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Study on Accurate Calculation Method of Model Attitude on Wind Tunnel Test

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Abstract : The accurate of model attitude angel plays an important role on the aerodynamic test results in the wind tunnel test. The original method applies the spherical coordinate system transformation to obtain attitude angel calculation.The model attitude angel is obtained by coordinate transformation and spherical surface mapping applying the nominal attitude angel (the balance attitude angel in the wind tunnel coordinate system) indicated by the mechanism. First, the coordinate transformation of this method is not only complex but also difficult to establish the transformed relationship between the space coordinate systems especially after many steps of coordinate transformation, moreover it cannot realize the iterative calculation of the interference relationship between attitude angels; Second, during the calculate process to solve the problem the arc is approximately used to replace the straight line, the angel for the tangent value, and the inverse trigonometric function is applied. Therefore, in the calculation of attitude angel, the process is complex and inaccurate, which can be solved approximately when calculating small attack angel. However, with the advancing development of modern aerodynamic unsteady research, the aircraft tends to develop high or super large attack angel and unsteady research field.According to engineering practice and vector theory, the concept of vector angel coordinate system is proposed for the first time, and the vector angel coordinate system of attitude angel is established.With the iterative correction calculation and avoiding the problem of approximate and inverse trigonometric function solution, the model attitude calculation process is carried out in detail, which validates that the calculation accuracy and accuracy of model attitude angels are improved.Based on engineering and theoretical methods, a vector angel coordinate system is established for the first time, which gives the transformation and angel definition relations between different flight attitude coordinate systems, that can accurately calculate the attitude angel of the corresponding coordinate system and determine its direction, especially in the channel coupling calculation, the calculation of the attitude angel between the coordinate systems is only related to the angel, and has nothing to do with the change order s of the coordinate system, which simplifies the calculation process.

Key words : attitude angel; angel Vector Coordinate System; Iterative calculation; Spherical Coordinate System; Wind Tunnel Test

1. Introduction

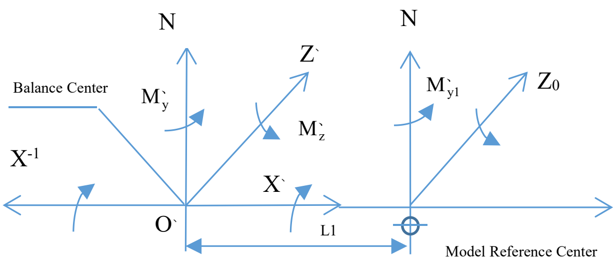
The calculation accuracy of the model attitude angel has a great influence on the calculation results when calculating the wind tunnel test data, that is an important component of the wind tunnel test data processing and accuracy improving. "Methods to improve the accuracy of wind tunnel experimental data" mentioned that the attack angel error arouses more than twenty-five percent error results of the resistance coefficient C for the GMB-04 model in the span-supersonic at α =4°x. If there is no error in the balance and pressure measurement the error of $\Delta\alpha = \pm 0.01°$ corresponds to resistance coefficient error the $\Delta Cx = \pm 0.0001$ can induce one percent of the payload for the subsonic transport model on the cruise state Cy=0.50 in the remote cruise[1]. Actually there are errors in the balance and pressure measurement.

Therefore, the error $\Delta\alpha$ that can be assigned to the attack angel must be less than 0.01° to let ΔC_x less than ± 0.0001[2]. At present, the elastic angel of the model is calculated by the elastic angel calculation formula from the lift and pitch torque measured by the balance. The attack angel error of model experiment is generally ± 0.05°, as responding the α is 0.02 domestically, which cannot meet the requirements of high-precision experimental data[2,3,4]. Foreign wind tunnel experiments have generally installed the attack angel and the elastic angel sensor in the model to measure the real attack angel directly. The threshold range of the attack angel sensor is 0.001°~0.003°, whose accuracy can reach 0.01°. The Boeing transonic wind tunnel utilizes the laser gradiometer to measure the model attitude angel, whose measurement accuracy can reaches ± 0.005°[5,6,7,8] which is an order of magnitude smaller than ± 0.05 that China can reach. At present, there is no case of installing the angel sensors in the model of wind tunnel in China due to the development of test technology and test technology. It is not widely applied because of the high cost of angel sensor and the inconvenient model design. When the model is installed, the installation angle error and elastic angles errors are superimposed on each other, and there is no special device to measure the attitude angle of each component. This effect unconsidered among the interaction components of the balance system is great at large angles of attack over a large distance, and also under atmospheric dynamics the mutual interference among pitching α, side slip β, and rolling γ will increase greatly[9,10]. Therefore, it is necessary to study the balance system.

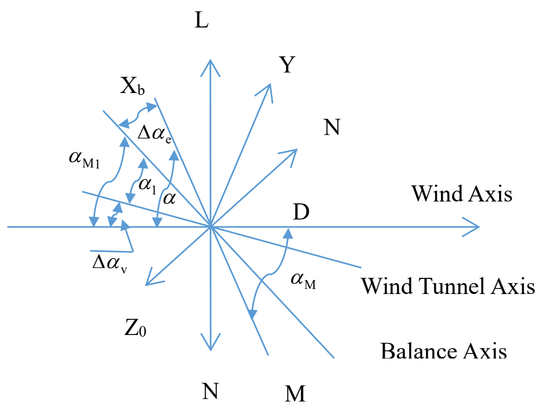
When the Euler angle describes the attitude angle of the aircraft, the method is simple and the physical meaning is intuitive, but for flying aircraft, a singularity occurs when the pitch angle is close to ninety degrees, which is that the Euler angle differential equation describing the attitude motion diverges is divergent. Therefore, the directional cosine and four-element methods are commonly used when actually solving the attitude angle of the aircraft, and then solved according to the Euler angle relationship, but in the coordinate system conversion, the attitude angle calculation error will continue to accumulate resulting in an increase in the calculation error[11,12]. According to the problems of the attitude angel calculation, the vector angel coordinate system is established for the wind tunnel experiment and the vehicle motion.

2. The method of spherical coordinate system calculation

The geometry of any space is formed by the intersection of many planes (or lines). The angle size of the two planes (or lines) does not change with the different positions of the planes (or lines) after translation at the same time. Therefore, the planes(or lines) and the angle, that makes up the space, can translate to the center of the spherical coordinates, and divide with the spherical surface to obtain several spherical triangles on the spherical surface that draws a spherical map, in which all angles in the spherical diagram have "authenticity", and then borrow the spherical triangular formula for calculation. This method is called the spherical coordinate method [13,14,15].First, define the coordinate system as shown in Figure 1. "X_1, Y_1, Z_1", "X′, Y′, Z′", "X, Y, Z", "X_b, Y_b, Z_b", "α, β, γ", "$\Delta\alpha_e$, $\Delta\beta_e$, $\Delta\gamma_e$" , "α_m, β_m, γ_m", "$\Delta\alpha_v$", represent the wind coordinate system, the balance coordinate system, the wind tunnel coordinate system (ground coordinate system) , the model coordinate system, the attitude angle, the elastic angle of the balance ,the nominal pose angle, the air flow deflection angle respectively [1,16].



(a) Coordinate Conversion



(b) System axis

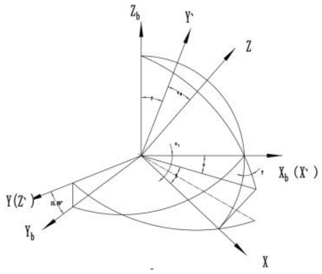
Figure 1 Schematic diagram of the axis system coordinate conversion

Take the transverse axis test as an example: Fan Jiechuan et al. Made a detailed derivation of the calculation process for model pre-roll γ, model pre-bias slip β and double axis test in the "Wind

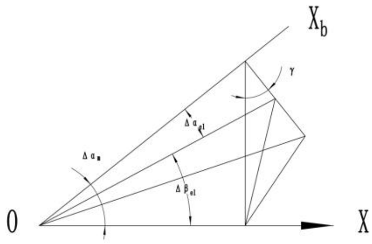
Tunnel Test Manual". Taking the aerodynamic calculation of each coordinate axis system in the model prefabricated roll angle γ transverse test as an example to effectuate the aerodynamic calculation of the double axial system of the model body[17].

2.1. Calculation of model axis

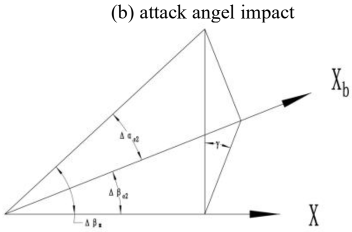
If the balance is statically calibrated with the earth coordinate, omitting the influence of the elastic deformation of the balance and the strut (not considerate the elastic deformation angle of the balance), or the balance is still calibrated with the body coordinate, the conversion aerodynamic force and the torque relationship between the balance and the body coordinate. Figure 2 below shows the relationship.



(a) Static calibration axis system



(b) attack angel impact



(c) Effect of side slip angel

Figure 2 angel relationship of turn γ angel transverse test

Normal force:

$$N = Y'cos\gamma + Z'sin\gamma \quad (1)$$

Transverse force:

$$Y = Z'cos\gamma - Y'sin\gamma \quad (2)$$

Axis force:

$$A = X'l = X' - \Delta X_b \quad (3)$$

Pitch torque:

$$M_b = M'_{z1}cos\gamma - M'_{y1}sin\gamma \quad (4)$$

Yawing torque:

$$N_b = -M'_{y1}cos\gamma - M'_{z1}sin\gamma \quad (5)$$

Rolling moment:

$$L_b = M'_{x1} = M'_x + Z'\Delta l_N \quad (6)$$

2.2. Model attack angel and side slip angel increment calculation

The elastic deformation of the balance system after being subjected to the aerodynamic action of the model is generated, and the elastic deformation angles $\Delta\alpha_m$ 、 $\Delta\beta_m$ and $\Delta\gamma_m$ generate in the balance coordinate coordinate system respectively. Since the model is rotated an angle relative to the balance, there is also an angle of attack increment in the symmetrical plane of the model and a slip angle increment in the corresponding slip plane. Because the rolling angle effect is small, it is usually ignored. The increments of the resulting model angle of attack and the side slip angle are $\Delta\alpha_{e1}$ and $\Delta\beta_{e1}$ generated by $\Delta\alpha_m$, the increments of the model attack and side slip angle are $\Delta\alpha_{e2}$ and $\Delta\beta_{e2}$ generated by $\Delta\beta_m$, and the increments of the model angle of attack and slip angle are as shown in Equation 7, Equation 8 shown:

$$\Delta\alpha_e = \Delta\alpha_{e1} + \Delta\alpha_{e2} \quad (7)$$

$$\Delta\beta_e = \Delta\beta_{e1} + \Delta\beta_{e2} \quad (8)$$

As from Figure 1 (b) and (c):

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$$\begin{cases} tg\Delta\alpha_{e1} = tg\Delta\alpha_m cos\gamma \\ sin\Delta\beta_{e1} = sin\Delta\alpha_m sin\gamma \\ tg\Delta\alpha_{e2} = -tg\Delta\beta_m sin\gamma \\ sin\Delta\beta_{e2} = sin\Delta\beta_m cos\gamma \end{cases} \quad (9)$$

In the above formula, the parameters of $\Delta\alpha_{e1}$、$\Delta\beta_{e1}$、$\Delta\alpha_{e2}$、$\Delta\beta_{e2}$ can abbrev as Equation 10 shown respectively,

$$\begin{cases} \Delta\alpha_{e1} \approx arctg(tg\Delta\alpha_m cos\gamma) \\ \Delta\beta_{e1} \approx arcsin(sin\Delta\alpha_m sin\gamma) \\ \Delta\alpha_{e2} \approx -arctg(tg\Delta\beta_m sin\gamma) \\ \Delta\beta_{e2} \approx arcsin(sin\Delta\beta_m cos\gamma) \end{cases} \quad (10)$$

Because the angels $\Delta\alpha_m$, $\Delta\beta_m$ are very small, the upper formula can be simplified to as Equation 11 shown:

$$\begin{cases} \Delta\alpha_{e1} \approx \Delta\alpha_m cos\gamma \\ \Delta\beta_{e1} \approx \Delta\alpha_m sin\gamma \\ \Delta\alpha_{e2} \approx -\Delta\beta_m sin\gamma \\ \Delta\beta_{e2} \approx \Delta\beta_m cos\gamma \end{cases} \quad (11)$$

General Equation (10), (11) (7), (8):

$$\begin{cases} \Delta\alpha_e = \Delta\alpha_m cos\gamma - \Delta\beta_m sin\gamma \\ \Delta\beta_e = \Delta\beta_m cos\gamma + \Delta\alpha_m sin\gamma \end{cases} \quad (12)$$

2.3. Model attack angel and side-slip angel calculation

The model attitude rolling angel and mechanism angel change γ , α_m respectively. Referring to figure 1 (b), the model attack angel α_{m1} and side slip angel β_{m1} are similar to Equation (9) defined in Equation [13]:

$$\begin{cases} tg\alpha_{m1} = tg\alpha_m cos\gamma \\ sin\beta_{m1} = sin\alpha_m sin\gamma \end{cases} \quad (13)$$

Where α_{m1} , β_{m1} were defined in Equation (14) respectively:

$$\begin{cases} \alpha_{m1} = aretg(tg\alpha_m cos\gamma) \\ \beta_{m1} = arcsin(sin\alpha_m sin\gamma) \end{cases} \quad (14)$$

The attack angel α and lateral slip angel β of the model are obtained in Equation (15) and Equation (16)

$$\begin{aligned} \alpha &= \alpha_{m1} + \Delta\alpha_e \\ &= arctg(tg\alpha_m cos\gamma) + \Delta\alpha_m cos\gamma - \Delta\beta_m sin\gamma \end{aligned} \quad (15)$$

$$\begin{aligned} \beta &= \beta_{m1} + \Delta\beta_e \\ &= arcsin(sin\alpha_m sin\gamma) + \Delta\beta_m cos\gamma + \Delta\alpha_m sin\gamma \end{aligned} \quad (16)$$

Supposing $sin\Delta \approx \Delta, \tan \approx \Delta$ during calculation above, the error does not affect the calculation results when the Δ is small , but with the angel increasing the error will greatly affect the calculation results. Especially, it is difficult to find the attitude angels correspondence between the wind axis system and the model coordinate system attitude angels using the spherical coordinate system conversion in the two-axis test. And each transformation requires an approximation of the reverse triangle function, which makes the calculation imprecise.

3. Vector angel coordinate system establishment and transformation

Defining three linear independent vectors $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ in linear space V over the domain F , (representing the three angle vectors of the pitch,the side slide, and the roll, respectively), $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ are perpendicular to each other,with" $|\alpha|$、$|\beta|$、$|\gamma|$ " and " k_1, k_2, k_3 "represent their magnitude, and their direction vectors in coordinates respectively, then the angle vectors of $\vec{\alpha} = |\alpha|(0,0,1)$, $\vec{\beta} = |\beta|(0,1,0)$, $\vec{\gamma} = |\gamma|(1,0,0)$ coordinate system is established as shown in Figure 3. Let the roll size change positive $\Delta\gamma$, then the vectors $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ rotate $\Delta\gamma$.

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Conforming to the right-hand rule, it obtains a new coordinate system, as shown in the Figure 3 [18,19]:

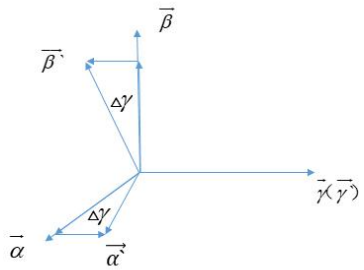

Figure 3 Vector angel coordinate system
Fig 3Vectorangularcoordinatesystem

Where suppose δ is any one vector in the linear space V , then:

$$\delta = k_1\vec{\gamma} + k_2\vec{\beta} + k_3\vec{\alpha} = (\vec{\gamma},\vec{\beta},\vec{\alpha})\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \quad (17)$$

When the $\vec{\gamma}$ size changes $\Delta\gamma$ is equivalent to the linear transformation of the base coordinate that describes as $P(\gamma)$. According to the vector transformation define, figure 4 show vector angel transformation.

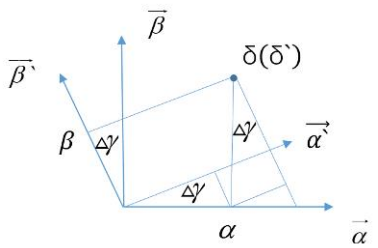

Figure 4 Vector angel transformation
Fig 4Vectorangular shift

Available by Figure 4:

$$\begin{aligned} O\beta` &= O\beta\cos\Delta\gamma + O\alpha\sin\Delta\gamma \\ O\alpha` &= O\alpha\cos\Delta\gamma - O\beta\sin\Delta\gamma \end{aligned} \quad (18)$$

Write it in a matrix form:

$$\delta` = \begin{bmatrix} \cos\Delta\gamma & \sin\Delta\gamma \\ -\sin\Delta\gamma & \cos\Delta\gamma \end{bmatrix}\delta \quad (19)$$

Write it to 3D space:

$$\delta` = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Delta\gamma & \sin\Delta\gamma \\ 0 & -\sin\Delta\gamma & \cos\Delta\gamma \end{bmatrix}\delta \quad (20)$$

According to the basis transformation and coordinate transformation theorems obtain Equation (21), Equation (22) and Equation (23), which can deduce Equation(24).

$$(\vec{\gamma}`,\vec{\beta}`,\vec{\alpha}`) = (\vec{\gamma},\vec{\beta},\vec{\alpha})P(\gamma) \quad (21)$$

$$\delta = (\vec{\gamma},\vec{\beta},\vec{\alpha})\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = (\vec{\gamma}`,\vec{\beta}`,\vec{\alpha}`)\begin{bmatrix} k`_1 \\ k`_2 \\ k`_3 \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = P(\gamma)\begin{bmatrix} k`_1 \\ k`_2 \\ k`_3 \end{bmatrix} \quad (23)$$

$$p(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \quad (24)$$

Equation(25) and Equation(26) can be obtained according to the derivation of $p(\gamma)$.

$$P(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (25)$$

$$P(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad (26)$$

According to the vector angel coordinate system in figure 4, when the $\Delta\gamma$ is generated due to the elastic angel, it equals that the balance coordinate system and the model coordinate system are transformed as $p(\gamma)$, and the vectors of the model coordinate system are obtained in Equation(27):

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = P(\gamma)\begin{bmatrix} k`_1 \\ k`_2 \\ k`_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}\begin{bmatrix} k`_1 \\ k`_2 \\ k`_3 \end{bmatrix} \quad (27)$$

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Where k_1, k_2, k_3 in Equation(27) represent the cosine value of the angels corresponding to the vector and the coordinate system. Equation (28) can be obtained when vector $\vec{\beta} = |\beta|[0,1,0]$ rotates $\Delta\gamma$.

$$\vec{\beta}' = P(\gamma)\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & \cos\gamma & -\sin\gamma\\0 & \sin\gamma & \cos\gamma\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\\cos\gamma\\\sin\gamma\end{bmatrix} \tag{28}$$

It is available that the column vectors of $P(\gamma)$ represent the three unit vectors of the converted base coordinates respectively as shown in Equation(29).

$$[\gamma \quad \beta \quad \alpha]^T = \begin{bmatrix}1 & 0 & 0\\0 & \cos\gamma_m & -\sin\gamma_m\\0 & \sin\gamma_m & \cos\gamma_m\end{bmatrix}\begin{bmatrix}|\gamma_m| & 0 & 0\\0 & |\beta_m| & 0\\0 & 0 & |\alpha_m|\end{bmatrix} \tag{29}$$

The model preset roll angel γ_{m} test is equivalent to that the γ_m axis transformation $P(\gamma_m)$ of the model base coordinate relative to the wind tunnel base coordinate in Equation(30).

$$\begin{aligned}&[\gamma_1 \quad \beta_1 \quad \alpha_1]^T_{\gamma}\\&= P(\gamma_m)\mathrm{diag}(|\gamma_{\mathrm{m}}| \quad 1 \quad 1)\end{aligned} \tag{30}$$

The model preset side slip β_m test simultaneously is equivalent to that the β_m axis transformation $P(\beta_m)$ of the model base coordinate relative to the wind tunnel base coordinate in Equation(31).

$$\begin{aligned}&[\gamma_2 \quad \beta_2 \quad \alpha_2]^T_{\gamma-\beta}\\&= P(\gamma_m)P(\beta_m)P^{-1}(\gamma_m)[\gamma_1 \quad \beta_1 \quad \alpha_1]^T_{\gamma}\\&= P(\gamma_m)P(\beta_m)\mathrm{diag}[|\gamma_{\mathrm{m}}| \quad |\beta_{\mathrm{m}}| \quad 1]\end{aligned} \tag{31}$$

When the attack angel α changes, it is equivalent to the balance base coordinate and the model base coordinate were transformed $P(\alpha)$ around the base axis α of the wind tunnel coordinate system as shown in Equation(32), and simplify as Equation(34).

$$\begin{aligned}&[\gamma_b \quad \beta_b \quad \alpha_b]^T_{\gamma-\beta-\alpha}\\&= P(\gamma_m)P(\beta_m)P(\alpha_m)P^{-1}(\beta_m)P^{-1}(\gamma_m)[\gamma_1 \quad \beta_1 \quad \alpha_1]^{\mathrm{T}}_{\gamma}\\&= P(\gamma_m)P(\beta_m)P(\alpha_m)\mathrm{diag}[|\gamma_{\mathrm{m}}| \quad |\beta_{\mathrm{m}}| \quad |\alpha_{\mathrm{m}}|]\end{aligned} \tag{32}$$

Equation(33) can be obtained by Equation(30), (31), (32).

$$\begin{aligned}&[\gamma_b \quad \beta_b \quad \alpha_b]^{\mathrm{T}}_{\gamma-\beta-\alpha}\\
&= \begin{bmatrix}1 & 0 & 0\\0 & \cos\gamma_{\mathrm{m}} & -\sin\gamma_{\mathrm{m}}\\0 & \sin\gamma_{\mathrm{m}} & \cos\gamma_{\mathrm{m}}\end{bmatrix}\begin{bmatrix}\cos\beta_{\mathrm{m}} & 0 & \sin\beta_{\mathrm{m}}\\0 & 1 & 0\\-\sin\beta_{\mathrm{m}} & 0 & \cos\beta_{\mathrm{m}}\end{bmatrix}\\
&\begin{bmatrix}\cos\alpha_{\mathrm{m}} & -\sin\alpha_{\mathrm{m}} & 0\\\sin\alpha_{\mathrm{m}} & \cos\alpha_{\mathrm{m}} & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}|\gamma_{\mathrm{e}}+\gamma_{\mathrm{m}}| & 0 & 0\\0 & |\beta_e+\beta_m| & 0\\0 & 0 & |\alpha_e+\alpha_{\mathrm{m}}|\end{bmatrix}\\
&= \begin{bmatrix}\cos\beta_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & -\cos\beta_{\mathrm{m}}\sin\alpha_{\mathrm{m}} & \sin\beta_{\mathrm{m}}\\\sin\gamma_{\mathrm{m}}\sin\beta_{\mathrm{m}}\cos\alpha_{\mathrm{m}}+\cos\gamma_{\mathrm{m}}\sin\alpha_{\mathrm{m}} & -\sin\alpha_{\mathrm{m}}\sin\gamma_{\mathrm{m}}\sin\beta_{\mathrm{m}}+\cos\gamma_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & -\sin\gamma_{\mathrm{m}}\cos\beta_{\mathrm{m}}\\\sin\gamma_{\mathrm{m}}\sin\alpha_{\mathrm{m}}-\sin\beta_{\mathrm{m}}\cos\gamma_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & \sin\alpha_{\mathrm{m}}\sin\beta_{\mathrm{m}}\cos\gamma_{\mathrm{m}}+\sin\gamma_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & \cos\beta_{\mathrm{m}}\cos\gamma_{\mathrm{m}}\end{bmatrix}\\
&\begin{bmatrix}|\gamma_{\mathrm{e}}+\gamma_{\mathrm{m}}| & 0 & 0\\0 & |\beta_e+\beta_m| & 0\\0 & 0 & |\alpha_e+\alpha_{\mathrm{m}}|\end{bmatrix}\end{aligned} \tag{33}$$

$$P = P(\gamma_m)P(\beta_m)P(\alpha_m) =$$

$$\begin{bmatrix}\cos\beta_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & -\cos\beta_{\mathrm{m}}\sin\alpha_{\mathrm{m}} & \sin\beta_{\mathrm{m}}\\\sin\gamma_{\mathrm{m}}\sin\beta_{\mathrm{m}}\cos\alpha_{\mathrm{m}}+\cos\gamma_{\mathrm{m}}\sin\alpha_{\mathrm{m}} & -\sin\alpha_{\mathrm{m}}\sin\gamma_{\mathrm{m}}\sin\beta_{\mathrm{m}}+\cos\gamma_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & -\sin\gamma_{\mathrm{m}}\cos\beta_{\mathrm{m}}\\\sin\gamma_{\mathrm{m}}\sin\alpha_{\mathrm{m}}-\sin\beta_{\mathrm{m}}\cos\gamma_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & \sin\alpha_{\mathrm{m}}\sin\beta_{\mathrm{m}}\cos\gamma_{\mathrm{m}}+\sin\gamma_{\mathrm{m}}\cos\alpha_{\mathrm{m}} & \cos\beta_{\mathrm{m}}\cos\gamma_{\mathrm{m}}\end{bmatrix} \tag{34}$$

Where the three column vectors of P represent the base axes of the model pose. There are two methods to calculate the attitude angel, which are spherical coordinate system method and conversion matrix method. Spherical method in the coordinate system is complex, especially when the third coordinate conversion, it is difficult to imagine the relationship between coordinates. The transformation matrix from the track coordinate system to wind axis-stability system and to model coordinate system conversion equals the matrix of the ground coordinate converse to the model coordinate which has nine equations, three parameters, but there are multiple solution or no solution. Where $\alpha_m = 60^\circ$ $\beta_m = -30^\circ$、$\gamma_m = 26^\circ$, it may seek different attitude angels when using different systems of equations.

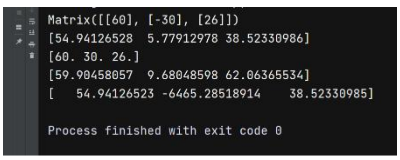

Figure 5. Comparison of the calculation results of the matrix equation

The attitude angle of the model wind coordinate system can be accurately obtained by using the vector coordinate system method. According to the definition of γ_s β_s α_t in the wind coordinate system, α_t is the angle between the projection of the velocity vector on the symmetrical plane of the aircraft and γ_b. β_s is the angle between the velocity vector and the symmetrical surface of the aircraft, and γ_s is the angle between the vector α_w ,which is perpendicular to the velocity vector in the symmetrical plane of the aircraft, and the plumb plane $o\alpha\gamma$ containing the velocity vector . As shown in the figure 6 below, the base coordinate system of the wind γ β α and the base coordinate system of the model γ_b β_b α_b are defined, and where $\beta_{\alpha,\beta,\gamma}$ represents the angle between the vector and the base coordinate respectively [20,21].

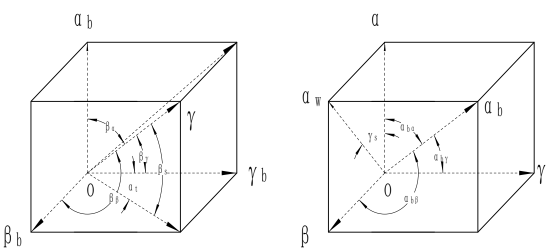

Figure 6 attitude angel conversion

Equation(35) can be obtained Using the triangle function:

$$\begin{cases} \alpha_t = ac\tan(\gamma_\beta / \gamma_\gamma) \\ \beta_s = 90 - ac\cos(\gamma_\alpha) \\ \gamma_s = ac\tan(\alpha_\beta / \alpha_\alpha) \end{cases} \tag{35}$$

Equation(36), Equation(37), Equation(38) can be deduced according to Equation(34)

$$\cos(\gamma_\gamma) = \cos\beta_{\mathrm{m}} \cos\alpha_{\mathrm{m}} \tag{36}$$

$$\cos(\gamma_\beta) = \sin\gamma_{\mathrm{m}} \sin\beta_{\mathrm{m}} \cos\alpha_{\mathrm{m}} + \cos\gamma_{\mathrm{m}} \sin\alpha_{\mathrm{m}} \tag{37}$$

$$\cos(\gamma_\alpha) = \sin\gamma_{\mathrm{m}} \sin\alpha_{\mathrm{m}} - \sin\beta_{\mathrm{m}} \cos\gamma_{\mathrm{m}} \cos\alpha_{\mathrm{m}} \tag{38}$$

Therefore, the problem of solving the triangle value approximation and solving the reverse triangle function in the iteration process are avoided when solve $\alpha_t, \beta_s, \gamma_s$. and it not only gives the size of $\alpha_t, \beta_s, \gamma_s$, but also the direction.

4. Computation results analysis

where $\gamma_m = -180° \sim 180°$ takes any values , $\alpha_m = -90° \sim 90°$ takes 5° interval order values, $\beta_m = -90° \sim 90°$ takes any values , the calculation results are shown in Figure 7 using the vector angel coordinate system . Among them, green, blue and red represent the calculation results of matrix equality, polynomial solution and spherical conversion formula respectively , the three methods will produce positive or negative errors and a certain amount of γ_m error, when the value range of β_m, α_m is greater than 90° or less than −90° . Each attitude angel of the wind coordinate system can be calculated accurately using the angular vector calculation method, which can make each equation of the conversion matrix equal in the Equation(34) above avoiding multiple solutions, singular solutions and other problems, and moreover it can judge the angular direction accurately as the black line shown in Figure 7. According to the calculation results, it is found that the vectors $\alpha_t, \beta_s, \gamma_s$ can be calculated equally using the conversion matrix equation solution method with the base coordinate of the vector α_m, γ_m, but the direction cannot be judged.

At present, the results is the same when carrying out the wind tunnel test $\gamma_m \sim \alpha_m$ (β_m=0) and $\beta_m \sim \alpha_m$ (γ_m=0) which is equivalent to one transformation .The calculation results using the

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spherical method that is difficult to derive the interrelation formula is inaccurate in the $\gamma_m, \beta_m \sim \alpha_m$ test, that is because the parameter γ is in rang of $-180° \sim 180°$. Provided where are the vectors γ_m, β_m, α_m, it can calculate the vectors γ_s, β_s, α_t accurately and quickly to determine its posture using the angle vector method. At the same time, if the vectors γ_m, β_m, α_m are replaced by A, N, Z ,the vectors L, D, C in the wind coordinate can be obtained , directly deduce with the angle coordinate equations.

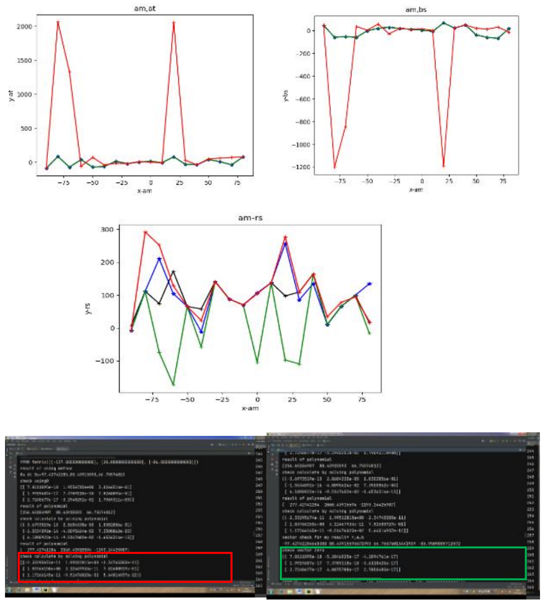

Figure 7. Comparison of the program calculation results

5. conclusion

According to the engineering practice and vector correlation theory, the concept of vector angel coordinate system is established, in which it gave the detail of the calculation deduce process, realized transformation of the model attitude angel from the wind system to track system. Both the approximate solution and the iterative process are avoided, and the correspondence of the attitude angels between each coordinate system is theoretically given. No matter how to transform from the wind system to the track system, the model attitude is easy to obtain by Equation(34). The formula of Equation(34) always hold regardless of how the model attitude is transformed from the wind coordinate system to the track coordinate system. Therefore, Equation(34) can be used directly to solve the aircraft attitude control process and as the loss function to avoid the local minimum and over-fitting problems of gradient descent when using artificial intelligence method to solve the three-channel fusion control strategy.

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