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The evolution of the majorana neutrino mass renormalization group in the super-weak theory
Эволюция ренормализационной группы масс майорановских нейтрино в сверхслабой теории

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Сверхслабое взаимодействие включает в себя три простых расширения стандартной модели: калибровочное расширение, фермионное расширение и скалярное расширение. На все эти расширения сильно влияет их сложная феноменология. Они могут объяснить ряд нерешенных вопросов физики элементарных частиц и космологии, включая генезис темной материи, космическую инфляцию, асимметрию материи и антиматерии, массы нейтрино и стабильность вакуума, если их объединить в единую структуру. Это расширение калибровочной группы стандартной модели $G_{\rm SM}$ на $G_{\rm SM} \otimes U(1)_Z$ без каких-либо аномалий. Мы исследуем последствия разработки общей майорановской группы перенормировки масс для нейтрино с массами в диапазоне 0,03 эВ и 0,1 эВ, которые попадают в недавно опубликованный диапазон, а также в диапазон, который предстоит исследовать в будущих
1 запланированных экспериментах.

The super-weak interaction includes three simple extensions of the standard model: gauge extension, fermionic extension, and scalar extension. All of these extensions are strongly influenced by their complex phenomenology. They can explain a number of unresolved questions in particle physics and cosmology, including the genesis of dark matter, cosmic inflation, asymmetry of matter and antimatter, neutrino masses, and vacuum stability, if combined into a single structure. This is an extension of the gauge group of the standard model $G_{\rm SM}$ by $G_{\rm SM} \otimes U(1)_Z$ without any anomalies. We investigate the implications of the development of a general Majorana mass renormalization group for neutrinos with masses in the range of $0.03\ eV$ and $0.1\ eV$, which fall within the recently published range as well as the
2 range to be explored in future planned experiments.

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1. Introduction

The standard model (SM) of elementary particle physics as applied to nature is widely recognized as unsatisfactory. Even if the SM works suspiciously well at describing particle interactions, it must be extended to include new interactions resulting from the discovery of neutrino oscillations in the vacuum and matter, among other phenomena. Even simple extensions can lead to rich phenomenology. Some of them are extensions of the gauge, fermionic, and scalar sectors. A heavy neutral lepton, a singlet scalar boson, or a tiny gauge group can be added to the model. A paradigm that takes into account and includes all three possible outcomes is called a super-weak model [1].

The big bang model predicts that there is a fixed ratio between the number of neutrinos and the number of photons in the cosmic microwave background, which imposes the strongest upper limit on neutrino masses. The universe would collapse due to excess mass if the total energy of three different types of neutrinos exceeded an average of 50 eV per neutrino [2]. It is possible to circumvent this limitation by assuming that the neutrino is unstable, although this is difficult to do due to the limitations of the SM. Therefore, we used the super-weak theory beyond the SM [1, 3–8]. At the top-quark scale, we will have strongest upper limit on the sum of neutrinos masses ($\sum m_\nu$) from the cosmology bound which starts from $\leq 0.09\ eV$ to $0.3\ eV$ [9–12].

2. Super-weak model

The SM group is extended by an extra U(1) group (Gauge extensions):

$$\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_z. \quad (1)$$

The kinetic terms of the $\mathrm{U}(1)_Y \otimes \mathrm{U}(1)_z$ sector of the group can be described with the Lagrangian density:

$$\mathcal{L}^{\mathrm{U}(1)} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - \frac{\varepsilon}{2}F^{\mu\nu}F'_{\mu\nu}, \quad (2)$$

where $F_{\mu\nu}$ and $F'_{\mu\nu}$ correspond to the field strength tensors of $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_z$.

The covariant derivative acting on the fermion field f:

$$\mathcal{D}_\mu^{U(1)} = \partial_\mu - \mathrm{i}(y^f g_y B_\mu + z^f g_z B'_\mu). \quad (3)$$

The y^f and z^f are the hypercharge and $\mathrm{U}(1)_z$ charge of f.

The mass eigenstates (A_μ, Z_μ, Z'_μ) with a rotation:

$$\begin{pmatrix} \hat{B}_\mu \\ W^3_\mu \\ \hat{B'}_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\cos\theta_Z \sin\theta_W & -\sin\theta_Z \sin\theta_W \\ \sin\theta_W & \cos\theta_Z \cos\theta_W & \cos\theta_W \sin\theta_Z \\ 0 & -\sin\theta_Z & \cos\theta_Z \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}, \quad (4)$$

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where θ_W is the weak mixing angle and θ_Z is the $Z - Z'$ mixing angle. Scalar and Goldstone mixing angles (Scalar extension):

The scalar sector of the SM Higgs SU(2) doublet ϕ with charges $(y_\phi, z_\phi) = (1/2, 1)$ and a complex singlet scalar χ with charges $(y_\chi, z_\chi) = (0, -1)$. The relevant Lagrangian is:

$$\mathcal{L}_{\rm scalar} = |D_\mu\phi|^2 + |D_\mu\chi|^2 - \mu_\phi^2|\phi|^2 - \mu_\chi^2|\chi|^2 - \lambda_\phi|\phi|^4 - \lambda_\chi|\chi|^4 - \lambda|\phi|^2|\chi|^2. \quad (5)$$

Parametrizing the fields after spontaneous symmetry breaking (SSB):

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\sqrt{2}\sigma^+ \\ v + h' + i\sigma_\phi \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}}\left(w + s' + i\sigma_\chi\right), \quad (6)$$

where $v \simeq 246.22\ GeV$ and w are the vacuum expectation values and the fields h', s', σ_χ and σ_ϕ are real. The fields σ_ϕ and σ_χ correspond to the Goldstone bosons. The Scalar and Goldstone mixing angles are:

$$\sin\theta_S = -\frac{\lambda v w}{\lambda_\phi v^2 - \lambda_\chi w^2}, \quad \tan\theta_G = \frac{M_{Z'}}{M_Z}\tan\theta_Z. \quad (7)$$

The fermion sector of the super-weak model is extended with three sterile massive Majorana neutrinos $N_R = (\nu_4, \nu_5, \nu_6)$ (Fermion extension):

The gauge invariant Yukawa interactions of the neutrinos are given by Lagrangian density:

$$\mathcal{L}_Y^\nu = -\overline{N_R}Y_\nu\varepsilon_{\alpha\beta}L_{L\alpha}\phi_\beta - \frac{1}{2}\overline{N_R}Y_N(N_R)^c\chi + \text{h.c.}, \quad (8)$$

where α and β are $\text{SU}(2)_L$ indices and $\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

The 3×3 Dirac and Majorana mass matrices are:

$$M_D = \frac{v}{\sqrt{2}}Y_\nu, \quad M_N = \frac{w}{\sqrt{2}}Y_N. \quad (9)$$

The light neutrino mass matrix $M_L = -M_D M_N^{-1} M_D^\dagger + \text{h.c.}$ can be obtained by "block-diagnonalizing" the full 6×6 neutrino mass matrix M via a unitary matrix P_{unit}:

$$P_{unit}^T M P_{unit} = P_{unit}^T \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} P_{unit} = M_{\rm diag} = \text{diag}(m_1, \ldots, m_6). \quad (10)$$

3. Renormalization group equations for Majorana neutrino mass eigenvalues

In order to directly employ the RGEs for the physical observables, specifically the Majorana mass eigenvalues m_i where $(i = 1, 2, 3)$, we will then do the "diagonalize and run" approach for the neutrino parameters. The

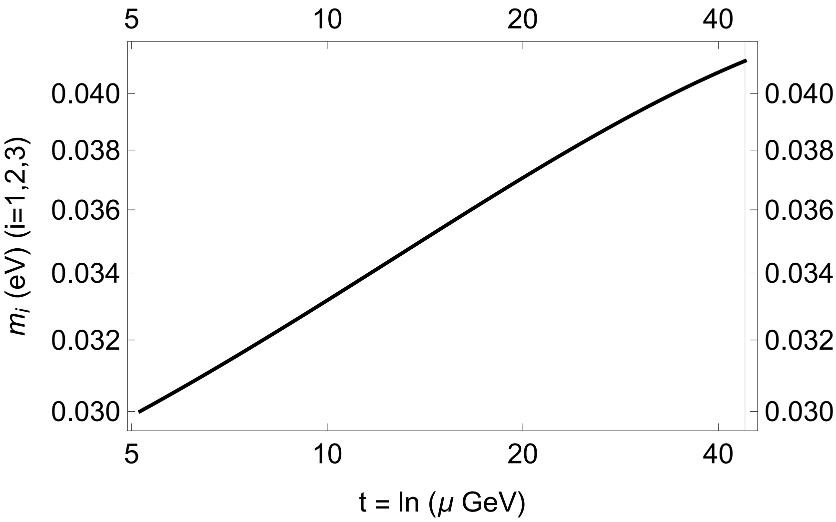

Fig. 1. Evolution of the Majorana neutrino mass eigenvalues from the top-quark pole mass scale to the Planck mass scale relative to the initial values of $m_i = 0.03$ eV, where $(i = 1,\, 2,\, 3)$.

same CP is also assumed to be shared by the neutrino mass eigenstates, and CP-violating phases in the mixing matrix are ignored.

Specifically, the real neutrino mixing matrix is:

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - c_{23}s_{13}s_{12} & c_{13}c_{23} \end{bmatrix}, \tag{11}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ $(i, j = 1, 2, 3)$. The mass matrix M in the flavor basis is diagonalized by U using $U^T M U = \mathrm{diag}(m_1, m_2, m_3)$. Based on the mass eigenvalues of the SM [13–15], the RGEs for the Majorana neutrino mass eigenvalues for the super-weak theory can be formulated as:

$$\frac{dm_i}{dt} = \frac{m_i}{16\pi^2}\left(2\lambda_{vw} + 6Y_t^2 + 2Y_\tau^2 - 3Y_t^2 U_{\tau i}^2 - 3g_2^2\right), (i = 1, 2, 3)\,. \tag{12}$$

4. Results

Numerical run of coupled RGEs with initial quasi-degenerate or almost identical values of m_i, 0.03 eV and 0.1 eV at top-quark pole mass (173.1 GeV) up to the Planck scale (1.22×10^{19} GeV) are shown in Fig. 1 and Fig. 2, respectively. Regardless of any initial values of m_i, we see an increase of approximately $\simeq 31.5\%$ at the Planck scale Fig. 3.

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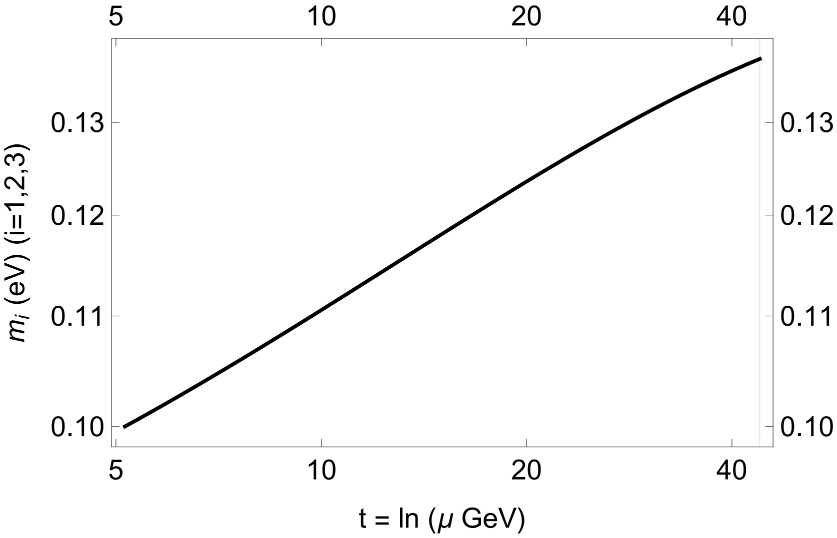

Fig. 2. Identical to Fig. 1, but with initial values of $m_i = 0.1\ eV$.

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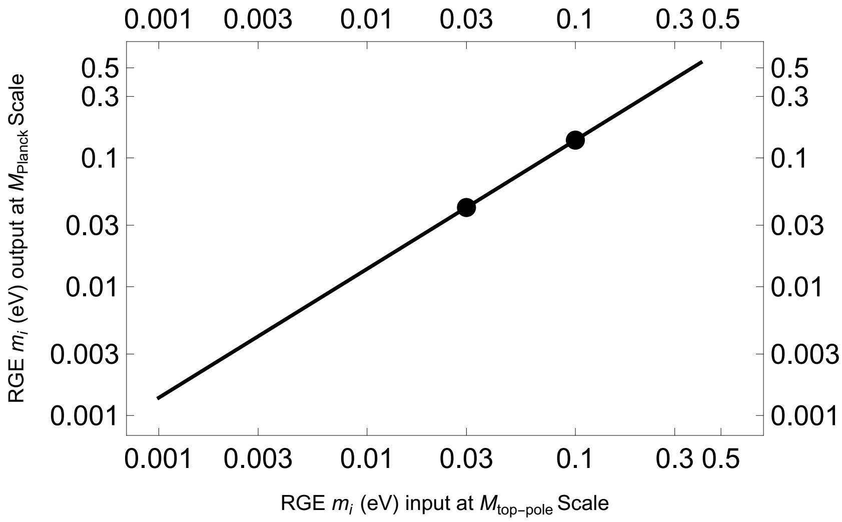

Fig. 3. The X-axis represents the initial Majorana neutrino mass eigenvalues at the top-quark pole mass scale, and the Y-axis represents the final Majorana neutrino mass eigenvalues at the Planck scale. Regardless of the Majorana neutrino mass eigenvalues used as input at the beginning of evolution, the increase in mass from the top-quark pole mass scale to the Planck scale is consistently around $\approx 31.5\%$. The lower dot and upper dot correspond to Fig. 1 and Fig. 2, respectively.

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