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Imaginary dimensions in physics require an imaginary set of base natural units and some negative parameter c_n corresponding to the speed of light in vacuum c . The second, negative fine-structure constant $\alpha_2^{-1} \approx -140.178$ is present in Fresnel coefficients for the normal incidence of electromagnetic radiation on monolayer graphene, leading to these imaginary natural units, and it establishes $c_n \approx -3.06 \times 10^8$ [m/s]. It follows that electric charges are the same in real and imaginary dimensions. We model neutron stars and white dwarfs, emitting perfect black-body radiation, as *objects* having energy exceeding their mass-energy equivalence ratios. We define complex energies in terms of real and imaginary natural units. Their imaginary parts, inaccessible for direct observation, store the excess of these energies. It follows that black holes are fundamentally uncharged, charged micro neutron stars and white dwarfs with masses lower than 5.7275×10^{-10} [kg] are inaccessible for direct observation, and the radii of white dwarfs' cores are limited to $R_{WD} < 3.3967 R_{BH}$, where R_{BH} is the Schwarzschild radius of a white dwarf mass. It is conjectured that the maximum atomic number $Z = 238$. A black-body *object* is in the equilibrium of complex energies of masses, charges, and photons if its radius $R_{eq} \approx 1.3833 R_{BH}$, which is marginally greater than a locally negative energy density bound of $4/3 R_{BH}$. Complex Newton's law of universal gravitation, based on complex energies, leads to the black-body object's surface gravity and the generalized Hawking radiation temperature, which includes its charge. The proposed model takes into account the value(s) of the fine-structure constant(s), which is/are otherwise neglected in general relativity, and explains the registered (GWOSC) high masses of neutron stars' mergers and the associated fast radio bursts (CHIME) without resorting to any hypothetical types of exotic stellar *objects*.

Keywords: emergent dimensionality; imaginary dimensions; natural units; fine-structure constant; black holes; neutron stars; white dwarfs; patternless binary messages; complex energy; complex force; Hawking radiation; extended periodic table; general relativity; entropic gravity; gravitational observations; holographic principle; mathematical physics;

I. INTRODUCTION

The universe began with the Big Bang, which is a current prevailing scientific opinion. But this Big Bang was not an explosion of 4-dimensional spacetime, which also is a current prevailing scientific opinion, but an explosion of dimensions. More precisely, in the -1 -dimensional void, a 0-dimensional point appeared, inducing the appearance of countably infinitely other points indistinguishable from the first one. The breach made by the first operation of the *dimensional successor function* of the Peano axioms inevitably continued leading to the formation of 1-dimensional, real and imaginary lines allowing for an ordering of points using multipliers of real units (ones) or imaginary units ($a \in \mathbb{R} \Leftrightarrow a = 1b^1, a \in \mathbb{I} \Leftrightarrow a = ib, b \in \mathbb{R}$). Then out of two lines of each kind, crossing each other only at one initial point (0, 0), the dimensional successor function formed 2-dimensional $\mathbb{R}^2, \mathbb{I}^2$, and $\mathbb{R} \times \mathbb{I}$ Euclidean planes, with $\mathbb{I}^2$ being a mirror reflection of $\mathbb{R}^2$. And so on, forming n -dimensional Euclidean spaces $\mathbb{R}^a \times \mathbb{I}^b$ with $a \in \mathbb{N}$ real and $b \in \mathbb{N}$ imaginary lines, $n := a + b$, and the scalar product defined by

$$\begin{aligned} \mathbf{x} \cdot \mathbf{y} &= (x_1, \dots, x_a, ix'_1, \dots, ix'_b)(y_1, \dots, y_a, iy'_1, \dots, iy'_b) := \\ &:= \sum_{k=1}^a x_k y_k + \sum_{l=1}^b x'_l \overline{y'_l}, \end{aligned} \quad (1)$$

where $\mathbf{x}, \mathbf{y} \in \mathbb{R}^a \times \mathbb{I}^b$. With the onset of the first 0-dimensional point, information began to evolve [1–6].

However, dimensional properties are not uniform. Concerning regular convex n -polytopes in natural dimensions, for example, there are countably infinitely many regular convex polygons, five regular convex polyhedra (Platonic solids), six regular convex 4-polytopes, and only three regular convex n -polytopes if $n > 3$ [7]. In particular, 4-dimensional Euclidean space is endowed with a peculiar property known as exotic $\mathbb{R}^4$ [8], absent in other dimensionalities. Thanks to this property, $\mathbb{R}^3 \times \mathbb{I}$ space provides a continuum of homeomorphic but non-diffeomorphic differentiable structures. Each piece of individually memorized information is homeomorphic to the corresponding piece of individually perceived information but remains non-diffeomorphic (non-smooth). This allowed for variation of phenotypic traits within populations of individuals [9] and extended the evolution of information into biological evolution. Exotic $\mathbb{R}^4$ solves the problem of extra dimensions of nature. Each biological cell perceives emergent space of three real and one imaginary (time) dimension observer-dependently [10] and at present when $i0 = 0$ is *real*, and perceived space requires a natural number of dimensions [11]. This is the emergent dimensionality (ED) [5, 9, 12–14].

Each dimension requires certain units of measure. In real dimensions, the *natural units of measure* were derived by Max Planck in 1899 as "independent of special *bodies* or *substances*, thereby necessarily retaining their meaning for all times and for all civilizations, including extraterrestrial and non-human ones" [15]. Planck units utilize the Planck constant h that he introduced in his black-body radiation formula. However, already in 1881, George Stoney derived a system of natural units [16] based on the elementary charge e (Planck's constant was unknown at that time). The ratio of Stoney units to Planck units is $\sqrt{\alpha}$, where α is the fine-

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¹ This is, of course, a circular definition. But it is given for clarity.

structure constant. This study derives the complementary set of natural units applicable for imaginary dimensions, including the imaginary base units, based on the discovered negative fine-structure constant α_2 . Similarly, the ratio of α_2 -Stoney units to α_2 -Planck units is $\sqrt{\alpha_2}$. As the speed of electromagnetic radiation is the product of its wavelength and frequency and both these quantities are imaginary in imaginary dimensions, some real but negative parameter $c_n = v_i \lambda_i$ corresponding to the speed of light in vacuum c (i.e., the natural speed) is also necessary as $i^2 = -1$. It turns out that the imaginary Planck energy E_{Pi} and temperature T_{Pi} are larger in moduli than the Planck energy E_P and temperature T_P setting more favorable conditions for biological evolution to emerge in $\mathbb{R}^3 \times \mathbb{I}$ Euclidean space than in $\mathbb{I}^3 \times \mathbb{R}$ Euclidean one due to the minimum energy principle.

The study shows that the energies of neutron stars and white dwarfs exceed their mass-energy equivalences and that excess energy is stored in imaginary dimensions and is inaccessible to direct observations. This corrects the value of the photon sphere radius and results in the upper bound on the size-to-mass ratio of their cores, where the Schwarzschild radius sets the lower bound.

The paper is structured as follows. Section II shows that Fresnel coefficients for the normal incidence of electromagnetic radiation on monolayer graphene include the second, negative fine-structure constant α_2 as a fundamental constant of nature. Section III shows that by this second fine-structure constant nature endows us with the complementary set of α_2 -natural units. Section IV introduces the concept of a black-body *object* in thermodynamic equilibrium, emitting perfect black-body radiation, and reviews its necessary properties. Section V introduces complex energies of masses, charges, and photons expressed in terms of real and imaginary Planck units introduced in Section III and discusses equilibria formed by comparing their moduli. Section VI applies these equilibria to black-body *objects* to derive the range of their size-to-mass ratios and the equilibrium size-to-mass ratio. Section VII applies this range to the observed mergers of black-body *objects* to show that the observed data is explainable with no need to introduce hypothetical exotic stellar *objects*. Section VIII define complex forces to derive a black-body *object* surface gravity, and the generalized Hawking radiation temperature. Section IX summarizes the findings of this study. Certain prospects for further research are given in the appendices.

II. THE SECOND FINE-STRUCTURE CONSTANT

Numerous publications provide Fresnel coefficients for the normal incidence of electromagnetic radiation (EMR) on monolayer graphene (MLG), which are remarkably defined only by π and the fine-structure constant α

$$\alpha^{-1} = \left(\frac{q_P}{e}\right)^2 = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036, \quad (2)$$

where q_P is the Planck charge, ϵ_0 is vacuum permittivity (the electric constant), $\hbar$ is the reduced Planck constant, and e is

the elementary charge. Transmittance (T) of MLG

$$T = \frac{1}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 0.9775, \quad (3)$$

for normal EMR incidence was derived from the Fresnel equation in the thin-film limit [17] (Eq. 3), whereas spectrally flat absorptance (A) $A \approx \pi\alpha \approx 2.3\%$ was reported [18, 19] for photon energies between about 0.5 and 2.5 [eV]. T was related to reflectance (R) [20] (Eq. 53) as $R = \pi^2\alpha^2 T/4$, i.e.,

$$R = \frac{\frac{1}{4}\pi^2\alpha^2}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 1.2843 \times 10^{-4}, \quad (4)$$

The above equations for T and R, as well as the equation for the absorptance

$$A = \frac{\pi\alpha}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 0.0224, \quad (5)$$

were also derived [21] (Eqs. 29-31) based on the thin film model (setting $n_s = 1$ for substrate). The sum of transmittance (3) and the reflectance (4) at normal EMR incidence on MLG was derived [22] (Eq. 4a) as

$$\begin{aligned} T + R &= 1 - \frac{4\sigma\eta}{4 + 4\sigma\eta + \sigma^2\eta^2 + k^2\chi^2} = \\ &= \frac{1 + \frac{1}{4}\pi^2\alpha^2}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 0.9776, \end{aligned} \quad (6)$$

where η is the vacuum impedance

$$\eta = \frac{4\pi\alpha\hbar}{e^2} = \frac{1}{\epsilon_0 c} \approx 376.73 \text{ } [\Omega], \quad (7)$$

$\sigma = e^2/(4\hbar) = \pi\alpha/\eta$ is the MLG conductivity [23], and $\chi = 0$ is the electric susceptibility of vacuum. These coefficients are thus well-established theoretically and experimentally confirmed [17–19, 22, 24, 25].

As a consequence of the conservation of energy

$$(T + A) + R = 1. \quad (8)$$

In other words, the transmittance in the Fresnel equation describing the reflection and transmission of EMR at normal incidence on a boundary between different optical media is, in the case of the 2-dimensional (boundary) of MLG, modified to include its absorption.

The reflectance $R = 0.013\%$ (4) of MLG can be expressed as a quadratic equation with respect to α

$$\frac{1}{4}(R - 1)\pi^2\alpha^2 + R\pi\alpha + R = 0. \quad (9)$$

This quadratic equation has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx 137.036, \quad \text{and} \quad (10)$$

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx -140.178. \quad (11)$$

Therefore, the equation (9) includes the second, negative fine-structure constant α_2 . It happens that the sum of the reciprocals of these fine-structure constants (10) and (11)

$$\alpha^{-1} + \alpha_2^{-1} = \frac{\pi - \pi\sqrt{R} - \pi - \pi\sqrt{R}}{2\sqrt{R}} = \frac{-2\pi}{2} = -\pi, \quad (12)$$

is remarkably independent of the value of the reflectance R . The same result can only be obtained for $T + A$ (cf. Appendix A). This result is intriguing in the context of a peculiar algebraic expression for the fine-structure constant [26]

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036303776 \quad (13)$$

that contains a *free* π term and is very close to the physical definition (2) of α^{-1} , which according to the CODATA 2018 value is 137.035999084. Notably, the value of the fine-structure constant is not *constant* but increases with time [27–31]. Thus, the algebraic value given by (13) can be interpreted as the asymptote of the α increase.

Using relations (12) and (13), we can express the negative reciprocal of the 2nd fine-structure constant α_2^{-1} that emerged in the quadratic equation (9) also as a function of π only

$$\alpha_2^{-1} = -\pi - \alpha_1^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.177896429, \quad (14)$$

and this value can also be interpreted as the asymptote of the α_2 decrease, where the current value would amount to $\alpha_2^{-1} \approx -140.177591737$, assuming the rate of change is the same for α and α_2 .

Using relations (13) and (14), T (3), R (4), and A (5) of MLG for normal EMR incidence can be expressed just by π . Moreover, equation (9) includes two π -like constants for two surfaces with positive and negative Gaussian curvatures (cf. Appendix B).

III. SET OF α_2 -PLANCK UNITS

For further analysis, we have to use Planck units over the Stoney units as only the former incorporate α (and h). Planck units can be derived from numerous starting points [5, 32] (cf. Appendices C and D). The definition of the Planck charge $q_P = \sqrt{4\pi\epsilon_0\hbar c}$ can be solved for the speed of light yielding $c = q_P^2/(4\pi\epsilon_0\hbar)$. Furthermore, the ratio of charges in the definition of the fine-structure constant $\alpha = e^2/q_P^2$ (2) applied for the negative α_2 , requires an introduction of some imaginary Planck charge q_{Pi} so that its square would yield a negative value of α_2

$$\frac{q_{Pi}^2}{e^2} = \alpha_2^{-1} \approx -140.178, \quad (15)$$

and since the elementary charge e is real

$$q_{Pi} = \pm \sqrt{\frac{e^2}{\alpha_2}} = \pm \sqrt{4\pi\epsilon_0\hbar c_n}. \quad (16)$$

Among the physical constants of the $\sqrt{4\pi\epsilon_0\hbar c_n}$ term, almost all are positive². Only the $c_n = v_i\lambda_i$ parameter, corresponding to the speed of light c , is negative as both frequency ν_i and wavelength λ_i are imaginary in imaginary dimensions. Therefore, the equation (16) can be solved for c_n yielding

$$c_n = q_{Pi}^2/(4\pi\epsilon_0\hbar) \approx -3.066653 \times 10^8 \text{ [m/s]}, \quad (17)$$

which is greater than the speed of light in vacuum c in modulus³. We also note that c is defined by the electric constant ϵ_0 and the magnetic constant μ_0 as $c = 1/\sqrt{\epsilon_0\mu_0}$; a square root is bivalued and the value of μ_0 depends on α . Furthermore, c is defined by α -dependent vacuum impedance (7).

The negative parameter c_n (17) leads to the imaginary Planck charge q_{Pi} , length ℓ_{Pi} , mass m_{Pi} , time t_{Pi} , and temperature T_{Pi} that redefined by square roots containing c_n raised to odd (1, 3, 5) powers become imaginary and bivalued⁴

$$\begin{aligned} q_{Pi} &= \pm \sqrt{4\pi\epsilon_0\hbar c_n} = \pm q_P \sqrt{\frac{\alpha}{\alpha_2}} \approx \\ &\approx \pm i1.8969 \times 10^{-18} \text{ [C]} \quad (|q_{Pi}| > |q_P|), \end{aligned} \quad (18)$$

$$\begin{aligned} \ell_{Pi} &= \pm \sqrt{\frac{\hbar G}{c_n^3}} = \pm \ell_P \sqrt{\frac{\alpha_2^3}{\alpha^3}} \approx \\ &\approx \pm i1.5622 \times 10^{-35} \text{ [m]} \quad (|\ell_{Pi}| < |\ell_P|), \end{aligned} \quad (19)$$

$$\begin{aligned} m_{Pi} &= \pm \sqrt{\frac{\hbar c_n}{G}} = \pm m_P \sqrt{\frac{\alpha}{\alpha_2}} \approx \\ &\approx \pm i2.2012 \times 10^{-8} \text{ [kg]} \quad (|m_{Pi}| > |m_P|), \end{aligned} \quad (20)$$

$$\begin{aligned} t_{Pi} &= \pm \sqrt{\frac{\hbar G}{c_n^5}} = \pm t_P \sqrt{\frac{\alpha_2^5}{\alpha^5}} \approx \\ &\approx \pm i5.0942 \times 10^{-44} \text{ [s]} \quad (|t_{Pi}| < |t_P|), \end{aligned} \quad (21)$$

$$\begin{aligned} T_{Pi} &= \pm \sqrt{\frac{\hbar c_n^5}{G k_B^2}} = \pm T_P \sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \\ &\approx \pm i1.4994 \times 10^{32} \text{ [K]} \quad (|T_{Pi}| > |T_P|), \end{aligned} \quad (22)$$

and furthermore can be expressed, using the relation (31), in terms of base Planck units q_P , ℓ_P , m_P , t_P , and T_P .

² Vacuum permittivity ϵ_0 is the value of the absolute dielectric permittivity of classical vacuum. The Planck constant h is the uncertainty principle parameter and also the relation (55) shows that $\hbar$ is common for αc and $\alpha_2 c_n$. Thus, ϵ_0 and h cannot have negative counterparts.

³ Their average $(c + c_n)/2 \approx -3.436417 \times 10^6 \text{ [m/s]}$ is in the range of the Fermi velocity.

⁴ Contrary to the Stoney units containing c raised to even (4, 6) powers.

Planck units derived from the imaginary base units (19)-(21) are generally not imaginary. The α_2 Planck volume

$$\begin{aligned}\ell_{Pi}^3 &= \pm \left(\frac{\hbar G}{c_n^3} \right)^{3/2} = \pm \ell_P^3 \sqrt{\frac{\alpha_2^9}{\alpha^9}} \approx \\ &\approx \pm i3.8127 \times 10^{-105} [\text{m}^3] \quad (|\ell_{Pi}^3| < |\ell_P^3|),\end{aligned}\quad (23)$$

the α_2 Planck momentum

$$\begin{aligned}p_{Pi} &= \pm m_{Pi} c_n = \pm \sqrt{\frac{\hbar c_n^3}{G}} = \pm m_P c \sqrt{\frac{\alpha^3}{\alpha_2^3}} \approx \\ &\approx \pm i6.7504 [\text{kg m/s}] \quad (|m_{Pi} c_n| > |m_P c|),\end{aligned}\quad (24)$$

the α_2 Planck energy

$$\begin{aligned}E_{Pi} &= \pm m_{Pi} c_n^2 = \pm \sqrt{\frac{\hbar c_n^5}{G}} = \pm E_P \sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \\ &\approx \pm i2.0701 \times 10^9 [\text{J}] \quad (|E_{Pi}| > |E_P|),\end{aligned}\quad (25)$$

and the α_2 Planck acceleration

$$\begin{aligned}a_{Pi} &= \pm \frac{c_n}{t_{Pi}} = \pm \sqrt{\frac{c_n^7}{\hbar G}} = \pm a_P \sqrt{\frac{\alpha^7}{\alpha_2^7}} \approx \\ &\approx \pm i6.0198 \times 10^{51} [\text{m/s}^2] \quad (|a_{Pi}| > |a_P|),\end{aligned}\quad (26)$$

are imaginary and bivalued. However, the α_2 Planck force

$$\begin{aligned}F_{P2} &= \pm \frac{E_{Pi}}{\ell_{Pi}} = \pm \frac{c_n^4}{G} = \pm F_P \frac{\alpha^4}{\alpha_2^4} \approx \\ &\approx \pm 1.3251 \times 10^{44} [\text{N}] \quad (|F_{P2}| > |F_P|),\end{aligned}\quad (27)$$

and the α_2 Planck density

$$\begin{aligned}\rho_{P2} &= \pm \frac{m_{Pi}}{\ell_{Pi}^3} = \pm \frac{c_n^5}{\hbar G^2} = \pm \rho_P \frac{\alpha^5}{\alpha_2^5} \approx \\ &\approx \pm 5.7735 \times 10^{96} [\text{kg/m}^3] \quad (|\rho_{P2}| > |\rho_P|),\end{aligned}\quad (28)$$

are real and bivalued. On the other hand, the α_2 Planck area

$$\begin{aligned}\ell_{Pi}^2 &= \frac{\hbar G}{c_n^3} = \ell_P^2 \frac{\alpha_2^3}{\alpha^3} \approx \\ &\approx -2.4406 \times 10^{-70} [\text{m}^2] \quad (|\ell_{Pi}^2| < |\ell_P^2|),\end{aligned}\quad (29)$$

is strictly negative, while the Planck area ℓ_P^2 is strictly positive. In the following, we shall call the units (18)-(29) α_2 -Planck units.

Both α_2 and c_n lead to the second, negative vacuum impedance

$$\eta_2 = \frac{4\pi\alpha_2\hbar}{e^2} = \frac{1}{\epsilon_0 c_n} \approx -368.29 [\Omega] \quad (|\eta_2| < |\eta|). \quad (30)$$

Solving both impedances (7) and (30) for $4\pi\hbar\epsilon_0/e^2$ and comparing with each other yields the following important relation

between the speed of light in vacuum c , negative parameter c_n , and the fine-structure constants α , α_2

$$c\alpha = c_n\alpha_2 \quad (=v_e), \quad (31)$$

where, notably, v_e is the electron's velocity at the first circular orbit in the Bohr model of the hydrogen atom. This is not the only α to α_2 relation. Along with the two π -like constants π_1 , π_2 (relations (B8) and (B10), cf. Appendix B)

$$\frac{\alpha_2}{\alpha} = \frac{c}{c_n} = \frac{\pi_1}{\pi} = \frac{\pi}{\pi_2} \approx -0.9776. \quad (32)$$

The relations between time (21) and temperature (22) α_2 -Planck units are inverted, $\alpha^5 t_{Pi}^2 = \alpha_2^5 t_P^2$, $\alpha_2^5 T_{Pi}^2 = \alpha^5 T_P^2$, and saturate Heisenberg's uncertainty principle (energy-time version) taking energy from the equipartition theorem for one degree of freedom (or one bit of information [5, 33])

$$\frac{1}{2}k_B T_{Pi} t_P = \frac{1}{2}k_B T_P t_{Pi} = \frac{\hbar}{2}. \quad (33)$$

Furthermore, eliminating α and α_2 from the relations (18)-(20), yield

$$\frac{q_P^2}{m_P^2} = \frac{q_{Pi}^2}{m_{Pi}^2} = 4\pi\epsilon_0 G, \quad (34)$$

and

$$\ell_P m_P^3 = \ell_{Pi} m_{Pi}^3 \quad \text{and} \quad \ell_P q_P^3 = \ell_{Pi} q_{Pi}^3. \quad (35)$$

Base Planck and Stoney units themselves admit negative values as negative square roots. By choosing complex analysis, within the framework of ED, we enter into bivalence by the very nature of this analysis. All geometric *objects* have both positive and negative volumes and surfaces [14] equal in moduli. On the other hand, imaginary and negative physical quantities are the subject of research. In particular, the subject of scientific research is thermodynamics in the complex plane. Lee-Yang zeros, for example, have been experimentally observed [34, 35]. We note here that the imaginary Planck Units are not imaginary due to being multiplied by the imaginary unit i . They are imaginary due to the negativity of odd powers of c_n being the square root argument; thus, they define imaginary physical quantities inaccessible to direct measurements⁵. They do not apply only to the time dimension but to any imaginary dimension. However, in our four-dimensional Euclidean $\mathbb{R}^3 \times \mathbb{I}$ space-time, Planck units apply in general to the spatial dimensions, while the imaginary ones in general to the imaginary temporal dimension. All the α_2 -Planck units have physical meanings. However, some are elusive, like the negative area or imaginary volume, which require two or three orthogonal imaginary dimensions.

⁵ Quantum measurement outcomes are *real* eigenvalues of hermitian operators.

Planck charge relations (2), (16), and the charge conservation principle imply that the elementary charge e is the same both in real and imaginary dimensions since

$$e^2 = \alpha q_P^2 = \alpha_2 q_{Pi}^2 = q_S^2, \quad (36)$$

where q_S is the Stoney charge. There is no physically meaningful *elementary mass* $M_e = \pm 1.8592 \times 10^{-9}$ [kg] that would satisfy the relation (20)

$$M_e^2 = \alpha m_P^2 = \alpha_2 m_{Pi}^2. \quad (37)$$

Neither is there a physically meaningful *elementary* (and imaginary) *length* $L_e \approx \pm i 9.7382 \times 10^{-39}$ [m] satisfying the relation (29)

$$L_e^2 = \alpha^3 \ell_P^2 = \alpha_2^3 \ell_{Pi}^2, \quad (38)$$

(which in modulus is almost 1660 times smaller than the Planck length), or an *elementary temperature* $T_e \approx \pm 6.4450 \times 10^{26}$ [K] abiding to (22)

$$T_e^2 = \alpha^5 T_P^2 = \alpha_2^5 T_{Pi}^2, \quad (39)$$

and close to the Hagedorn temperature of grand unified string models.

Thus, as to the modulus, charges are the same in real and imaginary dimensions, while masses, lengths, temperatures, and other derived quantities that can vary with time, may differ (the dimensional character of the charges is additionally emphasized by the real $\sqrt{\alpha}$ multiplied by i in the imaginary charge energy (56) and imaginary $\sqrt{\alpha_2}$ in the real charge energy (57)). We note that the same form of the relations (36) and (37) reflect the same form of Coulomb's law and Newton's law of gravity, which are inverse-square laws.

In the following, where deemed appropriate, we shall express the physical quantities by Planck units

$$\begin{aligned} M &:= mm_P, & M_i &:= m_i m_{Pi}, & m, m_i &\in \mathbb{R}, \\ Q &:= qe, & Q_i &:= iQ = iqe, & q &\in \mathbb{Z} \\ \lambda &:= l\ell_P, & \lambda_i &:= l_i \ell_{Pi}, & l, l_i &\in \mathbb{R}, \\ D &:= d\ell_P, & D_i &:= d_i \ell_{Pi}, & d, d_i &\in \mathbb{R}, \\ R &:= r\ell_P, & R_i &:= r_i \ell_{Pi} & r, r_i &\in \mathbb{R} \end{aligned} \quad (40)$$

where uppercase letters M , Q , λ , D , and R denote respectively masses, charges, wavelengths, diameters, and radii (or lengths), lowercase letters (with few exceptions) denote Planck units or their multipliers, and the subscripts i refer to imaginary quantities. We note that the discretization of charges by integer multipliers q of the elementary charge e is far-fetched, considering the fractional charges of *quasiparticles*.

IV. BLACK BODY OBJECTS

There are only three observable *objects* in nature that emit perfect black-body radiation: unsupported black holes (BHs, the densest), neutron stars (NSs), supported, as it is accepted,

by neutron degeneracy pressure, and white dwarfs (WDs), supported, as it is accepted, by electron degeneracy pressure (the least dense). We shall collectively call them black-body *objects* (BBOs). This term is not used in standard cosmology, but standard cosmology scrunches under embarrassingly significant failings, not just *tensions* as is sometimes described, as if to somehow imply that a resolution will eventually be found [36]. Entropic gravity [33] explains galaxy rotation curves without resorting to dark matter, has been experimentally confirmed [37], and is decoherence-free [38]. It has recently been experimentally confirmed that the so-called *accretion instability* is a fundamental physical process [39]. We conjecture that this process is common for all BBOs. Also James Webb Space Telescope data show multiple galaxies that grew too massive too soon after the Big Bang, which is a strong discrepancy with the Λ cold dark matter model (Λ CDM) expectations on how galaxies formed at early times at both redshifts, even when considering observational uncertainties [40]. This is an important unresolved issue indicating that fundamental changes to the reigning Λ CDM model of cosmology is needed [40].

Therefore, the term *object* as a collection of *matter* is a misnomer, as it neglects quantum nonlocality [41] that is independent of the entanglement among the *particles* [42]. Thus we use emphasis for (indistinguishable) *particle* and (distinguishable) *object*, as well as for *matter* and *distance*. These terms have no absolute meaning in ED. In particular, given the recent observation of *quasiparticles* in classical systems [43]. Within the framework of ED no *object* is *enclosed* in *space*. Also, the ugly duckling theorem [44, 45] asserts that every two *objects* we perceive are equally similar (or equally dissimilar), however ridiculous that may sound.

As black-body radiation is radiation of global thermodynamic equilibrium, it is patternless (thermal noise) radiation that depends only on one parameter. In the case of BHs, this is known as Hawking radiation and this parameter is the BH temperature $T_{BH} = T_P / (2\pi d_{BH})$ corresponding to the BH diameter [5] $D_{BH} = d_{BH} \ell_P$, where $d_{BH} \in \mathbb{R}$. As black-body radiation is patternless, the triangulated [5] BBOs contain a balanced number of Planck area triangles, each carrying binary potential $\delta\varphi_k = -c^2 \cdot \{0, 1\}$, as it has been shown for BHs⁶ [5], based on Bekenstein-Hawking (BH) entropy $S_{BH} = k_B N_{BH}/4$. BH entropy can be derived from the Bekenstein bound

$$S \leq \frac{2\pi k_B R E}{\hbar c}, \quad (41)$$

which defines an upper limit on the thermodynamic entropy S that can be contained within a sphere of radius R having energy E . After plugging the BH (Schwarzschild) radius $R_{BH} = 2GM_{BH}/c^2$ and mass-energy equivalence $E_{BH} = M_{BH}c^2$, where M_{BH} is the BH mass, into the bound (41), it reduces BH entropy. In other words, BH entropy saturates the Bekenstein bound (41).

⁶ Perhaps the only meaningful *spatial* concept is the Planck area triangle, encoding one bit of classical information and its curvature.

The patternless nature of the perfect black-body radiation was derived [5] by comparing BH entropy with the binary entropy variation $\delta S = k_B N_1/2$ ([5] Eq. (55)), valid for any entropy variation sphere (EVS), where $N_1 \in \mathbb{N}$ denotes the number of active Planck triangles bearing binary potential $\delta\varphi_k = -c^2$. Thus, the entropy of all BBOs is

$$S_{\text{BBO}} = \frac{1}{4} k_B N_{\text{BBO}}, \quad (42)$$

where $N_{\text{BBO}} := 4\pi R_{\text{BBO}}^2 / \ell_P^2 = \pi d_{\text{BBO}}^2$ is the information capacity of the BBO surface, i.e., the $\lfloor N_{\text{BBO}} \rfloor \in \mathbb{N}$ Planck triangles (where " $\lfloor x \rfloor$ " is the floor function that yields the greatest integer less than or equal to its argument x) corresponding to bits of information [33, 46, 47], and the fractional part triangle(s) having the area $\{N_{\text{BBO}}\} \ell_P^2 = (N_{\text{BBO}} - \lfloor N_{\text{BBO}} \rfloor) \ell_P^2$ so small to carry a single bit of information. Furthermore, $N_1 = N_{\text{BBO}}/2$ confirms the patternless thermodynamic equilibrium of the BBOs by maximizing Shannon entropy [5].

We shall define the generalized radius of a BBO (this definition applies to all EVSs) having mass M_{BBO} as a function of GM_{BBO}/c^2 multiplier $k \in \mathbb{R}$ or the BH radius R_{BH} multiplier $\hat{k} = k/2$

$$R_{\text{BBO}} := k \frac{GM_{\text{BBO}}}{c^2} = \hat{k} R_{\text{BH}} \Rightarrow r = km, \quad r_i = km_i, \quad (43)$$

and the generalized BBO energy E_{BBO} as a function of $M_{\text{BBO}} c^2$ multiplier $a \in \mathbb{R}$ (this definition also applies to all EVSs)

$$E_{\text{BBO}} := a M_{\text{BBO}} c^2. \quad (44)$$

Plugging definitions (43) and (44) into the Bekenstein bound (41) it becomes

$$S \leq \frac{1}{2} k_B \frac{a}{k} N_{\text{BBO}}, \quad (45)$$

and equals the BBO entropy (42) if $\frac{a}{2k} = \frac{1}{4} \Rightarrow a = \frac{k}{2}$. Thus, the energy of all BBOs having a radius (43) is

$$E_{\text{BBO}} = \frac{k}{2} M_{\text{BBO}} c^2 = \hat{k} M_{\text{BBO}} c^2, \quad (46)$$

with $k = 2$ in the case of BHs and $k > 2$ setting the lower bound for other BBOs. We shall further call the coefficients $k, \hat{k}$ the *size-to-mass ratios* (STM). The upper bound shall be given in Section VI.

According to the no-hair theorem, all BHs general relativity (GR) solutions are characterized only by three parameters: mass, electric charge, and angular momentum. However, BHs are fundamentally uncharged since the parameters of any conceivable BH, in particular, charged (Reissner–Nordström) and charged-rotating (Kerr–Newman) BH, can be altered arbitrarily, provided that the BH area does not decrease [48] by means of Penrose processes [49, 50] to extract BH electrostatic and/or rotational energy [51]. Thus any BH is defined by only one real parameter: its diameter (cf. [5] Fig. 2(b)), mass, temperature, energy, etc., each corresponding to the other. We note that in the complex Euclidean $\mathbb{R}^3 \times \mathbb{I}$ space, an n -ball

($n \in \mathbb{C}$) is spherical only for a vanishing imaginary dimension [14]. As the interiors of the BBOs are inaccessible to an exterior observer [46], BBOs do not have interiors⁷, which makes them similar to interior-less mathematical points. Yet, a BH can embrace this defining parameter. That means that three points forming a Planck triangle corresponding to a bit of information on a BH surface can store this parameter and this is intuitively comprehensible: the area of a spherical triangle is larger than that of a flat triangle defined by the same vertices, providing the curvature is nonvanishing, and depends on this curvature, i.e., this additional parameter defines it.

On the other hand, it is accepted that in the case of NSs, electrons combine with protons to form neutrons so that NSs are composed almost entirely of neutrons. But it is never the case that all electrons and all protons of an NS become neutrons. WDs are charged by definition as they are accepted to be composed mostly of electron-degenerate *matter*. But how can a charged BBO store both the curvature and an additional parameter corresponding to its charge? Fortunately, the relation (36) ensures that charges are the same in real and imaginary dimensions. Therefore each *charged* Planck triangle of a BBO surface is associated with three $\mathbb{R} \times \mathbb{I}$ Planck triangles, each sharing a vertex or two vertices with this triangle in $\mathbb{R}^2$. And this configuration is capable of storing both the curvature and the charge. The Planck triangle ℓ_P^2 and the $\mathbb{R} \times \mathbb{I}$ imaginary

Planck triangle $\ell_P \ell_{Pi} = \ell_P^2 \sqrt{\alpha_2^3 / \alpha^3}$, which has a smaller area in modulus, can be considered in a polyspherical coordinate system, in which gravitation/acceleration acts in a radial direction (with the entropic gravitation acting inwardly and acceleration acting in both radial directions) [5], while electrostatics act in a tangential direction.

Contrary to the no-hair theorem, we characterize BBOs only by mass and charge, neglecting the angular momentum, since the latter introduces the notion of time, which we find redundant in the BBO description of a patternless thermodynamical equilibrium.

Not only BBOs are perfectly spherical. Also, their mergers, to which we shall return in Section VII, are perfectly spherical, as it has been recently experimentally confirmed [52] based on the registered gravitational event GW170817. One can hardly expect a collision of two perfectly spherical, patternless thermal noises to produce some aspherical pattern instead of another perfectly spherical patternless noise. Where would the information about this pattern come from at the moment of the collision? From the point of impact? No point of impact is distinct on a patternless surface.

The hitherto considerations may be unsettling for the reader, as the energy (46) of BBOs other than BHs (i.e., for $k > 2$) exceeds mass-energy equivalence $E = Mc^2$, which is the limit of the maximum *real* energy. We note that mass-energy equivalence stems from Taylor expansion of the Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ around $v = 0$

$$Mc^2 \gamma \approx Mc^2 + \frac{Mv^2}{2} + \frac{3}{8} \frac{Mv^4}{c^2} + \frac{5}{16} \frac{Mv^6}{c^4} + \frac{35}{128} \frac{Mv^8}{c^6} + \dots, \quad (47)$$

⁷ Thus, the term *object* is a particularly staring misnomer if applied to BBOs.

where the 2nd term corresponds to the kinetic energy of mass M . Thus, the notion of time is included in countably infinite fractions of Taylor expansion (47). But Mc^2 is *timeless*. In the subsequent section, we shall show that a part of the energy of NSs and WDs, exceeding Mc^2 is imaginary and thus unmeasurable.

V. COMPLEX ENERGIES AND EQUILIBRIA

A complex energy formula

$$E_R := E_{M_R} + iE_{Q_R} = M_R c^2 + \frac{iQ_R}{2\sqrt{\pi\epsilon_0 G}} c^2, \quad (48)$$

where E_{M_R} and iE_{Q_R} represent respectively real and imaginary energy of an *object* having mass M_R and charge Q_R ⁸ was proposed in [53]. Equation (48) considers real (i.e., physically measurable) masses M_R and charges Q_R . We shall modify it to a form involving real and imaginary physical quantities, defining the following six complex energies, the complex energy of real mass and imaginary charge

$$\begin{aligned} E_{MQ_i} &:= E_M + E_{Q_i} = Mc^2 + \frac{Q_i}{2\sqrt{\pi\epsilon_0 G}} c^2 = \\ &= (mm_P + iq\sqrt{\alpha}m_P) c^2 = (m + iq\sqrt{\alpha}) E_P, \end{aligned} \quad (49)$$

of real charge and imaginary mass

$$\begin{aligned} E_{QM_i} &:= E_Q + E_{M_i} = \frac{Q}{2\sqrt{\pi\epsilon_0 G}} c_n^2 + M_i c_n^2 = \\ &= (q\sqrt{\alpha_2}m_{Pi} + m_i m_{Pi}) c_n^2 = \frac{\alpha^2}{\alpha_2^2} \left(q\sqrt{\alpha} + \sqrt{\frac{\alpha}{\alpha_2}} m_i \right) E_P, \end{aligned} \quad (50)$$

of real photon (energy or frequency ν) and imaginary mass

$$E_{FM_i} := h\nu + M_i c_n^2 = \left(f + \sqrt{\frac{\alpha^5}{\alpha_2^5}} m_i \right) E_P, \quad (51)$$

of real photon and imaginary charge

$$E_{FQ_i} := h\nu + \frac{Q_i}{2\sqrt{\pi\epsilon_0 G}} c^2 = (f + iq\sqrt{\alpha}) E_P, \quad (52)$$

of real mass and imaginary photon (with frequency $\nu_i = c_n/\lambda_i$)

$$E_{MF_i} := Mc^2 + \frac{hc_n}{\lambda_i} = \left(m + \sqrt{\frac{\alpha^5}{\alpha_2^5}} f_i \right) E_P, \quad (53)$$

and of real charge and imaginary photon

$$E_{QF_i} := \frac{Q}{2\sqrt{\pi\epsilon_0 G}} c_n^2 + h\nu_i = \frac{\alpha^2}{\alpha_2^2} \left(q\sqrt{\alpha} + \sqrt{\frac{\alpha}{\alpha_2}} f_i \right) E_P, \quad (54)$$

where $h\nu = 2\pi\hbar\frac{c}{\lambda} = \frac{2\pi}{\lambda} E_P := f E_P$, $h\nu_i := f_i E_{Pi}$, $f, f_i \in \mathbb{R}$, we used relations (20), (25), (31) (40), and

$$\alpha\hbar = \alpha_2 c_n \hbar = \frac{e^2}{4\pi\epsilon_0}. \quad (55)$$

Complex energies (49)-(54) link mass, charge, and photon energies within the framework of ED. We note in passing that using the different speed of light parameters in energies (49) and (50) yields a contradiction (cf. Appendix E).

Energies (49), (50), (52), and (54) yield two different quanta of the charge energies corresponding to the elementary charge, the imaginary quantum

$$E_{Q_i}(q = \pm 1) = \pm i\sqrt{\alpha} E_P \approx \pm i1.6710 \times 10^8 \text{ [J]}, \quad (56)$$

and the - larger in modulus - real quantum

$$E_Q(q = \pm 1) = \pm \sqrt{\alpha_2} E_{Pi} \approx \pm 1.7684 \times 10^8 \text{ [J]}. \quad (57)$$

Furthermore, $\forall q, \alpha^2 E_{Q_i} = i\alpha_2^2 E_Q$. We note that photon energy vanishes for the infinite wavelength.

The squared moduli of the energies (49)-(54) are

$$|E_{MQ_i}|^2 = (M^2 + q^2 \alpha m_P^2) c^4 = (m^2 + q^2 \alpha) E_P^2, \quad (58)$$

$$|E_{QM_i}|^2 = \frac{\alpha^4}{\alpha_2^4} (q^2 \alpha m_P^2 - M_i^2) c^4 = \frac{\alpha^4}{\alpha_2^4} \left(q^2 \alpha - \frac{\alpha}{\alpha_2} m_i^2 \right) E_P^2, \quad (59)$$

$$|E_{FM_i}|^2 = \left(f^2 - \frac{\alpha^5}{\alpha_2^5} m_i^2 \right) E_P^2, \quad (60)$$

$$|E_{MF_i}|^2 = \left(m^2 - \frac{\alpha^5}{\alpha_2^5} f_i^2 \right) E_P^2. \quad (61)$$

$$|E_{FQ_i}|^2 = (f^2 + q^2 \alpha) E_P^2, \quad (62)$$

$$|E_{QF_i}|^2 = \left(\frac{\alpha^4}{\alpha_2^4} q^2 \alpha - \frac{\alpha^5}{\alpha_2^5} f_i^2 \right) E_P^2. \quad (63)$$

Postulating that the squared moduli (58) and (59) are equal

$$\begin{aligned} |E_{MQ_i}|^2 &= |E_{QM_i}|^2, \\ \alpha_2^4 (M^2 + q^2 \alpha m_P^2) &= \alpha^4 (q^2 \alpha m_P^2 - M_i^2), \end{aligned} \quad (64)$$

we demand a mass-charge energy equilibrium condition from which we can obtain the value of the imaginary mass M_i as a function of mass M and charge Q in this equilibrium

$$M_i = \pm \sqrt{q^2 \alpha m_P^2 \left(1 - \frac{\alpha^4}{\alpha^4} \right) - \frac{\alpha^4}{\alpha^4} M^2}. \quad (65)$$

⁸ Charges in the cited study are defined in CGS units. Here we adopt SI.

In particular for $q = 0$ this yields

$$M_i \alpha^2 = \pm i M \alpha_2^2 \quad \text{or} \quad M_i = \pm i \frac{\alpha_2^2}{\alpha^2} M \approx \pm 0.9557 i M. \quad (66)$$

Since mass M_i is imaginary by definition, the argument of the square root in the relation (65) must be negative. Thus

$$M > |q| m_P \sqrt{\alpha \left(\frac{\alpha^4}{\alpha_2^4} - 1 \right)} \approx |q| 5.7275 \times 10^{-10} \text{ [kg]}. \quad (67)$$

This means that masses of uncharged micro BHs ($q = 0$) in thermodynamic equilibrium can be arbitrary. However, micro NSs and micro WDs, also in thermodynamic equilibrium, are inaccessible for direct observation, as they cannot achieve a net charge $Q = 0$. Even a single elementary charge of a white dwarf renders its mass $M_{\text{WD}} = 5.7275 \times 10^{-10} \text{ [kg]}$ comparable to the mass of a grain of sand.

We note here that only the masses satisfying $M < 2\pi m_P \approx 1.3675 \times 10^{-7} \text{ [kg]}$ have Compton wavelengths larger than the Planck length [5]. We note in passing that a classical description has been recently ruled out at the microgram ($1 \times 10^{-9} \text{ [kg]}$) mass scale [54]. Comparing this bound with the bound (67) yields the charge multiplier q corresponding to an atomic number

$$Z = \left\lfloor \frac{2\pi}{\sqrt{\alpha \left(\frac{\alpha^4}{\alpha_2^4} - 1 \right)}} \right\rfloor = 238, \quad (68)$$

of a hypothetical element, which - as we conjecture - sets the limit on an extended periodic table and is a little higher than the accepted limit of $Z = 184$ (unoctadium). More massive elements would have Compton wavelengths smaller than the Planck length, which is physically implausible.

Postulating that the squared moduli (62) and (63) are equal

$$|E_{FQ_i}|^2 = |E_{QF_i}|^2, \quad (69)$$

$$\alpha_2^4 (f^2 + q^2 \alpha) = \alpha^4 \left(q^2 \alpha - \frac{\alpha}{\alpha_2} f_i^2 \right),$$

we demand a photon-charge energy equilibrium condition from which we can obtain the value of the imaginary photon energy $h\nu_i$ corresponding to the real photon energy $h\nu$ and charge Q in this equilibrium

$$f_i = \pm \sqrt{\frac{\alpha_2^5}{\alpha^5}} \sqrt{q^2 \alpha \left(\frac{\alpha^4}{\alpha_2^4} - 1 \right)} - f^2. \quad (70)$$

Since $\sqrt{\alpha_2^5/\alpha^5}$ is imaginary, we demand $q^2 \alpha (\alpha^4/\alpha_2^4 - 1) < f^2$ to ensure that $f_i \in \mathbb{R}$. Thus

$$h\nu = f E_P > \pm q \sqrt{\alpha \left(\frac{\alpha^4}{\alpha_2^4} - 1 \right)} E_P \approx \pm q 5.1477 \times 10^7 \text{ [J]}, \quad (71)$$

which, using mass-energy equivalence, corresponds to the bound (67). We can also obtain the maximum wavelength in

this equilibrium corresponding to the charge. For $q^2 = 1$ it is $\lambda < 3.8589 \times 10^{-33} \text{ [m]}$ with $l < 238.7580$ corresponding to the bound (68).

It seems that no meaningful conclusions can be derived by postulating the equality of the squared moduli (60) and (61). Such a mass-photon energy equilibrium is an equation with four unknowns. Neither physically meaningful elementary mass (37) nor length (38) is common for real and imaginary dimensions.

Postulating the equality of all the squared moduli (58)-(63) to some constant energy

$$|E_{MQ_i}|^2 = |E_{QM_i}|^2 = |E_{FM_i}|^2 = |E_{FQ_i}|^2 = |E_{MF_i}|^2 = |E_{QF_i}|^2 := A E_P^2, \quad A \in \mathbb{R}, \quad (72)$$

we demand a mass-charge-photon equilibrium condition, which can be solved for A . Subtracting moduli (58) and (62) yields $m^2 = f^2$, and similarly subtracting moduli (59) and (63) yields $m_i^2 = f_i^2$. This equates moduli (60) and (61). Substituting $m_i^2 = f_i^2$ into the modulus (63) and subtracting from the modulus (58) yields

$$m^2 + \frac{\alpha}{\alpha_2} m_i^2 = A \left(1 - \frac{\alpha^4}{\alpha_2^4} \right). \quad (73)$$

Subtracting this from (60) or (61) yields

$$m_i^2 = f_i^2 = \frac{-A \alpha_2^9}{\alpha^5 (\alpha^4 + \alpha_2^4)}, \quad (74)$$

which substituted into the relation (73) yields

$$m^2 = f^2 = \frac{A \alpha^4}{\alpha^4 + \alpha_2^4}. \quad (75)$$

Finally, substituting the relation (75) into the modulus (58) yields

$$q^2 \alpha = \frac{A \alpha_2^4}{\alpha^4 + \alpha_2^4}. \quad (76)$$

VI. BBO COMPLEX ENERGY EQUILIBRIA

We can interpret the modulus of the generalized energy of BBOs (46) as the modulus of the complex energy of real mass (58), taking the observable real energy $E_{\text{BBO}} = M_{\text{BBO}} c^2$ of the BBO as the real part of this energy. Thus

$$\left(\frac{k}{2} M_{\text{BBO}} c^2 \right)^2 = (M_{\text{BBO}}^2 + q_{\text{BBO}}^2 \alpha m_P^2) c^4, \quad (77)$$

leads to

$$q_{\text{BBO}} = \pm \frac{M_{\text{BBO}}}{m_P} \sqrt{\frac{1}{\alpha} \left(\frac{k^2}{4} - 1 \right)}, \quad (78)$$

representing a charge surplus energy exceeding $M_{\text{BBO}} c^2$. For $k = 2$, q_{BBO} vanishes, confirming the vanishing net charge of

BHs. Similarly, we can interpret the modulus of the generalized energy of BBOs (46) as the modulus of the complex energy of real charge (59). Thus

$$\begin{aligned} \frac{k^2}{4} M_{\text{BBO}}^2 &= \frac{\alpha^4}{\alpha_2^4} (q_{\text{BBO}}^2 \alpha m_{\text{P}}^2 - M_{\text{iBBO}}^2), \\ M_{\text{iBBO}}^2 &= q_{\text{BBO}}^2 \alpha m_{\text{P}}^2 - \frac{\alpha^4}{\alpha_2^4} \frac{k^2}{4} M_{\text{BBO}}^2. \end{aligned} \quad (79)$$

Substituting q_{BBO}^2 from the relation (78) into the relation (79) turns the equilibrium condition (65) into a function of the STM k instead of the charge q

$$\begin{aligned} M_{\text{iBBO}}^2 &= \left[\frac{k^2}{4} \left(1 - \frac{\alpha_2^4}{\alpha^4} \right) - 1 \right] M_{\text{BBO}}^2, \\ M_{\text{iBBO}} &= \pm M_{\text{BBO}} \sqrt{\frac{k^2}{4} \left(1 - \frac{\alpha_2^4}{\alpha^4} \right) - 1}, \end{aligned} \quad (80)$$

which for BHs ($k = 2$) also corresponds to the relation (66) between uncharged masses M and M_i , where no assumptions concerning the BBO energy were made.

Furthermore, the argument of the square root in the relation (80) must be negative, as mass M_i is imaginary by definition. This leads to the maximum STM ratio

$$k_{\text{max}} = \pm \frac{2}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \approx 6.7933, \quad \hat{k}_{\text{max}} \approx \pm 3.3967, \quad (81)$$

where $k < |k_{\text{max}}|$ satisfies the mass equilibrium (80). Relations (78) and (80) are shown in Figure 1.

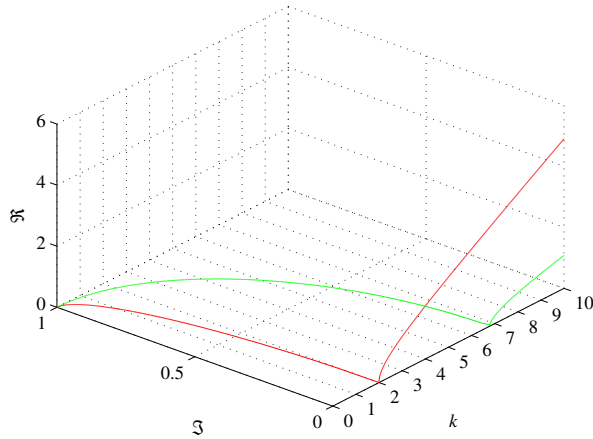


Figure 1. Ratios of imaginary mass M_{iBBO} to real mass M_{BBO} (green) and real charge $q_{\text{BBO}} \alpha m_{\text{P}} \sqrt{\alpha}$ to M_{BBO} (red) of a BBO as a function of the size-to-mass ratio k : $0 \leq k \leq 10$. Mass M_{iBBO} is imaginary for $k \lesssim 6.79$. Charge q_{BBO} is real for $k \geq 2$.

The maximum STM ratio k_{max} (81) sets the bounds on the BBO energy (46), mass, and radius (43)

$$R_{\text{BH}} = \frac{2GM_{\text{BBO}}}{c^2} \leq R_{\text{BBO}} < \frac{k_{\text{max}} GM_{\text{BBO}}}{c^2}. \quad (82)$$

In particular, using relations (40), $2m_{\text{BBO}} \leq r_{\text{BBO}} < k_{\text{max}} m_{\text{BBO}}$ or $r_{\text{BBO}}/k_{\text{max}} < m_{\text{BBO}} \leq r_{\text{BBO}}/2$. As WDs are the least dense BBOs, this bounds define the maximum radius and mass of a WD core.

Furthermore, relations (67) and (81) set the bound on the BBO minimum mass in the equilibrium (64)

$$m_{\text{BBO}} > \max \left\{ q_{\text{BBO}} \sqrt{\alpha \left(\frac{\alpha^4}{\alpha_2^4} - 1 \right)}, \frac{d_{\text{BBO}}}{4} \sqrt{1 - \frac{\alpha^4}{\alpha_2^4}} \right\}, \quad (83)$$

where

$$q_{\text{BBO}} = \frac{1}{4} \sqrt{\frac{\alpha_2^4}{\alpha^5}} d_{\text{BBO}} \quad (84)$$

defines a condition in which neither q_{BBO} nor d_{BBO} can be further increased to reach its counterpart (defined respectively by d_{BBO} and q_{BBO}) in the bound (83). Thus, for example, 1-bit BBO ($d_{\text{BBO}} = 1/\sqrt{\pi}$) corresponds to $q_{\text{BBO}} > 1.5780$, π -bit BBO ($d_{\text{BBO}} = 1$) corresponds to $q_{\text{BBO}} > 2.7969$, while the conjectured heaviest element with atomic number q_{BBO} (68) corresponds to

$$d_{\text{BBO}} = \pm \frac{8\pi}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \approx 85.3666. \quad (85)$$

In the case of a BBO, we obtain the equilibrium condition (72) by comparing the squared moduli (58)-(63) of the energies (49)-(54) with the squared BBO energy (46) which yields a solvable system of six nonlinear equations with six unknowns k, q, m, m_i, f, f_i

$$\begin{aligned} |E_{MQ_i}|^2 &\Rightarrow m^2 + q^2 \alpha = \frac{k^2}{4} m^2, \quad q^2 \alpha = m^2 \left(\frac{k^2}{4} - 1 \right), \\ |E_{QM_i}|^2 &\Rightarrow \frac{\alpha^4}{\alpha_2^4} q^2 \alpha - \frac{\alpha^5}{\alpha_2^5} m_i^2 = \frac{k^2}{4} m^2, \\ |E_{FM_i}|^2 &\Rightarrow f^2 - \frac{\alpha^5}{\alpha_2^5} m_i^2 = \frac{k^2}{4} m^2, \\ |E_{FQ_i}|^2 &\Rightarrow f^2 + q^2 \alpha = \frac{k^2}{4} m^2, \\ |E_{MF_i}|^2 &\Rightarrow m^2 - \frac{\alpha^5}{\alpha_2^5} f_i^2 = \frac{k^2}{4} m^2, \quad m^2 \left(1 - \frac{k^2}{4} \right) = \frac{\alpha^5}{\alpha_2^5} f_i^2, \\ |E_{QF_i}|^2 &\Rightarrow \frac{\alpha^4}{\alpha_2^4} q^2 \alpha - f_i^2 \frac{\alpha^5}{\alpha_2^5} = \frac{k^2}{4} m^2. \end{aligned} \quad (86)$$

Substituting $q^2 \alpha = m^2 \left(\frac{k^2}{4} - 1 \right)$ from $|E_{MQ_i}|^2$ to $|E_{FQ_i}|^2$ recovers the Compton wavelength of the BBO, $\lambda_{\text{BBO}} = \frac{h}{M_{\text{BBO}} c}$, in its Planck units form $l^2 = \frac{4\pi^2}{m^2}$. Furthermore, by substituting $q^2 \alpha$ and the Compton mass $m^2 = \frac{4\pi^2}{l^2}$ into $|E_{QM_i}|^2$, and comparing the LHSs of $|E_{QM_i}|^2$ and $|E_{FM_i}|^2$ we obtain the BBO equilibrium STM ratio

$$\begin{aligned} \frac{k_{\text{eq}}^2}{4} &= \frac{\alpha_2^4}{\alpha^4} + 1 \Rightarrow \\ k_{\text{eq}} &= \pm 2 \sqrt{1 + \frac{\alpha_2^4}{\alpha^4}} \approx 2.7665, \quad \hat{k}_{\text{eq}} \approx 1.3833, \end{aligned} \quad (87)$$

where BBO gravity, charge, and photon energies remain at equilibrium. The STM ratio $k = k_{\text{eq}}$ satisfies the equilibrium condition (72) for

$$A = \frac{1}{4}k_{\text{eq}}^2 m_{\text{BBO}}^2 = \left(1 + \frac{\alpha^4}{\alpha^4}\right) m_{\text{BBO}}^2 \approx 1.9133 m_{\text{BBO}}^2. \quad (88)$$

The equilibrium k_{eq} (87) and the maximum k_{max} (81) STM ratios are related as $k_{\text{eq}}^2 + 16/k_{\text{max}}^2 = 8$. Also, the following relations can be derived from the relations (86) for the BBO in the equilibrium k_{eq} (87)

$$m_i^2 = -\frac{\alpha_2^9}{\alpha^9} m^2 \Leftrightarrow M_{i\text{BBO}_{\text{eq}}} = \pm i \frac{\alpha_2^4}{\alpha^4} M_{\text{BBO}_{\text{eq}}}, \quad (89)$$

$$l_i^2 = -\frac{\alpha_2^9}{\alpha^9} l^2 \Leftrightarrow \lambda_{i\text{BBO}_{\text{eq}}} = \pm i \frac{\alpha_2^3}{\alpha^3} \lambda_{\text{BBO}_{\text{eq}}}, \quad (90)$$

$$m^2 = f^2 = \frac{4\pi^2}{l^2} \Leftrightarrow \lambda_{\text{BBO}_{\text{eq}}} = \frac{h}{M_{\text{BBO}_{\text{eq}}} c}, \quad (91)$$

$$q^2 \alpha = \frac{\alpha_2^4}{\alpha^4} m^2. \quad (92)$$

The BBO in the energy equilibrium k_{eq} bearing the elementary charge ($q^2 = 1$) would have mass $M_{\text{BBO}_{\text{eq}}} \approx \pm 1.9455 \times 10^{-9}$ [kg], imaginary mass $M_{i\text{BBO}_{\text{eq}}} \approx \pm i 1.7768 \times 10^{-9}$ [kg], wavelength $\lambda_{\text{BBO}_{\text{eq}}} \approx \pm 1.1361 \times 10^{-33}$ [m], and imaginary wavelength $\lambda_{i\text{BBO}_{\text{eq}}} \approx \pm i 1.2160 \times 10^{-33}$ [m].

We note that *objects* with radii $R_{\text{BH}} \leq R_* \leq 1.5R_{\text{BH}}$ are referred to in state of the art as *ultracompact* [55], where $1.5R_{\text{BH}}$ is a photon sphere radius⁹. Any *object* that undergoes complete gravitational collapse passes through an ultracompact stage [56], where $R_* < 1.5R_{\text{BH}}$. Collapse can be approached by gradual accretion, increasing the mass to the maximum stable value, or by the loss of angular momentum [56]. During the loss of angular momentum, the star passes through a sequence of increasingly compact configurations until it finally collapses to become a black hole. It was also pointed out [57] that for a neutron star of constant density, the pressure at the center would become infinite if $R_* < 1.125R_{\text{BH}}$, a radius of the maximal sustainable density for gravitating spherical *matter* given by Buchdahl's theorem. It was shown [58] that this limit applies to any well-behaved spherical star where density increases monotonically with radius. Furthermore, some observers would measure a locally negative energy density if $R_* < 1.3(3)R_{\text{BH}}$, thus breaking the dominant energy condition, although this may be allowed [59]. As the surface gravity grows, photons from further behind the NS become visible. At $R_* \approx 1.76R_{\text{BH}}$, the whole NS surface becomes visible

[60]. The relative increase in brightness between the maximum and minimum of a light curve are greater in the case of $R_* < 1.5R_{\text{BH}}$ than in the case of $R_* > 1.5R_{\text{BH}}$ [60]. Therefore the equilibrium STM ratio $R_{\text{eq}} = 1.3833 R_{\text{BH}}$ (87) is well within the range of radii of ultracompact *objects* researched in state-of-the-art within the framework of GR.

However, aside from the Schwarzschild radius, derivable from escape velocity $v_{\text{esc}}^2 = 2GM/R$ of mass M by setting $v_{\text{esc}}^2 = c^2$, and discovered in 1783 by John Michell [61], all the remaining significant radii of GR are only approximations. GR neglects the value of the fine-structure constants α and α_2 , which, similarly as π or the base of the natural logarithm, are the fundamental constants of nature.

VII. BBO MERGERS

As the entropy (Boltzmann, Gibbs, Shannon, von Neumann) of independent systems is additive, a merger of BBO_1 and BBO_2 having entropies (42) $S_{\text{BBO}_1} = \frac{1}{4}k_{\text{B}}N_{\text{BBO}_1}$ and $S_{\text{BBO}_2} = \frac{1}{4}k_{\text{B}}\pi d_{\text{BBO}_2}^2$, produces a BBO_C having entropy

$$S_{\text{BBO}_1} + S_{\text{BBO}_2} = S_{\text{BBO}_C} \Rightarrow d_{\text{BBO}_1}^2 + d_{\text{BBO}_2}^2 = d_{\text{BBO}_C}^2, \quad (93)$$

which shows that a merger of two primordial BHs, each having the Planck length diameter, the reduced Planck temperature $\frac{T_{\text{P}}}{2\pi}$ (the largest physically significant temperature [12]), and no tangential acceleration a_{LL} [5, 12], produces a BH having $d_{\text{BH}} = \pm\sqrt{2}$ which represents the minimum BH diameter allowing for the notion of time [12]. In comparison, a collision of the latter two BHs produces a BH having $d_{\text{BH}} = \pm 2$ having the triangulation defining only one precise diameter between its poles (cf. [5] Fig. 3(b)), which is also recovered from Heisenberg's Uncertainty Principle (cf. Appendix C).

Substituting the generalized radius (43) into the entropy relation (93) yields

$$k_{\text{BBO}_C}^2 M_{\text{BBO}_C}^2 = k_{\text{BBO}_1}^2 M_{\text{BBO}_1}^2 + k_{\text{BBO}_2}^2 M_{\text{BBO}_2}^2, \quad (94)$$

which establishes a Pythagorean relation between the generalized energies (46) of the merging components and the merger

$$\frac{k_{\text{BBO}_C}^2}{4} M_{\text{BBO}_C}^2 c^4 = \frac{k_{\text{BBO}_1}^2}{4} M_{\text{BBO}_1}^2 c^4 + \frac{k_{\text{BBO}_2}^2}{4} M_{\text{BBO}_2}^2 c^4, \quad (95)$$

valid both for $m_{\text{BBO}} \geq 0$ and $m_{\text{BBO}} \leq 0$.

It is accepted that gravitational events' observations alone are able to measure the masses of the merging components and set a lower limit on their compactness, but the results do not exclude mergers more compact than neutron stars such as quark stars, black holes, or more exotic *objects* [62]. We note in passing that describing the registered gravitational events as *waves* is misleading. Normal modulation of the gravitational potential, caused by rotating (in the merger case - inspiral) bodies, is wrongly interpreted as a gravitational wave understood as a carrier of gravity [63].

The accepted value of the Chandrasekhar WD mass limit, preventing its collapse into a denser form, is $M_{\text{Ch}} \approx$

⁹ At which, according to an accepted photon sphere definition, the strength of gravity *forces photons to travel in orbits*. The author wonders why photons would not *travel in orbits* at radius $R = GM/c^2$ corresponding to the orbital velocity $v_{\text{orb}}^2 = GM/R$. Obviously, photons do not *travel*.

1.4 $M_\odot$ [64] and the accepted value of the analogous Tolman–Oppenheimer–Volkoff NS mass limit is $M_{\text{TOV}} \approx 2.9 M_\odot$ [65, 66]. There is no accepted value of the BH mass limit. The conjectured value is $5 \times 10^{10} M_\odot$. The masses of most of the registered merging components are well beyond M_{TOV} . Of those that are not, most of the total or final masses exceed this limit. Therefore these mergers are classified as BH mergers. Only a few are classified otherwise, including GW170817, GW190425, GW200105, and GW200115. They are listed in Table I.

Table I. Selected BBO mergers discovered with LIGO and Virgo. Masses in $M_\odot$.

Event	M_{BBO_1}	M_{BBO_2}	M_{BBO_C}	k_{BBO_1}	k_{BBO_2}	k_{BBO_C}
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	2.8	4.39	4.39	3.03
GW190425	$2.00^{+0.6}_{-0.2}$	$1.4^{+0.3}_{-0.3}$	$3.4^{+0.3}_{-0.1}$	4.39	4.39	3.15
GW200105	$8.9^{+1.2}_{-1.2}$	$1.9^{+0.3}_{-0.3}$	$10.9^{+1.1}_{-1.1}$	2.76	4.39	2.38
GW200115	$5.7^{+1.3}_{-2.1}$	$1.5^{+0.7}_{-0.3}$	$7.1^{+1.5}_{-1.4}$	3	4.39	2.64

The relation (95) explains the measurements of large masses of the BBO mergers with at least one charged merging component without resorting to any hypothetical types of exotic stellar *objects* such as *quark stars*. Interferometric data, available online at the Gravitational Wave Open Science Center (GWOSC) portal¹⁰, indicate that the total mass of a merger is the sum of the masses of the merging components. Thus

$$\begin{aligned}
 m_{\text{BBO}_C} &= m_{\text{BBO}_1} + m_{\text{BBO}_2} \Rightarrow \\
 m_{\text{BBO}_C}^2 &= m_{\text{BBO}_1}^2 + m_{\text{BBO}_2}^2 + 2m_{\text{BBO}_1}m_{\text{BBO}_2} \Rightarrow \\
 m_{\text{BBO}_C}^2 &\begin{cases} > m_{\text{BBO}_1}^2 + m_{\text{BBO}_2}^2 & \text{if } m_{\text{BBO}_1}m_{\text{BBO}_2} > 0 \\ < m_{\text{BBO}_1}^2 + m_{\text{BBO}_2}^2 & \text{if } m_{\text{BBO}_1}m_{\text{BBO}_2} < 0 \end{cases} .
 \end{aligned} \quad (96)$$

We initially assume $m_{\text{BBO}} > 0 \Rightarrow m_{\text{BBO}_1}m_{\text{BBO}_2} > 0$, since negative masses, similarly to negative lengths, are inaccessible for direct observation, unlike charges.

We can use the BBO equilibrium relations (86) to derive some information about the merger from the relation (95). $|E_{MQ_i}|^2$ with the first inequality (96) lead to

$$\begin{aligned}
 m_{\text{BBO}_C}^2 + \alpha q_{\text{BBO}_C}^2 &= \\
 &= m_{\text{BBO}_1}^2 + \alpha q_{\text{BBO}_1}^2 + m_{\text{BBO}_2}^2 + \alpha q_{\text{BBO}_2}^2, \\
 m_{\text{BBO}_C}^2 &= \\
 &= \cancel{m_{\text{BBO}_1}^2} + \cancel{m_{\text{BBO}_2}^2} + \cancel{\alpha(q_{\text{BBO}_1}^2 + q_{\text{BBO}_2}^2)} - \cancel{\alpha q_{\text{BBO}_C}^2} \geq, \\
 &\geq \cancel{m_{\text{BBO}_1}^2} + \cancel{m_{\text{BBO}_2}^2} \Rightarrow q_{\text{BBO}_C}^2 \leq q_{\text{BBO}_1}^2 + q_{\text{BBO}_2}^2,
 \end{aligned} \quad (97)$$

which, by the charge conservation principle, implies mixed (positive and negative) charges of the merging components satisfying $q_{\text{BBO}_1}q_{\text{BBO}_2} \leq 0$. On the other hand, $|E_{MF_i}|^2$ with the first inequality (96) lead to

$$m_{\text{BBO}_C}^2 \geq m_{\text{BBO}_1}^2 + m_{\text{BBO}_2}^2 \Rightarrow f_{\text{BBO}_C}^2 \geq f_{\text{BBO}_1}^2 + f_{\text{BBO}_2}^2. \quad (98)$$

But $|E_{QF_i}|^2$ with the inequality (97) lead to an apparent contradiction

$$q_{\text{BBO}_C}^2 \leq q_{\text{BBO}_1}^2 + q_{\text{BBO}_2}^2 \Rightarrow f_{\text{BBO}_C}^2 \leq f_{\text{BBO}_1}^2 + f_{\text{BBO}_2}^2, \quad (99)$$

while $|E_{MF_i}|^2$ with the inequality (99) lead to

$$f_{\text{BBO}_C}^2 \leq f_{\text{BBO}_1}^2 + f_{\text{BBO}_2}^2 \Rightarrow m_{\text{BBO}_C}^2 \leq m_{\text{BBO}_1}^2 + m_{\text{BBO}_2}^2, \quad (100)$$

$|E_{QF_i}|^2$ with the inequality (98) lead to

$$f_{\text{BBO}_C}^2 \geq f_{\text{BBO}_1}^2 + f_{\text{BBO}_2}^2 \Rightarrow q_{\text{BBO}_C}^2 \geq q_{\text{BBO}_1}^2 + q_{\text{BBO}_2}^2, \quad (101)$$

and so on ($|E_{QM_i}|^2$, $|E_{FM_i}|^2$, $|E_{FQ_i}|^2$)

$$\begin{aligned}
 q_{\text{BBO}_C}^2 \geq q_{\text{BBO}_1}^2 + q_{\text{BBO}_2}^2 &\Rightarrow m_{\text{BBO}_C}^2 \geq m_{\text{BBO}_1}^2 + m_{\text{BBO}_2}^2 \\
 \Rightarrow f_{\text{BBO}_C}^2 \geq f_{\text{BBO}_1}^2 + f_{\text{BBO}_2}^2 &\Rightarrow q_{\text{BBO}_C}^2 \leq q_{\text{BBO}_1}^2 + q_{\text{BBO}_2}^2.
 \end{aligned} \quad (102)$$

Additivity of entropy (93) of statistically independent merging BBOs, both in global thermodynamic equilibrium, defined by their generalized radii (43), introduces the energy relation (95). This relation, equality of charges in real and imaginary dimensions (36), and the BBO equilibrium relations (86) induce not only mixed charges but also imaginary, negative, and mixed wavelengths and masses during the merger. A BBO merger spreads in all dimensions, not only the observable ones, as a gravitational event associated with a fast radio burst (FRB) event, as it has recently been reported [67] based on GW1904251 gravitational event and FRB 20190425A event¹¹.

In the observable dimensions during the merger, the STM ratio k_{BBO_C} decreases making the BBO_C denser until it becomes a BH for $k_{\text{BBO}_C} = 2$ and no further charge reduction is possible (cf. Fig 1). From the relation (94) and the first inequality (96) we see that this holds for

$$k_{\text{BBO}_C}^2 (M_{\text{BBO}_1}^2 + M_{\text{BBO}_2}^2) < k_{\text{BBO}_1}^2 M_{\text{BBO}_1}^2 + k_{\text{BBO}_2}^2 M_{\text{BBO}_2}^2. \quad (103)$$

Table I lists the mass-to-size ratios k_{BBO_C} calculated according to the relation (95) that provide the measured mass M_{BBO_C} of the merger and satisfy the inequality (103). Mass-to-size ratios k_{BBO_1} and k_{BBO_2} of the merging components were arbitrarily selected based on their masses, taking into account the M_{TOV} NS mass limit.

The meaning of f and f_i photon energy multipliers in the relations (98)-(102) requires further research. We conjecture that f relates to the spectrum of measured FRBs of the mergers.

VIII. BBO COMPLEX GRAVITY AND TEMPERATURE

Complex energies (49)-(54) define complex forces (similarly to the complex energy of real masses and charges (48),

¹⁰ <https://www.gw-openscience.org/eventapi/html/allevnts>

¹¹ Data available online at the Canadian Hydrogen Intensity Mapping Experiment (CHIME) portal <https://www.chime-frb.ca/catalog>

[53] Eq. (7)) acting over real and imaginary *distances* R, R_i . Using the relations (40), we obtain the following products

$$\begin{aligned} E_{1mq_i} E_{2mq_i} &:= E_{1MQ_i} E_{2MQ_i} / E_P^2 = \\ &= m_1 m_2 - q_1 q_2 \alpha + i \sqrt{\alpha} (m_1 q_2 + m_2 q_1), \\ E_{1qm_i} E_{2qm_i} &:= E_{1QM_i} E_{2QM_i} / E_P^2 = \\ &= \frac{\alpha^5}{\alpha_2^4} \left(q_1 q_2 + \frac{1}{\alpha_2} m_{i1} m_{i2} + \frac{1}{\sqrt{\alpha_2}} (q_1 m_{i2} + q_2 m_{i1}) \right), \end{aligned} \quad (104)$$

$$\begin{aligned} E_{1fm_i} E_{2fm_i} &:= E_{1FM_i} E_{2FM_i} / E_P^2 = \\ &= f_1 f_2 + \frac{\alpha^5}{\alpha_2^5} m_{i1} m_{i2} + \sqrt{\frac{\alpha^5}{\alpha_2^5}} (f_1 m_{i2} + f_2 m_{i1}), \\ E_{1mf_i} E_{2mf_i} &:= E_{1MF_i} E_{2MF_i} / E_P^2 = \\ &= m_1 m_2 + \frac{\alpha^5}{\alpha_2^5} f_{i1} f_{i2} + \sqrt{\frac{\alpha^5}{\alpha_2^5}} (m_1 f_{i2} + m_2 f_{i1}), \end{aligned} \quad (105)$$

$$\begin{aligned} E_{1qf_i} E_{2qf_i} &:= E_{1QF_i} E_{2QF_i} / E_P^2 = \\ &= \frac{\alpha^4}{\alpha_2^4} q_1 q_2 \alpha + \frac{\alpha^5}{\alpha_2^5} f_{i1} f_{i2} + \frac{\alpha^5}{\sqrt{\alpha_2^9}} (f_{i2} q_1 + f_{i1} q_2), \\ E_{1fq_i} E_{2fq_i} &:= E_{1FQ_i} E_{2FQ_i} / E_P^2 = \\ &= f_1 f_2 - q_1 q_2 \alpha + i \sqrt{\alpha} (f_1 q_2 + f_2 q_1), \end{aligned} \quad (106)$$

defining six complex forces acting over a real *distance* $R := r\ell_P, r \in \mathbb{R}$

$$F_{AB_i} = \frac{G}{c^4 R^2} E_{1AB_i} E_{2AB_i} = \frac{F_P}{r^2} E_{1ab_i} E_{2ab_i}, \quad (107)$$

and six complex forces acting over an imaginary *distance* $R_i := r_i \ell_{Pi}, r_i \in \mathbb{R}$

$$\tilde{F}_{AB_i} = \frac{G}{c_n^4 R_i^2} E_{1AB_i} E_{2AB_i} = \frac{\alpha_2}{\alpha} \frac{F_P}{r_i^2} E_{1ab_i} E_{2ab_i}, \quad (108)$$

where $A, B \in \{M, Q, F\}$ and $a, b \in \{m, q, f\}$, and

$$\alpha_2 r^2 F_{AB_i} = \alpha r_i^2 \tilde{F}_{AB_i}. \quad (109)$$

We exclude mixed forces (based on real and imaginary masses/charges/photons) as real and imaginary dimensions are orthogonal.

With a further simplifying assumption of $r^2 = r_i^2$, the forces acting over a real *distance* R are stronger and opposite to the corresponding forces acting over an imaginary *distance* R_i even though the Planck force is lower in modulus than the (real) α_2 -Planck force (27). This is a strong assumption but seemingly correct. General radius (43) and energy (46) are the same in Planck units, and α_2 -Planck units; STM remains the same.

In particular, we can use the complex force F_{MQ_i} (107) with (104) (i.e., complex Newton's law of universal gravitation)

to calculate the BBO surface gravity g_{BBO} , assuming an uncharged ($q_2 = 0$) test mass m_2

$$\begin{aligned} \frac{F_P}{r_{BBO}^2} (m_{BBO} m_2 + i \sqrt{\alpha} m_2 q_{BBO}) &= \\ &= M_2 g_{BBO} = m_2 m_P \hat{g}_{BBO} a_P, \\ \hat{g}_{BBO} &= \frac{1}{r_{BBO}^2} (m_{BBO} + i \sqrt{\alpha} q_{BBO}), \end{aligned} \quad (110)$$

where $g_{BBO} = \hat{g}_{BBO} a_P, \hat{g}_{BBO} \in \mathbb{R}$. Substituting the BBO equilibrium relation (78) and the generalized BBO radius (43) $r_{BBO} = k m_{BBO}$ into the relation (110) yields

$$g_{BBO} = \frac{a_P}{k r_{BBO}} \left(1 \pm i \sqrt{\frac{k^2}{4} - 1} \right), \quad (111)$$

which reduces to BH surface gravity for $k = 2$, in modulus

$$\hat{g}_{BBO}^2 = \frac{1}{k^2 r_{BBO}^2} \left(1 + i \sqrt{\frac{k^2}{4} - 1} \right) \left(1 - i \sqrt{\frac{k^2}{4} - 1} \right) = \frac{1}{4 r_{BBO}^2}, \quad (112)$$

equals to a squared BH surface gravity for all k , and in particular,

$$g_{BBO}(k_{\max}) = \pm \frac{a_P}{d_{BBO}} (0.2944 \pm 0.9557i), \quad (113)$$

$$g_{BBO}(k_{eq}) = \pm \frac{a_P}{d_{BBO}} (0.7229 \pm 0.6909i). \quad (114)$$

As the BBO potential is [5]

$$\delta\varphi_{BBO} = -\frac{N_1}{N_{BBO}} c^2 = -\frac{1}{2} c^2, \quad (115)$$

we conjecture that its complex form is

$$\delta\varphi_{BBO} = \pm \frac{c^2}{k} \left(1 \pm i \sqrt{\frac{k^2}{4} - 1} \right) \quad (116)$$

and only its negative modulus equals $-c^2/2$.

The BBO surface gravity (111) leads to the generalized complex Hawking blackbody-radiation equation

$$T_{BBO} = \frac{\hbar}{2\pi c k_B} g_{BBO} = \frac{T_P}{k\pi d_{BBO}} \left(1 \pm i \sqrt{\frac{k^2}{4} - 1} \right), \quad (117)$$

describing the BBO temperature¹² by including its charge in the imaginary part, which also in modulus equals squared BH temperature $\forall k$. In particular,

$$\begin{aligned} T_{BBO}(k_{\max}) &= \pm \frac{T_P}{2\pi d_{BBO}} \left(\frac{\sqrt{\alpha^4 - \alpha_2^4}}{\alpha^2} \pm i \frac{\alpha_2^2}{\alpha^2} \right), \\ &= \pm \frac{T_P}{2\pi^3 d_{BBO}} \left(\sqrt{\pi^4 - \pi_1^4} \pm i \pi_1^2 \right), \\ &= \pm \frac{T_P}{2\pi \pi_2^2 d_{BBO}} \left(\sqrt{\pi_2^4 - \pi^4} \pm i \pi^2 \right), \end{aligned} \quad (118)$$

¹² In a commonly used form it is $T_{BBO} = \frac{\hbar c^3}{2k^2 \pi G M_{BBO} k_B} \left(1 \pm i \sqrt{\frac{k^2}{4} - 1} \right)$.

$$\begin{aligned}
T_{\text{BBO}}(k_{\text{eq}}) &= \pm \frac{T_P}{2\pi d_{\text{BBO}}} \frac{\alpha^2 \pm i\alpha_2^2}{\sqrt{\alpha^4 + \alpha_2^4}}, \\
&= \pm \frac{T_P}{2\pi d_{\text{BBO}}} \frac{\pi^2 \pm i\pi_1^2}{\sqrt{\pi^4 + \pi_1^4}}, \\
&= \pm \frac{T_P}{2\pi d_{\text{BBO}}} \frac{\pi_2^2 \pm i\pi^2}{\sqrt{\pi_2^4 + \pi^4}},
\end{aligned} \tag{119}$$

reduce to a BH temperature for $\alpha_2 = 0$. We note that for $d_{\text{BBO}} = 1$, $\text{Re}(T_{\text{BBO}}(k_{\text{max}})) \approx 6.6387 \times 10^{30}$ [K] (where $T_P/(2\pi) \approx 2.2549 \times 10^{31}$ [K]) has the magnitude of the Hagedorn temperature of strings.

It seems, therefore, that a universe without imaginary dimensions (i.e., with $\alpha_2 = 0$) would be a black hole. Hence, the evolution of information [1–6] requires imaginary time. And we cannot zero α_2 as we would have to neglect graphene.

IX. DISCUSSION

The reflectance of graphene under the normal incidence of electromagnetic radiation expressed as the quadratic equation for the fine-structure constant α includes the 2nd negative fine-structure constant α_2 . The sum of the reciprocal of this 2nd fine-structure constant α_2 with the reciprocal of the fine-structure constant α (2) is independent of the reflectance value R and remarkably equals simply $-\pi$. Particular algebraic definition of the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$, containing the free π term, can be interpreted as the asymptote of the CODATA value α^{-1} , the value of which varies with time. The negative fine-structure constant α_2 leads to the set of α_2 -Planck units applicable to imaginary dimensions, including imaginary α_2 -Planck units (18)–(26). Real and imaginary mass and charge units (34), length and mass units (35) units, and temperature and time units (33) are directly related to each other. Also, the elementary charge e is common for real and imaginary dimensions (36).

Applying the α_2 -Planck units to a complex energy formula [53] yields complex energies (49), (50) setting the atomic number $Z = 238$ as the limit on an extended periodic table. The generalized energy (46) of all perfect black-body *objects* (black holes, neutron stars, and white dwarfs) having the generalized radius $R_{\text{BBO}} = \hat{k}R_{\text{BH}}$ exceed mass-energy equivalence if $\hat{k} > 1$. Complex energies (49), (50) allow for storing the excess of this energy in their imaginary parts, inaccessible for direct observation. The results show that the perfect black-body *objects* other than black holes cannot have masses lower than 5.7275×10^{-10} [kg] and that the STM ratios of their cores cannot exceed $\hat{k}_{\text{max}} \approx 3.3967$ defined by the relation (81). It is further shown that a black-body *object* is in the equilibrium of complex energies if its radius $R_{\text{eq}} \approx 1.3833 R_{\text{BH}}$ (87). BBO fluctuations for k_{eq} and k_{max} are briefly discussed in Appendix G. Appendix F presents BBO quantum statistics. The proposed model explains the registered (GWOSC) high masses of the neutron stars mergers without resorting to any hypothetical types of exotic stellar *objects*.

In the context of the results of this study, monolayer graphene, a truly 2-dimensional material with no thickness¹³, is a *keyhole* to other, unperceivable, dimensionalities. Graphene history is also instructive. Discovered in 1947 [69], graphene was long considered an *academic material* until it was eventually pulled from graphite in 2004 [70] by means of ordinary Scotch tape¹⁴. These fifty-seven years, along with twenty-nine years (1935-1964) between the condemnation of quantum theory as *incomplete* [71] and Bell's mathematical theorem [72] asserting that it is not true, and the fifty-eight years (1964-2022) between the formulation of this theorem and 2022 Nobel prize in physics for its experimental *loophole-free* confirmation, should remind us that Max Planck, the genius who discovered Planck units, has also discovered Planck's principle.

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Appendix A: Other quadratic equations

The quadratic equation for the sum of transmittance (3) and absorptance (5) of MLG under normal incidence of EMR, putting $C_{\text{TA}} := T + A$, is

$$\frac{1}{4}C_{\text{TA}}\pi^2\alpha^2 + (C_{\text{TA}} - 1)\pi\alpha + (C_{\text{TA}} - 1) = 0, \tag{A1}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{C_{\text{TA}}\pi}{2(1 - C_{\text{TA}} + \sqrt{1 - C_{\text{TA}}})} \approx 137.036, \tag{A2}$$

and

$$\alpha_2^{-1} = \frac{C_{\text{TA}}\pi}{2(1 - C_{\text{TA}} - \sqrt{1 - C_{\text{TA}}})} \approx -140.178, \tag{A3}$$

whereas their sum $\alpha^{-1} + \alpha_2^{-1} = -\pi$ is, similarly as the relation (12), also independent of T and A .

Other quadratic equations do not feature this property. For example, the sum of $T + R$ (6) expressed as the quadratic equation and putting $C_{\text{TR}} := T + R$, is

$$\frac{1}{4}(C_{\text{TR}} - 1)\pi^2\alpha^2 + C_{\text{TR}}\pi\alpha + (C_{\text{TR}} - 1) = 0, \tag{A4}$$

¹³ Thickness of MLG is reported [68] as 0.37 [nm] with other reported values up to 1.7 [nm]. However, considering that 0.335 [nm] is the established inter-layer *distance* and consequently the thickness of bilayer graphene, these results do not seem credible: the thickness of bilayer graphene is not $2 \times 0.37 + 0.335 = 1.075$ [nm].

¹⁴ Introduced into the market in 1932.

and has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi(C_{\text{TR}} - 1)}{-2C_{\text{TR}} + 2\sqrt{2C_{\text{TR}} - 1}} \approx 137.036, \quad (\text{A5})$$

and

$$\alpha_{\text{TR}}^{-1} = \frac{\pi(C_{\text{TR}} - 1)}{-2C_{\text{TR}} - 2\sqrt{2C_{\text{TR}} - 1}} \approx 0.0180, \quad (\text{A6})$$

whereas their sum

$$\alpha_{\text{TR}_1}^{-1} + \alpha_{\text{TR}_2}^{-1} = \frac{-\pi C_{\text{TR}}}{C_{\text{TR}} - 1} \approx 137.054 \quad (\text{A7})$$

is dependent on T and R.

Appendix B: Two π -like constants

With algebraic definitions of α (13) and α_2 (14), T (3), R (4) and A (5) of MLG for normal EMR incidence can be expressed just by π . For $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (13) they become

$$T(\alpha) = \frac{4(4\pi^2 + \pi + 1)^2}{(8\pi^2 + 2\pi + 3)^2} \approx 0.9775, \quad (\text{B1})$$

$$A(\alpha) = \frac{4(4\pi^2 + \pi + 1)}{(8\pi^2 + 2\pi + 3)^2} \approx 0.0224, \quad (\text{B2})$$

while for $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi$ (14) they become

$$T(\alpha_2) = \frac{4(4\pi^2 + \pi + 2)^2}{(8\pi^2 + 2\pi + 3)^2} \approx 1.0228, \quad (\text{B3})$$

$$A(\alpha_2) = -\frac{4(4\pi^2 + \pi + 2)}{(8\pi^2 + 2\pi + 3)^2} \approx -0.0229, \quad (\text{B4})$$

with

$$R(\alpha) = R(\alpha_2) = \frac{1}{(8\pi^2 + 2\pi + 3)^2} \approx 1.2843 \times 10^{-4}. \quad (\text{B5})$$

$(T(\alpha) + A(\alpha)) + R(\alpha) = (T(\alpha_2) + A(\alpha_2)) + R(\alpha_2) = 1$ as required by the law of conservation of energy (8), whereas each conservation law is associated with a certain symmetry, as asserted by Noether's theorem. $A(\alpha) > 0$ and $A(\alpha_2) < 0$ imply a *sink* and a *source* respectively, while the opposite holds true for T, as illustrated schematically in Fig 2. Perhaps, the negative A and T exceeding 100% for α_2 (11) or (14) could be explained in terms of spontaneous graphene emission.

The quadratic equation (9) describing the reflectance R of MLG under the normal incidence of EMR (or alternatively (A1)) can also be solved for π yielding two roots

$$\pi(R, \alpha_*)_1 = \frac{2\sqrt{R}}{\alpha_*(1 - \sqrt{R})}, \quad \text{and} \quad (\text{B6})$$

$$\pi(R, \alpha_*)_2 = \frac{-2\sqrt{R}}{\alpha_*(1 + \sqrt{R})}, \quad (\text{B7})$$

dependent on R and α_* , where α_* indicates α or α_2 . This can be further evaluated using the MLG reflectance R (4) or (B5) (which is the same for both α and α_2), yielding four, yet only three distinct possibilities

$$\pi_1 = \pi(\alpha)_1 = -\pi \frac{4\pi^2 + \pi + 1}{4\pi^2 + \pi + 2} = \pi \frac{\alpha_2}{\alpha} \approx -3.0712, \quad (\text{B8})$$

$$\pi(\alpha)_2 = \pi(\alpha_2)_1 = \pi \approx 3.1416, \quad \text{and} \quad (\text{B9})$$

$$\pi_2 = \pi(\alpha_2)_2 = -\pi \frac{4\pi^2 + \pi + 2}{4\pi^2 + \pi + 1} = \pi \frac{\alpha}{\alpha_2} \approx -3.2136. \quad (\text{B10})$$

The modulus of π_1 (B8) corresponds to a convex surface hav-

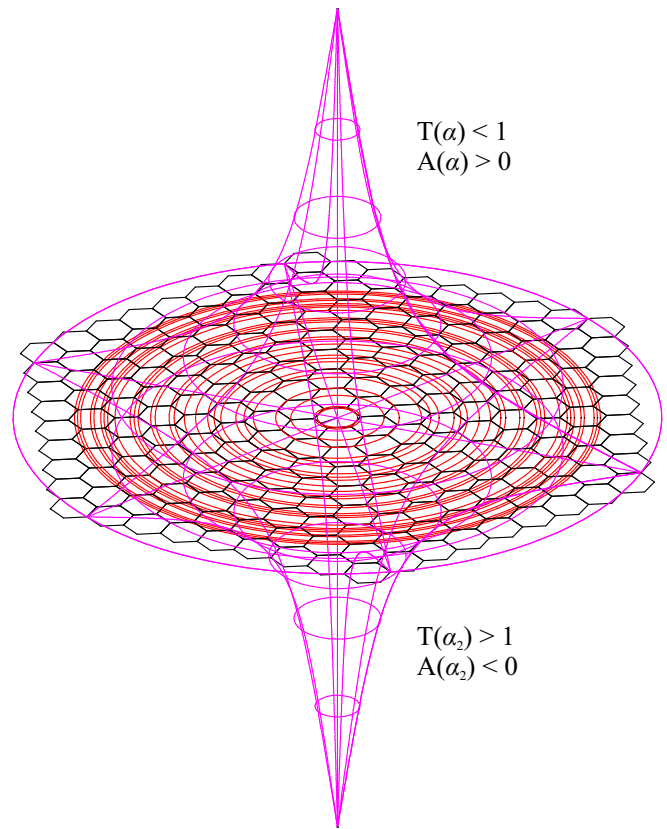


Figure 2. Illustration of the concepts of negative absorptance and excessive transmittance of EMR under normal incidence on MLG.

ing a positive Gaussian curvature, whereas the modulus of π_2 (B10) - to a negative Gaussian curvature. Their product $\pi_1\pi_2 = \pi^2$ is independent of α_* , their quotient $\pi_1/\pi_2 = \alpha_2^2/\alpha^2$ is not directly dependent of π , and $|\pi_1 - \pi| \neq |\pi - \pi_2|$. It remains to be found whether each of these π -like constants describes the ratio of the circumference of a circle drawn on the respective surface to its diameter (π_c) or the ratio of the area of this circle to the square of its radius (π_a). These definitions produce different results on curved surfaces, whereas $\pi_a > \pi_c$ on convex surfaces, while $\pi_a < \pi_c$ on saddle surfaces [75].

Appendix C: Planck units and HUP

Perhaps the simplest derivation of the squared Planck length is based on Heisenberg's uncertainty principle

$$\delta P_{\text{HUP}} \delta R_{\text{HUP}} \geq \frac{\hbar}{2} \quad \text{or} \quad \delta E_{\text{HUP}} \delta t_{\text{HUP}} \geq \frac{\hbar}{2}, \quad (\text{C1})$$

where δP_{HUP} , δR_{HUP} , δE_{HUP} , and δt_{HUP} denote momentum, position, energy, and time uncertainties, by replacing energy uncertainty $\delta E_{\text{HUP}} = \delta M_{\text{HUP}} c^2$ with mass uncertainty and time uncertainty with position uncertainty, using mass-energy equivalence and $\delta t_{\text{HUP}} = \delta R / c_{\text{HUP}}$ [32], which yields

$$\delta M_{\text{HUP}} \delta R_{\text{HUP}} \geq \frac{\hbar}{2c}. \quad (\text{C2})$$

Plugging $\delta M_{\text{HUP}} = \delta R_{\text{HUP}} c^2 / (2G)$ for BH mass into (C2) we arrive at $\delta R_{\text{HUP}}^2 = \ell_P^2 \Rightarrow \delta D_{\text{HUP}} = \pm 2\ell_P$ and recover [5] BH diameter $d_{\text{BH}} = \pm 2\ell_P$.

However, using the same procedure but inserting the BH radius, instead of the BH mass, into the uncertainty principle (C2) leads to $\delta M_{\text{HUP}}^2 = \frac{1}{4} \hbar c / G = \frac{1}{4} m_P^2$. In general, using the generalized radius (43) in both procedures, one obtains

$$\delta M_{\text{HUP}}^2 = \frac{1}{2k} m_P^2 \quad \text{and} \quad \delta R_{\text{HUP}}^2 = \frac{k}{2} \ell_P^2. \quad (\text{C3})$$

Thus, if k increases mass δM_{HUP} decreases, and δR_{HUP} increases and the factor is the same for $k = 1$ i.e., for *orbital speed radius* $\delta R = G\delta M/c^2$ or the *orbital speed mass* $\delta M = \delta R c^2 / G$.

Appendix D: The Stoney units drivation

We assume that the elementary charge is the unit of charge $q_s = e$ and that the speed of light is the quotient of the unit of length and time $c = l_s / t_s$. Next, we compare the Coulomb force between two elementary charges and units of masses m_s with Newton's law of gravity, acting over the same distance

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{R^2} = G \frac{m_s^2}{R^2} \Rightarrow m_s = \sqrt{\frac{e^2}{4\pi\epsilon_0 G}}. \quad (\text{D1})$$

Finally, we compare the inertial force of the unit of mass with Newton's law of gravity

$$m_s \frac{l_s}{t_s^2} = G \frac{m_s^2}{l_s^2} \Rightarrow l_s = \sqrt{\frac{G e^2}{4\pi\epsilon_0 c^4}}. \quad (\text{D2})$$

Appendix E: A mixed speeds hypothesis

Let us define the mass/charge energies (49), (50) with different speeds of light, i.e., the charge part of the energy E_{MQ_i} with c_n and the charge part of the energy E_{QM_i} with c

$$\begin{aligned} \hat{E}_{MQ_i} &:= M c^2 + \frac{Q_i}{2\sqrt{\pi\epsilon_0 G}} c_n^2 = M c^2 \pm i q \sqrt{a} m_P \frac{\alpha^2}{\alpha_2^2} c^2, \\ \hat{E}_{QM_i} &:= \frac{Q}{2\sqrt{\pi\epsilon_0 G}} c^2 + M_i c_n^2 = \pm q \sqrt{a} m_P c^2 + M_i \frac{\alpha^2}{\alpha_2^2} c^2, \end{aligned} \quad (\text{E1})$$

Demanding equality of their moduli

$$\begin{aligned} M^2 + q^2 \alpha m_P^2 \frac{\alpha^4}{\alpha_2^4} &= q^2 \alpha m_P^2 - M_i^2 \frac{\alpha^4}{\alpha_2^4}, \\ M_i &= \pm \sqrt{q^2 \alpha m_P^2 \left(\frac{\alpha_2^4}{\alpha^4} - 1 \right) - \frac{\alpha^4}{\alpha_2^4} M^2}. \end{aligned} \quad (\text{E2})$$

For $q = 0$ this relation corresponds to the relation (66). However, since mass M_i is imaginary, the argument of the square root in the relation (E2) must be negative, i.e.,

$$|M| \not\geq |q| m_P \sqrt{\alpha \left(1 - \frac{\alpha^4}{\alpha_2^4} \right)}. \quad (\text{E3})$$

But $\alpha^4 > \alpha_2^4$, yielding imaginary M , while M is real by definition. The same result would be obtained if mass-energy E_{QM_i} was parametrized with c_n and E_{QM_i} with c , since

$$\begin{aligned} \sqrt{\alpha \left(\frac{\alpha_2^4}{\alpha^4} - 1 \right)} &\in \mathbb{I}, & \sqrt{\alpha \left(1 - \frac{\alpha_2^4}{\alpha^4} \right)} &\in \mathbb{R}, \\ \sqrt{\alpha \left(\frac{\alpha^4}{\alpha_2^4} - 1 \right)} &\in \mathbb{R}, & \sqrt{\alpha \left(1 - \frac{\alpha^4}{\alpha_2^4} \right)} &\in \mathbb{I}. \end{aligned} \quad (\text{E4})$$

Therefore, complex energies E_{MQ_i} (49) and E_{QM_i} (50) must be parametrized respectively by c and c_n .

Appendix F: BBO quantum statistics

BH Bose-Einstein (BE), Fermi-Dirac (FD), and Maxwell-Boltzmann (MB) quantum statistics ([5], Eqs. (132)-(134), Figs. 12-14), with degeneracy interpreted as the BBO information capacity, can be extended to BBOs using the BBO temperature (117), relations (40), and

$$\exp\left(\frac{hc}{\lambda k_B T_{\text{BBO}}}\right) = \exp\left(\frac{8\pi^2 d_{\text{BBO}} \left(i \left(\frac{k^2}{4} - 1\right) \mp 1\right)}{kl}\right). \quad (\text{F1})$$

Using (F1) they become

$$N_{\text{BBO}_{\text{BE}}}(k, d_{\text{BBO}}, l) = \frac{\pi d^2}{\exp\left(\frac{8\pi^2 d_{\text{BBO}} \left(i \sqrt{\frac{k^2}{4} - 1} \mp 1\right)}{kl}\right) - 1}, \quad (\text{F2})$$

$$N_{\text{BBO}_{\text{MB}}}(k, d_{\text{BBO}}, l) = \frac{\pi d^2}{\exp\left(\frac{8\pi^2 d_{\text{BBO}} \left(i \sqrt{\frac{k^2}{4} - 1} \mp 1\right)}{kl}\right)}, \quad (\text{F3})$$

$$N_{\text{BBO}_{\text{FD}}}(k, d_{\text{BBO}}, l) = \frac{\pi d^2}{\exp\left(\frac{8\pi^2 d_{\text{BBO}} \left(i \sqrt{\frac{k^2}{4} - 1} \mp 1\right)}{kl}\right) + 1}. \quad (\text{F4})$$

To emit/absorb $N > 0$ photons or correspondingly increase/decrease $N > 0$ electron energy levels on average, $N(k, d_{\text{BBO}}, l)$ must be greater or equal to N and the statistics (F2)-(F4) yield the following wavelength thresholds

$$l_{\text{BE}}(d, N) \geq \frac{8\pi^2 d \left(i \sqrt{\frac{k^2}{4} - 1} \mp 1 \right)}{k (\ln(\pi d^2 + N) - \ln(N))}, \quad (\text{F5})$$

$$l_{\text{MB}}(d, N) \geq \frac{8\pi^2 d \left(i \sqrt{\frac{k^2}{4} - 1} \mp 1 \right)}{k (\ln(\pi d^2) - \ln(N))}, \quad (\text{F6})$$

$$l_{\text{FD}}(d, N) \geq \frac{8\pi^2 d \left(i \sqrt{\frac{k^2}{4} - 1} \mp 1 \right)}{k (\ln(\pi d^2 - N) - \ln(N))}. \quad (\text{F7})$$

Appendix G: Fluctuations of the BBOs

A relation describing a BH information capacity, having an initial information capacity $N_m(k) = \pi d_m^2$, after absorption (+) or emission (−) of a *particle* having the Compton wavelength l can be generalized (cf. [5], Appendix 3), using the generalized radius (43), to all EVSs, including BBOs as

$$N_{m+1}^{A/E}(k, d_m, l) = 16k^2 \pi^3 \frac{1}{l^2} \pm 8k\pi^2 \frac{d_m}{l} + \pi d_m^2. \quad (\text{G1})$$

Thus,

$$\Delta N^{A/E}(k, d_m, l) := N_{m+1} - N_m = 16k^2 \pi^3 \frac{1}{l^2} \pm 8k\pi^2 \frac{d_m}{l},$$

$$\Delta N^A \begin{cases} > 0 & l > -\frac{2k\pi}{d_m} \\ = 0 & l = -\frac{2k\pi}{d_m} \\ < 0 & l < -\frac{2k\pi}{d_m} \end{cases}, \quad \Delta N^E \begin{cases} > 0 & l < \frac{2k\pi}{d_m} \\ = 0 & l = \frac{2k\pi}{d_m} \\ < 0 & l > \frac{2k\pi}{d_m} \end{cases}. \quad (\text{G2})$$

The wavelength of a *particle* emitted from a BH that does not change the BH diameter corresponds to half of the BH Compton wavelength ($l_{\text{BH}} = 8\pi/d_{\text{BH}}$). Accordingly, the wavelength of a *particle* absorbed by a BH that does not change its diameter is $l_{\text{BHconst}} = -4\pi/d_{\text{BH}}$. We note in passing that three

spatial dimensions set the minimum for such conditions to occur (cf. [5], Table III). In general, $l_{\text{BBOconst}} = \mp 2k\pi/d_{\text{BBO}}$. In particular, for k_{eq} (87)

$$l_{\text{const}}(k_{\text{eq}}) = \mp \frac{4\pi \sqrt{1 + \frac{\alpha^4}{\alpha^4}}}{d} \approx \mp 1.3832 \frac{4\pi}{d}, \quad (\text{G3})$$

(with “−” for absorption and “+” for emission) reduces to l_{BHconst} for $\alpha_2 = 0$. For k_{max} (81)

$$l_{\text{const}}(k_{\text{max}}) = \mp \frac{4\pi}{d \sqrt{1 - \frac{\alpha^4}{\alpha^4}}} \approx \mp 3.3966 \frac{4\pi}{d}, \quad (\text{G4})$$

also reduces to l_{BHconst} for $\alpha_2 = 0$.

The relation (G1) is remarkably similar to the algebraic definitions of the inverses of α (13) and α_2 (14) also containing π^3 , π^2 , and π terms. This raises the question of whether the fine-structure constants’ inverses correspond to the number of bits. The floor function of the inverse of the fine-structure constant α represents the threshold on the atomic number (137) of a hypothetical element *feynmanium* that, in the Bohr model of the atom, still allows the 1s orbital electrons to travel slower than the speed of light. Furthermore, the fine-structure constant has been reported as the quantum of rotation [76]. Thus, two *alphas* between $\alpha^{-1} \approx 137.0363$ and $\alpha_2^{-1} \approx -140.1779$ hinted by the relations (13), (14), and (G1)

$$\tilde{\alpha}^{-1} = 4\pi^3 - \pi^2 + \pi \approx 117.2971,$$

$$\tilde{\alpha}_2^{-1} = -4\pi^3 + \pi^2 - 2\pi \approx -120.4387, \quad (\text{G5})$$

seem intriguing.

It was shown that the spectral density in the phenomenon of sonoluminescence, light emission by sound-induced collapsing gas bubbles in fluids, has the same frequency dependence as black-body radiation [77, 78]. Thus, the sonoluminescence, and in particular *shrimpluminescence* [79], is emitted by collapsing micro-BBOs. A micro-BH induced in glycerin by modulating acoustic waves was recently reported [80]. For example, the relation (G1) yields the wavelength $l = 8\pi/(d_{\text{BH}} \pm 1)$ required for collapsing a BH to the π -bit BH (i.e., to the reduced Planck temperature limit [12], to $d_{\text{BH}} = \pm 1$). Demanding $|l| \geq 1$ we obtain $|d_{\text{BH}}| \leq 8\pi \pm 1$.

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