A Numerical Quadrature Solution for Time Period of Simple Pendulum under magnetic action

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Abstract

In the present study, a simple approximation expression is given for the relationship between the period and amplitude of a simple pendulum under magnetic action. The analytical solution presented for the given problem. Two numerical quadrature methods Simpson's and Boole's method were utilized to demonstrate a new approximation of the problem. The results of the numerical quadrature have been compared to the exact solution. Absolute and relative mistakes of the problem have been presented. The Matlab program 2013R has created a numerical method that is used to analyze the outcome, It has been determined that the comparison's outcomes attest to the method's suitability and correctness. Moreover, the results show that numerical solution is suitable for the problem.

Keywords: simple pendulum, time period, Magnetic action, numerical integration, error analysis

Introduction

In last decades, differential equations have been applied for many problems in engineering, finance, physics and seismology [1-5]. They have several approximation methods which are different from each other [6-10]. Many numerical methods have been applied for solving linear and non-linear differential equations [11-13]. One of the most popular physical models encountered in undergraduate courses is the simple pendulum and the differential equation describing its motion [12-20]. Historically, the equation arises when studying the oscillations of a pendulum clock, but also appears in various other areas of physics, since problems often can be reduced to a differential equation similar to that describing the pendulum [18, 19]. The exact solution to the equation of motion of the undamped pendulum is well known in the literature and involves the Jacobi elliptic functions [13, 18, 19].

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Simple pendulum is a simple mechanical system in terms of setup, but it is difficult to calculate the factors that act on its motion, such as time period, amplitude, angle of oscillation, acting forces, and energy [20]. This simple mechanical system oscillates with a symmetric force due to gravity acting on it as a restoring force, as illustrated in (Fig. 1)[16]. Its equation of motion is given by:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} sin\theta = 0 \qquad (1)$$

The present paper numerically describes the solution of the time period of a simple pendulum under magnetic field.

The method

Ma and Zhang in [19], presented a periodic solution for the pendulum under magnetic action[21]. They have modelled pendulum under magnetic action as follow:

$$T = \frac{4}{\sqrt{1-A^2}} \int_0^{\frac{\pi}{2}} (1 + k\sin^2 t)^{-\frac{1}{2}} dt \qquad (2)$$

Where $k = \frac{A^2}{2(1-A^2)}$

In this study, we will examine two different numerical quadrature methods for (2), after that we will compare numerical results with exact solution that have been presented in [19] A numerical solution can be found and compared with the results in [19].

There are many numerical integration methods to evaluate composite integrals; in this paper, we use two numerical quadrature methods, Simpsons 3/8 method, and Boole's method [22-26].

If we set, $c = \frac{4}{\sqrt{1-A^2}}$, $f(t) = (1 + k\sin^2 t)^{-\frac{1}{2}}$ for the integral in Eq. (2), and applying Simpson's 3/8 method, we obtain:

$$c\int_0^{\frac{\pi}{2}} f(t)\,dt = c\frac{3h}{8}\left[\sum_{i=1}^{n/3} f_{3i-3} + 3\left(f_{3i-2} + f_{3i-1}\right) + f_{3i}\right] + O(h^4) \qquad (3)$$

where $O(h^4) = -\frac{\theta_M}{80} h^4 f^4(\zeta)$, where $0 \le \zeta \le \theta_M$.

Similarly, by applying Boole's method, we get:

$$c\int_0^{\frac{\pi}{2}} f(t)\,dt = c\frac{2h}{4}\left[\sum_{i=1}^{n/4} 7(f_{4i-4} + f_{4i}) + 32(f_{4i-3} + f_{4i-1}) + 12f_{4i-2}\right] + O(h^6) \qquad (4)$$

where $O(h^6) = -\frac{2\theta_M}{945} h^6 f^6(\zeta)$, where $0 \le \zeta \le \theta_M$.

The present work focused on the time period of simple pendulum under magnetic action as a function of its starting amplitude at a large angle numerically., for both integral equations (3) and (4), the results of Simpson's 3/8 and Boole's method will be compared with the exact results in [19] which is an analytical solution of the problem, and absolute errors (E_A) and relative errors (R_A) are calculated by the following,:

$$E_A = |Exact\ value - Numerical\ value| \quad and \quad R_A = \frac{E_A}{Exact\ Value}$$

The Matlab program have been implemented for comparison between exact and approximation solutions [18, 19, 27, 28],absolute error and relative error have been calculated, the Table 1. Shows the comparison between present study and the results in [19].

Table 1. The table presents absolute errors and relative errors of comparison between numerical results and results in paper [19]

	A=0.1	**A=0.5**	**A=0.9**
Numerical Result	6.30690135098058	6.97876252252756	10.6233569517113
Results in [19]	6.3069	6.9783	10.6192
Absolute error	1.35098057985061e-06	0.00006252252756	0.00415695171132135
Relative error	2.14206754483282e-07	6.62801151512013e-05	0.000391456203039905

Conclusion

In this paper, approximation solution of Simple Pendulum under magnetic action has been presented. Two different numerical quadrature methods, namely Simpson's and Boole's method have been examined. The analytical solution has been compared with the numerical solution and the agreement is found to be very good. Matlab software have been implemented for calculation. Absolute error and Relative error have been calculated. The results guarantee the accurate and stability of both methods.

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