incompressible liquids. In N. Bohr's liquid drop model of complex nuclei, the nucleus is treated as a droplet of continuous medium ([19], p. 91). P. A. M. Dirac suggested a spherical model of electron ([14], p. 203).

In VM [7], the $\Omega(1)$ substratum is proposed to act as the concept of dark energy. We introduce the following assumption.

Assumption 1

$$\rho_{vac} = \rho_1, \tag{8}$$

$$\phi_{vac} = \phi_1, \tag{9}$$

where ρ_{vac} is the mass density of vacuum, ρ_1 is the mass densities of the $\Omega(1)$ substratum, ϕ_1 is the energy densities of the $\Omega(1)$ substratum.

Inspired by the concept of nuclear drop in the history [18], we introduce the following assumption.

Assumption 2. *Vacuum is approximately modelled as a vapor phase, which is composed of the $\Omega(1)$ particles. An electron is approximately modelled as a spherical liquid droplet in vacuum, which is also composed of the $\Omega(1)$ particles.*

Figure 1 is an illustration of the liquid droplet model of an electron.

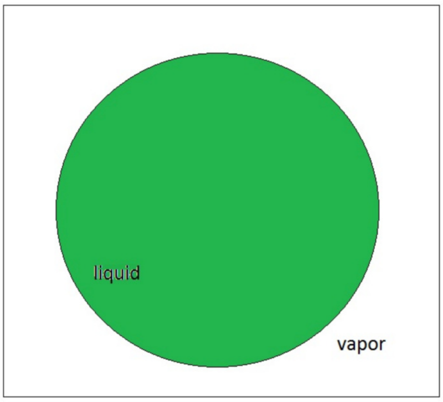

Figure 1. An illustration of the liquid droplet model of an electron.

3. The Laplace equation of an electron based on the theory of capillarity

A relation between the pressures of liquid phase and vapor phase of a liquid droplet was derived from mechanical equilibrium conditions of the liquid-vapor interface in 1805 independently by T. Young [20] and by P. S. Laplace [21]. Lord Kelvin also obtained this equation in 1858-1859 [22,23]. This equation can be written as ([24], p. 7)

$$p_e - p_v = \frac{2\sigma}{R}, \tag{10}$$

where p_e and p_v are the pressures of liquid phase and vapor phase respectively, R is radius of the spherical droplet, σ is the surface tension of the liquid-vapor interface.

Eq.(10) is called the Laplace's equation ([24], p. 7). The aforementioned theories are called mechanical theory of capillary phenomena ([24], p. 7).

In order to keep an electron from exploding, P. A. M. Dirac's model of electron should be supplied with a non-Maxwellian surface force ([14], p. 204). Such a surface force was introduced by Poincare in pre-relativistic electron theories in 1906 ([14], p. 204).

Poincare's concept of surface force of an electron is inspiring. According to Assumption 2, the pressures of the vapor phase of an electron is the pressure p_{vac} of vacuum. Applying the Laplace's equation (10) to an electron, we have

$$p_e - p_{vac} = \frac{2\sigma_e}{R_e}, \tag{11}$$

where p_e is the pressure of the liquid phase in the spherical electron, p_{vac} is the pressure of vacuum, σ_e is the surface tension of the liquid-vapor interface of the spherical electron, R_e is the radius of the spherical electron.

4. Derivation of Zeldovich's formula of CC

Next, we need to calculate the quantities p_e, p_{vac} and σ_e in the Laplace's equation (11).

Firstly, we calculate the quantity p_e. According to Assumption 2, an electron is composed of the $\Omega(1)$ particles. The gravitational constant G is a constant in the general theory of relativity (GR) [8,25,26]. However, in VM [27], the gravitational constant G depends on the density ρ_0 of the $\Omega(0)$ substratum. If ρ_0 depend on time and position in space, then G may also depend on time and position. Based on VM [27], there exists the following gravitational interaction between two $\Omega(1)$ particles in an electron

$$\mathbf{F}_{12}(t) = -G_e(t)\frac{m_1^2(t)}{r^2}\hat{\mathbf{r}}_{12}, \tag{12}$$

where

$$G_e(t) = \frac{\rho_{0e} q_0^2}{4\pi m_0^2(t)}, \tag{13}$$

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$m_0(t)$ is the mass of monad at time t, $-q_0(q_0 > 0)$ is the strength of a monad, $m_1(t)$ is the mass of a $\Omega(1)$ particle at time t, ρ_{0e} is the density of the $\Omega(0)$ substratum in the electron, $\hat{\mathbf{r}_{12}}$ is the unit vector directed along the line from a $\Omega(1)$ particle at $\hat{\mathbf{r}_2}$ to another at $\hat{\mathbf{r}_1}$, $\mathbf{F}_{12}$ is the force acting on the $\Omega(1)$ particle at $\hat{\mathbf{r}_1}$ by another at $\hat{\mathbf{r}_2}$.

The gravitational potential energy U_e of the $\Omega(1)$ particles in an electron is

$$U_e = -\int_0^{R_e} G_e \frac{m(r)dm}{r}, \tag{14}$$

where

$$m(r) = \frac{4\pi r^3 \rho_e}{3}, \tag{15}$$

ρ_e is the mass density of the liquid phase in the electron, r is the distance between the field point and the center of the electron.

Putting Eq. (15) into Eq. (14), we have

$$U_e = -\frac{3G_e m_e^2}{5R_e}. \tag{16}$$

The gravitational potential energy density ϕ_e of the $\Omega(1)$ particles in an electron is

$$\phi_e = \frac{U_e}{V_e}, \tag{17}$$

where $V_e = 4\pi R_e^3/3$ is the volume of the electron.

Using Eq. (16), Eq. (17) can be written as

$$\phi_e = -\frac{9G_e m_e^2}{20\pi R_e^4}, \tag{18}$$

Inspired by Eq. (7), we introduce the following assumption

Assumption 3.

$$p_e = -w_e \phi_e, \tag{19}$$

where $w_e > 0$ is a dimensionless parameter.

Comparing Eq. (18) and Eq. (19), we have

$$p_e = \frac{9w_e G_e m_e^2}{20\pi R_e^4}. \tag{20}$$

Secondly, we calculate the quantity p_{vac}. Since the hypothetical $\Omega(2)$ substratum in Ref. [7] seems to be unnecessary, we set $\phi_2 = 0$ in Eq. (104) of Ref. [7] and obtain the following relationship

$$\phi_1 = \frac{\Lambda c^4}{8\pi G}. \tag{21}$$

Noticing Eq. (9), Eq. (21) can be written as

$$\phi_{vac} = \frac{\Lambda c^4}{8\pi G}. \tag{22}$$

Putting Eq. (22) into Eq. (7), we have

$$p_{\text{vac}} = -\frac{\Lambda c^4}{8\pi G}. \tag{23}$$

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Thirdly, we calculate the surface tension σ_e. The liquid droplet model of electron in Assumption 2 is established by an analogy of the liquid drop model of nuclear in the history. According to the theory of capillary phenomena, surface tension can be regarded as the surface energy per unit area ([24], p. 5). Thus, the surface energy S_e of an electron is

$$S_e = 4\pi R_e^2 \sigma_e. \tag{24}$$

Assumption 4. *The surface energy S_e of an electron is proportional to negative value of the gravitational potential energy U_e of the $\Omega(1)$ particles in the electron, i.e.,*

$$S_e = -\lambda_0 U_e, \tag{25}$$

where $\lambda_0 > 0$ is a dimensionless parameter.

Putting Eq. (24) and Eq. (16) into Eq. (25), we have

$$\sigma_e = \frac{3\lambda_0 G_e m_e^2}{20 R_e^3}. \tag{26}$$

Noticing Eq. (20), Eq. (26) can be written as

$$\sigma_e = \frac{\lambda_0 R_e}{3 w_e} p_e. \tag{27}$$

Putting Eq. (27), Eq. (23) and Eq. (20) into Eq. (11), we obtain

$$\frac{\Lambda c^4}{8\pi G} = \left(\frac{2\lambda_0}{3 w_e} - 1 \right) p_e. \tag{28}$$

We use the following notation

$$G_e = \beta_0 G, \tag{29}$$

where $\beta_0 > 0$ is a dimensionless parameter.

Noticing Eq. (20) and Eq. (29), Eq. (28) can be written as

$$\Lambda = \gamma_0 \frac{G^2 m_e^2}{c^4 R_e^4}, \tag{30}$$

where

$$\gamma_0 = \frac{18\lambda_0 \beta_0}{5} \left(\frac{2\lambda_0}{3 w_e} - 1 \right). \tag{31}$$

The electron radius in the international system of units (SI) is ([28], p. 1183)

$$R_e = \frac{\hbar \alpha}{m_e c}, \tag{32}$$

where

$$\alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c} \tag{33}$$

is the fine-structure constant, e is the electron charge, ε_0 is the permittivity constant of vacuum, $\hbar = h/2\pi$, h is Planck's quantum of action.

Putting Eq. (32) into Eq. (30), we obtain

$$\Lambda = \theta_0 \frac{G^2 m_e^6}{\hbar^4}, \tag{34}$$

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where $\theta_0 = \gamma_0/\alpha^4$ is a dimensionless parameter.

We introduce the following notation

$$m_p = b_0 m_e, \tag{35}$$

where $b_0 > 0$ is a dimensionless parameter.

If we suppose that $k_0 = \theta_0/b_0^6$, then Eq. (34) is Zeldovich's formula Eq. (1).

Using Eq. (35), Eq. (34) can be written as

$$\Lambda = \frac{\theta_0}{b_0^6}\frac{G^2 m_p^6}{\hbar^4}. \tag{36}$$

If we suppose that $k_0 = \theta_0/b_0^6$, then Eq. (36) is Zeldovich's formula Eq. (1).

5. Derivation of Weinberg's relation of CC

We will explore the possibility of a derivation of Weinberg's relation Eq. (6). According to the Friedmann model ($\Lambda = 0$) of the universe, the sign of curvature of the spacetime is determined by the mass density of the universe and the Hubble constant H_0 ([29], p. 398). Therefore, there exists a critical mass density ρ_c ([29], p. 398)

$$\rho_c = \frac{3H_0^2}{8\pi G}, \tag{37}$$

where ρ_c is the critical mass density of the universe.

Thus, the critical energy density of the universe $\phi_c = \rho_c c^2$ is

$$\phi_c = \frac{3c^2 H_0^2}{8\pi G}, \tag{38}$$

where ρ_c is the critical mass density of the universe.

We introduce the following notation

$$\phi_c = \xi_0 \phi_{vac}, \tag{39}$$

where $\xi_0 > 0$ is a dimensionless parameter.

Using Eq. (39) and Eq. (22), Eq. (38) can be written as

$$H_0^2 = \frac{\xi_0 \Lambda c^2}{3}. \tag{40}$$

Applying Zeldovich's formula Eq. (34), Eq. (40) can be written as

$$m_e^3 = f_0 \frac{\hbar^2 H_0}{Gc}, \tag{41}$$

where $f_0 = \sqrt{\xi_0 \theta_0/3}$ is a dimensionless parameter.

We introduce the following notation

$$m_e = \zeta_0 m_\pi, \tag{42}$$

where $\zeta_0 > 0$ is a dimensionless parameter.

Using Eq. (42), Eq. (41) can be written as

$$m_\pi^3 = \frac{f_0}{\zeta_0^3}\frac{\hbar^2 H_0}{Gc}. \tag{43}$$

If we use the notation $w_0 = f_0/\zeta_0^3$, then Eq. (43) is Weinberg's relation Eq. (6).

6. Discussion

P. A. M. Dirac's dimensionless large numbers in the Centimeter-Gram-Second (CGS) system of units can be written as ([30], p21)

$$\frac{T_u}{\frac{e^2}{m_e c^3}} \approx 2 \times 10^{39}, \tag{44}$$

$$\frac{e^2}{G m_e m_p} \approx 0.7 \times 10^{39}, \tag{45}$$

where T_u is the age of the universe.

P. A. M. Dirac speculates that these two large numbers Eqs. (44-45) are related to the age of the universe ([31], p74). However, there are no convincing explanations of why these mysterious Dirac's large umbers Eqs. (44-45) should appear.

Y. B. Zeldovich points out that Dirac's large numbers can be derived based on two assumptions [1,2]. The first assumption is Zeldovich's formula Eq. (1). The second assumption is the following relationship [1,2]

$$\frac{R_u}{\Lambda^{-1/2}} = n \sim 1, \tag{46}$$

where R_u is the radius of the universe.

Although Dirac's version of the varying gravitational constant hypothesis seems to be ruled out by experiments ([30], p. 141), the general idea that some of the fundamental constants of nature may vary in time continues to be studied by physicists and cosmologists ([30], p. 162).

7. Conclusion

The observed time delay of $+1.74 \pm 0.05s$ between GRB 170817A and GW170817 shows that the speed of GW equals the speed of light in vacuum. Then the $\Omega(1)$ substratum, or we call the electromagnetic ether, is the medium which propagates the tensorial potential of gravitational fields. Thus, the hypothetical $\Omega(2)$ substratum in a previous model of vacuum seems to be unnecessary. Therefore, vacuum is approximately modelled as a vapor phase, which is composed of the $\Omega(1)$ particles. An electron is approximately modelled as a spherical liquid droplet in vacuum. The Zeldovich's formula of CC is derived by applying the Laplace's equation. Weinberg's relation may be a possible clue which suggests that Dirac's large numbers may be determined partly by the influence of the whole universe. Thus, the Weinberg's relation is derived based on the critical mass density formula in the Friedmann model of the universe and Zeldovich's formula.

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