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Imaginary dimensions in physics require an imaginary set of base Planck units and some negative parameter c_n corresponding to the speed of light in vacuum c. The second, negative fine-structure constant $\alpha_2^{-1} \approx -140.178$ is present in Fresnel coefficients for the normal incidence of electromagnetic radiation on monolayer graphene, leading to these imaginary Planck units, and it establishes $c_n \approx -3.06 \times 10^8$ [m/s]. It follows that electric charges are the same in real and imaginary dimensions. We model neutron stars and white dwarfs, emitting perfect black-body radiation, as *objects* having energy exceeding their mass-energy equivalence ratios. We define complex energies in terms of real and imaginary Planck units. Their imaginary parts, inaccessible for direct observation, store the excess of these energies. It follows that black holes are fundamentally uncharged, charged micro neutron stars and white dwarfs with masses lower than 5.7275×10^{-10} [kg] are inaccessible for direct observation, and the radii of white dwarfs' cores are limited to $R_{\text{WD}} < 6.7933\, GM_{\text{WD}}/c^2$. It is conjectured that the maximum atomic number $Z = 238$. A black-body *object* is in the equilibrium of complex energies of masses, charges, and photons if its radius $R_{\text{eq}} \approx 2.7665\, GM_{\text{BBO}}/c^2$, which corrects the value of the photon sphere radius $R_{\text{ps}} = 3GM/c^2$, by taking into account the value(s) of the fine-structure constant(s), which is otherwise neglected in general relativity. Complex Newton's law of universal gravitation, based on complex energies, leads to the black-body object's surface gravity and the generalized Hawking radiation temperature, which includes its charge. The proposed model explains the registered (GWOSC) high masses of neutron stars' mergers without resorting to any hypothetical types of exotic stellar *objects*.

Keywords: emergent dimensionality; imaginary dimensions; Planck units; fine-structure constant; black holes; neutron stars; white dwarfs; patternless binary messages; complex energy; complex force; Hawking radiation; extended periodic table; photon sphere; general relativity; entropic gravity; gravitational observations; holographic principle; mathematical physics;

I. INTRODUCTION

The universe began with the Big Bang, which is a current prevailing scientific opinion. But this Big Bang was not an explosion of 4-dimensional spacetime, which also is a current prevailing scientific opinion, but an explosion of dimensions. More precisely, in the -1-dimensional void, a 0-dimensional point appeared, inducing the appearance of countably infinitely other points indistinguishable from the first one. The breach made by the first operation of the *dimensional successor function* of the Peano axioms inevitably continued leading to the formation of 1-dimensional, real and imaginary lines allowing for an ordering of points using multipliers of real units (ones) or imaginary units ($a \in \mathbb{R} \Leftrightarrow a = 1b$[1]$, a \in \mathbb{I} \Leftrightarrow a = ib, b \in \mathbb{R}$). Then out of two lines of each kind, crossing each other only at one initial point $(0,0)$, the dimensional successor function formed 2-dimensional $\mathbb{R}^2$, $\mathbb{I}^2$, and $\mathbb{R}\times\mathbb{I}$ Euclidean planes, with $\mathbb{I}^2$ being a mirror reflection of $\mathbb{R}^2$. And so on, forming n-dimensional Euclidean spaces $\mathbb{R}^a \times \mathbb{I}^b$ with $a \in \mathbb{N}$ real and $b \in \mathbb{N}$ imaginary lines, $n := a + b$, and the scalar product defined by

$$\mathbf{x}\cdot\mathbf{y} = \left(x_1,\ldots,x_a, ix'_1,\ldots,ix'_b\right)\left(y_1,\ldots,y_a, iy'_1,\ldots,iy'_b\right) := \\ := \sum_{k=1}^{a} x_k y_k + \sum_{l=1}^{b} x'_l \overline{y'_l}, \tag{1}$$

where $\mathbf{x},\mathbf{y} \in \mathbb{R}^a \times \mathbb{I}^b$. With the onset of the first 0-dimensional point, information began to evolve [1–6].

However, dimensional properties are not uniform. Concerning regular convex n-polytopes in natural dimensions, for example, there are countably infinitely many regular convex polygons, five regular convex polyhedra (Platonic solids), six regular convex 4-polytopes, and only three regular convex n-polytopes if $n > 3$ [7]. In particular, 4-dimensional euclidean space is endowed with a peculiar property known as exotic $\mathbb{R}^4$ [8]. This property allowed for variation of phenotypic traits within populations of individuals [9] and extended the evolution of information into biological evolution. Each biological cell perceives emergent Euclidean $\mathbb{R}^3 \times \mathbb{I}$ space of three real and one imaginary (time) dimension observer-dependently [10] and at present when $i0 = 0$ is *real*; perceived space requires an integer dimensionality [11]. This is the emergent dimensionality (ED) [5, 9, 12–14].

Each dimension requires certain units of measure. In real dimensions, the *natural units of measure* were derived by Max Planck in 1899 as "independent of special *bodies* or *substances*, thereby necessarily retaining their meaning for all times and for all civilizations, including extraterrestrial and non-human ones" [15]. This study derives the complementary set of Planck units applicable for imaginary dimensions, including the imaginary base units. As the speed of electromagnetic radiation is the product of its wavelength and frequency and both these quantities are imaginary in imaginary dimensions, some real but negative parameter $c_n = \nu_i \lambda_i$ corresponding to the speed of light in vacuum c (i.e., the Planck speed) is also necessary as $i^2 = -1$. It turns out that the imaginary Planck energy $E_{\text{P}i}$ and temperature $T_{\text{P}i}$ are larger in moduli than the Planck energy E_{P} and temperature T_{P} setting more favorable conditions for biological evolution to emerge in $\mathbb{R}^3 \times \mathbb{I}$ Euclidean space than in $\mathbb{I}^3 \times \mathbb{R}$ Euclidean one due to the minimum energy principle.

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[1] This is, of course, a circular definition, but it is given for clarity.

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The study shows that the energies of neutron stars and white dwarfs exceed their mass–energy equivalences and that excess energy is stored in imaginary dimensions and is inaccessible to direct observations. This corrects the value of the photon sphere radius and results in the upper bound on the size-to-mass ratio of their cores, where the Schwarzschild radius sets the lower bound.

The paper is structured as follows. Section II shows that Fresnel coefficients for the normal incidence of electromagnetic radiation on monolayer graphene include the second, negative fine-structure constant α_2 as a fundamental constant of nature. Section III shows that by this second fine-structure constant nature endows us with the complementary set of α_2-Planck units. Section IV introduces the concept of a black-body *object* in thermodynamic equilibrium, emitting perfect black-body radiation, and reviews its necessary properties. Section V introduces complex energies of masses, charges, and photons expressed in terms of real and imaginary Planck units introduced in Section III and discusses equilibria formed by comparing their moduli. Section VI applies these equilibria to black-body *objects* to derive the range of their size-to-mass ratios and the equilibrium ratio. Section VII applies this range to the observed mergers of black-body *objects* to show that the observed data is explainable with no need to introduce hypothetical exotic stellar *objects*. Section VIII define complex forces to derive a black-body *object* surface gravity, and the generalized Hawking radiation temperature. Section IX summarizes the findings of this study. Certain prospects for further research are given in the appendices.

II. THE SECOND FINE-STRUCTURE CONSTANT

Numerous publications provide Fresnel coefficients for the normal incidence of electromagnetic radiation (EMR) on monolayer graphene (MLG), which are remarkably defined only by π and the fine-structure constant α

$$\alpha^{-1} = \left(\frac{q_{\mathrm{P}}}{e}\right)^2 = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036, \tag{2}$$

where q_{P} is the Planck charge, ϵ_0 is vacuum permittivity (the electric constant), $\hbar$ is the reduced Planck constant, and e is the elementary charge. Transmittance (T) of MLG

$$T = \frac{1}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.9775, \tag{3}$$

for normal EMR incidence was derived from the Fresnel equation in the thin-film limit [16] (Eq. 3), whereas spectrally flat absorptance (A) $A \approx \pi\alpha \approx 2.3\%$ was reported [17, 18] for photon energies between about 0.5 and 2.5 [eV]. T was related to reflectance (R) [19] (Eq. 53) as $R = \pi^2\alpha^2 T/4$, i.e,

$$R = \frac{\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 1.2843\times 10^{-4}, \tag{4}$$

The above equations for T and R, as well as the equation for the absorptance

$$A = \frac{\pi\alpha}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.0224, \tag{5}$$

were also derived [20] (Eqs. 29-31) based on the thin film model (setting $n_s = 1$ for substrate). The sum of transmittance (3) and the reflectance (4) at normal EMR incidence on MLG was derived [21] (Eq. 4a) as

$$\begin{aligned} T + R &= 1 - \frac{4\sigma\eta}{4 + 4\sigma\eta + \sigma^2\eta^2 + k^2\chi^2} \\ &= \frac{1+\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.9776, \end{aligned} \tag{6}$$

where η is the vacuum impedance

$$\eta = \frac{4\pi\alpha\hbar}{e^2} = \frac{1}{\epsilon_0 c} \approx 376.73\ \ [\Omega], \tag{7}$$

$\sigma = e^2/(4\hbar) = \pi\alpha/\eta$ is the MLG conductivity [22], and $\chi = 0$ is the electric susceptibility of vacuum. These coefficients are thus well-established theoretically and experimentally confirmed [16–18, 21, 23, 24].

As a consequence of the conservation of energy

$$(T + A) + R = 1. \tag{8}$$

In other words, the transmittance in the Fresnel equation describing the reflection and transmission of EMR at normal incidence on a boundary between different optical media is, in the case of the 2-dimensional (boundary) of MLG, modified to include its absorption.

The reflectance $R = 0.013\%$ (4) of MLG can be expressed as a quadratic equation with respect to α

$$\frac{1}{4}(R-1)\pi^2\alpha^2 + R\pi\alpha + R = 0. \tag{9}$$

This quadratic equation has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx 137.036, \quad \text{and} \tag{10}$$

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx -140.178. \tag{11}$$

Therefore, the equation (9) includes the second, negative fine-structure constant α_2. It happens that the sum of the reciprocals of these fine-structure constants (10) and (11)

$$\alpha^{-1} + \alpha_2^{-1} = \frac{\pi - \pi\sqrt{R} - \pi - \pi\sqrt{R}}{2\sqrt{R}} = -\pi, \tag{12}$$

is remarkably independent of the value of the reflectance R. The same result can only be obtained for $T + A$ (cf. Appendix

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A). This result is intriguing in the context of a peculiar algebraic expression for the fine-structure constant [25]

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036303776 \quad (13)$$

that contains a *free* π term and is very close to the physical definition (2) of α^{-1}, which according to the CODATA 2018 value is 137.035999084. Notably, the value of the fine-structure constant is not *constant* but increases with time [26–30]. Thus, the algebraic value given by (13) can be interpreted as the asymptote of the α increase.

Using relations (12) and (13), we can express the negative reciprocal of the 2nd fine-structure constant α_2^{-1} that emerged in the quadratic equation (9) also as a function of π only

$$\alpha_2^{-1} = -\pi - \alpha_1^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.177896429, \quad (14)$$

and this value can also be interpreted as the asymptote of the α_2 decrease, where the current value would amount to $\alpha_2^{-1} \approx -140.177591737$, assuming the rate of change is the same for α and α_2.

Using relations (13) and (14), transmittance T (3), reflectance R (4), and absorptance A (5) of MLG for normal EMR incidence can be expressed just by π. Moreover, equation (9) includes two π-like constants for two surfaces with positive and negative Gaussian curvatures (cf. Appendix B).

III. SET OF α_2-PLANCK UNITS

Planck units can be derived from numerous starting points [5, 31] (cf. Appendix C). The definition of the Planck charge $q_P = \sqrt{4\pi\epsilon_0\hbar c}$ can be solved for the speed of light yielding $c = q_P^2/(4\pi\epsilon_0\hbar)$. Furthermore, the ratio of charges in the definition of the fine-structure constant $\alpha = e^2/q_P^2$ (2) applied for the negative α_2, requires an introduction of some imaginary Planck charge q_{Pi} so that its square would yield a negative value of α_2

$$\frac{q_{Pi}^2}{e^2} = \alpha_2^{-1} \approx -140.178, \quad (15)$$

and since the elementary charge e is real

$$q_{Pi} = \pm\sqrt{\frac{e^2}{\alpha_2}} = \pm\sqrt{4\pi\epsilon_0\hbar c_n}. \quad (16)$$

Among the physical constants of the $\sqrt{4\pi\epsilon_0\hbar c_n}$ term, almost all are positive[2]. Only the $c_n = \nu_i\lambda_i$ parameter, corresponding to the speed of light c, is negative as both frequency ν_i and wavelength λ_i are imaginary in imaginary dimensions. Therefore, the equation (16) can be solved for c_n yielding

$$c_n = q_{Pi}^2/(4\pi\epsilon_0\hbar) \approx -3.066653 \times 10^8 \text{ [m/s]}, \quad (17)$$

which is greater than the speed of light in vacuum c in modulus[3]. We also note that c is defined by the electric constant ϵ_0 and the magnetic constant μ_0 as $c = 1/\sqrt{\epsilon_0\mu_0}$; a square root is bivalued and the value of μ_0 depends on α. Furthermore, c is defined by α-dependent vacuum impedance (7).

The negative parameter c_n (17) leads to the imaginary Planck charge q_{Pi}, length ℓ_{Pi}, mass m_{Pi}, time t_{Pi}, and temperature T_{Pi} that redefined by square roots containing c_n raised to an odd (1, 3, 5) power become imaginary and bivalued

$$q_{Pi} = \pm\sqrt{4\pi\epsilon_0\hbar c_n} = \pm q_P\sqrt{\frac{\alpha}{\alpha_2}} \approx \approx \pm i1.8969 \times 10^{-18} \text{ [C]} \quad (|q_{Pi}| > |q_P|), \quad (18)$$

$$\ell_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^3}} = \pm\ell_P\sqrt{\frac{\alpha_2^3}{\alpha^3}} \approx \approx \pm i1.5622 \times 10^{-35} \text{ [m]} \quad (|\ell_{Pi}| < |\ell_P|), \quad (19)$$

$$m_{Pi} = \pm\sqrt{\frac{\hbar c_n}{G}} = \pm m_P\sqrt{\frac{\alpha}{\alpha_2}} \approx \approx \pm i2.2012 \times 10^{-8} \text{ [kg]} \quad (|m_{Pi}| > |m_P|), \quad (20)$$

$$t_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^5}} = \pm t_P\sqrt{\frac{\alpha_2^5}{\alpha^5}} \approx \approx \pm i5.0942 \times 10^{-44} \text{ [s]} \quad (|t_{Pi}| < |t_P|), \quad (21)$$

$$T_{Pi} = \pm\sqrt{\frac{\hbar c_n^5}{G k_B^2}} = \pm T_P\sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \approx \pm i1.4994 \times 10^{32} \text{ [K]} \quad (|T_{Pi}| > |T_P|), \quad (22)$$

and furthermore can be expressed, using the relation (31), in terms of base Planck units q_P, ℓ_P, m_P, t_P, and T_P.

Planck units derived from the imaginary base units (19)-(21) are generally not imaginary. The α_2 Planck volume

$$\ell_{Pi}^3 = \pm\left(\frac{\hbar G}{c_n^3}\right)^{3/2} = \pm\ell_P^3\sqrt{\frac{\alpha_2^9}{\alpha^9}} \approx \approx \pm i3.8127 \times 10^{-105} \text{ [m}^3\text{]} \quad \left(|\ell_{Pi}^3| < |\ell_P^3|\right), \quad (23)$$

the α_2 Planck momentum

$$p_{Pi} = \pm m_{Pi}c_n = \pm\sqrt{\frac{\hbar c_n^3}{G}} = \pm m_P c\sqrt{\frac{\alpha^3}{\alpha_2^3}} \approx \approx \pm i6.7504 \text{ [kg m/s]} \quad (|m_{Pi}c_n| > |m_P c|), \quad (24)$$

[2] Vacuum permittivity ϵ_0 is the value of the absolute dielectric permittivity of classical vacuum. Thus, ϵ_0 cannot be negative. The Planck constant h is the uncertainty principle parameter. Thus, it cannot be negative.

[3] Their average $(c + c_n)/2 \approx -3.436417 \times 10^6$ [m/s] is in the range of the Fermi velocity.

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the α_2 Planck energy

$$E_{\mathrm{P}i} = \pm m_{\mathrm{P}i} c_n^2 = \pm\sqrt{\frac{\hbar c_n^5}{G}} = \pm E_{\mathrm{P}}\sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \approx \pm i 2.0701\times 10^{9}\ [\mathrm{J}] \quad (|E_{\mathrm{P}i}| > |E_{\mathrm{P}}|)\,, \tag{25}$$

and the α_2 Planck acceleration

$$a_{\mathrm{P}i} = \pm\frac{c_n}{t_{\mathrm{P}i}} = \pm\sqrt{\frac{c_n^7}{\hbar G}} = \pm a_{\mathrm{P}}\sqrt{\frac{\alpha^7}{\alpha_2^7}} \approx \approx \pm i 6.0198\times 10^{51}\ [\mathrm{m/s^2}] \quad (|a_{\mathrm{P}i}| > |a_{\mathrm{P}}|)\,, \tag{26}$$

are imaginary and bivalued. However, the α_2 Planck force

$$F_{\mathrm{P}2} = \pm\frac{E_{\mathrm{P}i}}{\ell_{\mathrm{P}i}} = \pm\frac{c_n^4}{G} = \pm F_{\mathrm{P}}\frac{\alpha^4}{\alpha_2^4} \approx \approx \pm 1.3251\times 10^{44}\ [\mathrm{N}] \quad (|F_{\mathrm{P}2}| > |F_{\mathrm{P}}|)\,, \tag{27}$$

and the α_2 Planck density

$$\rho_{\mathrm{P}2} = \pm\frac{m_{\mathrm{P}i}}{\ell_{\mathrm{P}i}^3} = \pm\frac{c_n^5}{\hbar G^2} = \pm\rho_{\mathrm{P}}\frac{\alpha^5}{\alpha_2^5} \approx \approx \pm 5.7735\times 10^{96}\ [\mathrm{kg/m^3}] \quad (|\rho_{\mathrm{P}2}| > |\rho_{\mathrm{P}}|)\,, \tag{28}$$

are real and bivalued. On the other hand, the α_2 Planck area

$$\ell_{\mathrm{P}i}^2 = \frac{\hbar G}{c_n^3} = \ell_{\mathrm{P}}^2\frac{\alpha_2^3}{\alpha^3} \approx \approx -2.4406\times 10^{-70}\ [\mathrm{m^2}] \quad \left(|\ell_{\mathrm{P}i}^2| < |\ell_{\mathrm{P}}^2|\right), \tag{29}$$

is strictly negative, while the Planck area ℓ_{P}^2 is strictly positive. In the following, we shall call the units (18)-(29) α_2-Planck units.

Both α_2 and c_n lead to the second, negative vacuum impedance

$$\eta_2 = \frac{4\pi\alpha_2\hbar}{e^2} = \frac{1}{\epsilon_0 c_n} \approx -368.29\ [\Omega] \quad (|\eta_2| < |\eta|)\,. \tag{30}$$

Solving both impedances (7) and (30) for $4\pi\hbar\epsilon_0/e^2$ and comparing with each other yields the following important relation between the speed of light in vacuum c, negative parameter c_n, and the fine-structure constants α, α_2

$$c\alpha = c_n\alpha_2 \quad (= v_e)\,, \tag{31}$$

where, notably, v_e is the electron's velocity at the first circular orbit in the Bohr model of the hydrogen atom. This is not the only α to α_2 relation. Along with the two π-like constants π_1, π_2 (relations (B8) and (B10), cf. Appendix B)

$$\frac{\alpha_2}{\alpha} = \frac{c}{c_n} = \frac{\pi_1}{\pi} = \frac{\pi}{\pi_2} \approx -0.9776. \tag{32}$$

The relations between time (21) and temperature (22) α_2-Planck units are inverted, $\alpha^5 t_{\mathrm{P}i}^2 = \alpha_2^5 t_{\mathrm{P}}^2$, $\alpha_2^5 T_{\mathrm{P}i}^2 = \alpha^5 T_{\mathrm{P}}^2$, and saturate Heisenberg's uncertainty principle (energy-time version) taking energy from the equipartition theorem for one degree of freedom (or one bit of information [5, 32])

$$\frac{1}{2}k_{\mathrm{B}}T_{\mathrm{P}}t_{\mathrm{P}} = \frac{1}{2}k_{\mathrm{B}}T_{\mathrm{P}i}t_{\mathrm{P}i} = \frac{\hbar}{2}. \tag{33}$$

Furthermore, eliminating α and α_2 from the relations (18)-(20), yield

$$\frac{q_{\mathrm{P}}^2}{m_{\mathrm{P}}^2} = \frac{q_{\mathrm{P}i}^2}{m_{\mathrm{P}i}^2} = 4\pi\epsilon_0 G, \tag{34}$$

and

$$\ell_{\mathrm{P}} m_{\mathrm{P}}^3 = \ell_{\mathrm{P}i} m_{\mathrm{P}i}^3 \quad \text{and} \quad \ell_{\mathrm{P}} q_{\mathrm{P}}^3 = \ell_{\mathrm{P}i} q_{\mathrm{P}i}^3. \tag{35}$$

Base Planck units themselves admit negative values as negative square roots. By choosing complex analysis, within the framework of ED, we enter into bivalence by the very nature of this analysis. All geometric *objects* have both positive and negative volumes and surfaces [14] equal in moduli. On the other hand, imaginary and negative physical quantities are the subject of research. In particular, the subject of scientific research is thermodynamics in the complex plane. Lee–Yang zeros, for example, have been experimentally observed [33, 34]. We note here that the imaginary Planck Units are not imaginary due to being multiplied by the imaginary unit i. They are imaginary due to the negativity of odd powers of c_n being the square root argument; thus, they define imaginary physical quantities inaccessible to direct measurements[4]. They do not apply only to the time dimension but to any imaginary dimension. However, in our four-dimensional Euclidean $\mathbb{R}^3\times\mathbb{I}$ space-time, Planck units apply in general to the spatial dimensions, while the imaginary ones in general to the imaginary temporal dimension. All the α_2-Planck units have physical meanings. However, some are elusive, like the negative area or imaginary volume, which require two or three orthogonal imaginary dimensions.

Planck charge relations (2) and (16) imply that the elementary charge e is the same both in real and imaginary dimensions since

$$e^2 = \alpha q_{\mathrm{P}}^2 = \alpha_2 q_{\mathrm{P}i}^2. \tag{36}$$

There is no physically meaningful *elementary mass* $M_e = \pm 1.8592\times 10^{-9}$ [kg] that would satisfy the relation (20)

$$M_e^2 = \alpha m_{\mathrm{P}}^2 = \alpha_2 m_{\mathrm{P}i}^2. \tag{37}$$

Neither is there a physically meaningful *elementary* (and imaginary) *length* $L_e \approx \pm i 9.7382\times 10^{-39}$ [m] satisfying the relation (29)

$$L_e^2 = \alpha^3\ell_{\mathrm{P}i}^2 = \alpha_2^3\ell_{\mathrm{P}}^2, \tag{38}$$

[4] Quantum measurement outcomes are *real* eigenvalues of hermitian operators.

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(which in modulus is almost 1660 times smaller than the Planck length), or an *elementary temperature* $T_e \approx \pm 6.4450 \times 10^{26}$ [K] abiding to (22)

$$T_e^2 = \alpha^5 T_P^2 = \alpha_2^5 T_{Pi}^2, \tag{39}$$

and close to the Hagedorn temperature of grand unified string models.

Thus, as to the modulus, charges are the same in real and imaginary dimensions, while masses, lengths, temperatures, and other derived quantities that can vary with time, may differ (the dimensional character of the charges is additionally emphasized by the real $\sqrt{\alpha}$ multiplied by i in the imaginary charge energy (54) and imaginary $\sqrt{\alpha_2}$ in the real charge energy (55)). We note that the same form of the relations (36) and (37) reflect the same form of Coulomb's law and Newton's law of gravity, which are inverse-square laws.

IV. BLACK BODY OBJECTS

There are only three observable *objects* in nature that emit perfect black-body radiation: unsupported black holes (BHs, the densest), neutron stars (NSs), supported, as it is accepted, by neutron degeneracy pressure, and white dwarfs (WDs), supported by electron degeneracy pressure (the least dense). We shall collectively call them black-body *objects* (BBOs). This term is not used in standard cosmology, but standard cosmology scrunches under embarrassingly significant failings, not just *tensions* as is sometimes described, as if to somehow imply that a resolution will eventually be found [35]. It has recently been experimentally confirmed that the so-called *accretion instability* is a fundamental physical process [36]. We conjecture that this process is common for all BBOs. Furthermore, the term *object* as a collection of *matter* is a misnomer, as it neglects quantum nonlocality [37] that is independent of the entanglement among the *particles* [38]. Thus we use emphasis for (indistinguishable) *particle* and (distinguishable) *object*, as well as for *matter* and *distance*. These terms have no absolute meaning in ED. In particular, given the recent observation of *quasiparticles* in classical systems [39].

As black-body radiation is radiation of global thermodynamic equilibrium, it is patternless (thermal noise) radiation that depends only on one parameter. In the case of BHs, this is known as Hawking radiation and this parameter is the BH temperature $T_{BH} = T_P/(2\pi d_{BH})$ corresponding to the BH diameter [5] $D_{BH} = d_{BH}\ell_P$, where $d_{BH} \in \mathbb{R}$. As black-body radiation is patternless, the triangulated [5] BBOs contain a balanced number of Planck area triangles, each carrying binary potential $\delta\varphi_k = -c^2 \cdot \{0, 1\}$, as it has been shown for BHs, based on Bekenstein-Hawking (BH) entropy. BH entropy can be derived from the Bekenstein bound

$$S \leq \frac{2\pi k_B R E}{\hbar c}, \tag{40}$$

which defines an upper limit on the thermodynamic entropy S that can be contained within a sphere of radius R having energy E. After plugging the BH (Schwarzschild) radius $R_{BH} = 2GM_{BH}/c^2$ and mass-energy equivalence $E_{BH} = M_{BH}c^2$, where M_{BH} is the BH mass, into the bound (40), it reduces BH entropy. In other words, BH entropy saturates the Bekenstein bound (40).

The patternless nature of the perfect black-body radiation was derived [5] by comparing BH entropy with the binary entropy variation $\delta S = k_B N_1/2$ ([5] Eq. (55)), valid for any holographic sphere, where $N_1 \in \mathbb{N}$ denotes the number of active Planck triangles with binary potential $\delta\varphi_k = -c^2$. Thus, the entropy of all BBOs is

$$S_{BBO} = \frac{1}{4} k_B N_{BBO}, \tag{41}$$

where $N_{BBO} := 4\pi R_{BBO}^2/\ell_P^2 = \pi d_{BBO}^2$ is the information capacity of the BBO surface, i.e., the $\lfloor N_{BBO} \rfloor \in \mathbb{N}$ Planck triangles (where "$\lfloor x \rfloor$" is the floor function that yields the greatest integer less than or equal to its argument x) corresponding to bits of information [32, 40, 41], and the fractional part triangle(s) having the area $\{N_{BBO}\}\ell_P^2 = (N_{BBO} - \lfloor N_{BBO} \rfloor)\ell_P^2$ to small to carry a single bit of information. Furthermore, $N_1 = N_{BBO}/2$.

We shall define the generalized radius of a BBO having mass M_{BBO} as a function of GM_{BBO}/c^2 multiplier $k \in \mathbb{R}$

$$R_{BBO} := k \frac{GM_{BBO}}{c^2}, \tag{42}$$

and the generalized BBO energy E_{BBO} as a function of $M_{BBO}c^2$ multiplier $a \in \mathbb{R}$

$$E_{BBO} := a M_{BBO} c^2. \tag{43}$$

Plugging definitions (42) and (43) into the Bekenstein bound (40) it becomes

$$S \leq \frac{1}{2} k_B \frac{a}{k} N_{BBO}, \tag{44}$$

and equals the BBO entropy (41) if $\frac{a}{2k} = \frac{1}{4} \Rightarrow a = \frac{k}{2}$. Thus, the energy of all BBOs having a radius (42) is

$$E_{BBO} = \frac{k}{2} M_{BBO} c^2, \tag{45}$$

with $k = 2$ in the case of BHs and $k > 2$ for NSs and WDs. We shall further call the coefficient k the *size-to-mass ratio*.

BHs are fundamentally uncharged since the parameters of any conceivable BH, in particular charged (Reissner–Nordström) and charged-rotating (Kerr–Newman) BH, can be altered arbitrarily, provided that the BH area does not decrease [42] by means of Penrose processes [43, 44] to extract BH electrostatic and/or rotational energy [45]. Thus any BH is defined by only one real parameter: its diameter (cf. [5] Fig. 2(b)), mass, temperature, energy, etc., each corresponding to the other. We note that in the complex Euclidean $\mathbb{R}^3 \times \mathbb{I}$ space, an n-ball ($n \in \mathbb{C}$) is spherical only for a vanishing imaginary dimension [14]. As the interiors of the BBOs are inaccessible to an exterior observer [40], BBOs do not have interiors[5], which makes them similar to interior-less mathematical points. Yet, a BH can embrace this defining parameter.

[5] Thus, the term *object* is a particularly staring misnomer if applied to BBOs.

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That means that three points forming a Planck triangle corresponding to a bit of information on a BH surface can store this parameter and this is intuitively comprehensible: the area of a spherical triangle is larger than that of a flat triangle defined by the same vertices, providing the curvature is nonvanishing, and depends on this curvature, i.e., this additional parameter defines it.

On the other hand, it is accepted that in the case of NSs, electrons combine with protons to form neutrons so that NSs are composed almost entirely of neutrons. But it is never the case that all electrons and all protons of an NS become neutrons. WDs are charged by definition as they are accepted to be composed mostly of electron-degenerate *matter*. But how can a charged BBO store both the curvature and an additional parameter corresponding to its charge? Fortunately, the relation (36) ensures that charges are the same in real and imaginary dimensions. Therefore each Planck triangle of a BBO surface is associated with three $\mathbb{R}\times\mathbb{I}$ Planck triangles, each sharing a vertex or two vertices with this triangle in $\mathbb{R}^2$. And this configuration is capable of storing both the curvature and the charge. The Planck triangle ℓ_P^2 and the $\mathbb{R}\times\mathbb{I}$ imaginary Planck triangle $\ell_P\ell_{Pi} = \ell_P^2\sqrt{\alpha_2^3/\alpha^3}$, which has a smaller area in modulus, can be considered in a polyspherical coordinate system, in which gravitation/acceleration acts in a radial direction (with the entropic gravitation acting inwardly and acceleration acting in both radial directions) [5], while electrostatics act in a tangential direction.

Not only BBOs are perfectly spherical. Also, their mergers, to which we shall return in Section VII, are perfectly spherical, as it has been recently experimentally confirmed [46] based on the registered gravitational event GW170817. One can hardly expect a collision of two perfectly spherical, patternless thermal noises to produce some aspherical pattern instead of another perfectly spherical patternless noise. Where would the information about this pattern come from at the moment of the collision? From the point of impact? No point of impact is distinct on a patternless surface.

The hitherto considerations may be unsettling for the reader, as the energy (45) of BBOs other than BHs (i.e., for $k > 2$) exceeds mass-energy equivalence $E = Mc^2$, which is the limit of the maximum *real* energy. In the subsequent section, we shall show that a part of the energy of NSs and WDs is imaginary and thus unmeasurable.

V. COMPLEX ENERGIES AND EQUILIBRIA

A complex energy formula

$$E_R := E_{M_R} + iE_{Q_R} = M_R c^2 + \frac{iQ_R}{2\sqrt{\pi\epsilon_0 G}}c^2, \tag{46}$$

where E_{M_R} and iE_{Q_R} represent respectively real and imaginary energy of an *object* having mass M_R and charge Q_R[6] was proposed in [47]. Equation (46) considers real (i.e., physically measurable) masses M_R and charges Q_R. We shall modify it to a form involving real and imaginary physical quantities expressing them, where deemed appropriate, by Planck units

$$\begin{aligned} M &:= m m_P, & M_i &:= m_i m_{Pi}, & m, m_i &\in \mathbb{R}, \\ Q &:= qe, & Q_i &:= iQ = iqe, & q &\in \mathbb{Z} \\ \lambda &:= l\ell_P, & \lambda_i &:= l_i \ell_{Pi}, & l, l_i &\in \mathbb{R}, \end{aligned} \tag{47}$$

where uppercase M, Q, and λ denote respectively masses, charges, and wavelengths, while the subscripts i refer to imaginary quantities. We note that the discretization of charges by integer multipliers q of the elementary charge e is far-fetched, considering the fractional charges of *quasiparticles*.

We define the following two complex energies, the complex energy of real mass and imaginary charge

$$\begin{aligned} E_{MQ_i} &:= E_M + E_{Q_i} = Mc^2 + \frac{Q_i}{2\sqrt{\pi\epsilon_0 G}}c^2 = \\ &= \left(m m_P + iq\sqrt{\alpha} m_P\right)c^2 = \left(m + iq\sqrt{\alpha}\right)E_P, \end{aligned} \tag{48}$$

of real charge and imaginary mass

$$\begin{aligned} E_{QM_i} &:= E_Q + E_{M_i} = \frac{Q}{2\sqrt{\pi\epsilon_0 G}}c_n^2 + M_i c_n^2 = \\ &= \left(q\sqrt{\alpha_2} m_{Pi} + m_i m_{Pi}\right)c_n^2 = \frac{\alpha^2}{\alpha_2^2}\left(q\sqrt{\alpha} + \sqrt{\frac{\alpha}{\alpha_2}} m_i\right)E_P, \end{aligned} \tag{49}$$

of real photon (energy or frequency ν) and imaginary mass

$$E_{FM_i} := h\nu + M_i c_n^2 = \left(f + \sqrt{\frac{\alpha^5}{\alpha_2^5}} m_i\right)E_P, \tag{50}$$

of real photon and imaginary charge

$$E_{FQ_i} := h\nu + \frac{Q_i}{2\sqrt{\pi\epsilon_0 G}}c^2 = \left(f + iq\sqrt{\alpha}\right)E_P, \tag{51}$$

of real mass and imaginary photon (with frequency $\nu_i = c_n/\lambda_i$)

$$E_{MF_i} := Mc^2 + \frac{hc_n}{\lambda_i} = \left(m + \sqrt{\frac{\alpha^5}{\alpha_2^5}} f_i\right)E_P, \tag{52}$$

and of real charge and imaginary photon

$$E_{QF_i} := \frac{Q}{2\sqrt{\pi\epsilon_0 G}}c_n^2 + h\nu_i = \frac{\alpha^2}{\alpha_2^2}\left(q\sqrt{\alpha} + \sqrt{\frac{\alpha}{\alpha_2}} f_i\right)E_P, \tag{53}$$

where $h\nu = 2\pi\hbar\frac{c}{\lambda} = \frac{2\pi}{l}E_P := fE_P$, $h\nu_i := f_i E_{Pi}$, $f \in \mathbb{R}$.

Complex energies (48)-(53) link mass, charge, and photon energies within the framework of ED. We note in passing that using the different speed of light parameters in energies (48) and (**??**) yields a contradiction (cf. Appendix D).

Energies (48), (49), (51), and (53) yield two different quanta of the charge energies corresponding to the elementary charge, the imaginary quantum

$$E_{Q_i}(q = \pm 1) = \pm i\sqrt{\alpha}E_P \approx \pm i1.6710\times 10^8 \text{ [J]}, \tag{54}$$

[6] Charges in the cited study are defined in CGS units. Here we adopt SI.

and the - larger in modulus - real quantum

$$E_Q(q=\pm 1) = \pm\sqrt{\alpha_2}E_{\mathrm{P}i} \approx \pm 1.7684\times 10^8 \text{ [J]}. \tag{55}$$

Furthermore, $\forall q$, $\alpha^2 E_{Qi} = i\alpha_2^2 E_Q$. We note that photon energy vanishes for the infinite wavelength.

The squared moduli of the energies (48)-(53) are

$$|E_{MQ_i}|^2 = \left(M^2 + q^2\alpha m_{\mathrm{P}}^2\right)c^4 = \left(m^2 + q^2\alpha\right)E_{\mathrm{P}}^2, \tag{56}$$

$$|E_{QM_i}|^2 = \frac{\alpha^4}{\alpha_2^4}\left(q^2\alpha m_{\mathrm{P}}^2 - M_i^2\right)c^4 = \frac{\alpha^4}{\alpha_2^4}\left(q^2\alpha - \frac{\alpha}{\alpha_2}m_i^2\right)E_{\mathrm{P}}^2, \tag{57}$$

$$|E_{FM_i}|^2 = \left(f^2 - \frac{\alpha^5}{\alpha_2^5}m_i^2\right)E_{\mathrm{P}}^2, \tag{58}$$

$$|E_{MF_i}|^2 = \left(m^2 - \frac{\alpha^5}{\alpha_2^5}f_i^2\right)E_{\mathrm{P}}^2. \tag{59}$$

$$|E_{FQ_i}|^2 = \left(f^2 + q^2\alpha\right)E_{\mathrm{P}}^2, \tag{60}$$

$$|E_{QF_i}|^2 = \left(\frac{\alpha^4}{\alpha_2^4}q^2\alpha - \frac{\alpha^5}{\alpha_2^5}f_i^2\right)E_{\mathrm{P}}^2, \tag{61}$$

where we used relations (20), (25), (31), and (47).

Postulating that the squared moduli (56) and (57) are equal

$$\begin{gathered}|E_{MQ_i}|^2 = |E_{QM_i}|^2,\\ \alpha_2^4\left(M^2 + q^2\alpha m_{\mathrm{P}}^2\right) = \alpha^4\left(q^2\alpha m_{\mathrm{P}}^2 - M_i^2\right),\end{gathered} \tag{62}$$

we demand a mass-charge energy equilibrium condition from which we can obtain the value of the imaginary mass M_i as a function of mass M and charge Q in this equilibrium

$$M_i = \pm\sqrt{q^2\alpha m_{\mathrm{P}}^2\left(1 - \frac{\alpha_2^4}{\alpha^4}\right) - \frac{\alpha_2^4}{\alpha^4}M^2}. \tag{63}$$

In particular for $q=0$ this yields

$$M_i\alpha^2 = \pm iM\alpha_2^2 \quad \text{or} \quad M_i = \pm i\frac{\alpha_2^2}{\alpha^2}M \approx \pm 0.9557iM. \tag{64}$$

Since mass M_i is imaginary by definition, the argument of the square root in the relation (63) must be negative

$$M > |q|m_{\mathrm{P}}\sqrt{\alpha\left(\frac{\alpha^4}{\alpha_2^4} - 1\right)} \approx |q|5.7275\times 10^{-10} \text{ [kg]}. \tag{65}$$

This means that masses of uncharged micro BHs ($q=0$) in thermodynamic equilibrium can be arbitrary. However, micro NSs and micro WDs, also in thermodynamic equilibrium, are inaccessible for direct observation, as they cannot achieve a net charge $Q=0$. Even a single elementary charge of a white dwarf renders its mass $M_{\mathrm{WD}} = 5.7275\times 10^{-10}$ [kg] comparable to the mass of a grain of sand.

We note here that only the masses satisfying $M < 2\pi m_{\mathrm{P}} \approx 1.3675\times 10^{-7}$ [kg] have Compton wavelengths larger than the Planck length [5]. Comparing this bound with the bound (65) yields the charge multiplier q corresponding to an atomic number

$$Z = \left\lfloor \frac{2\pi}{\sqrt{\alpha\left(\frac{\alpha^4}{\alpha_2^4} - 1\right)}} \right\rfloor = 238, \tag{66}$$

of a hypothetical element, which - as we conjecture - sets the limit on an extended periodic table and is a little higher than the accepted limit of $Z=184$ (unoctquadium). More massive elements would have Compton wavelengths smaller than the Planck length, which is physically implausible.

Postulating that the squared moduli (60) and (61) are equal

$$\begin{gathered}|E_{FQ_i}|^2 = |E_{QF_i}|^2,\\ \alpha_2^4\left(f^2 + q^2\alpha\right) = \alpha^4\left(q^2\alpha - \frac{\alpha}{\alpha_2}f_i^2\right),\end{gathered} \tag{67}$$

we demand a photon-charge energy equilibrium condition from which we can obtain the value of the imaginary photon energy $h\nu_i$ corresponding to the real photon energy $h\nu$ and charge Q in this equilibrium

$$f_i = \pm\sqrt{\frac{\alpha_2^5}{\alpha^5}}\sqrt{q^2\alpha\left(\frac{\alpha^4}{\alpha_2^4} - 1\right) - f^2}. \tag{68}$$

Since $\sqrt{\alpha_2^5/\alpha^5}$ is imaginary, we demand $q^2\alpha(\alpha^4/\alpha_2^4 - 1) < f^2$ to ensure that $f_i \in \mathbb{R}$. Thus

$$h\nu = fE_{\mathrm{P}} > \pm q\sqrt{\alpha\left(\frac{\alpha^4}{\alpha_2^4} - 1\right)}E_{\mathrm{P}} \approx \pm q5.1477\times 10^7 \text{ [J]}, \tag{69}$$

which, using mass-energy equivalence, corresponds to the bound (65). We can also obtain the maximum wavelength in this equilibrium corresponding to the charge. For $q^2 = 1$ it is $\lambda < 3.8589\times 10^{-33}$ [m] with $l < 238.7580$ corresponding to the bound (66).

It seems that no meaningful conclusions can be derived by postulating the equality of the squared moduli (58) and (59). Such a mass-photon energy equilibrium is an equation with four unknowns. Neither physically meaningful elementary mass (37) nor length (38) is common for real and imaginary dimensions.

Postulating the equality of all the squared moduli (56)-(61) to some constant energy

$$\begin{gathered}|E_{MQ_i}|^2 = |E_{QM_i}|^2 = |E_{FM_i}|^2 =\\ = |E_{FQ_i}|^2 = |E_{MF_i}|^2 = |E_{QF_i}|^2 := AE_{\mathrm{P}}^2, \quad A\in\mathbb{R},\end{gathered} \tag{70}$$

we demand a mass-charge-photon equilibrium condition, which can be solved for A. Subtracting moduli (56) and (60)

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yields $m^2 = f^2$, and similarly subtracting moduli (57) and (61) yields $m_i^2 = f_i^2$. This equates moduli (58) and (59). Substituting $m_i^2 = f_i^2$ into the modulus (61) and subtracting from the modulus (56) yields

$$m^2 + \frac{\alpha}{\alpha_2} m_i^2 = A\left(1 - \frac{\alpha_2^4}{\alpha^4}\right). \tag{71}$$

Subtracting this from (58) or (59) yields

$$m_i^2 = f_i^2 = \frac{-A\alpha_2^9}{\alpha^5(\alpha^4 + \alpha_2^4)}, \tag{72}$$

which substituted into the relation (71) yields

$$m^2 = f^2 = \frac{A\alpha^4}{\alpha^4 + \alpha_2^4}. \tag{73}$$

Finally, substituting the relation (73) into the modulus (56) yields

$$q^2\alpha = \frac{A\alpha_2^4}{\alpha^4 + \alpha_2^4}. \tag{74}$$

VI. BBO COMPLEX ENERGY EQUILIBRIA

We can interpret the modulus of the generalized energy of BBOs (45) as the modulus of the complex energy of real mass (56), taking the observable real energy $E_{\rm BBO} = M_{\rm BBO}c^2$ of the BBO as the real part of this energy. Thus

$$\left(\frac{k}{2} M_{\rm BBO} c^2\right)^2 = \left(M_{\rm BBO}^2 + q_{\rm BBO}^2 \alpha m_{\rm P}^2\right) c^4, \tag{75}$$

leads to

$$q_{\rm BBO} = \pm \frac{M_{\rm BBO}}{m_{\rm P}} \sqrt{\frac{1}{\alpha}\left(\frac{k^2}{4} - 1\right)}, \tag{76}$$

representing a charge surplus energy exceeding $M_{\rm BBO}c^2$. For $k = 2$, $q_{\rm BBO}$ vanishes, confirming the vanishing net charge of BHs. Similarly, we can interpret the modulus of the generalized energy of BBOs (45) as the modulus of the complex energy of real charge (57). Thus

$$\begin{aligned} \frac{k^2}{4} M_{\rm BBO}^2 &= \frac{\alpha^4}{\alpha_2^4}\left(q_{\rm BBO}^2 \alpha m_{\rm P}^2 - M_{i\rm BBO}^2\right), \\ M_{i\rm BBO}^2 &= q_{\rm BBO}^2 \alpha m_{\rm P}^2 - \frac{\alpha_2^4}{\alpha^4}\frac{k^2}{4} M_{\rm BBO}^2. \end{aligned} \tag{77}$$

Substituting $q_{\rm BBO}^2$ from the relation (76) into the relation (77) turns the equilibrium condition (63) into a function of the size-to-mass ratio k instead of the charge q

$$\begin{aligned} M_{i\rm BBO}^2 &= \left[\frac{k^2}{4}\left(1 - \frac{\alpha_2^4}{\alpha^4}\right) - 1\right] M_{\rm BBO}^2, \\ M_{i\rm BBO} &= \pm M_{\rm BBO} \sqrt{\frac{k^2}{4}\left(1 - \frac{\alpha_2^4}{\alpha^4}\right) - 1}, \end{aligned} \tag{78}$$

which for BHs ($k = 2$) also corresponds to the relation (64) between uncharged masses M and M_i, where no assumptions concerning the BBO energy were made.

Furthermore, the argument of the square root in the relation (78) must be negative, as mass M_i is imaginary by definition. This leads to the maximum size-to-mass ratio

$$k_{\rm max} = \pm \frac{2}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \approx 6.7933, \tag{79}$$

where $k < |k_{\rm max}|$ satisfies the mass equilibrium (78). Relations (76) and (78) are shown in Fig 1.

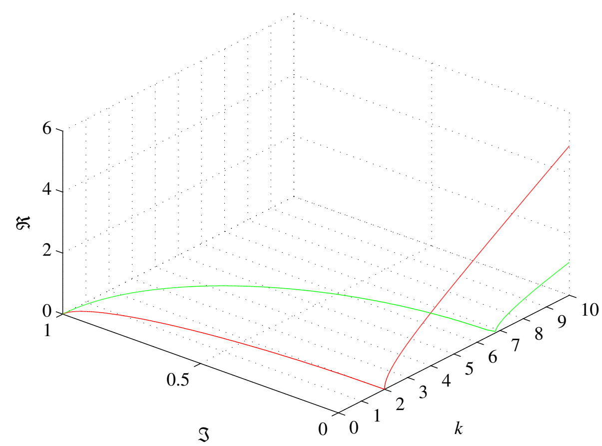

Figure 1. Ratios of imaginary mass $M_{i\rm BBO}$ to real mass $M_{\rm BBO}$ (green) and real charge $q_{\rm BBO} m_{\rm P}\sqrt{\alpha}$ to $M_{\rm BBO}$ (red) of a BBO as a function of the size-to-mass ratio $k : 0 \leq k \leq 10$. Mass $M_{i\rm BBO}$ is imaginary for $k \lessapprox 6.79$. Charge $q_{\rm BBO}$ is real for $k \geq 2$.

The maximum size-to-mass ratio $k_{\rm max}$ (79) sets the bounds on the BBO energy (45), mass, and radius (42)

$$R_{\rm BH} = \frac{2GM_{\rm BBO}}{c^2} \leq R_{\rm BBO} < \frac{k_{\rm max} G M_{\rm BBO}}{c^2}. \tag{80}$$

In particular, using relations (47), $2m_{\rm BBO} \leq r_{\rm BBO} < k_{\rm max} m_{\rm BBO}$ or $r_{\rm BBO}/k_{\rm max} < m_{\rm BBO} \leq r_{\rm BBO}/2$. As WDs are the least dense BBOs, this bounds define the maximum radius and mass of a WD core.

Furthermore, relations (65) and (79) set the bound on the BBO minimum mass in the equilibrium (62)

$$m_{\rm BBO} > \max\left\{ q_{\rm BBO}\sqrt{\alpha\left(\frac{\alpha^4}{\alpha_2^4} - 1\right)}, \frac{d_{\rm BBO}}{4}\sqrt{1 - \frac{\alpha^4}{\alpha_2^4}} \right\}, \tag{81}$$

where

$$q_{\rm BBO} = \frac{1}{4}\sqrt{\frac{\alpha_2^4}{\alpha^5}} d_{\rm BBO} \tag{82}$$

defines a condition in which neither $q_{\rm BBO}$ nor $d_{\rm BBO}$ can be further increased to reach its counterpart (defined respectively

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by d_{BBO} and q_{BBO}) in the bound (81). Thus, for example, 1-bit BBO ($d_{\mathrm{BBO}} = 1/\sqrt{\pi}$) corresponds to $q_{\mathrm{BBO}} > 1.5780$, π-bit BBO ($d_{\mathrm{BBO}} = 1$) corresponds to $q_{\mathrm{BBO}} > 2.7969$, while the maximum atomic number q_{BBO} (66) corresponds to

$$d_{\mathrm{BBO}} = \pm\frac{8\pi}{\sqrt{1-\frac{\alpha_2^4}{\alpha^4}}} \approx 85.3666. \tag{83}$$

In the case of a BBO, we obtain the equilibrium condition (70) by comparing the squared moduli (56)-(61) of the energies (48)-(53) with the squared BBO energy (45) which yields a solvable system of six nonlinear equations with six unknowns k, q, m, m_i, f, f_i

$$\begin{aligned}
|E_{MQ_i}|^2 &\Rightarrow m^2 + q^2\alpha = \frac{k^2}{4}m^2,\\
& q^2\alpha = m^2\left(\frac{k^2}{4}-1\right),\\
|E_{QM_i}|^2 &\Rightarrow \frac{\alpha^4}{\alpha_2^4}q^2\alpha - \frac{\alpha^5}{\alpha_2^5}m_i^2 = \frac{k^2}{4}m^2,\\
|E_{FM_i}|^2 &\Rightarrow f^2 - \frac{\alpha^5}{\alpha_2^5}m_i^2 = \frac{k^2}{4}m^2,\\
|E_{FQ_i}|^2 &\Rightarrow f^2 + q^2\alpha = \frac{k^2}{4}m^2,\\
|E_{MF_i}|^2 &\Rightarrow m^2\left(1-\frac{k^2}{4}\right) = \frac{\alpha^5}{\alpha_2^5}f_i^2,\\
|E_{QF_i}|^2 &\Rightarrow \frac{\alpha^4}{\alpha_2^4}q^2\alpha - f_i^2\frac{\alpha^5}{\alpha_2^5} = \frac{k^2}{4}m^2.
\end{aligned} \tag{84}$$

Substituting $q^2\alpha = m^2\left(\frac{k^2}{4}-1\right)$ from $|E_{MQ_i}|^2$ to $|E_{FQ_i}|^2$ recovers the Compton wavelength of the BBO, $\lambda_{\mathrm{BBO}} = \frac{h}{M_{\mathrm{BBO}}c}$, in its Planck units form $l^2 = \frac{4\pi^2}{m^2}$. Furthermore, by substituting $q^2\alpha$ and the Compton mass $m^2 = \frac{4\pi^2}{l^2}$ into $|E_{QM_i}|^2$, and comparing the LHSs of $|E_{QM_i}|^2$ and $|E_{FM_i}|^2$ we obtain the BBO equilibrium size-to-mass ratio

$$\frac{k_{\mathrm{eq}}^2}{4} = \frac{\alpha_2^4}{\alpha^4}+1 \Rightarrow k_{\mathrm{eq}} = \pm 2\sqrt{1+\frac{\alpha_2^4}{\alpha^4}} \approx 2.7665, \tag{85}$$

where $k = k_{\mathrm{eq}}$ satisfies the equilibrium condition (70) for

$$A = \frac{1}{4}k_{\mathrm{eq}}^2 m_{\mathrm{BBO}}^2 = \left(1+\frac{\alpha_2^4}{\alpha^4}\right)m_{\mathrm{BBO}}^2 \approx 1.9133 m_{\mathrm{BBO}}^2. \tag{86}$$

The equilibrium k_{eq} (85) and the maximum $k_{\max}$ (79) size-to-mass ratios are related as $k_{\mathrm{eq}}^2 + 16/k_{\max}^2 = 8$. Also, the following relations can be derived from the relations (84) for the BBO in the equilibrium k_{eq} (85)

$$m_i^2 = -\frac{\alpha_2^9}{\alpha^9}m^2 \quad\Leftrightarrow\quad M_{i\mathrm{BBO_{eq}}} = \pm i\frac{\alpha_2^4}{\alpha^4}M_{\mathrm{BBO_{eq}}}, \tag{87}$$

$$l_i^2 = -\frac{\alpha^9}{\alpha_2^9}l^2 \quad\Leftrightarrow\quad \lambda_{i\mathrm{BBO_{eq}}} = \pm i\frac{\alpha^3}{\alpha_2^3}\lambda_{\mathrm{BBO_{eq}}}, \tag{88}$$

$$m^2 = f^2 = \frac{4\pi^2}{l^2} \quad\Leftrightarrow\quad \lambda_{\mathrm{BBO_{eq}}} = \frac{h}{M_{\mathrm{BBO_{eq}}}c}, \tag{89}$$

$$q^2\alpha = \frac{\alpha_2^4}{\alpha^4}m^2. \tag{90}$$

The BBO in the energy equilibrium k_{eq} bearing the elementary charge ($q^2 = 1$) would have mass $M_{\mathrm{BBO_{eq}}} \approx \pm 1.9455 \times 10^{-9}$ [kg], imaginary mass $M_{i\mathrm{BBO_{eq}}} \approx \pm i1.7768 \times 10^{-9}$ [kg], wavelength $\lambda_{\mathrm{BBO_{eq}}} \approx \pm 1.1361 \times 10^{-33}$ [m], and imaginary wavelength $\lambda_{i\mathrm{BBO_{eq}}} \approx \pm i1.2160 \times 10^{-33}$ [m].

These results show that the radius (42) of charged BBOs is a continuous function of $k \in \mathbb{R} : 2 < k < k_{\max}$ satisfying the BBO entropy relation (41), a necessary condition of patternless perfect black body radiation [5].

Notably, $2.25 < k_{\mathrm{eq}} < 3$, where $9/4$ is the size-to-mass ratio of a radius of the maximal sustainable density for gravitating spherical *matter* given by Buchdahl's theorem, and 3 is the size-to-mass ratio of a BH photon sphere radius[7]. This hints that $k_{\mathrm{eq}} \approx 2.766$ is a true photon sphere radius, where BBO gravity, charge, and photon energies remain at equilibrium. Aside from the Schwarzschild radius (derivable from escape velocity $v_{esc}^2 = 2GM/R$ of mass M by setting $v_{esc}^2 = c^2$), all the remaining thresholds of general relativity, such as Buchdahl's threshold or a photon sphere radius, are only crude approximations. General relativity neglects the value of the fine-structure constants α and α_2, which, similarly as π or the base of the natural logarithm, are the fundamental constants of nature.

VII. BBO MERGERS

As the entropy of independent systems is additive, a merger of BBO_1 and BBO_2 having entropies (41) $S_{\mathrm{BBO}_1} = \frac{1}{4}k_B N_{\mathrm{BBO}_1}$ and $S_{\mathrm{BBO}_2} = \frac{1}{4}k_B\pi d_{\mathrm{BBO}_2}^2$, produces a BBO_C having entropy

$$S_{\mathrm{BBO}_1} + S_{\mathrm{BBO}_2} = S_{\mathrm{BBO}_C} \quad\Rightarrow d_{\mathrm{BBO}_1}^2 + d_{\mathrm{BBO}_1}^2 = d_{\mathrm{BBO}_C}^2, \tag{91}$$

which shows that a merger of two primordial BHs, each having the Planck length diameter, the reduced Planck temperature $\frac{T_P}{2\pi}$ (the largest physically significant temperature [12]), and no tangential acceleration a_{LL} [5, 12], produces a BH having $d_{\mathrm{BH}} = \pm\sqrt{2}$ which represents the minimum BH diameter allowing for the notion of time [12]. In comparison, a collision of the latter two BHs produces a BH having $d_{\mathrm{BH}} = \pm 2$ having the triangulation defining only one precise diameter between its poles (cf. [5] Fig. 3(b)), which is also recovered from Heisenberg's Uncertainty Principle (cf. Appendix C).

Substituting the generalized radius (42) into the entropy relation (91) yields

$$k_{\mathrm{BBO}_C}^2 M_{\mathrm{BBO}_C}^2 = k_{\mathrm{BBO}_1}^2 M_{\mathrm{BBO}_1}^2 + k_{\mathrm{BBO}_2}^2 M_{\mathrm{BBO}_2}^2, \tag{92}$$

[7] At which, according to an accepted photon sphere definition, the strength of gravity *forces photons to travel in orbits*. The author wonders why photons would not *travel in orbits* at radius $R = GM/c^2$ corresponding to the *orbital* velocity $v_{\mathrm{orb}}^2 = GM/R$. Obviously, photons do not *travel*.

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which establishes a Pythagorean relation between the generalized energies (45) of the merging components and the merger

$$\frac{k_{\mathrm{BBO}_C}^2}{4} M_{\mathrm{BBO}_C}^2 c^4 = \frac{k_{\mathrm{BBO}_1}^2}{4} M_{\mathrm{BBO}_1}^2 c^4 + \frac{k_{\mathrm{BBO}_2}^2}{4} M_{\mathrm{BBO}_2}^2 c^4. \quad (93)$$

The relation (93) explains the measurements of large masses of the BBO mergers with at least one charged merging component without resorting to any hypothetical types of exotic stellar *objects* such as *quark stars*. We note in passing that describing the registered gravitational events as *waves* is misleading. Normal modulation of the gravitational potential, caused by rotating (in the merger case - inspiral) bodies, is wrongly interpreted as a gravitational wave understood as a carrier of gravity [48]. Interferometric data, available online at the Gravitational Wave Open Science Center (GWOSC) portal[8], indicate that the total mass of a merger is the sum of the masses of the merging components. Thus[9]

$$\begin{aligned} &M_{\mathrm{BBO}_C} \approx M_{\mathrm{BBO}_1} + M_{\mathrm{BBO}_2} \Rightarrow \\ &M_{\mathrm{BBO}_C}^2 \approx M_{\mathrm{BBO}_1}^2 + M_{\mathrm{BBO}_2}^2 + 2M_{\mathrm{BBO}_1} M_{\mathrm{BBO}_2} \Rightarrow \\ &M_{\mathrm{BBO}_C}^2 > M_{\mathrm{BBO}_1}^2 + M_{\mathrm{BBO}_2}^2. \end{aligned} \quad (94)$$

The accepted value of the Chandrasekhar WD mass limit, preventing its collapse into a denser form, is $M_{\mathrm{Ch}} \approx 1.4\ M_\odot$ [49] and the accepted value of the analogous Tolman–Oppenheimer–Volkoff NS mass limit is $M_{\mathrm{TOV}} \approx 2.9\ M_\odot$ [50, 51]. There is no accepted value of the BH mass limit. The conjectured value is $5 \times 10^{10}\ M_\odot$. The masses of most of the registered merging components are well beyond M_{TOV}. Of those that are not, most of the total or final masses exceed this limit. Therefore these mergers were classified as BH mergers. Only a few were classified otherwise, including GW170817, GW190425, GW200105, and GW200115. They are listed in Table I.

Table I. Selected BBO mergers discovered with LIGO and Virgo. Masses in $M_\odot$.

Event	M_{BBO_1}	M_{BBO_2}	M_{BBO_C}	k_{BBO_1}	k_{BBO_2}	k_{BBO_C}
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	2.8	4.39	4.39	3.03
GW190425	$2.00^{+0.6}_{-0.2}$	$1.4^{+0.3}_{-0.3}$	$3.4^{+0.3}_{-0.1}$	4.39	4.39	3.15
GW200105	$8.9^{+1.2}_{-1.5}$	$1.9^{+0.3}_{-0.2}$	$10.9^{+1.1}_{-1.2}$	2.76	4.39	2.38
GW200115	$5.7^{+1.8}_{-2.1}$	$1.5^{+0.7}_{-0.3}$	$7.1^{+1.5}_{-1.4}$	3	4.39	2.64

We can use the BBO equilibrium relations (84) to derive some information from the relation (93). For example, substituting the squared energy modulus $|E_{MQ_i}|^2$ into the relation (93) and using the inequality (94), based on GWOSC data, yields

$$\begin{aligned} &m_{\mathrm{BBO}_C}^2 + q_{\mathrm{BBO}_C}^2 \alpha = m_{\mathrm{BBO}_1}^2 + m_{\mathrm{BBO}_2}^2 + \left(q_{\mathrm{BBO}_1}^2 + q_{\mathrm{BBO}_2}^2\right)\alpha, \\ &q_{\mathrm{BBO}_C}^2 \alpha = m_{\mathrm{BBO}_1}^2 + m_{\mathrm{BBO}_2}^2 + \left(q_{\mathrm{BBO}_1}^2 + q_{\mathrm{BBO}_2}^2\right)\alpha - m_{\mathrm{BBO}_C}^2, \\ &m_{\mathrm{BBO}_C}^2 = \cancel{m_{\mathrm{BBO}_1}^2} + \cancel{m_{\mathrm{BBO}_2}^2} + \left(q_{\mathrm{BBO}_1}^2 + q_{\mathrm{BBO}_2}^2\right)\cancel{\alpha} - q_{\mathrm{BBO}_C}^2\cancel{\alpha} \\ &> \cancel{m_{\mathrm{BBO}_1}^2} + \cancel{m_{\mathrm{BBO}_2}^2}, \\ &q_{\mathrm{BBO}_C}^2 < q_{\mathrm{BBO}_1}^2 + q_{\mathrm{BBO}_2}^2. \end{aligned} \quad (95)$$

On the other hand, substituting the squared energy modulus $|E_{QM_i}|^2$ from the relation (84) and $q_{\mathrm{BBO}_C}^2 \alpha$ from the relation (95) into the relation (93), and using the inequality (94) yields

$$\begin{aligned} &q_{\mathrm{BBO}_C}^2 \alpha - \frac{\alpha}{\alpha_2} m_{\mathrm{iBBO}_C}^2 = \\ &= \left(q_{\mathrm{BBO}_1}^2 + q_{\mathrm{BBO}_2}^2\right)\alpha - \frac{\alpha}{\alpha_2}\left(m_{\mathrm{iBBO}_1}^2 + m_{\mathrm{iBBO}_2}^2\right), \\ &m_{\mathrm{BBO}_1}^2 + m_{\mathrm{BBO}_2}^2 + \cancel{\left(q_{\mathrm{BBO}_1}^2 + q_{\mathrm{BBO}_2}^2\right)}\alpha - m_{\mathrm{BBO}_C}^2 - \frac{\alpha}{\alpha_2} m_{\mathrm{iBBO}_C}^2 = \\ &= \cancel{\left(q_{\mathrm{BBO}_1}^2 + q_{\mathrm{BBO}_2}^2\right)}\alpha - \frac{\alpha}{\alpha_2}\left(m_{\mathrm{iBBO}_1}^2 + m_{\mathrm{iBBO}_2}^2\right), \\ &m_{\mathrm{BBO}_C}^2 = \\ &= \cancel{m_{\mathrm{BBO1}}^2} + \cancel{m_{\mathrm{BBO2}}^2} + \frac{\cancel{\alpha}}{\cancel{\alpha_2}}\left(m_{\mathrm{iBBO}_1}^2 + m_{\mathrm{iBBO}_2}^2\right) - \frac{\cancel{\alpha}}{\cancel{\alpha_2}} m_{\mathrm{iBBO}_C}^2 \\ &> \cancel{m_{\mathrm{BBO}_1}^2} + \cancel{m_{\mathrm{BBO}_2}^2}, \\ &m_{\mathrm{iBBO}_C}^2 < m_{\mathrm{iBBO}_1}^2 + m_{\mathrm{iBBO}_2}^2. \end{aligned} \quad (96)$$

Similarly, the squared energy modulus $|E_{MF_i}|^2$ (84) and the relations (93), (94) yield

$$f_{\mathrm{iBBO}_C}^2 > f_{\mathrm{iBBO}_1}^2 + f_{\mathrm{iBBO}_2}^2. \quad (97)$$

Therefore, the size-to-mass ratio k_{BBO_C} decreases making the BBO_C denser until it becomes a BH for $k_{\mathrm{BBO}_C} = 2$ and no further charge reduction is possible (cf. Fig 1). From the relation (92) and the inequality (94) we see that this holds for

$$k_{\mathrm{BBO}_C}^2 \left(M_{\mathrm{BBO}_1}^2 + M_{\mathrm{BBO}_2}^2\right) < k_{\mathrm{BBO}_1}^2 M_{\mathrm{BBO}_1}^2 + k_{\mathrm{BBO}_2}^2 M_{\mathrm{BBO}_2}^2. \quad (98)$$

From inequalities (94)-(97) we also conjecture that $|q_{\mathrm{BBO}_C}| = ||q_{\mathrm{BBO}_1}| - |q_{\mathrm{BBO}_2}||$, $m_{\mathrm{iBBO}_C} = |m_{\mathrm{iBBO}_1} - m_{\mathrm{iBBO}_2}|$, and $f_{\mathrm{iBBO}_C} = |f_{\mathrm{iBBO}_1} + f_{\mathrm{iBBO}_2}|$. In other words, the merger's real mass and the imaginary photon energy are sums of the merging components' masses and imaginary photon energies. In contrast, the charge and imaginary mass are absolute differences of their charges and imaginary masses.

Table I lists the mass-to-size ratios k_{BBO_C} calculated according to the relation (93) that provide the measured mass M_{BBO_C} of the merger and satisfy the inequality (98). Mass-to-size ratios k_{BBO_1} and k_{BBO_2} of the merging components were arbitrarily selected based on their masses, taking into account the M_{TOV} NS mass limit.

[8] https://www.gw-openscience.org/eventapi/html/allevents
[9] We assume $M > 0$. Negative masses are inaccessible for direct observation, unlike charges.

VIII. BBO COMPLEX GRAVITY AND TEMPERATURE

Complex energies (48)-(53) define complex forces (similarly to the complex energy of real masses and charges (46), [47] Eq. (7)) acting over real and imaginary *distances* R, R_i. Using the relations (47), we obtain the following products

$$\begin{aligned} &E_{1mq_i}E_{2mq_i} := E_{1MQ_i}E_{2MQ_i}/E_P^2 = \\ &= m_1m_2 - q_1q_2\alpha + i\sqrt{\alpha}(m_1q_2 + m_2q_1), \\ &E_{1qm_i}E_{2qm_i} := E_{1QM_i}E_{2QM_i}/E_P^2 = \\ &= \frac{\alpha^5}{\alpha_2^4}\left(q_1q_2 + \frac{1}{\alpha_2}m_{i1}m_{i2} + \frac{1}{\sqrt{\alpha_2}}(q_1m_{i2} + q_2m_{i1})\right), \end{aligned} \tag{99}$$

$$\begin{aligned} &E_{1fm_i}E_{2fm_i} := E_{1FM_i}E_{2FM_i}/E_P^2 = \\ &= f_1f_2 + \frac{\alpha^5}{\alpha_2^5}m_{i1}m_{i2} + \sqrt{\frac{\alpha^5}{\alpha_2^5}}(f_1m_{i2} + f_2m_{i1}), \\ &E_{1mf_i}E_{2mf_i} := E_{1MF_i}E_{2MF_i}/E_P^2 = \\ &= m_1m_2 + \frac{\alpha^5}{\alpha_2^5}f_{i1}f_{i2} + \sqrt{\frac{\alpha^5}{\alpha_2^5}}(m_1f_{i2} + m_2f_{i1}), \end{aligned} \tag{100}$$

$$\begin{aligned} &E_{1qf_i}E_{2qf_i} := E_{1QF_i}E_{2QF_i}/E_P^2 = \\ &= \frac{\alpha^4}{\alpha_2^4}q_1q_2\alpha + \frac{\alpha^5}{\alpha_2^5}f_{i1}f_{i2} + \frac{\alpha^5}{\sqrt{\alpha_2^9}}(f_{i2}q_1 + f_{i1}q_2), \\ &E_{1fq_i}E_{2fq_i} := E_{1FQ_i}E_{2FQ_i}/E_P^2 = \\ &= f_1f_2 - q_1q_2\alpha + i\sqrt{\alpha}(f_1q_2 + f_2q_1), \end{aligned} \tag{101}$$

defining six complex forces acting over a real *distance* $R := r\ell_P$, $r \in \mathbb{R}$

$$F_{AB_i} = \frac{G}{c^4R^2}E_{1AB_i}E_{2AB_i} = \frac{F_P}{r^2}E_{1ab_i}E_{2ab_i}, \tag{102}$$

and six complex forces acting over an imaginary *distance* $R_i := r_i\ell_{Pi}$, $r_i \in \mathbb{R}$

$$\tilde{F}_{AB_i} = \frac{G}{c_n^4R_i^2}E_{1AB_i}E_{2AB_i} = \frac{\alpha_2}{\alpha}\frac{F_P}{r_i^2}E_{1ab_i}E_{2ab_i}, \tag{103}$$

where $A, B \in \{M, Q, F\}$ and $a, b \in \{m, q, f\}$, and

$$\alpha_2 r^2 F_{AB_i} = \alpha r_i^2 \tilde{F}_{AB_i}. \tag{104}$$

With a simplifying assumption of $r^2 = r_i^2$, the forces acting over a real *distance* R are stronger and opposite to the corresponding forces acting over an imaginary *distance* R_i even though the Planck force is lower in modulus than the (real) α_2-Planck force (27). We excluded mixed forces (based on real and imaginary masses/charges/photons) as real and imaginary dimensions are orthogonal.

In particular, we can use the complex force F_{MQ_i} (102) with (99) (i.e., complex Newton's law of universal gravitation) to calculate the BBO surface gravity g_{BBO}, assuming an uncharged ($q_2 = 0$) test mass m_2

$$\begin{aligned} &\frac{F_P}{r_{BBO}^2}\left(m_{BBO}m_2 + i\sqrt{\alpha}m_2q_{BBO}\right) = M_2g_{BBO} = \\ &= m_2m_P\hat{g}_{BBO}a_P, \\ &\hat{g}_{BBO} = \frac{1}{r_{BBO}^2}\left(m_{BBO} + i\sqrt{\alpha}q_{BBO}\right), \end{aligned} \tag{105}$$

where $g_{BBO} = \hat{g}_{BBO}a_P$, $\hat{g}_{BBO} \in \mathbb{R}$. Substituting the BBO equilibrium relation (76) and the generalized BBO radius (42) $r_{BBO} = km_{BBO}$ into the relation (105) yields

$$g_{BBO} = \frac{a_P}{kr_{BBO}}\left(1 \pm i\sqrt{\frac{k^2}{4} - 1}\right), \tag{106}$$

which reduces to BH surface gravity for $k = 2$, in modulus

$$\hat{g}_{BBO}^2 = \frac{1}{k^2r_{BBO}^2}\left(1 + i\sqrt{\frac{k^2}{4} - 1}\right)\left(1 - i\sqrt{\frac{k^2}{4} - 1}\right) = \frac{1}{4r_{BBO}^2}, \tag{107}$$

equals to a squared BH surface gravity for all k, and in particular,

$$g_{BBO}(k_{max}) = \pm\frac{a_P}{d_{BBO}}(0.2944 \pm 0.9557i), \tag{108}$$

$$g_{BBO}(k_{eq}) = \pm\frac{a_P}{d_{BBO}}(0.7229 \pm 0.6909i). \tag{109}$$

The BBO surface gravity (106) leads to the generalized complex Hawking blackbody-radiation equation

$$T_{BBO} = \frac{\hbar}{2\pi ck_B}g_{BBO} = \frac{T_P}{k\pi d_{BBO}}\left(1 \pm i\sqrt{\frac{k^2}{4} - 1}\right), \tag{110}$$

describing the BBO temperature[10] by including its charge in the imaginary part, which also in modulus equals squared BH temperature $\forall k$. In particular,

$$T_{BBO}(k_{max}) = \pm\frac{T_P}{2\pi d_{BBO}}\left(\frac{\sqrt{\alpha^4 - \alpha_2^4}}{\alpha^2} \pm i\frac{\alpha_2^2}{\alpha^2}\right), \tag{111}$$

$$T_{BBO}(k_{eq}) = \pm\frac{T_P}{2\pi d_{BBO}}\frac{\alpha^2 \pm i\alpha_2^2}{\sqrt{\alpha^4 + \alpha_2^4}}, \tag{112}$$

reduce to a BH temperature for $\alpha_2 = 0$. We note that for $d_{BBO} = 1$, $\mathrm{Re}(T_{BBO}(k_{max})) \approx 6.6387 \times 10^{30}$ [K] (where $T_P/(2\pi) \approx 2.2549 \times 10^{31}$ [K]) has the magnitude of the Hagedorn temperature of strings.

It seems, therefore, that a universe without imaginary dimensions (i.e., with $\alpha_2 = 0$) would be a black hole. Hence, the evolution of information [1–6] requires imaginary time.

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[10] In a commonly used form it is $T_{BBO} = \frac{\hbar c^3}{2k^2\pi GM_{BBO}k_B}\left(1 \pm i\sqrt{\frac{k^2}{4} - 1}\right)$.

IX. DISCUSSION

The reflectance of graphene under the normal incidence of electromagnetic radiation expressed as the quadratic equation for the fine-structure constant α includes the 2^{nd} negative fine-structure constant α_2. The sum of the reciprocal of this 2^{nd} fine-structure constant α_2 with the reciprocal of the fine-structure constant α (2) is independent of the reflectance value R and remarkably equals simply $-\pi$. Particular algebraic definition of the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$, containing the free π term, can be interpreted as the asymptote of the CODATA value α^{-1}, the value of which varies with time. The negative fine-structure constant α_2 leads to the set of α_2-Planck units applicable to imaginary dimensions, including imaginary α_2-Planck units (18)-(26). Real and imaginary mass and charge units (34), length and mass units (35) units, and temperature and time units (33) are directly related to each other. Also, the elementary charge e is common for real and imaginary dimensions (36).

Applying the α_2-Planck units to a complex energy formula [47] yields complex energies (48), (49) setting the atomic number $Z = 238$ as the limit on an extended periodic table. The generalized energy (45) of all perfect black-body *objects* (black holes, neutron stars, and white dwarfs) having the generalized radius $R_{\text{BBO}} = kGM/c^2$ exceed mass-energy equivalence if $k > 2$. Complex energies (48), (49) allow for storing the excess of this energy in their imaginary parts, inaccessible for direct observation. The results show that the perfect black-body *objects* other than black holes cannot have masses lower than 5.7275×10^{-10} [kg] and that the size-to-mass ratios of their cores cannot exceed $k_{\max} \approx 6.7933$ defined by the relation (79). It is further shown that a black-body *object* is in the equilibrium of complex energies if its radius $R_{\text{eq}} \approx 2.7665\ GM_{\text{BBO}}/c^2$ (85). It is conjectured that this is the correct value of the photon sphere radius. BBO fluctuations for k_{eq} and $k_{\max}$ are briefly discussed in Appendix E. The proposed model explains the registered (GWOSC) high masses of the neutron stars mergers without resorting to any hypothetical types of exotic stellar *objects*.

In the context of the results of this study, monolayer graphene, a truly 2-dimensional material with no thickness[11], is a *keyhole* to other, unperceivable, dimensionalities. Graphene history is also instructive. Discovered in 1947 [53], graphene was long considered an *academic material* until it was eventually pulled from graphite in 2004 [54] by means of ordinary Scotch tape[12]. These fifty-seven years, along with twenty-nine years (1935-1964) between the condemnation of quantum theory as *incomplete* [55] and Bell's mathematical theorem [56] asserting that it is not true, and the fifty-eight years (1964-2022) between the formulation of this theorem and 2022 Nobel prize in physics for its experimental *loophole-free* confirmation, should remind us that Max Planck, the genius who discovered Planck units, has also discovered Planck's principle.

[11] Thickness of MLG is reported [52] as 0.37 [nm] with other reported values up to 1.7 [nm]. However, considering that 0.335 [nm] is the established inter-layer *distance* and consequently the thickness of bilayer graphene, these results do not seem credible: the thickness of bilayer graphene is not $2 \times 0.37 + 0.335 = 1.075$ [nm].

[12] Introduced into the market in 1932.

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Appendix A: Other quadratic equations

The quadratic equation for the sum of transmittance (3) and absorptance (5), putting $C_{TA} \coloneqq T + A$, is

$$\frac{1}{4}C_{TA}\pi^2\alpha^2 + (C_{TA} - 1)\pi\alpha + (C_{TA} - 1) = 0, \tag{A1}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} + \sqrt{1 - C_{TA}}\right)} \approx 137.036, \tag{A2}$$

and

$$\alpha_2^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} - \sqrt{1 - C_{TA}}\right)} \approx -140.178, \tag{A3}$$

whereas their sum $\alpha^{-1} + \alpha_2^{-1} = -\pi$ is, similarly as the relation (12), also independent of T and A.

Other quadratic equations do not feature this property. For example, the sum of $T + R$ (6) expressed as the quadratic equation and putting $C_{TR} \coloneqq T + R$, is

$$\frac{1}{4}(C_{TR} - 1)\pi^2\alpha^2 + C_{TR}\pi\alpha + (C_{TR} - 1) = 0, \tag{A4}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} + 2\sqrt{2C_{TR} - 1}} \approx 137.036, \tag{A5}$$

and

$$\alpha_{TR}^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} - 2\sqrt{2C_{TR} - 1}} \approx 0.0180, \tag{A6}$$

whereas their sum

$$\alpha_{TR_1}^{-1} + \alpha_{TR_2}^{-1} = \frac{-\pi C_{TR}}{C_{TR} - 1} \approx 137.054 \tag{A7}$$

is dependent on T and R.

Appendix B: Two π-like constants

With algebraic definitions of α (13) and α_2 (14), transmittance T (3), reflectance R (4) and absorptance A (5) of MLG for normal EMR incidence can be expressed just by π. For $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (13) they become

$$T(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)^2}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 0.9775, \tag{B1}$$

$$A(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 0.0224, \tag{B2}$$

while for $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi$ (14) they become

$$T(\alpha_2) = \frac{4\left(4\pi^2 + \pi + 2\right)^2}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 1.0228, \tag{B3}$$

$$A(\alpha_2) = -\frac{4\left(4\pi^2 + \pi + 2\right)}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx -0.0229, \tag{B4}$$

with

$$R(\alpha) = R(\alpha_2) = \frac{1}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 1.2843 \times 10^{-4}. \tag{B5}$$

$(T(\alpha) + A(\alpha)) + R(\alpha) = (T(\alpha_2) + A(\alpha_2)) + R(\alpha_2) = 1$ as required by the law of conservation of energy (8), whereas each conservation law is associated with a certain symmetry, as asserted by Noether's theorem. $A(\alpha) > 0$ and $A(\alpha_2) < 0$ imply respectively a *sink* and a *source*, while the opposite holds true for the transmittance T, as illustrated schematically in Fig 2. Perhaps, the negative absorptance and transmittance exceeding 100% for α_2 (11) or (14) could be explained in terms of graphene spontaneous emission.

The quadratic equation (9) describing the reflectance R of MLG under normal incidence of EMR (or alternatively (A1)) can also be solved for π yielding two roots

$$\pi(R, \alpha_*)_1 = \frac{2\sqrt{R}}{\alpha_*(1 - \sqrt{R})}, \quad \text{and} \tag{B6}$$

$$\pi(R, \alpha_*)_2 = \frac{-2\sqrt{R}}{\alpha_*(1 + \sqrt{R})}, \tag{B7}$$

dependent on R and α_*, where α_* indicates α or α_2. This can be further evaluated using the MLG reflectance R (4) or (B5) (which is the same for both α and α_2), yielding four, yet only three distinct, possibilities

$$\pi_1 = \pi(\alpha)_1 = -\pi\frac{4\pi^2 + \pi + 1}{4\pi^2 + \pi + 2} = \pi\frac{\alpha_2}{\alpha} \approx -3.0712, \tag{B8}$$

$$\pi(\alpha)_2 = \pi(\alpha_2)_1 = \pi \approx 3.1416, \quad \text{and} \tag{B9}$$

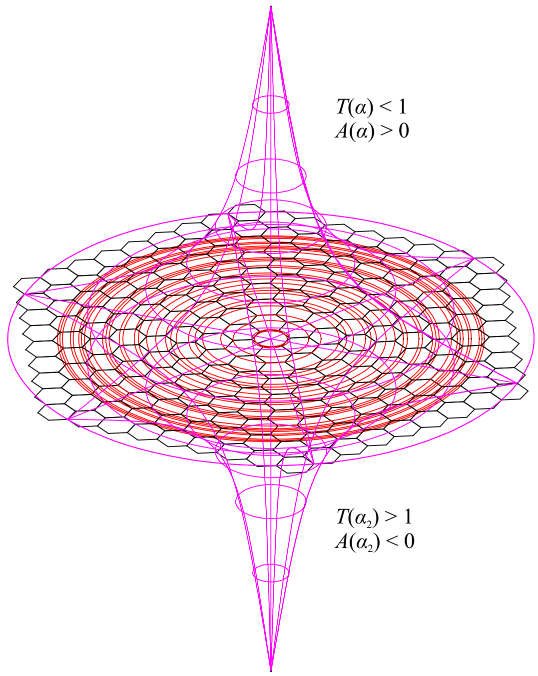

Figure 2. Illustration of the concepts of negative absorptance and excessive transmittance of EMR under normal incidence on MLG.

$$\pi_2 = \pi(\alpha_2)_2 = -\pi\frac{4\pi^2 + \pi + 2}{4\pi^2 + \pi + 1} = \pi\frac{\alpha}{\alpha_2} \approx -3.2136. \tag{B10}$$

The modulus of π_1 (B8) corresponds to a convex surface having a positive Gaussian curvature, whereas the modulus of π_2 (B10) - to a negative Gaussian curvature. Their product $\pi_1\pi_2 = \pi^2$ is independent of α_*, and their quotient $\pi_1/\pi_2 = \alpha_2^2/\alpha^2$ is not directly dependent of π. It remains to be found whether each of these π-like constants describes the ratio of the circumference of a circle drawn on the respective surface to its diameter (π_c) or the ratio of the area of this circle to the square of its radius (π_a). These definitions produce different results on curved surfaces, whereas $\pi_a > \pi_c$ on convex surfaces, while $\pi_a < \pi_c$ on saddle surfaces [59].

Appendix C: Planck units and HUP

Perhaps the simplest derivation of the squared Planck length is based on Heisenberg's uncertainty principle

$$\delta P_{\text{HUP}}\delta R_{\text{HUP}} \geq \frac{\hbar}{2} \quad \text{or} \quad \delta E_{\text{HUP}}\delta t_{\text{HUP}} \geq \frac{\hbar}{2}, \tag{C1}$$

where δP_{HUP}, δR_{HUP}, δE_{HUP}, and δt_{HUP} denote momentum, position, energy, and time uncertainties, by replacing energy uncertainty $\delta E_{\text{HUP}} = \delta M_{\text{HUP}} c^2$ with mass uncertainty and time uncertainty with position uncertainty, using mass-energy equivalence and $\delta t_{\text{HUP}} = \delta R / c_{\text{HUP}}$ [31], which yields

$$\delta M_{\text{HUP}}\delta R_{\text{HUP}} \geq \frac{\hbar}{2c}. \tag{C2}$$

Plugging $\delta M_{\mathrm{HUP}} = \delta R_{\mathrm{HUP}} c^2/(2G)$ for BH mass into (C2) we arrive at $\delta R^2_{\mathrm{HUP}} = \ell_{\mathrm{P}}^2 \Rightarrow \delta D_{\mathrm{HUP}} = \pm 2\ell_{\mathrm{P}}$ and recover [5] BH diameter $d_{\mathrm{BH}} = \pm 2$.

However, using the same procedure but inserting the BH radius, instead of the BH mass, into the uncertainty principle (C2) leads to $\delta M^2_{\mathrm{HUP}} = \frac{1}{4}\hbar c/G = \frac{1}{4} m_{\mathrm{P}}^2$. In general, using the generalized radius (42) in both procedures, one obtains

$$\delta M^2_{\mathrm{HUP}} = \frac{1}{2k} m_{\mathrm{P}}^2 \quad \text{and} \quad \delta R^2_{\mathrm{HUP}} = \frac{k}{2}\ell_{\mathrm{P}}^2. \tag{C3}$$

Thus, if k increases mass δM_{HUP} decreases, and δR_{HUP} increases and the factor is the same for $k = 1$ i.e., for *orbital speed radius* $\delta R = G\delta M/c^2$ or the *orbital speed mass* $\delta M = \delta R c^2/G$.

Appendix D: A *mixed speeds* hypothesis

Let us define the mass/charge energies (48), (49) with different speeds of light, i.e., the charge part of the energy E_{MQ_i} with c_n and the charge part of the energy E_{QM_i} with c

$$\begin{aligned} \hat{E}_{MQ_i} &:= Mc^2 + \frac{Q_i}{2\sqrt{\pi\epsilon_0 G}} c_n^2 = Mc^2 \pm iq\sqrt{\alpha} m_P \frac{\alpha^2}{\alpha_2^2} c^2, \\ \hat{E}_{QM_i} &:= \frac{Q}{2\sqrt{\pi\epsilon_0 G}} c^2 + M_i c_n^2 = \pm q\sqrt{\alpha} m_P c^2 + M_i \frac{\alpha^2}{\alpha_2^2} c^2, \end{aligned} \tag{D1}$$

Demanding equality of their moduli

$$\begin{aligned} M^2 + q^2\alpha m_P^2 \frac{\alpha^4}{\alpha_2^4} &= q^2 \alpha m_P^2 - M_i^2 \frac{\alpha^4}{\alpha_2^4}, \\ M_i &= \pm\sqrt{q^2\alpha m_{\mathrm{P}}^2\left(\frac{\alpha_2^4}{\alpha^4} - 1\right) - \frac{\alpha_2^4}{\alpha^4} M^2}. \end{aligned} \tag{D2}$$

For $q = 0$ this relation corresponds to the relation (64). However, since mass M_i is imaginary, the argument of the square root in the relation (D2) must be negative, i.e.,

$$|M| \ngtr |q| m_{\mathrm{P}} \sqrt{\alpha\left(1 - \frac{\alpha^4}{\alpha_2^4}\right)}. \tag{D3}$$

But $\alpha^4 > \alpha_2^4$, yielding imaginary M, while M is real by definition. The same result would be obtained if mass energy E_{QM_i} was parametrized with c_n and E_{QM_i} with c, since

$$\begin{aligned} &\sqrt{\alpha\left(\frac{\alpha_2^4}{\alpha^4} - 1\right)} \in \mathbb{I}, \quad \sqrt{\alpha\left(1 - \frac{\alpha_2^4}{\alpha^4}\right)} \in \mathbb{R}, \\ &\sqrt{\alpha\left(\frac{\alpha^4}{\alpha_2^4} - 1\right)} \in \mathbb{R}, \quad \sqrt{\alpha\left(1 - \frac{\alpha^4}{\alpha_2^4}\right)} \in \mathbb{I}. \end{aligned} \tag{D4}$$

Therefore, complex energies E_{MQ_i} (48) and E_{QM_i} (49) must be parametrized respectively by c and c_n.

Appendix E: Fluctuations of the BBOs

A relation describing a BH information capacity after absorption (+) or emission (−) of a *particle* having the wavelength l can be generalized (cf. [5], Appendix 3), using the generalized radius (42), to all holographic spheres, including BBOs as

$$N^{A/E}(d,l) = 16k^2\pi^3\frac{1}{l^2} \pm 8k\pi^2\frac{d}{l} + \pi d^2. \tag{E1}$$

The wavelength of a *particle* emitted from a BH that does not change the BH diameter corresponds to half of the BH Compton wavelength ($l_{\mathrm{BH}} = 8\pi/d_{\mathrm{BH}}$). Accordingly, the wavelength of a *particle* absorbed by a BH that does not change its diameter is $l_{\mathrm{BHconst}} = -4\pi/d_{\mathrm{BH}}$. We note in passing that three spatial dimensions set the minimum for such conditions to occur (cf. [5], Table III). In general, $l_{\mathrm{BBOconst}} = \mp 2k\pi/d_{\mathrm{BBO}}$. In particular, for k_{eq} the relation (E1) yields

$$\begin{aligned} &4\pi\left(1 + \frac{\alpha_2^4}{\alpha^4}\right) = \mp d l_{\mathrm{const}}\sqrt{1 + \frac{\alpha_2^4}{\alpha^4}}, \quad B := \frac{\alpha_2^4}{\alpha^4}, \\ &16\pi^2 B^2 + \left(32\pi^2 - d^2 l_{\mathrm{const}}^2\right) B + 16\pi^2 - d^2 l_{\mathrm{const}}^2 = 0, \\ &\sqrt{\Delta} = \pm d^2 l_{\mathrm{const}}^2, \quad B_{1,2} = \frac{d^2 l_{\mathrm{const}}^2 - 32\pi^2 \pm d^2 l_{\mathrm{const}}^2}{32\pi^2}. \end{aligned} \tag{E2}$$

The second solution is contradicting, as $\alpha_2^4 \neq -\alpha^4$. But the first one

$$l_{\mathrm{const}} = \mp\frac{4\pi\sqrt{1 + \frac{\alpha_2^4}{\alpha^4}}}{d} \approx \mp 1.3832\frac{4\pi}{d}, \tag{E3}$$

(with ”−” for absorption and ”+” for emission) reduces to l_{BHconst} for $\alpha_2 = 0$. For $k_{\max}$ the relation (E1) yields

$$\begin{aligned} &\frac{4\pi}{\left(1 - \frac{\alpha_2^4}{\alpha^4}\right)} = \mp\frac{d l_{\mathrm{const}}}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}}, \quad B := \frac{\alpha_2^4}{\alpha^4}, \\ &d^2 l_{\mathrm{const}}^2 B^2 + (16\pi^2 - 2d^2 l_{\mathrm{const}}^2) B + d^2 l_{\mathrm{const}}^2 - 16\pi^2 = 0, \\ &\sqrt{\Delta} = \pm 16\pi^2, \quad B_{1,2} = \frac{2d^2 l_{\mathrm{const}}^2 - 16\pi^2 \pm 16\pi^2}{2d^2 l_{\mathrm{const}}^2}. \end{aligned} \tag{E4}$$

The first solution is contradicting, but the second one

$$l_{\mathrm{const}} = \mp\frac{4\pi}{d\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \approx \mp 3.3966\frac{4\pi}{d}, \tag{E5}$$

also reduces to l_{BHconst} for $\alpha_2 = 0$.

The relation (E1) is remarkably similar to the algebraic definitions of the inverses of α (13) and α_2 (14) also containing π^3, π^2, and π terms. This raises the question of whether the fine-structure constants' inverses correspond to the number of bits[13]. Recently the fine-structure constant has been reported as the quantum of rotation [60]. Two *alphas* between

[13] The floor function of the inverse of the fine-structure constant α represents the threshold on the atomic number (137) of a hypothetical element *feynmanium* that, in the Bohr model of the atom, still allows the 1s orbital electrons to travel slower than the speed of light

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$\alpha^{-1} \approx 137.0363$ and $\alpha_2^{-1} \approx -140.1779$ hinted by the relations (13), (14), and (E1)

$$\tilde{\alpha}^{-1} = 4\pi^3 - \pi^2 + \pi \approx 117.2971,$$
$$\tilde{\alpha_2}^{-1} = -4\pi^3 + \pi^2 - 2\pi \approx -120.4387, \tag{E6}$$

are thus intriguing.

It was shown that the spectral density in the phenomenon of sonoluminescence, light emission by sound-induced collapsing gas bubbles in fluids, has the same frequency dependence as black-body radiation [61, 62]. Thus, the sonoluminescence, and in particular *shrimpoluminescence* [63], is emitted by collapsing micro-BBOs. For example, the relation (E1) yields the wavelength $l = 8\pi/(d_{\mathrm{BH}} \pm 1)$ required for collapsing a BH to the π-bit BH (i.e., to the reduced Planck temperature limit [12], to $d_{\mathrm{BH}} = \pm 1$). Demanding $|l| \geq 1$ we obtain $|d_{\mathrm{BH}}| \leq 8\pi \pm 1$.

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