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Negative Speed of Light, and the Imaginary Set of Base Planck Units

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The reflectance R of monolayer graphene for the normal incidence of electromagnetic radiation is known to be remarkably defined only by π and the fine structure constant α. It is shown in this paper that the reflectance (or the sum of transmittance and absorptance) of monolayer graphene, expressed as a quadratic equation with respect to the fine structure constant α must unsurprisingly introduce the 2nd fine structure constant α_2, as the 2nd root of this equation. It turns out that this 2nd fine structure constant is negative, and the sum of its reciprocal with the reciprocal of the *physical* fine structure constant α is independent of the reflectance value R and remarkably equals $-\pi$. Particular algebraic definition of the fine structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$, containing the free π term, when introduced to this sum, yields $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi < 0$. Assuming universal validity of the physical definition of α, α_2 defines the negative speed of light in vacuum c_n and introduces the imaginary set of base Planck units. The average of this speed and the speed of light in vacuum is in the range of the Fermi velocity (10^6 m/s).

Keywords: Planck units; the fine-structure constant; speed of light in vacuum

I. INTRODUCTION

Numerous publications provide Fresnel coefficients for the normal incidence of electromagnetic radiation (EMR) on monolayer graphene (MLG), which are remarkably defined only by π and the fine structure constant α having the reciprocal

$$\alpha^{-1} = \left(\frac{q_P}{e}\right)^2 = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036, \tag{1}$$

where e is the elementary charge, q_P is the Planck charge, ϵ_0 is vacuum permittivity, $\hbar$ is the reduced Planck constant, and c is the speed of light in vacuum.

Transmittance (T) of MLG

$$T = \frac{1}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 97.746\% \tag{2}$$

for normal EMR incidence was derived from the Fresnel equation in the thin-film limit [1] (Eq. 3), whereas spectrally flat absorptance (A) $A \approx \pi\alpha \approx 2.3\%$ was reported [2, 3] for photon energies between about 0.5 and 2.5 eV. T was related to reflectance (R) [4] (Eq. 53) as $R = \pi^2\alpha^2 T/4$, i.e,

$$R = \frac{\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.013\%, \tag{3}$$

The above formulas for T and R, as well as the formula for the absorptance

$$A = \frac{\pi\alpha}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 2.241\%, \tag{4}$$

were also derived [5] (Eqs. 29-31) based on the thin film model (setting $n_s = 1$ for substrate).

The sum of transmittance (2) and the reflectance (3) at normal EMR incidence on MLG was also derived [6] (Eq. 4a) as

$$\begin{aligned} T + R &= 1 - \frac{4\sigma\eta}{4 + 4\sigma\eta + \sigma^2\eta^2 + k^2\chi^2} \\ &= \frac{1+\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 97.759\%, \end{aligned} \tag{5}$$

where

$$\eta = \frac{4\pi\alpha\hbar}{e^2} = \frac{1}{\epsilon_0 c} \tag{6}$$

is the impedance of vacuum, $\sigma = e^2/(4\hbar) = \pi\alpha/\eta$ is the MLG conductivity [7], and $\chi = 0$ is the electric susceptibility of vacuum.

These coefficients are thus well-established theoretically and experimentally confirmed [1–3, 6, 8, 9].

As a consequence of the conservation of energy

$$(T + A) + R = 100\%. \tag{7}$$

In other words, the transmittance in the Fresnel equation describing the reflection and transmission of EMR at normal incidence on a boundary between different optical media is, in the case of the 2-dimensional (boundary) of MLG, modified to include its absorption.

The paper is structured as follows. Section II shows that Fresnel coefficients for the normal incidence of EMR on MLG introduce the second, negative fine structure constant α_2. Section III shows that this second fine structure constant introduces the negative speed of light in vacuum c_n. Section IV shows that the negativity of the α_2 and c_n introduces the imaginary set of base Planck units. Section V shows that Fresnel coefficients for the normal incidence of EMR on MLG can be expressed just by π. Section VI concludes and discusses the findings of this study.

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II. THE SECOND FINE STRUCTURE CONSTANT

The reflectance $R = 0.013\%$ (3) of MLG can be expressed as a quadratic equation with respect to α

$$\frac{1}{4}\pi^2 (R-1)\alpha^2 + R\pi\alpha + R = 0, \tag{8}$$

having two roots with reciprocals

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx 137.036, \tag{9}$$

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx -140.178. \tag{10}$$

Therefore, Equation (8) introduces the second, negative fine structure constant α_2.

The sum of the reciprocals of these fine structure constants (9) and (10)

$$\alpha^{-1} + \alpha_2^{-1} = \frac{\pi - \pi\sqrt{R} - \pi - \pi\sqrt{R}}{2\sqrt{R}} = -\pi, \tag{11}$$

is remarkably independent of the reflectance R. The same result can be obtained for the sum of T and A, as shown in Appendix .

Furthermore, this result is intriguing in the context of a peculiar algebraic definition of the fine structure constant [10]

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036 \tag{12}$$

that contains a *free* π term and agrees with the physical definition (1) of α to the 5$^{\text{th}}$ significant digit. Therefore, using Equations (11) and (12), we can express the negative reciprocal of the 2$^{\text{nd}}$ fine structure constant α_2^{-1} that emerged in Equation (8) also as a function of π only

$$\alpha_2^{-1} = -\pi - \alpha_1^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.178. \tag{13}$$

This result supports the validity of the algebraic definition (12).

But how can this negative value be interpreted physically?

III. NEGATIVE SPEED OF LIGHT

If $\alpha^{-1} = (q_P/e)^2$ (1) is valid also for the negative α_2^{-1} (10) or (13) then it requires an introduction of the imaginary Planck charge iq_{Pi}, so that its square would yield a negative number

$$i^2 q_{Pi}^2 = e^2\alpha_2^{-1} < 0. \tag{14}$$

Furthermore, almost all physical constants of $(4\pi\epsilon_0\hbar c)/e^2$ in the *physical* definition of the fine structure constant (1) are positive[1], whereas the charge e is squared. Only the velocity can be negative, as it is a *directional* quantity. Therefore, if

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} = \frac{4\pi\epsilon_0\hbar c}{e^2}, \tag{15}$$

then

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} = \frac{4\pi\epsilon_0\hbar c_n}{e^2}, \tag{16}$$

where c_n is the negative speed of light in vacuum that, using Equations (11) with (1) and (16), amounts

$$\begin{aligned} \frac{4\pi\epsilon_0\hbar c}{e^2} + \frac{4\pi\epsilon_0\hbar c_n}{e^2} &= -\pi \\ c_n = -\frac{e^2}{4\epsilon_0\hbar} - c &\approx -3.066653 \times 10^8 \text{ [m/s]}, \end{aligned} \tag{17}$$

which is greater that the speed of light in vacuum c in modulus, whereas their average

$$\frac{c + c_n}{2} \approx -3.436417 \times 10^6 \text{ [m/s]} \tag{18}$$

is in the range of the Fermi velocity.

Furthermore, we can express the $e^2/(4\epsilon_0\hbar)$ term in (17) using the impedance of vacuum (6) as $c_n = -c(\alpha\pi + 1)$ which, using the algebraic definitions of α (12) and α_2 (12), yields the following relation between the speed of light in vacuum c, negative speed of light c_n, the fine-structure constant α and the negative fine-structure constant α_2 (13)

$$c\alpha = c_n\alpha_2 \quad (= v_e), \tag{19}$$

where v_e is the electron's velocity in the first circular orbit of the Bohr model of the atom.

IV. IMAGINARY BASE PLANCK UNITS

Using the value of the elementary charge e in (14) and the value of the α_2 (10) or (13) , the imaginary Planck charge (14) amounts

$$q_{Pi} = \pm\sqrt{\frac{e^2}{\alpha_2}} = \pm q_P\sqrt{\frac{\alpha}{\alpha_2}} \approx \pm i1.8969 \times 10^{-18} \text{ [C]} \, (> iq_P). \tag{20}$$

Furthermore, the negative speed of light in vacuum c_n (17) introduces all the remaining base Planck units defined by square roots containing c raised to an odd (1, 3, 5) power, that redefined with $c_n < 0$ become imaginary:

$$\ell_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^3}} = \pm\ell_P\sqrt{\frac{\alpha_2^3}{\alpha^3}} \approx \pm i1.5622 \times 10^{-35} \text{ [m]} \, (< i\ell_P), \tag{21}$$

$$m_{Pi} = \pm\sqrt{\frac{\hbar c_n}{G}} = \pm m_P\sqrt{\frac{\alpha}{\alpha_2}} \approx \pm i2.2012 \times 10^{-8} \text{ [kg]} \, (> im_P), \tag{22}$$

[1] Vacuum permittivity ϵ_0 is a measure of how *dense* is an electric field; objects that do not change their measure with respect to orientation (as compared to volumes, for example) are densities. Thus, ϵ_0 cannot be negative. The Planck constant h is the uncertainty principle parameter. Thus, it cannot be negative; negative probabilities do not seem to withstand Occam's razor.

$$t_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^5}} = \pm t_P\sqrt{\frac{\alpha_2^5}{\alpha^5}} \approx \pm i5.0942\times 10^{-44}\ [\mathrm{s}]\,(< it_P)\,, \tag{23}$$

$$T_{Pi} = \pm\sqrt{\frac{\hbar c_n^5}{G k_B^2}} = \pm T_P\sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \pm i1.4994\times 10^{32}\ [\mathrm{K}]\,(> iT_P)\,, \tag{24}$$

and can be expressed, using (19), in terms of base Planck units ℓ_P, m_P, t_P, and T_P. We note in passing that base Planck units themselves admit negative values as negative square roots.

Both charge units $\alpha_2 q_{Pi}^2 = \alpha q_P^2$ (20) and mass units $\alpha_2 m_{Pi}^2 = \alpha m_P^2$ (22) are related with base Planck units in the same way. It is not surprising: both Coulomb's law and Newton's law of universal gravitation are inverse-square laws. We also note that the relations between time units (23) and between temperature units (24) are inverted: $\alpha^5 t_{Pi}^2 = \alpha_2^5 t_P^2$, whereas $\alpha_2^5 T_{Pi}^2 = \alpha^5 T_P^2$. Furthermore, eliminating α and α_2 from (21)-(24) yields

$$\frac{m_P^2}{m_{Pi}^2} = \frac{q_P^2}{q_{Pi}^2} \tag{25}$$

and

$$\frac{T_P^2}{T_{Pi}^2} = \frac{t_{Pi}^2}{t_P^2}. \tag{26}$$

Planck units derived from imaginary base units (21)-(24) are not imaginary in general.

The 2nd Planck volume

$$\ell_{Pi}^3 = \mp\left(\frac{\hbar G}{c_n^3}\right)^{3/2} \approx \mp i3.8127\times 10^{-105}\ [\mathrm{m}^3]\ \left(< \ell_P^3\right), \tag{27}$$

the 2nd Planck momentum

$$m_{Pi}c_n = \pm\sqrt{\frac{\hbar c_n^3}{G}} \approx \pm i6.7504\ [\mathrm{kg\ m/s}]\,(> m_P c)\,, \tag{28}$$

the 2nd Planck energy

$$E_{Pi} = m_{Pi}c_n^2 = \pm\sqrt{\frac{\hbar c_n^5}{G}} \approx \pm i2.0701\times 10^{9}\ [\mathrm{J}]\,(> E_P)\,, \tag{29}$$

and the 2nd Planck acceleration

$$a_{Pi} = \frac{c_n}{t_{Pi}} = \pm\sqrt{\frac{c_n^7}{\hbar G}} \approx \pm i6.0198\times 10^{51}\ [\mathrm{m/s^2}]\,(> a_P)\,, \tag{30}$$

are imaginary and bivalued.

However, the 2nd Planck force

$$F_{Pi} = \pm\frac{E_{Pi}}{\ell_{Pi}} \approx \pm 1.3251\times 10^{44}\ [\mathrm{N}]\,(> F_P)\,, \tag{31}$$

and the 2nd Planck density

$$\rho_{Pi} = \pm\frac{m_{Pi}}{\ell_{Pi}^3} \approx \pm 5.7735\times 10^{96}\ [\mathrm{kg/m^3}]\,(> \rho_P)\,, \tag{32}$$

are real and bivalued, whereas the 2nd Planck density is only negative, if defined by $c_n^5/(\hbar G^2)$, while the 2nd Planck force is only positive, if defined by c_n^4/G.

The 2nd Planck area

$$\ell_{Pi}^2 = \frac{\hbar G}{c_n^3} \approx -2.4406\times 10^{-70}\ [\mathrm{m}^2]\left(< \ell_P^2\right), \tag{33}$$

is negative. According to the holographic principle, each bit of information is physically represented on the holographic boundary by the Planck area [11–13] (having a positive value), whereas these areas are triangular [14][2]. Notably, out of all omnidimensional (i.e. present in all complex dimensions n) convex n-polytopes and n-balls, only n-simplices (i.e., triangles for $n = 2$) are bivalued, admitting both positive and negative volumes and surfaces [15].

V. ALGEBRAIC FRESNEL COEFFICIENTS FOR THE NORMAL INCIDENCE OF EMR ON MLG

With algebraic definitions of α (12) and α_2 (13) transmittance T (2), reflectance R (3) and absorptance A (4) of MLG for normal EMR incidence can be expressed just by π.

For $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (12) they become

$$T(\alpha) = \frac{4\left(4\pi^2+\pi+1\right)^2}{\left(8\pi^2+2\pi+3\right)^2} \approx 97.746\%, \tag{34}$$

$$A(\alpha) = \frac{4\left(4\pi^2+\pi+1\right)}{\left(8\pi^2+2\pi+3\right)^2} \approx 2.241\%, \tag{35}$$

while for $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi$ (13) they become

$$T(\alpha_2) = \frac{4\left(4\pi^2+\pi+2\right)^2}{\left(8\pi^2+2\pi+3\right)^2} \approx 102.279\%, \tag{36}$$

$$A(\alpha_2) = -\frac{4\left(4\pi^2+\pi+2\right)}{\left(8\pi^2+2\pi+3\right)^2} \approx -2.292\%, \tag{37}$$

with

$$R(\alpha) = R(\alpha_2) = \frac{1}{\left(8\pi^2+2\pi+3\right)^2} \approx 0.013\%. \tag{38}$$

Obviously $(T(\alpha)+A(\alpha))+R(\alpha) = (T(\alpha_2)+A(\alpha_2))+R(\alpha_2) = 1$ as required by the law of conservation of energy (7), whereas each conservation law is associated with a certain symmetry, as asserted by Noether's theorem. Nonetheless, physical interpretation of $T(\alpha_2) > 1$ and $A(\alpha_2) < 0$ requires further research. $A(\alpha) > 0$ implies a *sink*, whereas $A(\alpha_2) < 0$ implies a

[2] This is also compatible with the Causal Dynamical Triangulation (CDT) approach, which does not assume a pre-existence of a dimensional space, but focuses on the evolution of the spacetime as such.

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source, whereas the opposite holds true for the transmittance T, as illustrated schematically in Fig 1. Perhaps, the negative absorptance and transmittance exceeding 100% for α_2 (10) or (13) could be explained in terms of graphene spontaneous emission.

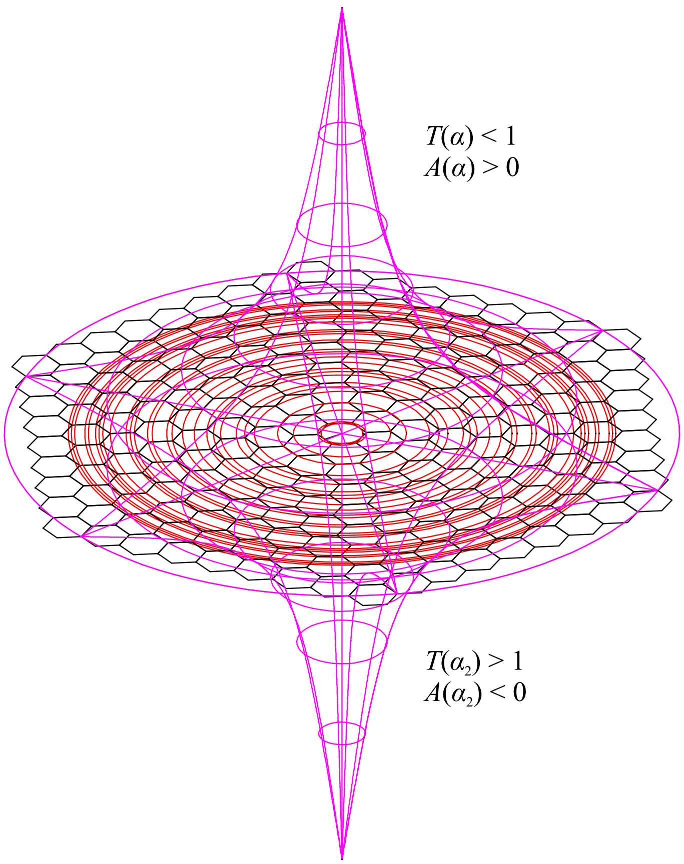

Figure 1. Illustration of the concepts of negative absorptance and excessive transmittance of EMR under normal incidence on MLG.

VI. DISCUSSION

We have shown that the reflectance of graphene under the normal incidence of electromagnetic radiation (EMR), expressed as the quadratic equation with respect to the fine structure constant α must introduce the 2^{nd} negative fine structure constant α_2.

It is shown that the sum of the reciprocal of this 2^{nd} fine structure constant α_2 with the reciprocal of the *physical* fine structure constant α (1) is independent of the reflectance value R and remarkably equals simply $-\pi$.

Particular algebraic definition of the *physical* fine structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (12), containing the free π term, when introduced to this sum, yields $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi < 0$.

Assuming universal validity of the physical definition of the fine structure constant α (1), the 2^{nd} fine structure constant α_2 (13) defines the negative speed of light c_n (17), the average of which and the speed of light in vacuum is in the range of the Fermi velocity (10^6 m/s).

The negative fine structure constant α_2 introduces the imaginary set of five base Planck units (20)-(24), whereas mass and time units (25), as well as temperature and time units (26) are related to base Planck units.

The Planck volume (27), momentum (28), energy (29), and acceleration (30) derived from imaginary base units (21)-(24) are imaginary and bivalued, whereas the Planck force (31) and density (32) are real and bivalued.

The square of the imaginary Planck length (21) introduces the negative Planck area (33). If Planck areas correspond to bits of information [11–13] and if they are triangular [14], then the fact that among square, disk, and triangle, only the latter admits negative surface [15], suggests a physical interpretation for this negative Planck area (33).

The findings of this study require further research. Particularly in the context of emergent dimensionality [14–17].

This paper is a cleanup of the research presented in [18] and [19].

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Appendix: Other Quadratic Equations

The quadratic equation for the sum of transmittance (2) and absorptance (4), putting $C_{TA} \doteq T + A$, is

$$\frac{1}{4} C_{TA} \pi^2 \alpha^2 + (C_{TA} - 1)\pi\alpha + (C_{TA} - 1) = 0, \tag{A.1}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} + \sqrt{1 - C_{TA}}\right)} \approx 137.036, \tag{A.2}$$

and

$$\alpha_2^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} - \sqrt{1 - C_{TA}}\right)} \approx -140.178, \tag{A.3}$$

whereas their sum $\alpha^{-1} + \alpha_2^{-1} = -\pi$ is also independent of T and A.

Other quadratic equations do not feature this property. For example, the sum of $T + R$ (5) expressed as the quadratic equation and putting $C_{TR} \doteq T + R$, is

$$\frac{1}{4} (C_{TR} - 1)\pi^2\alpha^2 + C_{TR}\pi\alpha + (C_{TR} - 1) = 0, \tag{A.4}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} + 2\sqrt{2C_{TR} - 1}} \approx 137.036, \tag{A.5}$$

and

$$\alpha_{TR}^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} - 2\sqrt{2C_{TR} - 1}} \approx 0.0180, \tag{A.6}$$

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whereas their sum

$$\alpha_{TR_1}^{-1} + \alpha_{TR_2}^{-1} = \frac{-\pi C_{TR}}{C_{TR} - 1} \approx 137.054 \tag{A.7}$$

is dependent on T and R.

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