Article

An Axiomatic Formulation of Dimensionless Constants in Physical Sciences Beyond their Ascertained Invariance

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Abstract: Two meters is the ratio of the distance between two points according to the standard 1-meter ruler saved somewhere in France, and this comparative ratio is really equal to dimensionless number 2. We can easily repeat such comparative ratios for kilograms and any other quantities that modern physics believes they carry units. When we think about it, all physical quantities are dimensionless comparative ratios referred to standard units saved somewhere in France (never mind more complicated, derived units, they still depend on the particular items saved in France). Once this important fact is established for dimensional units, we realize how we should deal with dimensionless constants in physics: we need to determine one such constant by experiment, and then all other related dimensionless constants fall in place by simple ratios, just as has been done for dimensional units over many years in the past. For the forces of nature, we advocate the fine-structure constant as the dimensionless quantity that will serve as the baseline for other dimensionless constants representing the coupling of forces. Then, we can extend gravity in the atomic world and quantum physics into large-scale cosmology, both quite flawlessly.

Keywords: Cosmology; Elementary particles; Gravitation

1. Introduction

The distance between me and my friend is 2 meters. What does this number/unit really mean? It means that the purported distance is twice as long as the standard 1-m ruler saved in France. This distance is really a dimensionless ratio, and the value assumes superficial units only by comparison to that standard ruler saved in France. The artificiality of such comparisons pervades physics [1]. People think that units, such as meters and kilograms, are meaningful; and these subjective, concocted (arbitrary) units acquire a dimension of their own in our lives. Everyone uses them, as if they were household items, although most scientists have never visited those arbitrary standards saved somewhere in France.

In physics, we also encounter dimensionless quantities, and we do not really know what to do with them. To declare that dimensionless quantities are invariant in all systems of units [2] is, of course, a trivial matter. But can we say/do anything more useful about such important physical constants? Of course, we can!

Physics is a science of comparisons. All quantities must be referred to their respective standards, dimensional or not [3]. This simple fact has not been realized until now, and there are no works that instill this concept to their readers. In fact, we bring up our students believing that 2 meters or 3 kilograms are "standard" and meaningful quantities, although most students have never experienced the 1-m rod or the 1-kg cylinder, the standardized items guarded somewhere in France. Instead, we all learn their meaning from every-day chores and encounters.

In the next section, we describe how we can use the most famous dimensionless constant, the fine-structure constant α_h, to normalize the other force-related constants in nature. We choose α_h because it can be measured by experiment, whereas other related constants, such as the gravitational coupling constant α_G, will never be measured by experiment, simply because they are too small in the (sub)atomic world. In the process, we correct Dirac's error [4–6], known as the "universal constant $\hbar = h/(2\pi)$", where h is Planck's constant. Planck's constant h is the true universal constant, because the geometry-dependent 2π term carries the units of radians [3,7,8], and the two-dimensional geometry implied by 2π certainly obscures and exarcerbates the physics relating to the subatomic world [9]. In the last section, we summarize our conclusions.

2. The correct treatment of physical dimensionless constants

We dismiss Dirac's error of the quantum universal constant $\hbar$ [4,5] at the outset. It is absolutely wrong to introduce two-dimensional geometry in Planck's constant h, such as that in Dirac's $\hbar = h/(2\pi)$. In quantum physics, the only universal constant is Planck's celebrated constant h [10,11].

Using Planck's h, we define the fine-structure constant α_h by the equation

$$\alpha_h \equiv \frac{e^2/(4\pi\epsilon_0)}{hc} = \frac{1}{861}\,, \tag{1}$$

and the gravitational coupling constant α_G by the equation

$$\alpha_G \equiv \frac{Gm_{\rm e}^2}{hc} = 2.7881 \times 10^{-46}\,, \tag{2}$$

where e is the fundamental charge, ϵ_0 is the permittivity of the vacuum, c is the speed of light, G is the Newtonian gravitational constant, and $m_{\rm e}$ is the mass of the electron. The denominator 861 in equation (1) is the well-known value 137 multiplied by 2π, in order to remove the artificial two-dimensional geometry dependence introduced by $\hbar$ at modern times.

We also define the comparative ratio $\beta_G \equiv \alpha_G/\alpha_h$, viz.

$$\beta_G = \frac{Gm_{\rm e}^2}{e^2/(4\pi\epsilon_0)} = 2.4006 \times 10^{-43}\,, \tag{3}$$

that is clearly independent of h and c.

The above equations, with $\hbar$ in place of h, have seduced and tormented many of the great minds of the past [see, e.g., 4,12–17]. The error in using $\hbar$ rather than Planck's h could not be overcome by the greatest thinkers over the past two centuries [see Ref. 17, for a broad discussion of the problem]. With h being prominently displayed in equations (1) and (2), we can now rectify this very old conundrum. The h-dependent equations exhibit the following indisputable physical properties:

(a) The fine-structure constant is clearly dependent on three-dimensional geometry because of the 4π term that is introduced by the electric field.
(b) The gravitational coupling constant is not at all dependent on geometry.
(c) These dependencies were reversed at modern times by the use of $\hbar$ instead of h. Dirac's $\hbar$ eliminates the geometry of the electric field in α_h, and it introduces spurious geometry in the gravitational-field constant α_G. This erroneous reversal is the blockade that does not allow us to make progress nowadays.
(d) The ratio β_G restores sanity because h or $\hbar$ cancel out completely!

The ratio β_G has never been seriously considered in the past, besides the usual pronouncement that gravity is too weak in the atomic world, and Dirac's attempt to somehow reset its value to 1 in the distant cosmological past (the early universe). But this is precisely the quantity that should have been at the center of all older investigations.

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Having gotten rid of $\hbar$ and h, the geometric error has been resolved, and this ratio shows us how we should truly approach dimensionless constants in physics—in ratios. Just like we have always done for dimensional constants. The only difference is that here we cannot define an arbitrary dimensionless scale and save it somewhere in France, as we have done for all major dimensional scales. Instead, the dimensionless reference value must be supplied by a measured dimensionless constant, and the fine-structure constant is appropriate for this purpose. Therefore, β_G is effectively normalized to α_h, and it tells us that gravity is indeed too weak in the (sub)atomic world, as it should be since (sub)atomic masses are tiny by all standards.

It is well-known that gravity is an extremely weak force (equation (3)), and it can only grow and dominate only if enormous amounts of mass can be accumulated in the same region of space. Dirac's attempt to amplify the force in the atomic world was doomed to failure from the outset, simply because (sub)atomic masses are ridiculously small. Rather than concoct pretences to override this fact, we adopted this unshakeable result [3], and we investigated where it leads us. As is now known, this investigation leads us to a connection between the Planck scale and the atomic world. But the fact remains, that this connection was found only because we treated the dimensionless constants correctly—in ratios.

3. Conclusions

In this work, we have tried to describe in simple terms how dimensionless constants should be treated in the physical sciences—in ratios, relative to a fundamental dimensionless constant that can be measured by experiment [18]. Our conclusions rely on the results of [3].

The working example in the present case is the famous fine-structure constant 137 that, unfortunately, depends on Dirac's $\hbar$ in modern times. We have shown that the true universal fine-structure constant is

$$861$$

when Planck's constant h is used in the definition of the fine-structure constant rather than $\hbar$, where $861 = 137(2\pi)$. We believe that our results are not debatable, because they find strong support from recent seminal works [7,8]. Furthermore, we have discovered a physical explanation for 861, the only one ever proposed [for details, see 3]: the dimensionless factor $\sqrt{861} \approx 30$ is a scale factor used by the Higgs field to assign much lower masses [to the bottom quark and below; see 9,15]. The only other scale factor used by the Higgs field is Koide's constant of 2/3 [19], which is quite small compared to the $\sqrt{861}$ scale.

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