Improved CUSUM Schemes for Monitoring Processes Mean

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Abstract

In recent times, there has been a growing interest in the use of control charts as a feedback process monitoring technique. Particularly, CUSUM schemes have been proven to be efficient for monitoring processes where the magnitude of the sustained mean shifts is small or moderate. However, because of the constant control limits, the detection ability of CUSUM schemes becomes slow at the initial set-up of the process. The fast initial response FIR feature has often been used to enhance it at the process startup. Meanwhile, the dynamism of real-life problems has always encouraged a more sensitive CUSUM scheme capable of detecting process shifts more rapidly. In this paper, an improved CUSUM scheme for monitoring process mean is proposed. This scheme will be substantial for monitoring processes whose observations are obtained at a distant time interval for example hourly, daily or weekly and where the sustained shifts are assumed to be small or moderate. To demonstrate the practical applications of the proposed scheme, we present its real-life application using datasets from a bottling company and petroleum refinery laboratory.

Keywords: CUSUM; fast initial response; generalized fast initial response (GFIR); Average run length (ARL)

1 Introduction

In recent years, process control has attracted the attention of researchers in sciences and engineering. Because of its great importance and widespread applications, process control has become a fundamental problem evolving in sciences and engineering. The primary objective of process control is to increase process output while maintaining a target level of product quality and safety as well as to make the process more cost-effective while satisfying the environmental and product quality requirements.

The common goal of any controller who monitors a process is to ensure that the process is free from any random or unnatural occurrences that can arise from either internal or external factors.

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Such unnatural occurrences are often called disturbances. Because of the complexity of the plants, their operations are very much uneasy and such plants usually function with some disturbances which propagate from one point to the other of the interconnected components or units. Process disturbances are also known as process shifts. Its effects on a controlled variable are to deviate it from its set point or reference point or target value or process mean thereby requiring a control action so that the process can be brought to normalcy. Some chemical processes are very unsteady, i.e the process naturally exhibits dynamical behaviour or transient process behaviour which usually occurs at the process startups and shutdowns. Seborg et al. (2003) noted that such processes are usually accompanied by process disturbances at the transit of the products from one grade to the other.

Process disturbances can arise as a result of internal factors and external factors which cause the controlled variable to deviate from the target value. By process disturbance by internal factors, we mean process changes influenced by the factors within the plant. For example, loosening of bolts and nuts, loosing of oil seals, wear and tear of some parts of a machine etc. Also, Process disturbance by external factors is influenced by the environments where the process is taking place. A typical example of external factors is rainfall and wind. The occurrence of process disturbances can have patterns of occurrence. Also, they can occur randomly but they cannot be influenced by the control engineer or quality practitioner. In whatever case, the deviation from the process target i.e the error between the shifted controlled variables and its predefined targets can be minimized when prompt corrective actions are taken by the controller.

Although there exist several tools for monitoring processes, statistical process control tools such as control charts were found to be robust, reliable and powerful monitoring tools. Because of its easy implementation and unambiguous interpretations, many control practitioners have adopted the use of control charts for monitoring industrial processes. The two subdivisions of control charts are; memory type such as Shewhart$-\bar{X}$ charts and memoryless such as cumulative sum control charts (CUSUM) first proposed by Page (1954). Another example of memory-type control charts is the exponentially weighted moving average (EWMA) control chart which was first suggested by Roberts (1959). In this paper, we shall focus mainly on the CUSUM schemes. Since the introduction of CUSUM schemes, it has widely been used to detect process shifts or disturbances, especially when the magnitude of the process shifts or disturbances is small or moderate. To better understand the detection ability of CUSUM schemes, several authors have investigated its average run length (ARL) properties. For example, Page (1954); Lucas and Crosier (1982); Montgomery (2009).

Because of the constant control limit for CUSUM schemes, they are insensitive at the process startup thereby posing a huge risk to processes that may be characterized by startup problems. Many authors have put forward some enhancement techniques such as headstart or fast initial response (FIR) to improve CUSUM schemes for monitoring process mean. Such techniques have been proven to be effective in enhancing CUSUM schemes at the startup. For example, see Lucas and Crosier (1982), Lucas and Saccucci (1990a), Steiner (1999) and Ajibade et al. (2022). In this work, we utilize the recently proposed generalized fast initial response (GFIR) to further enhance the CUSUM schemes for monitoring process mean. This will particularly be important when the quality variable to be monitored is characterized by startup issues and where the magnitude of process shifts is small or moderate. The rest of the paper is arranged as follows; in Section 2, we present a brief introduction of CUSUM schemes and its enhancements. The proposed chart is presented in Section 3. Section 4 contains the performance measures and its analysis. In Section 5, we demonstrate the real-life application of the proposed scheme. Lastly, Section 6 contains the conclusive remarks.

2 Cumulative sum (CUSUM) schemes and its enhancements

Let q_t denote the quality variable of interest at time t. Let μ_0 and R denote the target and reference values respectively. The one-sided upper and lower CUSUMs for monitoring process mean are respectively given as

$$C_t^+ = \max[0, q_t - (\mu_0 + R) + C_{t-1}^+] \tag{1a}$$

$$C_t^- = \max[0, (\mu_0 - R) - q_t + C_{t-1}^-]. \tag{1b}$$

Where $C_0^+ = C_0^-$ are the starting values that are usually set to zero. Note that $\max[a, b]$ returns the maximum of $[a, b]$ for any $a, b \in \Re$. Let H denote the control threshold or the decision parameter, or the control limit. The process will be out-of-control when C_t^+ or C_t^- exceeds H.

Despite CUSUM's effectiveness at detecting small process changes, its constant control limit has been a major shortcoming, especially at the process startup. This usually renders it less sensitive at the process startup and consequently posed a high economical risk for industrial production as any undetected disturbance(s) in the process may result in huge financial penalties (please see Montgomery (2009)). In other to increase the sensitivity of CUSUM schemes at the initial setup of the process, Lucas and Crosier (1982) proposed the use of the headstart or fast initial response (FIR) features for CUSUM schemes to further improve its sensitivity at the process startup. They introduced nonzero starting values for CUSUM schemes i.e $C_0^+ = C_0^- \neq 0$. In some applications, C_0^+ and C_0^- are usually set to half of the decision parameter H i.e $C_0^+ = C_0^-$ $= H/2$. This is regarded as a 50% headstart. Note that this FIR or headstart is constant. Lucas and Saccucci (1990b) used this concept to enhance EWMA schemes based on asymptotic control limits. However, Montgomery (2009) and Rhoads et al. (2007) noted that the control limits for EWMA should be time-varying but not asymptotic. So, they utilized the FIR feature coupled with the time-varying control limits to improve the performance of EWMA schemes.

However, the effects of time-varying control limits for the CUSUM schemes are almost the same as when the FIR feature suggested by Lucas and Crosier (1982) is used. The purpose of the FIR feature in a charting structure is to reduce the out-of-control ARL. Although, research has shown that the effects of FIR feature on the in-control ARL are small.

To further enhance the detection ability of EWMA control charts, Steiner (1999) suggested the use of the time-varying FIR feature of the form

$$\mathrm{F}_{ir} = (1 - (1 - f)^{1+\psi(t-1)}) \tag{2}$$

where he set the parameter ψ as

$$\psi = \frac{-1}{19}\left(1 + \frac{2}{\log_{10}(1-f)}\right). \tag{3}$$

The slightly modified version was put forward by Haq et al. (2013) by taking $1+1/t$ exponentiation of F_{ir}. These FIR features have been used majorly to enhance the sensitivity of the EWMA and CUSUM schemes at the process startup. For example, Ajadi et al. (2016) have used the time-varying FIR features to increase the sensitivity of mixed EWMA–CUSUM control charts for location parameters. Also, Knoth (2005a) compared the performance of FIR features on EWMA schemes by using different change point locations. His approach provides the user with a clearer insight into the performance of FIR-based EWMA schemes. In addition, the use of taut strings (TS) for the univariate process monitoring was proposed by Pokojovy and Jobe (2021). Their approach equally improves the detection ability of CUSUM schemes. To have a general form of the fast initial response, Ajibade (2016) had put forward a generalized time-varying FIR (GFIR) feature. This FIR feature was analyzed further by Ajibade et al. (2022). It has been shown that the GFIR feature has special cases where other previously proposed FIR features can be retrieved. In this paper, we utilize the highly sensitive property of GFIR to obtain a new CUSUM scheme for monitoring process mean. This new scheme will highly be desired by quality control practitioners because of its simplicity to use and implementation. Also, there are no ambiguities in the interpretations of the resulting charts. This proposed scheme is presented in the next section.

3 Design of the improved CUSUM schemes

Let $q_t, t \in \Omega$ be an independent and identical quality variable of interest indexed by the finite index set $\Omega = \{t_1, t_2, ..., t_s, ..., t_n\}$ for some $s, n \in \mathbb{N}$. Note that the index set Ω contains uniform spaced time t at which each of the quality variables q_t is realized. Assume that q_t is with constant variance. Let μ_q denote the process mean that is obtained by finding the mean of the quality variable q_t. Our interest is to promptly detect the time t when the process q_t deviates from μ_q. Let t' denote the time t at which the process deviates from μ_q. To monitor the process mean i.e μ_q, the decision structure is formulated as follows;

$$H_0 : \mu(t) = \mu_q \quad \text{if } t < t',\ t \in \Omega \tag{4a}$$

$$H_1 : \text{There exists } t'_j \in \Omega \text{ such that } \mu(t'_j) \neq \mu_q,\ t > t',\ \mu(t') = \mu_1 \tag{4b}$$

$$j = 1, 2, 3, ... \tag{4c}$$

where $\mu(.)$ denote the mean of the quality variable at each time t. This structure utilizes a stopping rule which produces a run length R_L or stopping time for the process $q_t, t \in \Omega$ such that the average run length (ARL) is given as

$$\text{ARL}(\mu) = \mathbb{E}_\mu(R_L) \tag{5}$$

where $\mathbb{E}$ denote the average or mean of any random number R_L with respect to the magnitude of the process mean μ. Before a run length can be determined, control limits must have been predefined. One of the most important part of any charting structure is its control limits. Their careful design and enhancements pave the way for quicker detection of process changes. The standard control limit for CUSUM is

$$H = h\sigma \tag{6}$$

where σ is the standard deviation and $h \in \Re^+$ is the control limit multiplier. Since σ is constant, it turns out that H is also constant. One way to enhance the detection ability of the CUSUM scheme without violating its statistical properties is to introduce time-varying FIR adjustments to its control limit. Considering the generalized FIR feature proposed by Ajibade (2016), we can rewrite equation (6) as the following

$$H_{gfir} = h\sigma(1 - (1 - f)^{(\phi+\psi)t-\psi}) \tag{7}$$

where ψ is as given in equation (3) and ϕ denotes the switching parameter. It indicates how G_{fir} functions. For example, for classical CUSUM and FIR-CUSUM control limits, ϕ is given as $\phi = t + N_c$, $N_c \in \mathbb{N}$ ($N_c \geq 400$) and $\phi = 1/t$ respectively. Please, see Ajibade et al.Ajibade et al. (2022) for details. To obtain improved CUSUM charts, we set ϕ equal to

$$\phi = \frac{1+\psi}{t^2}\left(1 - t\right) \tag{8}$$

The performance comparisons and analysis of CUSUM, FIR-CUSUM, and GFIR-CUSUM schemes are presented in the next section.

4 Performance measures and analysis

The performance of a charting structure is usually evaluated by its average run length (ARL) properties. The ARL is the average number of observations to be taken before a chart flags a signal. The in-control ARL (ARL_0) and out-of-control ARL (ARL_1) are two subdivisions of ARL. ARL_0 is the average number of observations to be taken before an out-of-control signal is detected when the process is in the in-control state while ARL_1 is the average number of observations to be taken when the process is shifted to an out-of-control state. During the computation of ARL, the control limit multiplier h is repeatedly adjusted to achieve the desired ARL_0. It has recently been shown by Ajibade et al. (2022) that the steady state ARL for the classical EWMA chart and FIR-based EWMA schemes are the same. Taking into account that the GFIR is also asymptotic, its influence on the control limits of the CUSUM will also be limited to the observations at the process startup. Consequently, the performance measure will only be based on the zero-state ARL.

Let δ denote the magnitude of the sustained shifts in the mean of a process q_t, and let σ_q denote the standard deviation, we consider a suitably large number of observations (50000) generated from standard normal Gaussian process i.e $q_t \sim \mathcal{N}(\mu_q + \delta\sigma_q, \sigma_q)$ where $\mu_q = 0, \sigma_q = 1$, and $\mathcal{N}$ denotes the standard normal Gaussian process. We consider δ on the interval $0 \leq \delta \leq 2$ (note that $\delta = 0$ refers to in-control state and $\delta > 0$ refers to out-of-control state). We compute the CUSUM statistics C_i^+ and C_i^- for these observations using equation (1). The control limits given in equation (7) are then applied on C_i^+ and C_i^- for each t. This process is repeated for 50000 times and the time t at which either C_i^+ or C_i^- falls out of the controlling limit is recorded. This process is generally regarded to as the Monte Carlo simulation. The ARL and SDRL of the proposed scheme and its counterparts are presented in Table 1 and Table 2 respectively. Also, we present the graphical display of the ARL and $\log_{10}$(ARL) in Figure 1a and 1b respectively.

Table 1: ARL of CUSUM, FIR-CUSUM and GFIR-CUSUM control schemes when $k = 0.5$, $f = 0.5$ and $\text{ARL}_0 = 500$.

Schemes $\rightarrow$	CUSUM	FIR-CUSUM	MFIR-CUSUM	GFIR-CUSUM
$\delta \downarrow h \rightarrow$	5.0695	5.0969	5.1124	5.2182
0	500.7657	497.2087	500.1286	500.4596
0.25	145.3633	143.9577	140.2112	131.3499
0.5	39.0193	36.6709	35.4726	30.5819
0.75	17.3358	15.5043	14.7130	11.7891
1	10.5227	8.8259	8.2007	6.3044
1.25	7.5013	5.8445	5.2952	4.0819
1.5	5.8202	4.3200	3.8487	3.0358
1.75	4.7733	3.3570	2.9604	2.4957
2	4.0572	2.7676	2.4343	2.1712

It is seen in Table 1 that the proposed scheme has the smallest ARL_1 values among its counterparts. That means the proposed scheme will be quicker in detecting small or moderate shifts in a process mean. The direct interpretation of the ARL Table 1 is; if the magnitude of the shifts in a process mean is 0.25, it will take about 145 observations for classical CUSUM to detect the shift. Also, for the same magnitude of the mean shift, it will take approximately 143 and 140 observations for FIR-CUSUM and MFIR-CUSUM to detect the shift. Lastly, for the same magnitude of the process mean shift, it will take approximately 131 observations for GFIR-CUSUM to detect the shift. That means the newly proposed scheme is 9 observations faster than MFIR-CUSUM scheme, and 14 observations faster than the classical CUSUM scheme. For processes whose observations are taking in the interval of hours, days or weeks, the newly proposed scheme will highly be desirable

Table 2: SDRL of CUSUM, FIR-CUSUM and GFIR-CUSUM control schemes when $k = 0.5$, $f = 0.5$ and $\text{ARL}_0 = 500$.

Schemes $\rightarrow$	CUSUM	FIR-CUSUM	MFIR-CUSUM	GFIR-CUSUM
$\delta \downarrow h \rightarrow$	5.0695	5.0969	5.1124	5.2182
0	493.9445	504.5208	516.2349	570.4192
0.25	137.3213	142.2208	142.2959	149.2789
0.5	31.8721	32.2563	32.4024	33.3897
0.75	11.2144	11.6814	11.9014	11.8155
1	5.4935	5.9603	6.1314	5.7489
1.25	3.3335	3.5964	3.7037	3.1905
1.5	2.2672	2.4642	2.4854	1.8828
1.75	1.6674	1.7745	1.7054	1.2254
2	1.3002	1.3685	1.2160	0.8464

to monitor them because any undetected persistent shifts may cause unprecedented economical consequences. The same analysis applies to other process mean shifts reported in Table 1. The SDRL value of the proposed scheme is slightly higher than its counterparts. This is a result of its high sensitivities. Also, the interpretation given above can better be understood via the ARL and $\log_{10}(\text{ARL})$ curves in Figure 1a and Figure 1b respectively.

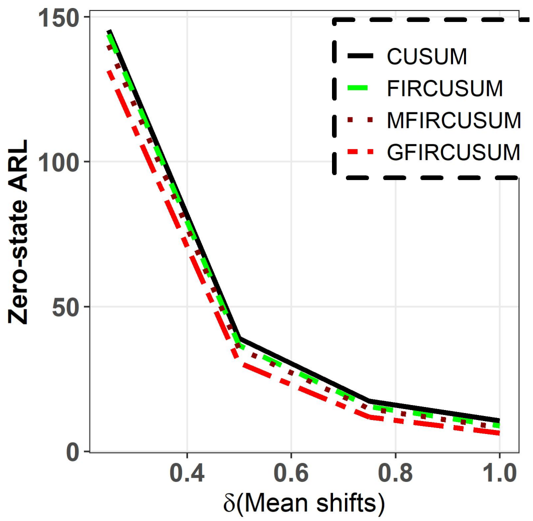
5 Applications

In this section, we present the practical application of the newly proposed scheme by using two different datasets namely; the filling heights of bottles and Di-glycol amine (DGA) datasets. The first dataset is presented as a case study I while the second dataset is presented as a case study II.

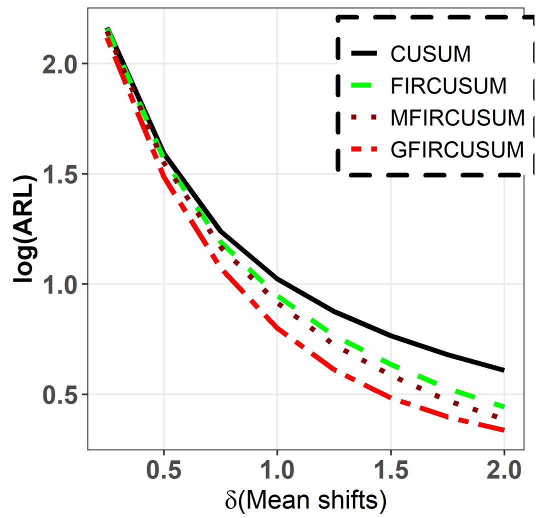
5.1 Case study I: The filling heights of bottles

The filling heights of empty bottles with soft drinks is a problem arising in bottling companies when producing soft drinks. The fillers fill the drinks into 30 bottles at a time. The dataset is obtained from Ajadi et al. (2020). We monitor the dataset with the proposed scheme and its counterparts using the decision parameters; $k = 0.5$, $f = 0.5$ and the control limit multiplier that makes $\text{ARL}_0 = 500$. The graphical representations are displayed in Figures 2a-2d.

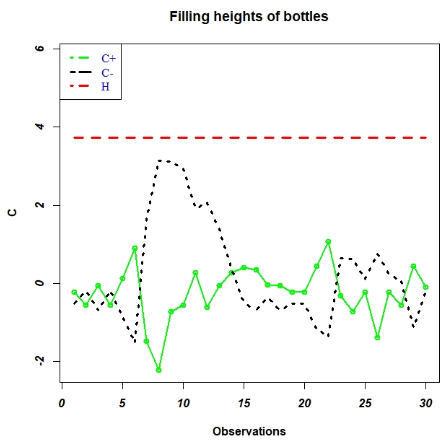
 doi:10.20944/preprints202212.0325.v1



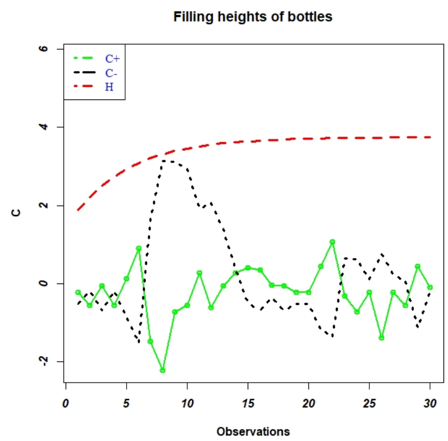
(a) Performance comparison of the ARLs



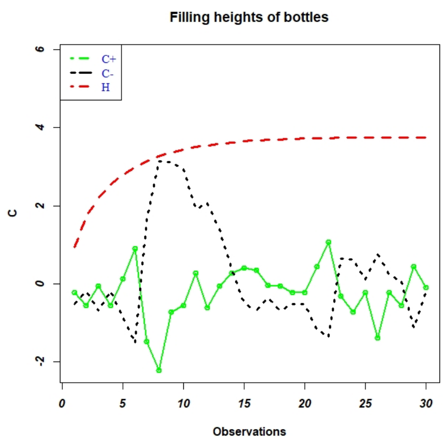
(b) Performance comparison of the ARLs in logarithmic scale



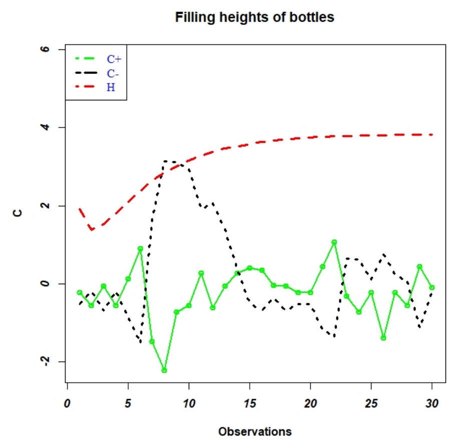
(a) CUSUM control chart



(b) FIR-CUSUM control chart

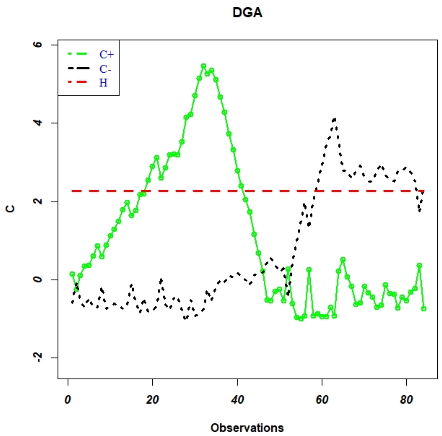


(c) MFIR-CUSUM control chart

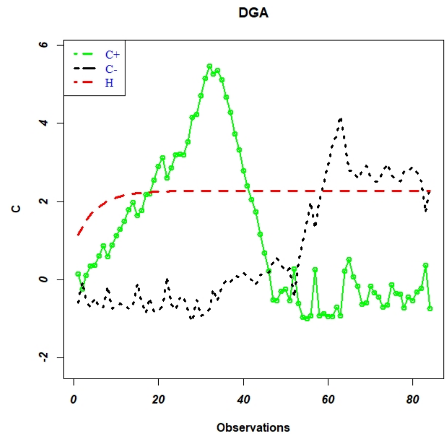


(d) GFIR-CUSUM control chart

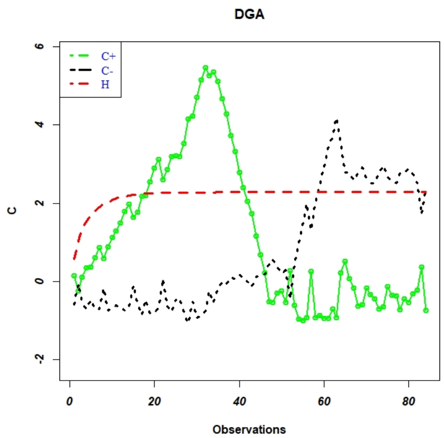
 doi:10.20944/preprints202212.0325.v1



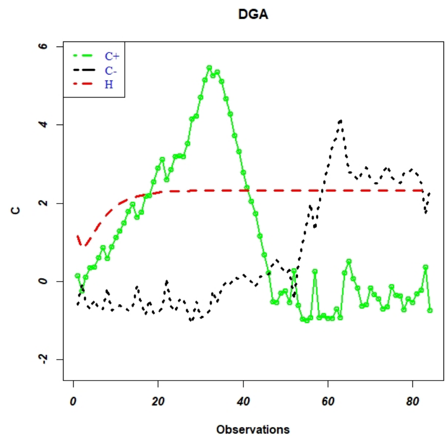
(a) CUSUM control chart



(b) FIR-CUSUM control chart



(c) MFIR-CUSUM control chart



(d) GFIR-CUSUM control chart

5.2 Case study II: Di-Glycol Amine (DGA)

Di-Glycol Amine (DGA) is an Amine compound used in Petroleum refineries to eliminate sulfur compounds from petroleum gases by using a chemical process called the gas sweetening process. The dataset monitors the purity of Di-Glycol Amine (DGA) analyzer performance. The description of the dataset can be found in Ajadi et al. (2016). We monitor the dataset with the proposed scheme and its counterparts using decision parameters $k = 0.5$, $f = 0.5$ and the control limit multiplier that makes $ARL_0 = 500$. The graphical representations are displayed in Figures 3a-3d.

5.3 Results and Discussion

It is seen that the process is in-control for CUSUM, FIR-CUSUM and MFIR-CUSUM. The only exception is the GFIR-CUSUM where the process goes out-of-control at the 8th and 9th observations. Because of the improved sensitivity, the proposed scheme returns two out-of-control scenarios. Comparing the control charts displayed in Figures 3a-3d, we observe that the classical CUSUM, FIR-CUSUM and MFIR-CUSUM schemes and the proposed scheme detect out-of-control

 doi:10.20944/preprints202212.0325.v1

between 19th-41st observations and 59th-84th observations each. This suggests that the dataset does not have startup issues. This result also agrees with the conclusion by Ajibade et al. (2022).
Remark: Note that the exact interpretation of the control charts in Figures 2a-2d and Figures 3a-3d can only be given by the quality controller who directly monitors the process in the laboratory or in the industry. We have only demonstrated (as a researcher) the method of application.

6 Conclusion

In this paper, we utilize generalized fast initial response features to improve CUSUM schemes. The newly obtained CUSUM scheme (GFIR-CUSUM) is highly sensitive to the small or moderate process mean shifts. We investigate its ARL and SDRL properties and present our results. The ARL values show that the proposed scheme is quicker in detecting shifts in a process mean than its counterparts. Also, we present the graphical display of the ARL and logarithmic to base 10 of the ARL for visualization. Lastly, we use real-life datasets from bottling company and petroleum refinery laboratories to demonstrate the practical application of the proposed scheme. The proposed scheme will be very substantial in monitoring processes that are characterized by startup issues, especially those whose observations are taking hours, days or weeks where slow detection of the process changes can have devastating consequences.

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