White Holes are Christmas Magic

Enrique Gaztanaga[1,2*]

[1*]Institute of Space Sciences , (ICE, CSIC), Campus UAB, Carrer de Can Magrans, s/n, Bellaterra, 08193, Barcelona, Spain.

[2]Institut d'Estudis Espacials de Catalunya, (IEEC), Edif. Nexus, Barcelona, 08034, Barcelona, Spain.

Corresponding author(s). E-mail(s): gaztanaga@darkcosmos.com;

Abstract

White Hole solutions in classical General Relativity refer to the time reverse Black Hole solutions that allow the crossing of the gravitational radius inside out. The Big Bang model is the most famous White Hole solution. But recent measurements of cosmic acceleration indicate that this solution is not a White Hole, but an expanding Black Hole. We present a general explanation of how this happens which solves the Dark Energy mystery and indicates that classical spherically symmetric White Holes do not exist.

Keywords: Cosmology;Dark Energy; General Relativity; Black Holes

1 Introduction

A Schwarzschild (SW) Black Hole (BH) solution:

$$ds^2 = -\left[1 - r_S/r\right]\, dt^2 + \frac{dr^2}{\left[1 - r_S/r\right]} + r^2 d^2\Omega, \quad (1)$$

represents a singular point source of mass M. The gravitational radius $r_S \equiv 2GM$ corresponds to an Event Horizon and prevent us from seeing inside r_S. The SW solution applies to the exterior of any BH, no matter what the interior solution is, as long as we can approximate the outside as empty space. The Hawking-Penrose's theorems [1, 2] tell us that nothing can come out of r_S and this has created the BH information lost paradox [3, 4]. One possible way around this is to introduce the concept of maximally extended SW solution using the Kruskal–Szekeres coordinates $T = T(t, r)$ and $X = X(t, r)$ (see Fig.1) where the future BH event horizon becomes the past White Hole (WH) horizon. Information can escape r_S in a WH. There are two disconnected exterior spaces which could be connected inside with an Einstein-Rosen bridge or SW wormhole [5].

If we throw a particle into a BH, the WH solution corresponds to the traveling of that particle back in time to us (from our past), before the particle was sent. Such trajectory might be formally possible (because there is no arrow of time at the fundamental level), but it violates causality, so it makes no physical sense as a classical solution (quantum mechanics effects might provide some way around this [6]). This is related to the example of retarded and advanced potentials in classical electrodynamics: both are mathematical solutions of the wave equations, but only one of them connects cause and effect. The mirror image of the top quadrant in Fig.1 has the arrow pointing downward and not upward. This shows that the time reverse solution (the mirror image) is still a BH (where the particle falls into the gravitational radius rS) and not a WH (as indicated in the figure).

Here we will study the more realistic case of classical LTB solutions, which include the FLRW

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2022 doi:10.20944/preprints202212.

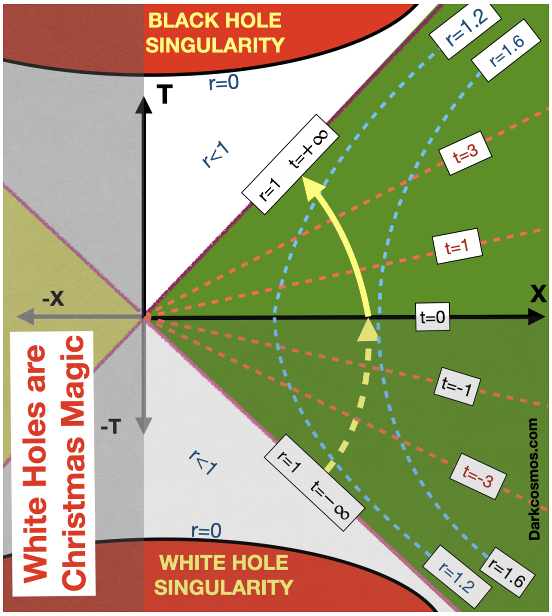

Fig. 1 Relation between the SW coordinates (t, r) (in units of $r_S = 1$) and the Kruskal–Szekeres coordinates (T, X). The top right quadrant $(T > 0,\ X > 0)$ is the regular BH solution (the green region is external to r_S). Lines of constant r are the cyan hyperbolic dashed lines. Lines of constant time t are orange dashed straight lines ($T = 0$ is also $t = 0$). If we throw a radial test particle at $t = 0$, it will follow the continuous yellow arrow. This solution can be formally extended into the bottom right quadrant, which corresponds to a WH, and is the time reverse (horizontal flip) of the BH solution. A particle can escape the event horizon of the WH (dashed yellow arrow), but only before it is thrown! This violates causality and is therefore not a physical solution. With a vertical flip, the solution could be maximally extended to a negative radius $X < 0$. But this generates a disconnected external space (yellow region), which is also not part of the original solution.

metric, the Oppenhaimer-Snyder BH [7] and the thin shell BH [8] as particular cases. For some reason, i.e. the difficulty of a black-to-white hole bounce (see [9, 10]), these solutions are usually investigated only as BH collapsing solutions. As we will show, the same solutions also exist, in principle, as WH solutions, the most famous of which is the expanding Big Bang model originally proposed by Friedman in 1922 [11] and Lemaitre in 1927 [12]. But at closer inspection, these solutions need to be modified to include a surface term. After that correction, we show that the WH correspond in fact to BH expanding solutions. Our argument is supported by considering surface terms in the Einstein-Hilbert action of classical GR and also by the recent observation that our cosmic expansion is accelerating.

2 LTB solutions

The most general metric with spherical symmetry in spherical coordinates $dx^\mu = (dt, dr, d\theta, d\phi)$ can be written as [13]:

$$ds^2 = -A(t,r)\ \ dt^2 + B(t,r)\ dr^2 + R^2(t,r) d^2\Omega. \qquad (2)$$

Other common notation is: $A \equiv e^\nu$, $B \equiv e^\lambda$ and $R^2 \equiv e^u$ [7, 14, 15]. An alternative to this uses proper time $dx^\mu = (d\tau, d\chi, d\theta, d\phi)$:

$$ds^2 = -d\tau^2 + e^{\lambda(\tau,\chi)} d\chi^2 + r^2(\tau,\chi) d\Omega^2, \qquad (3)$$

where the radial coordinate χ can be comoving or not (because its evolution can be encoded in the λ and r functions). This last metric is sometimes called the *Lemaitre-Tolman* metric [12, 14] or the *Lemaitre-Tolman-Bondi* or *LTB metric*. A metric, such as this one, expressed with $g_{00} = 1$ and $g_{0\mu} = 0$ is called *synchronous* (or in a *synchronous frame*) because time lines are geodesics. Either way, it is possible to express the spherical symmetric metric with 2 functions and the best form in each case depends on the energy content and the observer's frame. In all cases, this is a *local metric* around a reference central point in space which we have set to be the origin ($\vec{r} = 0$).

The advantage of using the proper time and an observer moving with a perfect fluid is that the stress tensor becomes diagonal: $T^\nu_\mu = diag[-\rho, p, p, p]$, where $\rho = \rho(\tau, \chi)$ is the energy density and $p = p(\tau, \chi)$ is the pressure. We will focus here in the matter dominated case $p = 0$ for simplicity, but we expect similar results to apply to more general situations (see [16]). The solution to the field equation $8\pi G T^1_0 = G^1_0 = 0$ is $\dot{\lambda} r' = 2\dot{r}'$ where dots and primes correspond to time τ and radial χ partial derivatives. This equation can be solved as: $e^\lambda = C r'^2$, where $C = C(\chi)$ is an arbitrary function of χ. The choice $C = 1$ corresponds to the particular flat geometry case:

$$ds^2 = -d\tau^2 + [\partial_\chi r]^2 d\chi^2 + r(\tau,\chi)^2 d\Omega^2, \qquad (4)$$

The solution for r in this case is easily found:

$$H^2 \equiv r_H^{-2} \equiv \left(\frac{\dot{r}}{r}\right)^2 = \frac{2GM}{r^3} \qquad (5)$$

$$M \equiv 4\pi \int_0^\chi \rho(\tau,\chi) r' r^2 d\chi = M(\chi) \qquad (6)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2022 doi:10.20944/preprints202212.

The above expression reproduces the Newtonian energy conservation in free fall: $\frac{1}{2}\dot{r}^2 = GM/r$ [17] and corresponds to an expanding or collapsing relativistic spherical ball. When $\rho = \rho(\tau)$ is uniform, we find $r = a(\tau)\chi$, so that Eq.4 and Eq.5 reproduce the flat FLRW metric and corresponding solution $3H^2 = 8\pi G\rho$ [1].

The next simplest solution to Eq.5 is that of the FLRW cloud with a fixed total mass M_T:

$$M_T \equiv \int_0^\infty \rho\, 4\pi r^2 (\partial_\chi r) d\chi \qquad (7)$$

The solution is $r = a\chi$ as in the standard FLRW metric but with a boundary at $R(\tau) \equiv a(\tau)\chi_*$ above which ($\chi > \chi_*$) we have empty space: $\rho = 0$.

This is a consequence of *Birkhoff's theorem* [18] (or *Gauss' law* in non relativistic mechanics), since a sphere cut out of an infinite uniform distribution has the same spherical symmetry. Thus, the FLRW metric is both a solution to a global homogeneous (i.e. $M_T = \infty$) uniform background and also to the inside of a local (finite M_T) uniform sphere centered around one particular point. The local solution is called the FLRW cloud (FLRW*) [16]. As we will show next, the LTB solution can in principle correspond to either a BH or a WH.

A timelike radial geodesic ($d\chi = 0$) has a mass-energy M inside χ which is independent of τ. A fixed coordinate $\chi = \chi_*$, corresponds to a system with a fixed mass M_T (see also [19]) which is expanding or collapsing following the Hubble-Lemaitre law of Eq.5. From Eq.5 we have $H = H_S(a/a_S)^{-3/2} = \pm 1/\tau$, where H_S is just the value at some arbitrary time ($a = a_S$), when R intersects r_S, so that $r_S = a_S\chi_* = 1/H_S = 2GM_T$. This solution is time reversible and the evolution can cross r_S. This is a well know solution which includes the Oppenheimer-Snyder BH collapse [7]. But note that when $R < r_S$ we have $R > r_H$ (or $\dot{R} > 1$) which creates a region between $R > r > r_H$ which is acausal during expansion (this is the well known horizon problem in the standard Big Bang cosmology). We can also reproduce the same LTB (or FLRW*) solution using junction conditions to verify that the outside of r_S is indeed a classical (SW) BH despite the look of Eq.4. This derivation [16] is reproduced in Appendix A (with some typos corrected) for reference.

To show that this solution crosses the gravitational radius r_S we can estimate the Event Horizon (EH), R_{EH}, of the FLRW* metric. This is the maxium distance that a photon emitted at time τ can travel (outgoing radial null geodesic, [20]):

$$R_{EH} = a(\tau) \int_\tau^\infty \frac{d\tau}{a(\tau)} = a \int_a^\infty \frac{da}{H(a)a^2} \qquad (8)$$

For $H \sim a^{-3/2}$ we have $R_{EH} \sim a^{3/2}$, which grows unbounded with a and therefore crosses r_S, as shown by the dashed red line in Fig.2.

The case $H_S < 0$ corresponds to a collapsing solution, and therefore a BH. This collapsing solution is protected by the Equivalence principle, as a free fall test particle placed at $r = R$ is equivalent to a particle moving in empty space and can therefore cross r_S. The case $H_S > 0$ represents an expanding solution and corresponds to a WH. It just corresponds to a fluid expanding inside r_S. But what is strange about this solution is that information can actually escape from the inside to the outside of r_S, which is contrary to all we have learned about BHs. How is that possible?

The standard objection to this paradox is that this expanding configuration can never be achieved. This is reflected in the fact that $R > r_H$ is not causally connected to its past (the horizon problem), which is a similar objection to the one for WH interpretation of the SW solution, as discussed in the Introduction. But note that this expanding solution corresponds to a matter dominated Big Bang solution, which is very close to current observations.[2]. This is why it is often said that the Big Bang is a WH [3].

Here we argue that this expanding WH solution is not correct. This is not because it can not be achieved (as illustrated by the existence of our own observed universe). The gravitational radius

[1] The non flat case can also be reproduced if we study the more general solution $e^\lambda = (r')^2/[1 + K(r)]$.

[2] This is the case only if we ignore cosmic acceleration or if we consider an observer in a galaxy far away (say at $z = 2$), when matter domination was an excellent approximation.

[3] Note that both a WH and a BH require a finite total mass. If M_T is infinitely large, then $r_S = \infty$ and there is no WH or BH. This in fact the standard Big Bang assumption. But does it makes any physical sense to have an infinite mass, spread uniformly over an infinite space for a Universe that is finite in time?

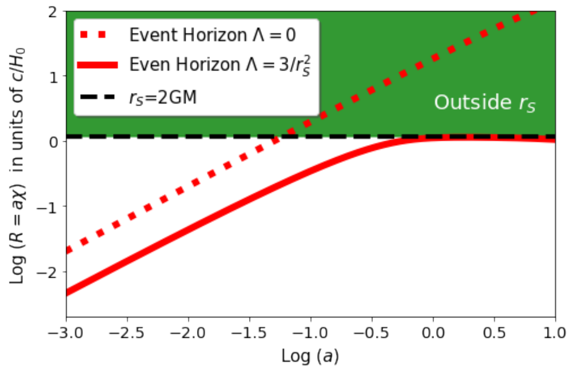

Fig. 2 Event Horizon in Eq.8 as a function of cosmic time (given by the scale factor a) for a matter dominated FLRW metric ($\Omega_m = 1$, $\Omega_\Lambda = 0$, dashed red line) and for one which also has a $\Lambda = 3/r_S$ term ($\Omega_m = 0.25$, $\Omega_\Lambda = 0,75$, continuous red line).

r_S should be interpreted as a boundary that separates the interior from the exterior manifold. This is strictly the case if the outside is empty (as we assume here). But even if the exterior is not totally empty and there is some small accretion from the outside, the value of r_S will slowly increase as the BH mass increases. But the r_S boundary still needs to be taken into account to evaluate the action inside. Such boundary requires that we change the GR field equations. Appendix B reproduces here the original calculation in [16, 21] that shows that the GHY boundary in the action corresponds to an effective Λ term: $\Lambda = 3/r_S^2$. We will show next how this boundary term transforms the WH solution into a BH solution.

2.1 How a WH turns into a BH

We will next review the derivation in Appendix A including an effective Λ term: $\Lambda = 3/r_S^2$ inside r_S. Such Λ term does not change the form of the FLRW metric itself, but (as is well known) it changes the solution to expansion rate $3H^2 = 8\pi G\rho + \Lambda$. The Λ term does change the form of the SW metric inside to deSitter–Schwarzschild metric: $F = 1 - r_S/R - R^2/r_S^2$. So to find the new junction we just need to replace F in the definition of β in Eq.14 . The new second junction condition then becomes:

$$R = \left[\frac{r_H^2 r_S^3}{r_S^2 - r_H^2}\right]^{1/3} \quad \text{or} \quad H^2 = \frac{r_S}{R^3} + \frac{1}{r_S^2} \qquad (9)$$

which is exactly the new Hubble law with $\Lambda = 3/r_S^2$ and a constant mass $M = 4/3\pi\rho R^3$ in Eq.6. This shows that the LTB (or FLRW*) expanding metric is also a solution to the new field equations with the r_S boundary. But this solution is no longer a WH, but it has become a BH. We can check this by estimating the new EH in Eq.8 including now the effective Λ term in H. The new estimation for R_{EH} is displayed as a red continuous line in Fig.2. As can be seem, the EH is trapped inside r_S, which indicates that no information can escape. The WH solution has now turn into a BH.

3 Conclusion

We have shown that classical WH solutions in GR can be turn into an expanding BH solutions once we account for the fact that the gravitational radius r_S corresponds to a boundary condition in the action of GR.

The matter dominated case study here is a very good approximation for our Universe, because in the later stages of its evolution it is totally dominated by mater and the effective $\Lambda = 3/r_S^2$. This could also be in general a good approximation for stellar or supermassive BHs with uniform density and pressure because as $a \to \infty$ inside, matter and Λ always dominate. The characteristic gravitational time is quite short:

$$\tau \sim GM \simeq 1.1 \times 10^{-13} \frac{M}{M_\odot} \mathrm{yr}\,, \qquad (10)$$

so even for a super massive BH ($M \sim 10^9 M_\odot$) time is measured in seconds or hours. In astronomical time-scales, the evolution is quickly dominated by the effective $\Lambda = 1/r_S^2$ term inside. This, by the way, explains the coincidence problem in our Universe [22].

If we think of experimental Cosmology before the year 2003 (i.e. ignore cosmic acceleration for a minute), the LTB expanding WH solution in Eq.5 (with $H > 0$) agrees very well with all the observations at that time, which favoured a matter dominated Universe (the so called EdS universe with $\Omega_m = 1$). This is why some people still say that the Big Bang is a WH. But today we know that the universe has an effective Λ term and this could indicate instead that we are inside a BH [16].

Here we interpret the observed Λ to be an effective term that corresponds to the gravitational radius $r_S = \sqrt{3/\Lambda} = 2GM$ of our local Universe. Such BH Universe (BHU) is within a larger background that may or may not be totally empty. In the later case, r_S will increase if there is accretion from outside. This case needs to be studied in more detail, but it would result an effective Λ term that decreases with time ($\omega > -1$).

We acknowledge grants from Spain (PGC2018-102021-B-100, CEX2020-001058-M) and the EU (LACEGAL 734374).

References

[1] Penrose, R.: Gravitational collapse and space-time singularities. Phys. Rev. Lett. **14**, 57–59 (1965)

[2] Dadhich, N.: Singularity: Raychaudhuri equation once again. Pramana **69**(1), 23 (2007) https://arxiv.org/abs/gr-qc/0702095 [gr-qc]

[3] Hawking, S.W.: Black holes and thermodynamics. PRD **13**(2), 191–197 (1976). https://doi.org/10.1103/PhysRevD.13.191

[4] Mathur, S.D.: The information paradox: a pedagogical introduction. Classical and Quantum Gravity **26**(22), 224001 (2009) https://arxiv.org/abs/0909.1038 [hep-th]. https://doi.org/10.1088/0264-9381/26/22/224001

[5] Maldacena, J., Susskind, L.: Cool horizons for entangled black holes. Fortschritte der Physik **61**(9), 781–811 (2013) https://arxiv.org/abs/1306.0533 [hep-th]. https://doi.org/10.1002/prop.201300020

[6] Haggard, H.M., Rovelli, C.: Quantum-gravity effects outside the horizon spark black to white hole tunneling. PRD **92**(10), 104020 (2015) https://arxiv.org/abs/1407.0989 [gr-qc]. https://doi.org/10.1103/PhysRevD.92.104020

[7] Oppenheimer, J.R., Snyder, H.: On Continued Gravitational Contraction. Physical Review **56**(5), 455–459 (1939)

[8] Israel, W.: Singular hypersurfaces and thin shells in general relativity. Nuovo Cimento B Serie **44**(1), 1–14 (1966). https://doi.org/10.1007/BF02710419

[9] Barceló, C., Carballo-Rubio, R., Garay, L.J.: Mutiny at the white-hole district. International Journal of Modern Physics D **23**(12), 1442022 (2014) https://arxiv.org/abs/1407.1391 [gr-qc]. https://doi.org/10.1142/S021827181442022X

[10] Ben Achour, J., Brahma, S., Mukohyama, S., Uzan, J.-P.: Towards consistent black-to-white hole bounces from matter collapse. JCAP **2020**(9), 020 (2020) https://arxiv.org/abs/2004.12977 [gr-qc]. https://doi.org/10.1088/1475-7516/2020/09/020

[11] Friedmann, A.: On the Curvature of Space. General Relativity and Gravitation **31**, 1991 (1999)

[12] Lemaître, G.: Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. Annales de la S.S. de Bruxelles **47**, 49–59 (1927)

[13] Padmanabhan, T.: Gravitation, Cambridge Univ. Press, (2010)

[14] Tolman, R.C.: Effect of Inhomogeneity on Cosmological Models. Proceedings of the National Academy of Science **20**(3), 169–176 (1934)

[15] Landau, L.D., Lifshitz, E.M.: The Classical Theory of Fields, (1971)

[16] Gaztañaga, E.: The Black Hole Universe, part I. Symmetry **14**(9), 1849 (2022)

[17] Faraoni, V., Atieh, F.: Turning a Newtonian analogy for FLRW cosmology into a relativistic problem. PRD **102**(4), 044020 (2020)

[18] Johansen, N., Ravndal, F.: On the discovery of birkhoff's theorem. General Relativity and Gravitation **38**, 537–540 (2006)

[19] Gaztanaga, E.: The Mass of Our Observable Universe. Preprints **202301.0337**(v1), 1–6 (2022). https://doi.org/10.20944/preprints202301.0337.v1

[20] Ellis, G.F.R., Rothman, T.: Lost horizons. American Journal of Physics **61**(10), 883–893 (1993)

[21] Gaztañaga, E.: The Cosmological Constant as Event Horizon. Symmetry **14**(2), 300 (2022) https://arxiv.org/abs/2202.00641

[22] Gaztañaga, E.: The Black Hole Universe, part II. Symmetry **14**(10), 1984 (2022)

[23] Hilbert, D.: Die Grundlage der Physick. Konigl. Gesell. d. Wiss. Göttingen, Math-Phys K **3**(7), 395–407 (1915)

[24] Weinberg, S.: Gravitation and Cosmology, John Wiley & Sons, NY, (1972)

[25] Weinberg, S.: Cosmology, Oxford University Press, (2008)

[26] Einstein, A.: Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik **354**(7), 769–822 (1916)

[27] Carroll, S.M.: Spacetime and Geometry. Addison-Wesley,, (2004)

[28] York, J.W.: Role of Conformal Three-Geometry in the Dynamics of Gravitation. PRL **28**(16), 1082–1085 (1972)

[29] Gibbons, G.W., Hawking, S.W.: Cosmological event horizons, thermodynamics, and particle creation. PRD **15**(10), 2738–2751 (1977)

[30] Hawking, S.W., Horowitz, G.T.: The gravitational Hamiltonian, action, entropy and surface terms. Class Quantum Gravity **13**(6), 1487 (1996)

A Timelike Junction

We start by choosing a timelike Σ fixed in comoving coordinates at some fix value χ_*. The spherical shell radius $R = a\chi_*$ follows a radial geodesic trajectory in the FLRW metric. This corresponds to a FLRW cloud of fixed mass M_T that is expanding or contracting. The induced 3D metric $h^-_{\alpha\beta}$ for $dy^\alpha = (d\tau, d\delta, d\theta)$ and fixed $\chi = \chi_*$, is:

$$ds^2_{\Sigma^-} = h^-_{\alpha\beta} dy^\alpha dy^\beta = -d\tau^2 + a^2(\tau)\chi_*^2 d\Omega^2 \quad (11)$$

The only free variable remaining is τ, the FLRW comoving time (the solid angle $d\Omega$ is the same in both metrics as we have spherical symmetry). For the outside SW frame, the same junction Σ^+ is described by some unknown functions $r = R(\tau)$ and $t = T(\tau)$, where t and r are the time and radial coordinates in the physical SW frame of Eq.1. We then have:

$$dr = \dot{R} d\tau \quad ; \quad dt = \dot{T} d\tau, \quad (12)$$

where the dot refers to derivatives with respect to τ. The induced metric h^+ estimated from the outside BH.SW metric (in Eq.1) becomes:

$$\begin{aligned} ds^2_{\Sigma^+} &= h^+_{\alpha\beta} dy^\alpha dy^\beta = -F dt^2 + \frac{dr^2}{F} + r^2 d\Omega^2 \\ &= -(F\dot{T}^2 - \dot{R}^2/F) d\tau^2 + R^2 d\Omega^2 \quad (13) \end{aligned}$$

where $F \equiv 1 - r_S/R$. Comparing Eq.11 with Eq.13, the first matching condition $h^- = h^+$ results in:

$$R(\tau) = a(\tau)\chi_* \quad ; \quad F\dot{T} = \sqrt{\dot{R}^2 + F} \equiv \beta \quad (14)$$

For any given $a(\tau)$ and χ_* we can find both $R(\tau)$ and $\beta(\tau)$. We also want the derivative of the metric to be continuous at Σ. For this, we estimate the extrinsic curvature $K^\pm$ normal to Σ from each side of the hypersurface ($\Sigma^\pm$) as:

$$K_{\alpha\beta} = -\left[\partial_a n_b - n_c \Gamma^c_{ab}\right] e^a_\alpha e^b_\beta \quad (15)$$

where $e^a_\alpha = \partial x^a/\partial y^\alpha$ and n_a is the 4D vector normal to Σ. The outward 4D velocity is $u^a = e^a_\tau = (1, 0, 0, 0)$ and the normal to Σ^- on the inside is then $n^- = (0, a, 0, 0)$. On the outside $u^a = (\dot{T}, \dot{R}, 0, 0)$ and $n^+ = (-\dot{R}, \dot{T}, 0, 0)$. It is straightforward to verify that: $n_a u^a = 0$ and $n_a n^a = +1$ (for a timelike surface) for both n^- and n^+. The extrinsic curvature estimated with the inside FLRW metric, i.e. K^- is:

$$\begin{aligned} K^-_{\tau\tau} &= -(\partial_\tau n^-_\tau - a\Gamma^\chi_{\tau\tau})e^\tau_\tau e^\tau_\tau = 0 \\ K^-_{\theta\theta} &= a\Gamma^\chi_{\theta\theta}e^\theta_\theta e^\theta_\theta = -a\chi_* = R \end{aligned} \quad (16)$$

where we have used Eq.14 and the following Christoffel symbols for the FLRW:

$$\begin{aligned} \Gamma^\tau_{\tau\tau} = \Gamma^\tau_{\tau\chi} = \Gamma^\chi_{\tau\tau} = \Gamma^\chi_{\chi\chi} = 0 \;;\; \Gamma^\tau_{\theta\theta} = -a^2\chi_*^2 H \\ \Gamma^\chi_{\tau\chi} = \Gamma^\tau_{\chi\chi}a^{-2} = -H \;;\; \Gamma^\chi_{\theta\theta} = \chi_* \end{aligned} \quad (17)$$

For the SW metric:

$$\begin{aligned} \Gamma^t_{tt} = \Gamma^r_{tr} = \Gamma^t_{\theta\theta} = 0 \;;\; \Gamma^r_{\theta\theta} = FR; \\ \Gamma^t_{tr} = -\Gamma^r_{rr} = \Gamma^r_{tt}F^{-2} = \frac{r_S}{2FR^2} \end{aligned} \quad (18)$$

which results in K^+:

$$\begin{aligned} K^+_{\tau\tau} &= \ddot{R}\dot{T} - \dot{R}\ddot{T} + \frac{\dot{T}r_S}{2R^2F}(\dot{T}^2F^2 - 3\dot{R}^2) = \frac{\dot{\beta}}{\dot{R}} \\ K^+_{\theta\theta} &= \dot{T}\Gamma^r_{\theta\theta} = \dot{T}FR = \beta R \end{aligned} \quad (19)$$

where we have used the definition of β in Eq.14. In both cases $K_{\delta\delta} = \sin^2\theta K_{\theta\theta}$, so that $K^-_{\delta\delta} = K^+_{\delta\delta}$ follows from $K^-_{\theta\theta} = K^+_{\theta\theta}$. Comparing Eq.16 with Eq.19, the matching conditions $K^-_{\alpha\beta} = K^+_{\alpha\beta}$ require $\beta = 1$, which using Eq.14 gives:

$$R = \left[r_H^2 r_S\right]^{1/3} \quad (20)$$

This just reproduces the LTB (or FLRW*) with M inside R in Eq.6.

B The GHY boundary term

Given the Einstein-Hilbert action [13, 23–25]:

$$S = S_4 \equiv \int_{V_4} dV_4 \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}\right], \quad (21)$$

where $dV_4 = \sqrt{-g}d^4x$ is the invariant volume element, V_4 is the volume of the 4D spacetime manifold, $R = R^\mu_\mu = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar curvature and $\mathcal{L}$ the Lagrangian of the energy-matter content. We can obtain Einstein's field equations for the metric field $g_{\mu\nu}$ from this action by requiring S to be stationary $\delta S = 0$ under arbitrary variations of the metric $\delta g^{\mu\nu}$. The solution is well known [13, 25, 26]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}},$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. This solution requires that boundary terms vanish (e.g. see [13, 15, 27]). Otherwise, we need to add a Gibbons-Hawking-York (GHY) boundary term [28–30] to the action $S = S_4 + S_{GHY}$, where:

$$S_{GHY} = \frac{1}{8\pi G}\oint_{\partial V_4} d^3y\sqrt{-h}\,K. \quad (22)$$

where K is the trace of the extrinsic curvature at the boundary ∂V_4 and h is the induced metric. The expansion inside an isolated BH is bounded by the event horizon $r < r_S$ and we need to add this GHY boundary term S_{GHY} to the action. The integral is over the induced metric at ∂V_4, i.e. Eq.11 with $\partial V_4 = \Sigma^-$ at $R = r_S$:

$$ds^2_{\partial V_4} = h_{\alpha\beta}dy^\alpha dy^\beta = -d\tau^2 + r_S^2 d\Omega^2 \quad (23)$$

So the only remaing degrees of freedom in the action are time τ and the angular coordinates. We can use this metric and Eq.16 to estimate K:

$$K = K^\alpha_\alpha = \frac{K_{\theta\theta}}{R^2} + \frac{K_{\delta\delta}}{R^2\sin^2\theta} = -\frac{2}{R} = -\frac{2}{r_S} \quad (24)$$

We then have

$$S_{GHY} = \frac{1}{8\pi G}\int d\tau\, 4\pi r_S^2\, K = -\frac{r_S}{G}\tau \quad (25)$$

The Λ contribution to the action in Eq.22 is:

$$S_\Lambda = -\frac{\Lambda}{8\pi G}V_4 = -\frac{r_S^3\Lambda}{3G}\tau \quad (26)$$

We have estimated the total 4D volume V_4 as that bounded by ∂V_4 inside $r < r_S$: $V_4 = 2V_3\tau$, where the factor 2 accounts for the fact that $V_3 = 4\pi r_S^3/3$ can be covered twice (during collapse and during expansion). Comparing the two terms we can see that we need $\Lambda = 3r_S^{-2}$ or equivalently $r_\Lambda = r_S$ to cancel the boundary term. In other words: evolution inside a BH event horizon induces a Λ term in the field equations even when there is no Λ term to start with.