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and the Imaginary Set of Base Planck Units

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The reflectance R of monolayer graphene for the normal incidence of electromagnetic radiation is known to be remarkably defined only by π and the fine-structure constant α. It is shown in this paper that the reflectance (or the sum of transmittance and absorptance) of monolayer graphene, expressed as a quadratic equation with respect to the fine-structure constant α must unsurprisingly introduce the 2nd fine-structure constant α_2, as the 2nd root of this equation, as well as two π-like constants for two surfaces with positive and negative Gaussian curvatures. It turns out that this 2nd fine-structure constant is negative, and the sum of its reciprocal with the reciprocal of the fine-structure constant α is independent of the reflectance value R and remarkably equals $-\pi$. Particular algebraic definition of the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036$, containing the free π term and agreeing with the physical definition of this dimensionless constant to the 5th significant digit, when introduced to this sum, yields $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.178$. Assuming universal validity of the physical definition of α, α_2 defines the negative speed of light in vacuum c_n. The average of this speed c_n and the speed of light in vacuum c is in the range of the Fermi velocity (10^6 m/s). Furthermore, the 2nd negative fine-structure constant α_2 alone introduces the imaginary set of base Planck units.

Keywords: Planck units; the fine-structure constant; speed of light in vacuum; emergent dimensionality

I. INTRODUCTION

Numerous publications provide Fresnel coefficients for the normal incidence of electromagnetic radiation (EMR) on monolayer graphene (MLG), which are remarkably defined only by π and the fine-structure constant α having the reciprocal

$$\alpha^{-1} = \left(\frac{q_P}{e}\right)^2 = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036, \tag{1}$$

where e is the elementary charge, q_P is the Planck charge, ϵ_0 is vacuum permittivity, $\hbar$ is the reduced Planck constant, and c is the speed of light in vacuum.

Transmittance (T) of MLG

$$T = \frac{1}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 97.746\% \tag{2}$$

for normal EMR incidence was derived from the Fresnel equation in the thin-film limit [1] (Eq. 3), whereas spectrally flat absorptance (A) $A \approx \pi\alpha \approx 2.3\%$ was reported [2, 3] for photon energies between about 0.5 and 2.5 eV. T was related to reflectance (R) [4] (Eq. 53) as $R = \pi^2\alpha^2 T/4$, i.e,

$$R = \frac{\frac{1}{4}\pi^2\alpha^2}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 0.013\%, \tag{3}$$

The above formulas for T and R, as well as the formula for the absorptance

$$A = \frac{\pi\alpha}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 2.241\%, \tag{4}$$

were also derived [5] (Eqs. 29-31) based on the thin film model (setting $n_s = 1$ for substrate).

The sum of transmittance (2) and the reflectance (3) at normal EMR incidence on MLG was also derived [6] (Eq. 4a) as

$$T + R = 1 - \frac{4\sigma\eta}{4 + 4\sigma\eta + \sigma^2\eta^2 + k^2\chi^2} = \frac{1 + \frac{1}{4}\pi^2\alpha^2}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 97.759\%, \tag{5}$$

where

$$\eta = \frac{4\pi\alpha\hbar}{e^2} = \frac{1}{\epsilon_0 c} \tag{6}$$

is the impedance of vacuum, $\sigma = e^2/(4\hbar) = \pi\alpha/\eta$ is the MLG conductivity [7], and $\chi = 0$ is the electric susceptibility of vacuum.

These coefficients are thus well-established theoretically and experimentally confirmed [1–3, 6, 8, 9].

As a consequence of the conservation of energy

$$(T + A) + R = 100\%. \tag{7}$$

In other words, the transmittance in the Fresnel equation describing the reflection and transmission of EMR at normal incidence on a boundary between different optical media is, in the case of the 2-dimensional (boundary) of MLG, modified to include its absorption.

The paper is structured as follows. Section II shows that Fresnel coefficients for the normal incidence of EMR on MLG introduce the second, negative fine-structure constant α_2. Section III shows that this second fine-structure constant introduces the negative speed of light in vacuum c_n, which in turn introduces the imaginary set of base Planck units. Section IV shows that the negativity of the α_2 alone is sufficient to introduce the α_2-set of Planck units. Section V presents Fresnel

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coefficients for the normal incidence of EMR on MLG expressed just by π. Section VI shows that Fresnel coefficients for the normal incidence of EMR on MLG introduce two π-like constants for a convex and a saddle surface having respectively positive and negative Gaussian curvatures. Section VII outlines certain prospects of further research, whereas Section VIII concludes the findings of this study.

II. THE SECOND FINE-STRUCTURE CONSTANT

The reflectance $R = 0.013\%$ (3) of MLG can be expressed as a quadratic equation with respect to α

$$\frac{1}{4}(R-1)\pi^2\alpha^2 + R\pi\alpha + R = 0, \tag{8}$$

having two roots with reciprocals

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx 137.036, \quad \text{and} \tag{9}$$

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx -140.178. \tag{10}$$

Therefore, the quadratic equation (8) introduces the second, negative fine-structure constant α_2.

The sum of the reciprocals of these fine-structure constants (9) and (10)

$$\alpha^{-1} + \alpha_2^{-1} = \frac{\pi - \pi\sqrt{R} - \pi - \pi\sqrt{R}}{2\sqrt{R}} = -\pi, \tag{11}$$

is remarkably independent of the reflectance R. The same result can be obtained for the sum of T and A, as shown in Appendix .

Furthermore, this result is intriguing in the context of a peculiar algebraic definition of the fine-structure constant [10]

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036 \tag{12}$$

that contains a *free* π term and agrees with the physical definition (1) of α to the 5th significant digit. Therefore, using Equations (11) and (12), we can express the negative reciprocal of the 2nd fine-structure constant α_2^{-1} that emerged in the quadratic equation (8) also as a function of π only

$$\alpha_2^{-1} = -\pi - \alpha_1^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.178. \tag{13}$$

This result supports the validity of the algebraic definition (12).

But how can this negative value be interpreted physically?

III. NEGATIVE SPEED OF LIGHT AND THE α_2-SET OF PLANCK UNITS BASED ON IT

Almost all physical constants of $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ in the physical definition of the fine-structure constant (1) are positive[1], whereas the charge e is squared. Only the velocity can be negative, as it is a *directional* quantity. Therefore, if

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036, \tag{14}$$

then

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} = \frac{4\pi\epsilon_0\hbar c_n}{e^2} \approx -140.178, \tag{15}$$

where c_n is the negative speed of light in vacuum that, using Equations (11) with (1) and (15), amounts

$$\begin{aligned} \frac{4\pi\epsilon_0\hbar c}{e^2} + \frac{4\pi\epsilon_0\hbar c_n}{e^2} &= -\pi \\ c_n = -\frac{e^2}{4\epsilon_0\hbar} - c &\approx -3.066653\times 10^8 \text{ [m/s]}, \end{aligned} \tag{16}$$

which is greater that the speed of light in vacuum c in modulus, whereas their average

$$\frac{c + c_n}{2} \approx -3.436417\times 10^6 \text{ [m/s]} \tag{17}$$

is in the range of the Fermi velocity.

Furthermore, we can express the $e^2/(4\epsilon_0\hbar)$ term in (16) using the impedance of vacuum (6) as $c_n = -c(\alpha\pi + 1)$ which, using the algebraic definitions of α (12) and α_2 (13), yields the following relation between the speed of light in vacuum c, negative speed of light c_n, the fine-structure constant α and the negative fine-structure constant α_2

$$c\alpha = c_n\alpha_2 \quad (= v_e), \tag{18}$$

where v_e is the electron's velocity at the first circular orbit in the Bohr model of the atom.

The concept of the negative speed of light in vacuum c_n might seem to be a *nonsence* or unsettling for the reader. If this is the case, the reader is advised to skip to the subsequent Section IV.

The negative speed of light in vacuum c_n (16) introduces the imaginary set of base Planck units $\{q_{Pi}, \ell_{Pi}, m_{Pi}, t_{Pi}, T_{Pi}\}$ that redefined by square roots containing $c_n < 0$ raised to an odd (1, 3, 5) power become imaginary and bivalued[2]:

$$\begin{aligned} q_{Pi} &= \pm\sqrt{4\pi\epsilon_0\hbar c_n} = \pm q_P\sqrt{\frac{\alpha}{\alpha_2}} \\ &\approx \pm i1.8969\times 10^{-18} \text{ [C]} \quad (> q_P), \end{aligned} \tag{19}$$

$$\begin{aligned} \ell_{Pi} &= \pm\sqrt{\frac{\hbar G}{c_n^3}} = \pm\ell_P\sqrt{\frac{\alpha_2^3}{\alpha^3}} \\ &\approx \pm i1.5622\times 10^{-35} \text{ [m]} \quad (< \ell_P), \end{aligned} \tag{20}$$

[1] Vacuum permittivity ϵ_0 is the value of the absolute dielectric permittivity of classical vacuum. Thus, ϵ_0 cannot be negative. The Planck constant h is the uncertainty principle parameter. Thus, it cannot be negative; negative probabilities do not seem to withstand Occam's razor.

[2] Base Planck units themselves admit negative values as negative square roots.

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$$m_{Pi} = \pm\sqrt{\frac{\hbar c_n}{G}} = \pm m_P \sqrt{\frac{\alpha}{\alpha_2}} \approx \pm i 2.2012 \times 10^{-8} \text{ [kg]} \quad (> m_P), \tag{21}$$

$$t_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^5}} = \pm t_P \sqrt{\frac{\alpha_2^5}{\alpha^5}} \approx \pm i 5.0942 \times 10^{-44} \text{ [s]} \quad (< t_P), \tag{22}$$

$$T_{Pi} = \pm\sqrt{\frac{\hbar c_n^5}{G k_B^2}} = \pm T_P \sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \pm i 1.4994 \times 10^{32} \text{ [K]} \quad (> T_P), \tag{23}$$

and can be expressed[3], using (18), in terms of base Planck units ℓ_P, m_P, t_P, and T_P.

We note in passing, that imaginary and negative physical quantities are the subject of research. In particular, thermodynamics in the complex plane is the subject of research. In particular, Lee–Yang zeros have been experimentally observed [11, 12].

Both charge $\alpha_2 q_{Pi}^2 = \alpha q_P^2$ (19) and mass α_2 Planck units $\alpha_2 m_{Pi}^2 = \alpha m_P^2$ (21) are related with base Planck units in the same way. It is not surprising: both Coulomb's law and Newton's law of gravity are inverse-square laws. We also note that the relations between time (22) and temperature α_2 Planck units (23) are inverted: $\alpha^5 t_{Pi}^2 = \alpha_2^5 t_P^2$, whereas $\alpha_2^5 T_{Pi}^2 = \alpha^5 T_P^2$. Furthermore, eliminating α and α_2 from (19)-(23) yields the following relations

$$\frac{q_P^2}{m_P^2} = \frac{q_{Pi}^2}{m_{Pi}^2} = 4\pi\epsilon_0 G, \tag{24}$$

$$T_P^2 t_P^2 = T_{Pi}^2 t_{Pi}^2 = \frac{\hbar^2}{k_B^2}, \tag{25}$$

and

$$\frac{\ell_P}{\ell_{Pi}} = \frac{q_{Pi}^3}{q_P^3} = \frac{m_{Pi}^3}{m_P^3}, \tag{26}$$

independent of the velocity of light.

The first relation (24) is interesting. A complex energy formula

$$E = E_M + iE_Q = (1 + i\beta) Mc^2, \tag{27}$$

where $E_M = Mc^2$ represents the real energy, M is the mass of a *particle*, E_Q represents the imaginary energy, and

$$\beta \doteq \frac{E_Q}{E_M} = \frac{Q}{2M\sqrt{\pi\epsilon_0 G}}, \tag{28}$$

is the imaginary-real energy ratio[4], where Q is the charge[5] of the *particle*, was proposed in [13] (Eqs. (1), (3), and (4)). Squared imaginary-real energy ratio (28) can be expressed as

$$\beta^2 = \frac{Q^2}{4\pi\epsilon_0 G M^2}, \quad \frac{Q^2}{\beta^2 M^2} = 4\pi\epsilon_0 G, \tag{29}$$

which certainly supports the complex energy formula (27). We see that for $\beta^2 = 1$, $Q^2/M^2 = q_P^2/m_P^2 = q_{Pi}^2/m_{Pi}^2$ (24). In such a case, the complex energy (27) is real-to-imaginary balanced, i.e., $E = (1 \pm i) Mc^2$. Such a balanced situation is also specific to black holes, the horizons of which comprise a balanced number of Planck areas with binary potentials equal to $-c^2$ and zero [14]. Furthermore, the complex energy (27) of an *antiparticle* is defined as a conjugate of the complex energy of its associated *particle*. The complex energy (27) also unifies Coulomb's law and Newton's law of gravity (cf. [13] Eqs. (7), (8)) between two *particles* with masses M_1, M_2, charges Q_1, Q_2, a distance of R apart, in a complex force formula

$$\begin{aligned} F &= \frac{G}{c^4}\frac{E_1 E_2}{R^2} = \\ &= G\frac{M_1 M_2}{R^2} - \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R^2} + i\frac{\sqrt{G}}{2\sqrt{\pi\epsilon_0}}\left(\frac{Q_1 M_2 + Q_2 M_1}{R^2}\right) = \\ &= G\frac{M_1 M_2}{R^2}\left[1 + i\left(\beta_1 + \beta_2\right)\right] - \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R^2}. \end{aligned} \tag{30}$$

We note that the imaginary term vanishes if $M_1 = M_2$ and $Q_1 = -Q_2$ (particle vs. antiparticle). Also, the quantum of the imaginary energy (corresponding to the elementary charge e) $E_Q(e) = ec^2/(2\sqrt{\pi\epsilon_0 G})$ ([13] Eq. (10)) can be expressed, using the impedance of vacuum (6), in terms of the fine-structure constant as

$$E_Q(e) = \sqrt{\alpha} k_B T_P. \tag{31}$$

Planck units derived from imaginary base units (20)-(23) are not imaginary in general.

The α_2 Planck volume

$$\ell_{Pi}^3 = \pm\left(\frac{\hbar G}{c_n^3}\right)^{3/2} = \pm\ell_P^3\sqrt{\frac{\alpha_2^9}{\alpha^9}} \approx \pm i 3.8127 \times 10^{-105} \text{ [m}^3\text{]} \quad \left(< \ell_P^3\right), \tag{32}$$

the α_2 Planck momentum

$$p_{Pi} = \pm m_{Pi} c_n = \pm\sqrt{\frac{\hbar c_n^3}{G}} = \pm m_P c\sqrt{\frac{\alpha^3}{\alpha_2^3}} \approx \pm i 6.7504 \text{ [kg m/s]} \quad (> m_P c), \tag{33}$$

[3] The notation $\ell_{Pi}(< \ell_P)$, for example, means that the absolute value of the imaginary Planck length (20) is lower than the absolute value of the Planck length $\ell_P = \sqrt{\hbar G/c^3}$.

[4] In the cited study it is called α, so we shall call it β to avoid confusion with the fine-structure constant α.

[5] Charges in the cited study are defined in CGS units; here we adopt SI.

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the α_2 Planck energy

$$E_{Pi} = \pm m_{Pi} c_n^2 = \pm\sqrt{\frac{\hbar c_n^5}{G}} = \pm E_P \sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \pm i 2.0701 \times 10^9 \text{ [J]} \quad (> E_P), \tag{34}$$

and the α_2 Planck acceleration

$$a_{Pi} = \pm\frac{c_n}{t_{Pi}} = \pm\sqrt{\frac{c_n^7}{\hbar G}} = \pm a_P \sqrt{\frac{\alpha^7}{\alpha_2^7}} \approx \pm i 6.0198 \times 10^{51} \text{ [m/s}^2\text{]} \quad (> a_P), \tag{35}$$

are imaginary and bivalued.

However, the α_2 Planck force

$$F_{P2} = \pm\frac{E_{Pi}}{\ell_{Pi}} = \pm\frac{c_n^4}{G} = \pm F_P \frac{\alpha^4}{\alpha_2^4} \approx \pm 1.3251 \times 10^{44} \text{ [N]} \quad (> F_P), \tag{36}$$

and the α_2 Planck density

$$\rho_{P2} = \pm\frac{m_{Pi}}{\ell_{Pi}^3} = \pm\frac{c_n^5}{\hbar G^2} = \pm\rho_P \frac{\alpha^5}{\alpha_2^5} \approx \pm 5.7735 \times 10^{96} \text{ [kg/m}^3\text{]} \quad (> \rho_P), \tag{37}$$

are real and bivalued.

On the other hand, the α_2 Planck area

$$\ell_{Pi}^2 = \frac{\hbar G}{c_n^3} = \ell_P^2 \frac{\alpha_2^3}{\alpha^3} \approx -2.4406 \times 10^{-70} \text{ [m}^2\text{]} \quad \left(< \ell_P^2\right), \tag{38}$$

is strictly negative. According to the holographic principle, each bit of information is physically represented on the holographic boundary by the Planck area [15–17] (having a positive value), whereas these areas are triangular [14][6]. Notably, out of all four omnidimensional (i.e., present in all complex dimensions n) convex n-polytopes and n-balls, only n-simplices (i.e., triangles for $n = 2$) are bivalued, admitting both positive and negative volumes and surfaces [18].

IV. ALTERNATIVE DERIVATION OF THE α_2-SET OF PLANCK UNITS WITHOUT THE NEGATIVE SPEED OF LIGHT

The α_2-set of Planck units, discussed in the preceding section, can also be derived without the negative speed of light c_n (16).

Using the definition of the fine-structure constant $\alpha = e^2/q_P^2$ (1) for the negative α_2 (10) or (13), we see that it requires an introduction of some imaginary Planck charge q_{Pi}, so that its square would yield a negative α_2

$$\frac{q_{Pi}^2}{e^2} = \alpha_2^{-1} \approx -140.178 < 0. \tag{39}$$

Thus, using the value of the elementary charge e in (39) and the value of the α_2 (10) or (13), the imaginary Planck charge (39) can be derived without the negative speed of light c_n as

$$q_{Pi} = \pm\sqrt{\frac{e^2}{\alpha_2}}. \tag{40}$$

Furthermore, the concept of the negative speed of light c_n is not required to derive the imaginary Planck mass, which, using (40), (24) and the values of the Planck mass and charge is

$$m_{Pi} = \pm\frac{q_{Pi} m_P}{q_P}. \tag{41}$$

In the derivation of (41), we have used the relation (24) that was derived on the basis of the negative speed of light, but this relation, as such, is independent of any particular value of the speed of light, be it positive or negative.

Knowing that the Planck charge is $q_P = \sqrt{4\pi\epsilon_0\hbar c}$ we can solve it for the speed of light to find $c = q_P^2/(4\pi\epsilon_0\hbar)$ and introduce it to the Planck force $F_P = \pm c^4/G$ making it is independent on the speed of light.

This relation must be valid also for the α_2 Planck force that therefore can be derived without the negative speed of light c_n and using the imaginary Planck charge (40), or the relation (24) (with the second term) as

$$F_{P2} = \pm\frac{1}{G}\left(\frac{q_{Pi}^2}{4\pi\epsilon_0\hbar}\right)^4 = \pm\frac{G^3}{\hbar^4} m_{Pi}^8. \tag{42}$$

Knowing the value of the α_2 Planck force (42) and mass (41) we can derive the α_2 Planck acceleration

$$a_{Pi} = \pm\frac{F_{P2}}{m_{Pi}}, \tag{43}$$

and the remaining α_2 Planck units, using the imaginary Planck charge (40) instead of the negative speed of light c_n.

V. ALGEBRAIC FRESNEL COEFFICIENTS FOR THE NORMAL INCIDENCE OF EMR ON MLG

With algebraic definitions of α (12) and α_2 (13) transmittance T (2), reflectance R (3) and absorptance A (4) of MLG for normal EMR incidence can be expressed just by π.

For $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (12) they become

$$T(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)^2}{(8\pi^2 + 2\pi + 3)^2} \approx 97.746\%, \tag{44}$$

$$A(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)}{(8\pi^2 + 2\pi + 3)^2} \approx 2.241\%, \tag{45}$$

[6] This is also compatible with the Causal Dynamical Triangulation (CDT) approach, which does not assume a pre-existence of a dimensional space, but focuses on the evolution of the spacetime as such.

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while for $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi$ (13) they become

$$T(\alpha_2) = \frac{4\left(4\pi^2 + \pi + 2\right)^2}{(8\pi^2 + 2\pi + 3)^2} \approx 102.279\%, \quad (46)$$

$$A(\alpha_2) = -\frac{4\left(4\pi^2 + \pi + 2\right)}{(8\pi^2 + 2\pi + 3)^2} \approx -2.292\%, \quad (47)$$

with

$$R(\alpha) = R(\alpha_2) = \frac{1}{(8\pi^2 + 2\pi + 3)^2} \approx 0.013\%. \quad (48)$$

Obviously $(T(\alpha)+A(\alpha))+R(\alpha) = (T(\alpha_2)+A(\alpha_2))+R(\alpha_2) = 1$ as required by the law of conservation of energy (7), whereas each conservation law is associated with a certain symmetry, as asserted by Noether's theorem. Nonetheless, physical interpretation of $T(\alpha_2) > 1$ and $A(\alpha_2) < 0$ invites further research. $A(\alpha) > 0$ implies a *sink*, whereas $A(\alpha_2) < 0$ implies a *source*, whereas the opposite holds true for the transmittance T, as illustrated schematically in Fig 1. Perhaps, the negative absorptance and transmittance exceeding 100% for α_2 (10) or (13) could be explained in terms of graphene spontaneous emission.

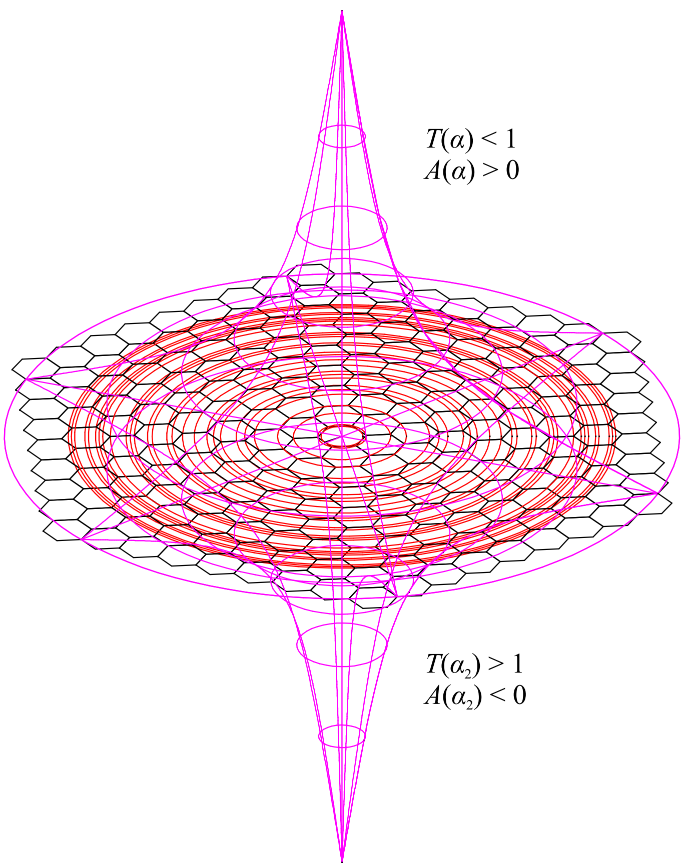

Figure 1. Illustration of the concepts of negative absorptance and excessive transmittance of EMR under normal incidence on MLG.

VI. FRESNEL COEFFICIENTS FOR THE NORMAL INCIDENCE OF EMR ON MLG INTRODUCE TWO π-LIKE CONSTANTS FOR TWO SURFACES WITH POSITIVE AND NEGATIVE GAUSSIAN CURVATURES

The quadratic equation (8) describing the reflectance R of MLG under normal incidence of EMR (or alternatively (A.1)) can also be solved for π yielding two roots

$$\pi(R, \alpha_*)_1 = \frac{2\sqrt{R}}{\alpha_*(1 - \sqrt{R})}, \quad \text{and} \quad (49)$$

$$\pi(R, \alpha_*)_2 = \frac{-2\sqrt{R}}{\alpha_*(1 + \sqrt{R})}, \quad (50)$$

dependent on R and α_*, where α_* indicates α or α_2. This can be further evaluated, using the algebraic definition of R (48) (which is the same for both α and α_2), yielding four, yet only three distinct, possibilities

$$\pi_1 = \pi(\alpha)_1 = -\pi\frac{4\pi^2 + \pi + 1}{4\pi^2 + \pi + 2} = \pi\frac{\alpha_2}{\alpha} \approx -3.0712, \quad (51)$$

$$\pi(\alpha)_2 = \pi(\alpha_2)_1 = \pi \approx 3.1416, \quad \text{and} \quad (52)$$

$$\pi_2 = \pi(\alpha_2)_2 = -\pi\frac{4\pi^2 + \pi + 2}{4\pi^2 + \pi + 1} = \pi\frac{\alpha}{\alpha_2} \approx -3.2136. \quad (53)$$

The absolute value of π_1 (51) corresponds to a convex surface having a positive Gaussian curvature, whereas the absolute value of π_2 (53) - to a negative Gaussian curvature. Their product $\pi_1\pi_2 = \pi^2$ is independent of α_*, and their quotient $\pi_1/\pi_2 = \alpha_2^2/\alpha^2$ is independent of π. It remains to be found, whether each of them describes the ratio of circumference of a circle drawn on the respective surface to its diameter (π_c) or the ratio of the area of this circle to the square of its radius (π_a). These definitions produce different results on curved surfaces, whereas $\pi_a > \pi_c$ on convex surfaces, while $\pi_a < \pi_c$ on saddle surfaces [19].

VII. DISCUSSION

A general relation describing the information capacity $N_{BH} = \pi d^2$ (i.e., the number of the triangular Planck areas at the black hole horizon, corresponding to bits of information and the fractional part triangle $\{\pi d^2\}$ to small to carry a bit) of a Schwarzschild black hole (BH) having the diameter D multiplier $d = D/\ell_P$ after absorption ($+16\pi^2 d/l$) or emission ($-16\pi^2 d/l$) of a *particle* having the wavelength λ multiplier $l = \lambda/\ell_P$ has been disclosed in [14] (Eq. (18)) as

$$N_{BH}^{A/E}(d, l) = 64\pi^3\frac{1}{l^2} \pm 16\pi^2\frac{d}{l} + \pi d^2. \quad (54)$$

This general geometric relation, originally derived only for *particles* having the Compton wavelength equal to the BH radius [20](p. 153), is remarkably similar to the algebraic definitions of the inverses of α (12) and α_2 (13) also containing

π^3, π^2, and π terms. This raises the question of whether the inverses of the fine-structure constants themselves correspond to the number of bits. Remarkably, the floor function of the inverse of the fine-structure constant α represents the threshold on the atomic number (137) of a hypothetical element *feynmanium* that, in the Bohr model of the atom, still allows the 1s orbital electrons to travel slower than the speed of light. On the other hand, the negative α_2 introduces the negative BH information capacity, which - taking into account the negative, triangular α_2 Planck area ℓ_{Pi}^2 (38) - could be in the form

$$N_{BH}(\alpha,\alpha_2) = \frac{\pi d^2 \ell_P^2}{\ell_{Pi}^2} = \pi d^2 \frac{\alpha^3}{\alpha_2^3} \approx -1.0704\pi d^2, \quad (55)$$

or

$$N_{BH}(\alpha_2,\alpha) = \frac{\pi d^2 \ell_{Pi}^2}{\ell_P^2} = \pi d^2 \frac{\alpha_2^3}{\alpha^3} \approx -0.8728\pi d^2. \quad (56)$$

To examine the conditions under which the BH information capacity would equal α (12) and α_2(13) after absorption or emission we compare (54) with (12), which yields

$$d_{A/E,\alpha} = \mp\frac{8\pi}{l} \pm \sqrt{4\pi^2+\pi+1}, \quad (57)$$

whereas comparing (54) with (13) yields

$$d_{A/E,\alpha_2} = \mp\frac{8\pi}{l} \pm i\sqrt{4\pi^2+\pi+2}, \quad (58)$$

in both cases with -8π for absorption and $+8\pi$ for emission. The emission first term in (57) and (58) corresponds to the BH Compton diameter multiplier [14] (Eq. (9)). This issue certainly requires further research. We note in passing that the fine-structure constant has recently been reported as the quantum of rotation [21].

VIII. CONCLUSIONS

We have shown that the reflectance of graphene under the normal incidence of electromagnetic radiation (EMR), expressed as the quadratic equation with respect to the fine-structure constant α must introduce the 2^{nd} negative fine-structure constant α_2.

The sum of the reciprocal of this 2^{nd} fine-structure constant α_2 with the reciprocal of the fine-structure constant α (1) is independent of the reflectance value R and remarkably equals simply $-\pi$.

Particular algebraic definition of the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (12), containing the free π term, when introduced to this sum, yields $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi < 0$.

Assuming universal validity of the physical definition of the fine-structure constant α (1), the 2^{nd} fine-structure constant α_2 (13) defines the negative speed of light c_n (16), the average of which and the speed of light in vacuum is in the range of the Fermi velocity (10^6 m/s).

Furthermore, the negative fine-structure constant α_2 alone introduces the imaginary set of five base Planck units (19)-(23), whereas mass and charge units (24), as well as temperature and time units (25) are directly related to base Planck units.

The Planck volume (32), momentum (33), energy (34), and acceleration (35) derived from imaginary base units (20)-(23) are imaginary and bivalued, whereas the Planck force (36) and density (37) are real and bivalued.

The square of the imaginary Planck length (20) introduces the negative Planck area (38). If Planck areas correspond to bits of information [15–17] and if they are triangular [14], then the fact that among four basic geometrical structures present in all complex dimensions: n-balls, n-cubes, n-orthoplices, and n-simplices only the latter have bivalued (positive and negative) volumes and surfaces equal to the modulus [18] for any real dimension n, suggests a physical interpretation for this negative Planck area (38).

The findings of this study, in particular, the meaning of the relation (25) (which clearly hints at Heisenberg's uncertainty principle) and the negative ratios π_1 (51) and π_2 (53), inquire further research in the context of information-theoretic approach [22–24] and emergent dimensionality [14, 18, 25, 26].

In the context of the results of this study, monolayer graphene, a truly 2-dimensional material with no thickness[7], is a *keyhole* to other, unperceivable [14], dimensionalities. Graphene history is also instructive. Discovered in 1947 [28], graphene was long considered *academic material* until it was eventually pulled from graphite in 2004 [29] by means of ordinary Scotch tape[8]. These fifty-seven years, along with twenty-nine years (1935-1964) between the condemnation of quantum theory as *incomplete* [30] and Bell's mathematical theorem [31] asserting that it is not true, and the fifty-eight years (1964-2022) between the formulation of this theorem and 2022 Nobel prize in physics for its experimental *loophole-free* confirmation, should remind us that Max Planck, the genius who discovered Planck units, has also discovered Planck's principle.

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[7] Thickness of MLG is reported [27] as 0.37 nm with other reported values up to 1.7 nm. However, taking into account that 0.335 nm is the established inter-layer distance, and thus the thickness of bilayer graphene, these results do not seem credible.

[8] Introduced into the market in 1932.

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Appendix: Other Quadratic Equations

The quadratic equation for the sum of transmittance (2) and absorptance (4), putting $C_{TA} \doteq T + A$, is

$$\frac{1}{4} C_{TA} \pi^2 \alpha^2 + (C_{TA} - 1)\,\pi\alpha + (C_{TA} - 1) = 0, \tag{A.1}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} + \sqrt{1 - C_{TA}}\right)} \approx 137.036, \tag{A.2}$$

and

$$\alpha_2^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} - \sqrt{1 - C_{TA}}\right)} \approx -140.178, \tag{A.3}$$

whereas their sum $\alpha^{-1} + \alpha_2^{-1} = -\pi$ is also independent of T and A.

Other quadratic equations do not feature this property. For example, the sum of $T + R$ (5) expressed as the quadratic equation and putting $C_{TR} \doteq T + R$, is

$$\frac{1}{4}\,(C_{TR} - 1)\,\pi^2\alpha^2 + C_{TR}\pi\alpha + (C_{TR} - 1) = 0, \tag{A.4}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} + 2\sqrt{2C_{TR} - 1}} \approx 137.036, \tag{A.5}$$

and

$$\alpha_{TR}^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} - 2\sqrt{2C_{TR} - 1}} \approx 0.0180, \tag{A.6}$$

whereas their sum

$$\alpha_{TR_1}^{-1} + \alpha_{TR_2}^{-1} = \frac{-\pi C_{TR}}{C_{TR} - 1} \approx 137.054 \tag{A.7}$$

is dependent on T and R.

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