Article

Application of RRAP Reliability Optimization as Test of Nature Inspired Algorithms

Adam Pieprzycki * and Bogusław Filipowicz

Posted Date: 6 January 2023

doi: 10.20944/preprints202301.0124.v1

Keywords: reliability optimization; RRAP; swarm intelligence; firefly algorithm FA; cuckoo search algorithm CS; ANOVA

 doi:10.20944/preprints202301.0124.v1

Article

Application of RRAP Reliability Optimization as A Test of Nature-Inspired Algorithms

Adam Pieprzycki †,‡,*, Bogusław Filipowicz ‡

Department of Computer Science /University of Applied Sciences in Tarnów, Poland; fil@agh.edu.pl
* Correspondence: a_pieprzycki@anstar.edu.pl; Tel.: +48-604-968-287
† Current address: ul. Mickiewicza 8, 33-100 Tarnów, Poland
‡ These authors contributed equally to this work.

Abstract: This paper considers the application of two selected algorithms using swarm intelligence (Cuckoo Search CS, Firefly Algorithm FA) to determine and maximize the reliability of two complex systems known from the literature. This system operates with resource constraints. In addition, several classical methods for determining the reliability of systems have been applied. Although the CS algorithm is known for its effectiveness as it uses the Lévy flight, for the presented optimization tasks the FA firefly algorithm in the considered parameter range proved to be more effective. The presented results broaden the existing discussion on the application of nature-inspired algorithms for solving RRAP (Reliability Redundancy Allocation Problem) problems. From this point of view, two described test scenarios can be considered as a suitable tool for validating other algorithms in RRAP problems. The paper presents metrics and ways to analyse and compare the performance of algorithms in RRAP optimisation. Not only the values of the criterion functions can be compared, but also other introduced parameters and statistical analyse of variance (ANOVA) with post-hoc RIR Tuckey test were discussed.

Keywords: reliability optimization; RRAP, swarm intelligence; firefly algorithm FA; cuckoo search algorithm CS; ANOVA; Lévy flies

---

1. Introduction

The reliability of a non–renewable object can be defined as the ability to retain such properties that allow it to be used for its intended purpose [1] The optimization of the considered topic may concern increasing reliability (increasing safety) or decreasing it in order to reduce costs. Increasing reliability may concern each component of the system or involve the introduction of redundancy (redundancy) in individual subsystems. For a extremely reliable system, it is necessary to design the system with redundant elements in a balance between: price, weight, volume or lifetime. This type of non–linear design with (resource) constraints is called RRAP (Reliability Redundancy Allocation Problem). The problem of reliability optimization considering redundancy can be qualified as a non–linear programming problem with one or more constraints. The solution of the optimization task consists in the optimal allocation of system components (number of elements, reliability level), maximizing the total reliability while satisfying the existing constraints. RRAP belongs to NP–hard problems. Such redundancy is desirable and provides protection in the event that part of the system fails. A critical component may be duplicated or several components (even three or more) used in parallel to avoid system breakdown. In engineering, this can be the parallel operation of several components (e.g., temperature sensor contacts, etc.), or redundant devices in the power grid. A classic example of increasing system reliability in computing is the use of RAID (Redundant Array of Independent Disks) disk arrays. The objective of reliability optimization is to select (calculate) the number of redundant components and their reliability values in each subsystem, and to maximize the total system reliability. The criterion functions presented in the following part of the article describe the reliability of the system (system), which has been taken as a quality indicator (criterion function) and is subject to maximization with the introduced constraints. In an attempt to solve the discussed reliability

 doi:10.20944/preprints202301.0124.v1

optimization problems of complex systems, the results obtained using two selected heuristic algorithms were considered and compared. These are algorithms based on the social behavior of animals, birds or insects. Two selected optimization algorithms have been applied: Firefly Algorithm FA [2] and Cuckoo Search CS [3]. In the analyzed class of problems, the FA firefly algorithm is known for its effectiveness. The use of another algorithm (CS cuckoo search algorithm) allowed us to compare their accuracy in solving two optimization problems [4–6]. Recent studies show that CS is potentially far more efficient than PSO (Particle Swarm Optimisation), genetic algorithms, and other algorithms [2]. In the meanwhile, scientists, especially theorists, ask the question of why animals do Lévy flight, which fascinates researchers from various disciplines from ecology to physics [7]. Application of artificial intelligence in reliability optimization of systems successfully previously applied also in power grid optimization (k-terminal grid model) using swarm optimization algorithms [8]. In that work, a reliability function in the form of BDD (Binary Decision Diagram) diagrams and a combined decomposition EED (Edge Expansion Diagrams) [9], CAE (Composition After Expansion) [10] and fixed-sink algorithm for k-terminal networks [11].

2. Problem Definition

The issue under discussion concerns the maximization of the criterion function:

$$max\left(F_c(\mathbf{r},\mathbf{n})\right), \tag{1}$$

within constraints:

$$g_y(\mathbf{r},\mathbf{n}) \leq b_y, \tag{2}$$

$$0 \leq r \leq 1,\ n \in Z^+,\ 1 \leq y \leq k,\ y \in Z^+,\ k \in Z^+, \tag{3}$$

where: $F_c(\mathbf{r},\mathbf{n})$ – system reliability function, $\mathbf{n}$ – vector of the number of elements in the various subsystems, $\mathbf{r}$ – the reliability vector of each element in each subsystem, g_y– the physical characteristics of the element in subsystem for constraint number y, b_y– upper limit for constraint number y, k– number of constraints.

3. Models of the RRAP System

The criterion functions for the scenarios considered have been derived in detail in the literature [3,4,12]. Another—not discussed in the article—test cases are: an overspeed system for a gas turbine (Figure 1) or a more complicated 15-unit system reliability problem with different combinations of parameters [12].

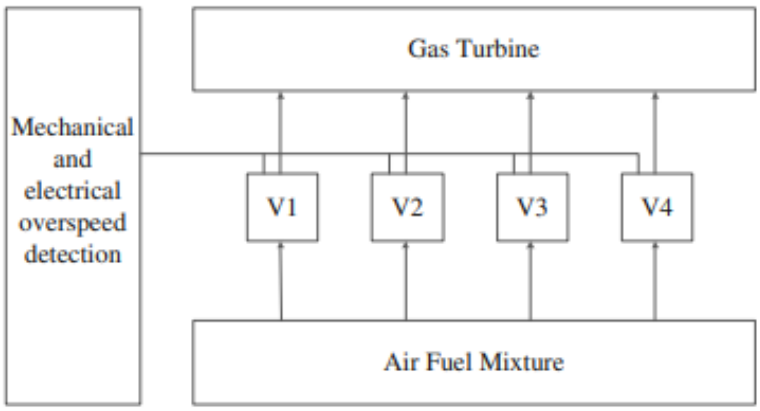

Figure 1. The schematic diagram of an overspeed system for a gas turbine [12].

 doi:10.20944/preprints202301.0124.v1

3.1. Scenario 1—Bridge System

In the first scenario, the five–element bridge system (Figure 2) [3,4,12] was considered.

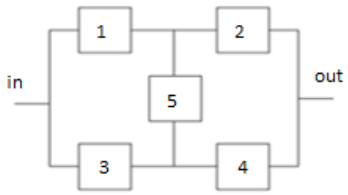

Figure 2. Diagram of the bridge system analysed in scenario 1.

The criterion function of a bridge system has been derived in detail in literature can be formulated as:

$$F_{c_1}(\mathbf{r},\mathbf{n}) = R_1 \cdot R_2 + R_3 \cdot R_4 + R_1 \cdot R_4 \cdot R_5 + R_2 \cdot R_3 \cdot R_5 - R_1 \cdot R_2 \cdot R_3 \cdot R_4 - R_1 \cdot R_2 \cdot R_3 \cdot R_5 + \\ - R_1 \cdot R_2 \cdot R_4 \cdot R_5 - R_1 \cdot R_3 \cdot R_4 \cdot R_5 - R_2 \cdot R_3 \cdot R_4 \cdot R_5 + 2 \cdot R_1 \cdot R_2 \cdot R_3 \cdot R_4 \cdot R_5, \quad (4)$$

where the individual reliability values R_i of each subsystem are calculated as:

$$R_i = 1 - (1 - r_i)^{n_i}, \ \forall\, i \in \{1, 2, \ldots, m_1\}. \quad (5)$$

For a bridged system occurs the optimization of 10 decision variables (five variables r_i and another five variables n_i with integer values). Three constraints (k_1=3) scenario 1 of total weight and volume (V), cost (C), lifetime (T) and system weight (W) were introduced (Table 1):

$$g_1(\mathbf{r},\mathbf{n}) = \sum_{i=1}^{m_1} w_i \cdot v_i^2 \cdot n_i^2 - V \leq 0, \quad (6)$$

$$g_2(\mathbf{r},\mathbf{n}) = \sum_{i=1}^{m_1} \alpha_i \cdot \left(-\frac{T}{ln(r_i)}\right)^{\beta} \cdot (n_i + e^{0.25 \cdot n_i}) - C \leq 0, \quad (7)$$

$$g_3(\mathbf{r},\mathbf{n}) = \sum_{i=1}^{m_1} w_i \cdot n_i \cdot e^{0.25 \cdot n_i} - W \leq 0, \quad (8)$$

$$0 \leq i \leq m_1, \ 0 \leq r_i \leq 1, \ n_i \in Z^+, \quad (9)$$

where: m_1 – number of subsystems in the whole system (m_1=5) at scenario 1, n_i – number of elements in i–th subsystem, r_i – the reliability of each element in i–th subsystem, R_i – reliability of i–th subsystem, α_i, β_i – physical characteristics of element in i–th subsystem, w_i, v_i, c_i – weight, volume and cost of the element in i–th subsystem.

In order to compare the results obtained with other solutions found in the literature, the parameter settings of the bridge system were adopted as shown in Table 2 [4]. Other methods of system reliability analysis are also used, such as optimization using the path method, for which the objective function takes the form:

$$F_{c_2}(\mathbf{r},\mathbf{n}) = 1 - (1 - R_1 \cdot R_2) \cdot (1 - R_3 \cdot R_4) \cdot (1 - R_1 \cdot R_4 \cdot R_5) \cdot (1 - R_2 \cdot R_3 \cdot R_5), \quad (10)$$

or a cutting method for which the criterion function can be expressed as:

$$F_{c_3}(\mathbf{r},\mathbf{n}) = (1 - (1 - R_1) \cdot (1 - R_3)) \cdot (1 - (1 - R_2) \cdot (1 - R_4)) \cdot (1 - (1 - R_2 \cdot (1 - R_3) \cdot (1 - R_5)) \cdot \\ \cdot (1 - (1 - R_1) \cdot (1 - R_4) \cdot (1 - R_5)). \quad (11)$$

 doi:10.20944/preprints202301.0124.v1

Table 1. Bridge system settings.

β_i	V	C	W	T [h]
1.5	110	175	200	1000

Table 2. Bridge system settings.

Subsystem i	$10^5 \cdot \alpha_i$	$w_i \cdot v_i^2$	w_i
1	2.330	1	7
2	1.450	2	8
3	0.541	3	8
4	8.050	4	6
5	1.950	2	9

The reliability of the system calculated with the method of minimum cuts is always smaller (11) (it is the lower limit) than the value of reliability calculated with the method of minimal path F_{c_2} (10)—it is the upper limit. This difference can be a space for exact optimization (Figure 3).

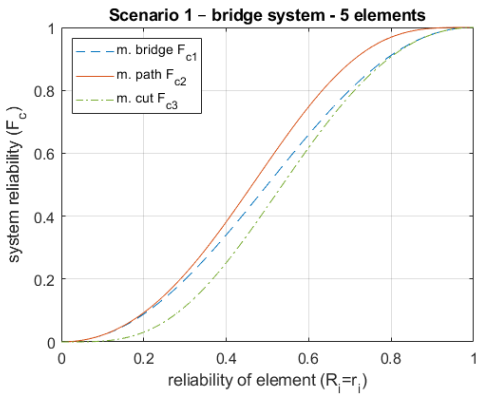

Figure 3. System reliability for different analysis methods with n=1 (set fixed number of redundant elements).

Analyzing the constraints adopted for the analyzed model, it can be noted that the second constraint (g_2 (r,n)) (7) significantly affects the permissible values of element reliability (Figure 4).

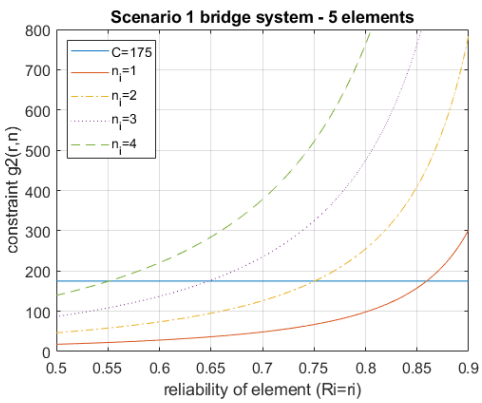

Figure 4. Analysis of the second constraint g_2 (r,n).

 doi:10.20944/preprints202301.0124.v1

3.2. Scenario 2—System Consisting 10 Elements

The criterion function of a ten elements system (Figure 5) is more complicated than bridge system [4,12] and can be formulated as:

$$\begin{aligned} F_{c_4}(\mathbf{r},\mathbf{n}) = R_1 \cdot R_2 \cdot R_3 \cdot R_4 + (R_1 \cdot R_2 \cdot R_6 \cdot R_{10}) \cdot (Q_3 + R_3 \cdot Q_4) + (R_1 \cdot R_5 \cdot R_9 \cdot R_{10}) \cdot (Q_2 + \\ + R_2 \cdot Q_3 \cdot Q_6 + R_2 \cdot R_3 \cdot Q_4 \cdot Q_6) + R_7 \cdot R_8 \cdot R_9 \cdot R_{10} \cdot (Q_1 + R_1 \cdot R_2 \cdot Q_3 \cdot Q_5 \cdot Q_6 + \\ + R_1 \cdot R_3 \cdot Q_4 \cdot Q_5 \cdot Q_6) + R_2 \cdot R_3 \cdot R_4 \cdot R_5 \cdot R_7 \cdot R_8 \cdot Q_1 \cdot (Q_9 + R_9 \cdot Q_{10}) + \\ + Q_1 \cdot R_3 \cdot R_4 \cdot R_6 \cdot R_7 \cdot R_8 \cdot Q_{10} \cdot (Q_2 + R_2 \cdot Q_5) + \\ + Q_1 \cdot Q_2 \cdot R_3 \cdot R_4 \cdot R_6 \cdot R_7 \cdot R_8 \cdot R_9 \cdot Q_{10} + R_1 \cdot Q_2 \cdot R_3 \cdot R_4 \cdot R_5 \cdot R_6 \cdot R_9 \cdot Q_{10} \cdot (Q_7 + R_7 \cdot Q_8) + \\ + Q_1 \cdot R_2 \cdot R_5 \cdot R_6 \cdot R_7 \cdot R_8 \cdot Q_9 \cdot R_{10} \cdot (Q_3 + R_3 \cdot Q_4), \end{aligned} \quad (12)$$

where: R_i is is defined the same as the expression (5), Q_i is defined as:

$$Q_i = 1 - R_i, \ \forall\, i \in \{1, 2, \ldots, m_2\}. \quad (13)$$

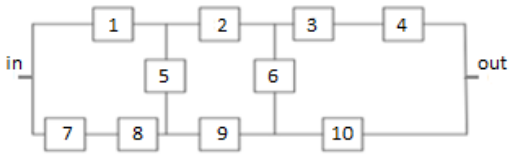

Figure 5. Diagram of the 10 elements system under analysis in scenario 2.

For scenario 2, the constraints were expressed as:

$$g_y(\mathbf{n}) = \sum_{i=1}^{m_2} c_{y_i} \cdot n_i \leq b_y, \quad (14)$$

$$y = 1, 2, \ldots, k_2; \ n_i \in Z^+, \quad (15)$$

where k_2. is number of constraints in scenario 2. For scenario 2, k_2=5 constraints were assumed.

The coefficients c_{y_i} are random numbers in the range [0, 100], the coefficient r is searched in the range $[lb_1, ub_1]$, and the parameter b_y is calculated as:

$$b_y = d_{rand} \cdot \sum_{i=1}^{m_2} c_{y_i}, \quad (16)$$

where: d_{rand}—random number with uniform distribution in the range (1.5, 3.5).

In order to compare the solutions obtained, the values of these parameters are taken according to Table 3, as in the work [2,4].

 doi:10.20944/preprints202301.0124.v1

Table 3. Parameters used in scenario 2.

i	c_{1i}	c_{2i}	c_{3i}	c_{4i}	c_{5i}
1	33.2468	35.6054	13.7848	44.1345	10.9891
2	27.5668	44.9520	96.7365	25.9855	68.0713
3	13.3800	28.6889	85.8783	19.2621	1.0164
4	0.4710	0.4922	63.0815	12.1687	29.4809
5	51.2555	39.6833	78.5364	23.9668	59.5441
6	82.9415	59.2294	11.8123	28.9889	46.5904
7	51.8804	78.4996	97.1872	47.8387	49.6226
8	77.9446	86.6633	45.0850	25.0545	59.2594
9	26.8835	7.8195	3.6722	76.9923	87.4070
10	85.8722	27.7460	55.3950	53.3007	55.3175

Table 4. Parameters used in scenario 2.

Parameters	d_{rand}
c_{1i}	3.1250
c_{2i}	3.4710
c_{3i}	3.3247
c_{4i}	2.6236
c_{5i}	3.4288

4. Selected Optimization Algorithms

More than 100 OPA (optimization algorithms) are recently known which using phenomena of different behavior observed in the world of plants and animals [13,14]. The development of optimization methods is due to the fact that there are no universal and efficient methods of searching for the global extreme of the analyzed function. Therefore, it is desirable to know (and use) more than one optimization method [15]. For the purposes of this paper, in the process of searching for the global extreme of the criteria functions: F_{c_1} (Equation (4)) and F_{c_4} (Equation (12)), two selected heuristic algorithms [16] were used. It is worth noting that Firefly Algorithm FA is known for its efficiency in optimizing RRAP problems [17].

4.1. Firefly Algorithm FA

In the 2007–dated FA firefly algorithm, developed by Xin–She Yang [3] of Cambridge University, the solution to the optimization task is based on the difference in light intensity, which is proportional to the value of the criterion function F_{c_1}. Each brighter firefly attracts other individuals to it, allowing for an intensified and therefore more efficient exploration of the search space. In the solution space reviewed, the k–th step during which a firefly with index i, located at position x_i, attempts to approach a "more attractive" individual with index j, located at position x_j can be expressed by the equation [2]:

$$x_i^k = x_i^{k-1} + \beta_0 \cdot e^{-\gamma \cdot d_{ij}^{k-1}} \cdot (x_j^{k-1} - x_i^{k-1}) + \alpha_{ff} \cdot (x_{rand} - 0.5), \tag{17}$$

where: x_{rand}—random number with uniform distribution in the range [0,1], d_{ij}^{k-1}—distance between fireflies with index i and j in the k–1 (previous) step.

 doi:10.20944/preprints202301.0124.v1

4.2. Cuckoo Search CS

The second algorithm chosen to search for the solution two RRAP problems is the CS Cuckoo Search algorithm [2]. It is an algorithm from 2009, which was proposed by Xin–She Yang and Suash Deb [18]. The algorithm under consideration models (mimics) the behavior of some cuckoo species that use the nests of other birds to raise their offspring. Randomly selecting the i–th cuckoo/nest and generating a new solution x_i^k, e.g., by using Lévy flight is expressed by the relation:

$$x_i^k = x_i^{k-1} + \alpha_{CS} \oplus s, \tag{18}$$

where: $\oplus$—is point-to-point multiplication (entry-wise product of two vectors), k—the step number, that is the next iteration, x_i^k—the solution obtained in the k–th step for the i–th cuckoo, α_{CS}—a scale factor the value depends on the size of the problem, s—the step length, determined by a Lévy probability distribution [2].

Here, $\alpha_{CS} > 0$ is the step size scaling factor, which should be related to the scales of the problem of interest. In most cases, we can use α_{CS} = O(L/10) or α_{CS} = O(L/100) in some cases, where L is the characteristic scale of the problem of interest [19]. The exploitation mechanism of the CS algorithm are local motions and the exploration mechanism are Lévy flights, based on the Lévy probability distribution [2], expressed by the formula:

$$L(s,\lambda) = \frac{\lambda \cdot \Gamma(1+\lambda) \cdot \sin\left(\frac{\pi \cdot \lambda}{2}\right)}{\pi} \cdot \frac{1}{|s|^{\lambda}}, s >> 0, 1 \leq \lambda \leq 3, \tag{19}$$

and for stable Lévy distribution ($\alpha = 0.5$) probability density function:

$$L(s,\alpha,\delta,\gamma_l) = \sqrt{\frac{\gamma_l}{2\pi}} \frac{1}{(s-\delta)^{1+\alpha}} exp\left(-\frac{\gamma_l}{2(s-\delta}\right), \alpha = 0.5, \delta < s < \infty, \tag{20}$$

where: α—first shape parameter $0 < \alpha \leq 2$, δ—location parameter $---\infty < \delta < \infty$, γ_l—scale parameter $0 < \gamma_l < \infty$, $\Gamma(\cdot)$—the gamma function :

$$\Gamma(x) = \int_0^{\infty} e^{-t} \cdot t^{x-1} dt. \tag{21}$$

The step length (size) was calculated according to Mantenga's algorithm [20] as:

$$s = \frac{U}{|V|^{\frac{1}{\lambda}}}, \tag{22}$$

where:

$$U = N\left(0, \sigma_v^2\right) \cdot \sigma_u, V \sim N\left(0, \sigma_v^2\right), \tag{23}$$

and:

$$\sigma_v = 1,\ \sigma_u = \left(\frac{\Gamma(1+\lambda) \cdot \sin\left(\frac{\pi \cdot \lambda}{2}\right)}{\Gamma(\frac{1+\lambda}{2}) \cdot \lambda \cdot 2^{\frac{\lambda-1}{2}}}\right)^{\frac{1}{\lambda}}, \lambda = 1+\alpha, 0 < \alpha \leq 2, \tag{24}$$

where: α—first shape parameter distribution.

Figure 6 shows an example implementation of a Lévy flight (visualization restricted to R^3 space), with the parameters listed in Table 5. Analyzing the steps generated in the CS algorithm used, it can be seen that among the large number of small steps of the algorithm, from time to time there are large jumps called Lévy flights, after the French mathematician Paul Pierre Lévy. A characteristic feature of this Lévy distribution [21] is the long "tails" that occur for large values – unlike the Gauss (Normal)

 doi:10.20944/preprints202301.0124.v1

distribution (Figure 7) [22]. In fact, the trajectory of a Lévy flight has fractal dimension $d_f = \lambda$ [23–25]. A search for the maximum of the criterion function F_{c_1} (4) and F_{c_4} (12) was performed for the two selected algorithms. For each combination of the selected control parameters of the algorithm, $N_{repcalc}$ (number of repetitions) calculations were performed (Table 5). The range of control parameters of the algorithms was chosen arbitrarily, limiting them to the most characteristic cases for the considered algorithm. In addition, the calculation parameters for the CS algorithm (number of iterations N_{iter}^{max} and number of repetitions $N_{repcalc}$) were chosen so that the calculation times were comparable.

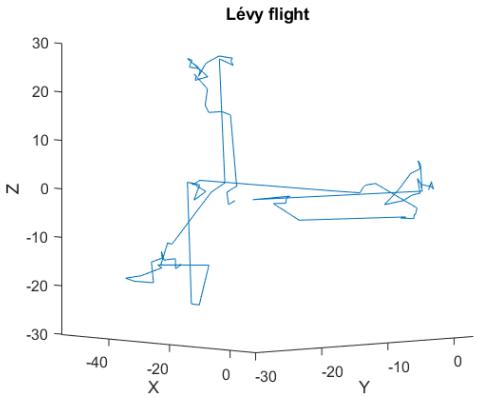

Figure 6. Visualization of an example Lévy flight (λ=1.3) in Euclidean space R^3.

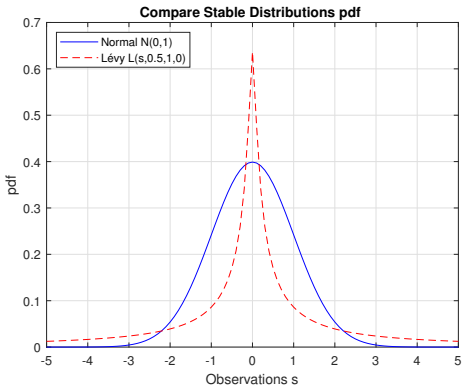

Figure 7. Compare Stable Distributions pdf.

 doi:10.20944/preprints202301.0124.v1

Table 5. Parameters of scenario 1 and scenario 2.

Parameters	Values
Problem dimension	$N_D = m_{sc} \cdot 2$; sc $\in \{1,2\}$; m_1=5, m_2=10
Maximum number of iterations	N_{iter}^{max} = 100 and 1000 (CS)
Number of repetitions (each case)	$N_{repcalc}$=10 and 10000 (CS)
Constraints values (lower bound, upper bound)	lb_1=0.65; ub_1=0.85; lb_2=1; ub_2=4
CS – Cuckoo Search Algorithm	
Number of nests	N_{nests} = 50
Probability of detecting a cuckoo's egg	$p_a = \{0.25, 0.26, \ldots, 0.50\}$
Lévy distribution parameter	$\lambda = \{1.1, 1.2, \ldots, 1.9\}$
Positive step size scaling factor	$\alpha_{CS} = \frac{1}{100} \cdot N(0,1) \cdot (x_i^{k-1} - x_i^{best})$
FA – Firefly Algorithm	
Number of fireflies	N_{ff}= 50
Randomization parameter	$\alpha_{ff} = \{0.1, 0.2, \ldots, 1.0\}$
Reference factor of 'attractiveness'	$\beta_0 = \{0.1, 0.2, \ldots, 1.0\}$
Absorption coefficient	$\gamma = \{0.01, 0.10, 1.00\}$

where: x_i^{best} - the best solution (nest with the best egg).

5. Calculation Results

For the control variables of the selected algorithms presented in Table 5, obtain results of calculations and solutions for scenario 1 and scenario 2 obtained with the two analyzed algorithms. For the CS [26] algorithm, the calculations concerned changes in the probability of detecting an egg tossed by the cuckoo p_a and Lévy distribution parameter λ (Table 5). For the FA algorithm [27], all combinations of the three parameters listed in Table 5 were considered: randomness α_{ff}, reference "attractiveness" factor β_0 and absorption coefficient γ. An improvement index is required to measure the improvement of the best solutions found by the FA algorithm in comparison with those given by CS. This index [12], which has been called Maximum Possible Improvement (MPI), is as follows:

$$MPI(\%) = \frac{F_c^{FA} - F_c^{alg}}{1 - F_c^{alg}} \tag{25}$$

where: F_c^{FA}, F_c^{alg}—the best system reliability for criteria function F_{c_1} (Equation (4)) or F_{c_4} (Equation (12)) obtained by the FA algorithm and other algorithm ($alg \notin FA$) e.g., CS algorithm.

The correlation coefficient between the results (criterion function values) of the two scenarios using the FA and CS algorithm was calculated according to the formulas:

$$r^{alg} = \frac{\sum\limits_{par1,par2,\ldots} \left(F_{c_1}^{alg} - \overline{F_{c_1}^{alg}}\right) \cdot \left(F_{c_4}^{alg} - \overline{F_{c_4}^{alg}}\right)}{\sqrt{\left(\sum\limits_{par1,par2,\ldots} \left(F_{c_1}^{alg} - \overline{F_{c_1}^{alg}}\right)^2\right)} \cdot \sqrt{\left(\sum\limits_{par1,par2,\ldots} \left(F_{c_4}^{alg} - \overline{F_{c_4}^{alg}}\right)^2\right)}}, \tag{26}$$

where: $\overline{F_{c_1}^{alg}}$, $\overline{F_{c_4}^{alg}}$—mean value of the criteria function F_{c_1} (Equation (4)) and F_{c_4}) (Equation (12)) obtained by algorithm (alg) FA or CS, $par1$, $par2$, ... parameters algorithms, e.g.,: FA: α_{ff}, β_0, γ or CS: p_a, λ.

 doi:10.20944/preprints202301.0124.v1

To determine the reasonable significant difference of average performance, test statistics of ANalysis Of VAriance (ANOVA) was used. Post hoc (RIR) Tukey test Honest Significant Difference (HSD) was conduced, to investigated which of the parameters are different from each other for tested algorithm.

Presented values (in tables and figures) of criterion functions and constraints are unitless.

5.1. Results for FA

For both scenarios, the best results have been achieved for the FA algorithm parameter set: α_{ff}=0.5, β_0=1.0, γ =0.01. For criterion function F_{c_1}, maximum value was 0.99995661 (Table 6). In scenario 2, the best solutions have been achieved with F_{c_4}=0.99992902 (Table 7) for FA algorithm.

Table 6. The best solution for scenario 1 for FA.

Parameters	$\alpha_{ff} = 0.5, \beta_0 = 1.0, \gamma$=0.01
Solutions F_c^{min}	0.99995653
Solutions F_c^{max}	**0.99995661**
Solutions F_c^{mean}	0.99995657
Solutions σ	2.27604463e–08
Calc. time [s]	216.0in

Table 7. The best solution for scenario 2 has been achieved using FA.

Parameters	$\alpha_{ff} = 0.5, \beta_0 = 1.0, \gamma$=0.01
Solutions F_c^{min}	0.99992644
Solutions F_c^{max}	**0.99992902**
Solutions F_c^{mean}	0.99992773
Solutions σ	9.85429365e–07
Calc. time [s]	189.3

Table 8. The best solution for scenario 2 by FA: $\alpha_{ff} = 0.5, \beta_0 = 1.0, \gamma$=0.01.

Parameters	Values	Parameters	Values
r_1	0.8500	n_1	4
r_2	0.8500	n_2	4
r_3	0.8500	n_3	4
r_4	0.8500	n_4	4
r_5	0.8500	n_5	4
r_6	0.7429	n_6	1
r_7	0.7989	n_7	2
r_8	0.6626	n_8	2
r_9	0.6902	n_9	2
r_{10}	0.8295	n_{10}	2

 doi:10.20944/preprints202301.0124.v1

Table 9. Post-hoc Tuckey RIR tests for FA scenario 1 and α_{ff}=0.4, β_0=0.5 and γ =0.1.

Parameters	p-values
α_{ff}=0.1, β_0 =0.1, γ =0.01	0.000116
α_{ff}=0.1, β_0 =0.2, γ =0.01	0.000116
α_{ff}=0.2, β_0 =0.1, γ =0.01	0.000116
α_{ff}=0.1, β_0 =0.1, γ =0.1	0.000116
α_{ff}=0.1, β_0 =0.2, γ =0.1	0.000116
α_{ff}=0.1, β_0 =0.3, γ =0.1	0.000116
α_{ff}=0.2, β_0 =0.1, γ =0.1	0.000116
α_{ff}=0.2, β_0 =0.2, γ =0.1	0.000116
α_{ff}=0.1, β_0 =0.1, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.2, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.3, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.4, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.5, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.6, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.7, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.8, γ =1.0	0.000116
α_{ff}=0.1, β_0 =0.9, γ =1.0	0.000116
α_{ff}=0.1, β_0 =1.0, γ =1.0	0.000117
α_{ff}=0.2, β_0 =0.1, γ =1.0	0.000116
α_{ff}=0.2, β_0 =0.2, γ =1.0	0.000116
α_{ff}=0.2, β_0 =0.3, γ =1.0	0.000116
α_{ff}=0.2, β_0 =0.4, γ =1.0	0.000116
α_{ff}=0.2, β_0 =0.5, γ =1.0	0.000116
α_{ff}=0.2, β_0 =0.6, γ =1.0	0.000116
α_{ff}=0.3, β_0 =0.1, γ =1.0	0.000116
α_{ff}=0.3, β_0 =0.2, γ =1.0	0.000116
α_{ff}=0.3, β_0 =0.3, γ =1.0	0.000116
α_{ff}=0.3, β_0 =0.4, γ =1.0	0.014025
α_{ff}=0.3, β_0 =0.5, γ =1.0	0.000116
α_{ff}=0.3, β_0 =0.6, γ =1.0	0.000144
α_{ff}=0.3, β_0 =0.7, γ =1.0	0.000116

Analysing the parameter space with the ad-hoc RIR Tuckey test (ANOVA), it can be seen that for the set of α_{ff}=0.4, β_0=0.8, γ =0.01 parameters, the hypothesis of no reasonable difference should be rejected for three cases.

 doi:10.20944/preprints202301.0124.v1

Table 10. Post-hoc RIR Tuckey tests for FA scenario 2 and α_{ff}=0.4, β_0=0.8 and γ =0.01.

Parameters	p-values
α_{ff}=0.1, β_0=0.1	0.000047
α_{ff}=0.2, β_0=0.1	0.000047
α_{ff}=0.1, β_0=0.2	0.000047
α_{ff}=0.1, β_0=0.3	0.483894
α_{ff}=0.3, β_0=0.7	0.997808
α_{ff}=0.1, β_0=0.8	0.857372
α_{ff}=0.2, β_0=0.8	0.987661
α_{ff}=0.3, β_0=0.8	0.999464
α_{ff}=0.1, β_0=0.9	0.991139
α_{ff}=0.2, β_0=0.9	0.999796
α_{ff}=0.3, β_0=0.9	0.952106
α_{ff}=0.1, β_0=1.0	0.278482
α_{ff}=0.2, β_0=1.0	0.832396
α_{ff}=0.3, β_0=1.0	0.999918
other cases	1.000000

5.2. Results for CS

Due to different solutions using the CS algorithm (Tables 11 and 12), the stopping criterion a and the number of iterations have been changed (Table 5).

The solutions for parameter N_{iter}^{max}=1000, $N_{repcalc}$=10000 (Table 13) were also checked.

Table 11. The best solution for scenario 1 by FA $\alpha_{ff} = 0.5, \beta_0 = 1.0$, γ=0.01.

Parameters	Values
r_1	0.78571
r_2	0.8500
r_3	0.8500
r_4	0.7520
r_5	0.6601
n_1	4
n_2	4
n_3	3
n_4	2
n_5	3

 doi:10.20944/preprints202301.0124.v1

Table 12. The best solution for scenario 1 by CS: p_a=0.27, $\lambda = 1.5$, $N_{iter}^{max} = 100$, $N_{repcalc} = 10$.

Parameters	Values
r_1	0.8298
r_2	0.8380
r_3	0.8500
r_4	0.6500
r_5	0.7265
n_1	4
n_2	4
n_3	2
n_4	3
n_5	2

Table 13. The best solution for scenario 1 by CS.

Parameters	p_a=0.30, $\lambda = 1.1$ $N_{iter}^{max} = 1000$, $N_{repcalc} = 10000$
Solutions F_c^{min}	0.99921071
Solutions F_c^{max}	**0.99995645**
Solutions F_c^{mean}	0.99984281
Solutions σ	6.41364325e–05
Calc. time [s]	346.4
MPI (%)	0.0037
$\|\mathbf{r}^{FA} - \mathbf{r}^{CS}\|_2$	0.0124
$\|\mathbf{n}^{FA} - \mathbf{n}^{CS}\|_2$	0

Table 14. The best solution for scenario 1 by CS: p_a=0.30, $\lambda = 1.1$, $N_{iter}^{max} = 1000$, $N_{repcalc} = 10000$.

Parameters	Values
r_1	0.7919
r_2	0.8500
r_3	0.8499
r_4	0.7483
r_5	0.6500
n_1	4
n_2	4
n_3	3
n_4	2
n_5	3

A more detailed analysis of the parameter space led to the largest values of the criterion function in both scenarios for α_{ff}=0.4, β_0=0.8, γ =0.01 and the exact maximum value was F_{c_1}=0.999956606987731. For the second scenario, the exact maximum value of the criterion function F_{c_4}=0.999929024736168 was reached in 28 cases (white boxes Figure 8). By reviewing the control parameter space of the FA algorithm (Figure 9) it is possible to observe a high compliance of the range of algorithm parameters

 doi:10.20944/preprints202301.0124.v1

leading to worse solutions. The highest values of correlation r (26) were achieved for (Figure 9): α_{ff}=0.5, r^{FA}=0.9912, α_{ff}=0.7, r^{FA}=0.9881 and α_{ff}=0.9, r^{FA}=0.9858.

By reviewing the control parameter space of the CS algorithm (Figure 8), similar (small value of MPI)—but "worse" solutions can be obtained (Tables 13 and 15). The highest values of correlation r^{CS} (26) were achieved for (Figure 8): p_a=0.39, r^{CS}=0.7855, p_a=0.25, r^{CS}=0.7105 and p_a=0.30, r^{CS}=0.6264.

For the first scenario, the usage of FA led to (Table 6) to better results (compared to the results for the CS), but use of the CS cuckoo algorithm led to a solution in much less time consumed (Table 16). All calculations were performed using Matlab package R2020b (also with Statistic and Machine Learning Toolbox) on Win 10 pro operating system and Intel(R) Core(TM) i5–7200U CPU @ 2.50GHz. The solutions obtained using the CS algorithm were different from those obtained using FA algorithm, for example the number of elements $(n_3 \div n_5)$ (Table 11 vs. Table 12) for scenario 1, number of elements (n_5) Table 10 vs. Table 17) for scenario 2 and of course values the reliability. In the investigated space of FA control parameters, difficulties in obtaining a solution satisfying the assumed constraints occurred in both scenarios for the set α_{ff}=0.1; β_0=1.0; γ =0.1, and additionally for second scenario α_{ff}=0.1, β_0=0.5, γ =0.1. In addition, the effect of changing the lambda parameter (λ) of the Lévy distribution on the solutions achieved was checked with an increased number of iterations (Table 5). For the CS algorithm (first scenario), it led to the highest value of the criterion function for the parameters – p_a = 0.27, λ = 1.5, N_{iter}^{max}=100, $N_{repcalc}$=10 (Table 16) and p_a= 0.30, λ=1.1 for N_{iter}^{max}=1000, $N_{repcalc}$=10000 (Table 13). Increasing the number of iterations N_{iter}^{max} and iterations of the algorithm $N_{repcalc}$ in scenario no. 1, not only increased the value of the criterion function but also reduced the Euclidean difference ($\|\mathbf{r}^{FA} - \mathbf{r}^{CS}\|_2$, $\|\mathbf{n}^{FA} - \mathbf{n}^{CS}\|_2$) to the best solution obtained with the FA algorithm (Tables 16 and 13). For the second scenario, the highest value of the criterion function, for the CS algorithm, was achieved for p_a = 0.33, λ=1.1 (Table 15). Comparing the solutions, despite the small difference in MPI (Tables 7 and 15), different values for the sought variables were achieved (Tables 10 and 17).

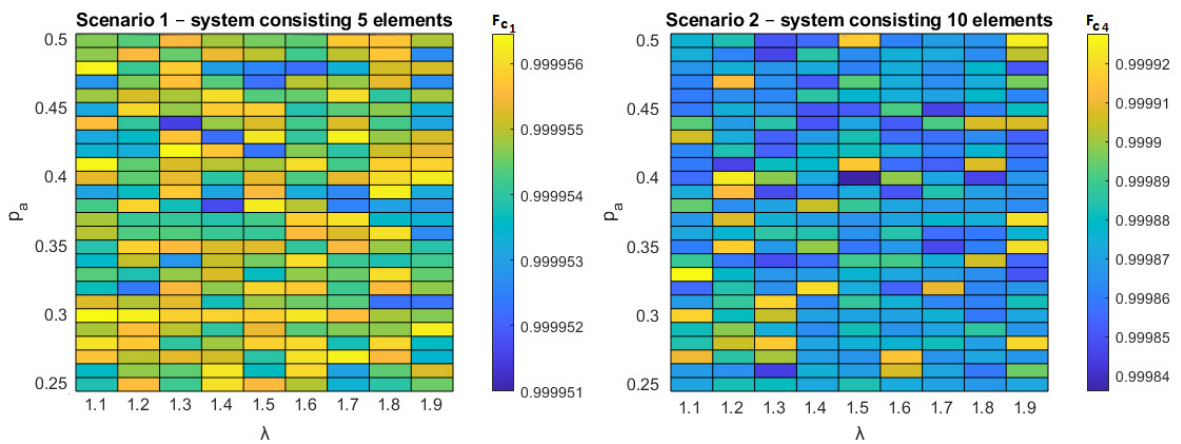

Figure 8. Solutions for scenario 1 and 2 has been achieved using CS algorithm.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 6 January 2023 doi:10.20944/preprints202301.0124.v1

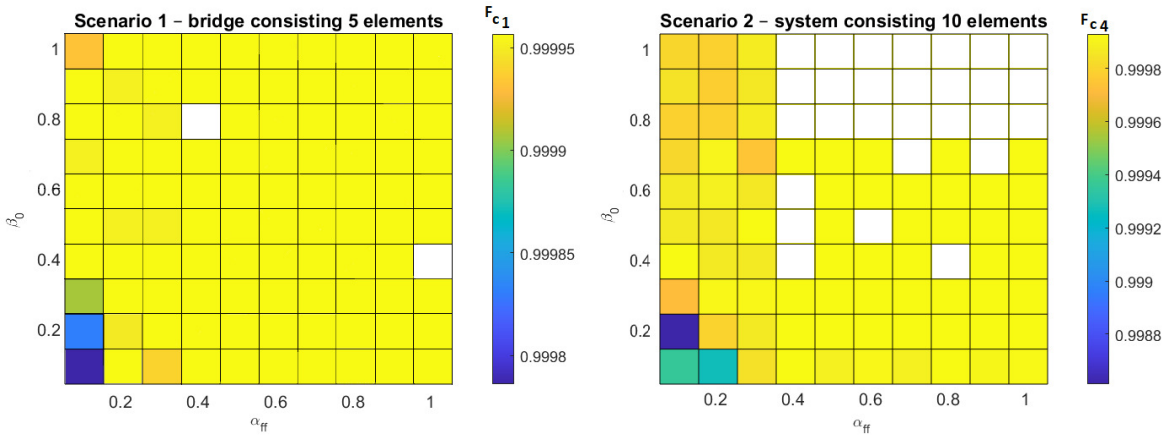

Figure 9. Solutions for scenario 1 and 2 has been achieved using FA algorithm for parameter γ =0.01.

Table 15. The best solution for scenario 2 by CS.

Parameters	p_a=0.33, $\lambda = 1.1$ $N_{iter}^{max} = 1000, N_{repcalc} = 10000$
Solutions F_c^{min}	0.99550799
Solutions F_c^{max}	**0.99992758**
Solutions F_c^{mean}	0.99892402
Solutions σ	5.51629947e-04
Calc. time [s]	387.5
MPI (%)	0.0198
$\|\mathbf{r}^{FA} - \mathbf{r}^{CS}\|_2$	0.1893
$\|\mathbf{n}^{FA} - \mathbf{n}^{CS}\|_2$	1

Table 16. The best solution for scenario 1 by CS.

Parameters	p_a=0.27, $\lambda = 1.5$ $N_{iter}^{max} = 100, N_{repcalc} = 10$
Solutions F_c^{min}	0.99972966
Solutions F_c^{max}	**0.0.99994780**
Solutions F_c^{mean}	0.99984451
Solutions σ	7.40185829e–05
Calc. time [s]	1.7
MPI (%)	0.1688
$\|\mathbf{r}^{FA} - \mathbf{r}^{CS}\|_2$	0.1300
$\|\mathbf{n}^{FA} - \mathbf{n}^{CS}\|_2$	1.7321

 doi:10.20944/preprints202301.0124.v1

Table 17. The best solution for scenario 2 by CS: p_a=0.33, $\lambda = 1.1$, $N_{iter}^{max} = 1000$, $N_{repcalc} = 10000$.

Parameters	Values	Parameters	Values
r_1	0.8500	n_1	4
r_2	0.8500	n_2	4
r_3	0.8500	n_3	4
r_4	0.8500	n_4	4
r_5	0.8500	n_5	3
r_6	0.7671	n_6	1
r_7	0.8453	n_7	2
r_8	0.8432	n_8	2
r_9	0.6973	n_9	2
r_{10}	0.8500	n_{10}	2

6. Conclusions

The presented results broaden the existing discussion on the application of nature-inspired algorithms for solving RRAP problems. From this point of view, two described test scenarios (and the Firefly Algorithm FA) can be considered as a suitable tool for validating other algorithms in RRAP problems. Although the CS algorithm is known for its effectiveness as it uses the Lévy flight (Figure 6), the FA firefly algorithm in the considered parameter range proved to be more effective. The use of the FA algorithm led to solutions with a higher value of the criterion function (Tables 6 and 7). It can also be notices that the best solutions using CS were achieved for λ values different from the 1.5 - as default value ($\alpha = 0.5$) for Lévy stable distribution used. It is worth noting that in both scenarios using the FA algorithm, the sigma value σ (standard deviation) was smaller than the solutions using CS.

Not only the values of the criterion functions can be compared, but also the MPI (Equation (25)), the Euclidean distance differences of both the reliability ($\|\mathbf{r}^{FA} - \mathbf{r}^{CS}\|_2$) and the number of redundant elements ($\|\mathbf{n}^{FA} - \mathbf{n}^{CS}\|_2$) obtained by carefully selecting the parameters of the CS algorithm.

The comparison of the values of the criterion function, the linear r-Pearson correlation coefficient and the data from the post-hoc RIR Tuckey test leads to the selection of the same (or similar) control parameters of the analysed algorithms. Therefore, the presented analysis methods can also be used to compare other optimization algorithms.

Such an approach can broaden the application the well-known test function benchmarks for global in the RRAP optimization (test functions e.g.: Michalewicz's, Rosenbrock's, De Jong's, Schwefel's, Ackley's, Rastring's, Easom's, Griewank's, Shubert's [18,28], Bohachenoticed Matyas's, Zakharov's, Goldstein-Prices [29], other functions [30,31] or other Tallard's test functions [32]). From the compilation of literature data [3,4], the use of algorithms: PSO, MPSO, ABC, CS–GA, BAT, ACO led to obtaining worse solutions than FA. The question remains open: are other algorithms lead to obtaining a different global maximum of the considered (or other RRAP) criterion functions?

Author Contributions: A.P. and B.F. equally contributed in all stages of the project. Conceptualization, A.P. and B.F.; methodology, A.P. and B.F.; software, A.P.; validation, A.P. and B.F.; formal analysis, A.P.; investigation, A.P. and B.F.; resources, B.F.; data curation, A.P.; writing—original draft preparation, A.P.; writing—review and editing, A.P. and B.F.; visualization, A.P.; supervision, B.F.; project administration, A.P. All authors have read and agreed to the published version of the manuscript. All authors reviewed the manuscript.

Funding: This research received no external funding.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Sample Availability: Samples of the compounds ... are available from the authors.

Abbreviations

The following abbreviations are used in this manuscript:

RRAP	Reliability Redundancy Allocation Problem
CS	Cuckoo Search
FA	Firefly Algorithm

References

1. Sobczak, W. *Podstawy Probabilistyczne Teorii SystemóW Informacyjnych*; WNT: Warsaw, Poland, 1981.
2. Yang, X.S. *Nature-Inspired Optimisation Algorithms*; Elsevier, 2014. https://doi.org/10.1016/C2013-0-01368-0.
3. Yang, X.S. Multiobjective firefly algorithm for continuous optimization. *Eng. Comput.* **2013**, *29*, 175–184. https://doi.org/10.1007/s00366-012-0254-1.
4. Kwiecień, J.; Filipowicz, B. Optymalizacja niezawodności zlozonych systemw za pomoca algorytmu świetlika. *Eksploat. Niezawodn. Maint. Reliab.* **2017**, *19*, 296–301. https://doi.org/10.17531/ein.2017.2.18.
5. Kwiecień, J. *Algorytmy stadne w rozwiazaniu wybranych zagadnień optymalizacji dyskretnej i kombinatorucznej*; AGH: Cracow, Poland, 2015.
6. Klempka, R.; Filipowicz, B. Comparison of using the genetic algorithm and cuckoo search for multicriteria optimisation with limitation. *Turk. J. Electr. Eng. Comput. Sci.* **2017**, *25*, 1300–1310. https://doi.org/10.3906/elk-1511-252.
7. Zhao, K.; Jurdak, R.; Liu, J.; Westcott, D.; Kusy, B.; Parry, H.; Sommer, P.; McKeown, A. Optimal Levy-flight foraging in a finite landscape. *J. R. Soc. Interface R. Soc.* **2015**, *12*. https://doi.org/10.1098/rsif.2014.1158.
8. Fuksa, A.K. Zastosowanie sztucznej inteligencji w optymalizacji niezawodnościowej systemów. PhD Thesis, AGH Akademia Górniczo-Hutnicza, Kraków, 2017.
9. Kuo, S.; Yeh, F.; Lin, H. Efficient and Exact Reliability Evaluation for Networks With Imperfect Vertices. *IEEE Trans. Reliab.* **2007**, *56*, 288–300. https://doi.org/10.1109/TR.2007.896770.
10. Kuo, S.Y.; Lu, S.K.; Yeh, F.M. Determining terminal-pair reliability based on edge expansion diagrams using OBDD. *IEEE Trans. Reliab.* **1999**, *48*, 234–246. https://doi.org/10.1109/24.799845.
11. Yeh, F.M.; Lu, S.K.; Kuo, S.Y. OBDD-based evaluation of k-terminal network reliability. *IEEE Trans. Reliab.* **2002**, *51*, 443–451. https://doi.org/10.1109/TR.2002.804736.
12. Valian, E. Solving Reliability Optimization Problems. In *Cuckoo Search and Firefly Algorithm Theory and Applications*; Yang, X.S., Ed.; Springer: London, 2014; pp. 195–216.
13. Pijarski, P. *Optymalizacja heurystyczna w ocenie warunków pracy i planowania rozwoju systemu elektroenergetycznego*; Politechnika Lubelska: Lublin, Poland, 2019.
14. R., D.; V., K. Power system oscillation damping controller design: A novel approach of integrated HHO-PSO algorithm. *Arch. Control. Sci.* **2021**, *31*, 553–591. https://doi.org/10.24425/acs.2021.138692.
15. Kusiak, J.; Danielewska-Tułecka, A.; Oprocha, P. *Optymalizacja Wybrane metody z przykladami zastosowań*; PWN: Warsaw, Poland, 2009.
16. Filipowicz, B.; Kwiecień, J. Algorytmy stadne w problemach optymalizacji. *Pomiary Autom. Robot.* **2011**, *15*, 152–157.
17. Fuksa, K. Zastosowanie sztucznej inteligencji w optymalizacji niezawodnościowej systemów. PhD Thesis, AGH, AGH Cracow, Poland, 2017.
18. Yang, X.S.; Deb, S. Cuckoo Search via Levy Flights **2010**.
19. Vázquez, R.; Sandoval, G.; Ambrosio Bastian, J. How to Generate the Input Current for Exciting a Spiking Neural Model Using the Cuckoo Search Algorithm; 2014; Vol. 516, pp. 155–178. https://doi.org/10.1007/978-3-319-02141-6_8.
20. Manteng, R. Fast, accurate algorithm for numerical simulation of Levy stable stochastic process. *Phys. Rev. E* **1994**, *5*, 4677–4683. https://doi.org/10.1103/PhysRevE.49.4677.
21. Viswanathan, G.; Afanasyev, V.; Buldyrev, S.V.; Havlin, S.; da Luz, M.; Raposo, E.; Stanley, H. Lévy flights in random searches. *Phys. A Stat. Mech. Its Appl.* **2000**, *282*, 1–12. https://doi.org/10.1016/S0378-4371(00)00071-6.

 doi:10.20944/preprints202301.0124.v1

22. Nolan, J. *Stable Distributions: Models for Heavy-Tailed Data*; Springer: New York, NY, USA, 2016.
23. Hughes, B. *Random Walks and Random Environments: Random walks*; Number t. 1 in Oxford science publications, Clarendon Press, 1995.
24. Chechkin, A.; Metzler, R.; Klafter, J.; Gonchar, V., Introduction to the Theory of Lévy Flights; 2008; pp. 129 – 162. https://doi.org/10.1002/9783527622979.ch5.
25. Bovet, A. An Introduction to Non-diffusive Transport Models. *ArXiv* **2015**.
26. Yang, X.S. Website of the: Cuckoo Search (CS) Algorithm, MATLAB Central File Exchange. 2020. Available online: https://www.mathworks.com/matlabcentral/fileexchange/29809-cuckoo-search-cs-algorithm (accessed on).
27. Yang, X.S. Website of the: Firefly Algorithm, MATLAB Central File Exchange. 2021. Available online: https://www.mathworks.com/matlabcentral/fileexchange/29693-firefly-algorithm (accessed on).
28. Roy, S.; Chaudhuri, S. Cuckoo Search Algorithm using Lèvy Flight: A Review. *Int. J. Mod. Educ. Comput. Sci.* **2013**, *5*, 10–15. https://doi.org/10.5815/ijmecs.2013.12.02.
29. Mareli, M.; Twala, B. An adaptive Cuckoo search algorithm for optimisation. *Appl. Comput. Inform.* **2017**, *14*. https://doi.org/10.1016/j.aci.2017.09.001.
30. Yang, X.S., Ed. *Nat.-Inspired Optim. Algorithms*; Elsevier: Oxford, 2014; p. i. https://doi.org/10.1016/B978-0-12-416743-8.00016-6.
31. Surjanovic, S.; Bingham, D. Virtual Library of Simulation Experiments: Test Functions and Datasets. Available online: http://www.sfu.ca/ ssurjano (accessed on 31 December 2022).
32. Taillard, E. Benchmarks for basic scheduling problems. *Eur. J. Oper. Res.* **1993**, *64*, 278–285.