Negative Speed of Light, and the Imaginary Set of Base Planck Units

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The reflectance R of monolayer graphene for the normal incidence of electromagnetic radiation is known to be remarkably defined only by π and the fine structure constant α. It is shown in this paper that the reflectance (or the sum of transmittance and absorptance) of monolayer graphene, expressed as a quadratic equation with respect to the fine structure constant α must unsurprisingly introduce the *2nd* fine structure constant α_2, as the root of this equation. It turns out that this 2nd fine structure constant is negative and the sum of its reciprocal with the reciprocal of the *physical* fine structure constant α is independent of the reflectance value R and remarkably equals $-\pi$. Particular algebraic definition of the fine structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$, containing the free π term, when introduced to this sum, yields $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi < 0$. Assuming universal validity of the physical definition of α, α_2 defines the negative speed of light in vacuum c_n and introduces the imaginary set of base Planck units. The average of this speed and the speed of light in vacuum is in the range of the Fermi velocity (10^6 m/s).

Keywords: Planck units, the fine-structure constant, speed of light in vacuum

I. INTRODUCTION

Numerous publications provide Fresnel coefficients for the normal incidence of electromagnetic radiation (EMR) on monolayer graphene, which are remarkably defined only by π and the fine structure constant α having the reciprocal

$$\alpha^{-1} = \left(\frac{q_P}{e}\right)^2 = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036, \tag{1}$$

where e is the elementary charge, q_P is the Planck charge, ϵ_0 is vacuum permittivity, $\hbar$ is the reduced Planck constant, and c is the speed of light in vacuum.

Transmittance (T) of monolayer graphene

$$T = \frac{1}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 97.746\% \tag{2}$$

for normal EMR incidence was derived from the Fresnel equation in the thin-film limit [1] (Eq. 3), whereas spectrally flat absorptance (A) $A \approx \pi\alpha \approx 2.3\%$ was reported [2, 3] for photon energies between about 0.5 and 2.5 eV. T was related to reflectance (R) [4] (Eq. 53) as $R = \pi^2\alpha^2 T/4$, i.e,

$$R = \frac{\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.013\%, \tag{3}$$

The above formulas for T and R, as well as the formula for the absorptance

$$A = \frac{\pi\alpha}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 2.241\%, \tag{4}$$

were also derived [5] (Eqs. 29-31) based on the thin film model (setting $n_s = 1$ for substrate).

The sum of transmittance (2) and the reflectance (3) at normal EMR incidence was also derived [6] (Eq. 4a) as

$$\begin{aligned} T + R &= 1 - \frac{4\sigma\eta}{4 + 4\sigma\eta + \sigma^2\eta^2 + k^2\chi^2} \\ &= \frac{1+\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 97.759\%, \end{aligned} \tag{5}$$

where $\eta = 4\pi\alpha\hbar/e^2 = 1/(\epsilon_0 c)$ is the impedance of vacuum, $\sigma = e^2/4\hbar$ is the monolayer graphene conductivity [7], and $\chi = 0$ is the electric susceptibility of vacuum.

These coefficients are thus well-established theoretically and experimentally confirmed [1–3, 6, 8, 9].

As a consequence of the conservation of energy

$$(T + A) + R = 100\%. \tag{6}$$

In other words, the transmittance in the Fresnel equation describing the reflection and transmission of EMR at normal incidence on a boundary between different optical media is, in the case of the 2-dimensional monolayer (boundary) of graphene, modified to include its absorption.

II. THE SECOND FINE STRUCTURE CONSTANT

The reflectance R (3) of monolayer graphene can be expressed as a quadratic equation with respect to α

$$\frac{1}{4}\pi^2 (R-1)\alpha^2 + R\pi\alpha + R = 0, \tag{7}$$

having two roots with reciprocals

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx 137.036, \tag{8}$$

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx -140.178. \tag{9}$$

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Therefore, Equation (7) introduces the second, negative fine structure constant α_2.

The sum of the reciprocals of these fine structure constants (8) and (9)

$$\alpha^{-1} + \alpha_2^{-1} = \frac{\pi - \pi\sqrt{R} - \pi - \pi\sqrt{R}}{2\sqrt{R}} = -\pi, \tag{10}$$

is remarkably independent of the reflectance R. The same result can be obtained for the sum of T and A, as shown in Appendix .

Furthermore, this result is intriguing in the context of a peculiar algebraic definition of the fine structure constant [10]

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036 \tag{11}$$

that contains a *free* π term and agrees with the physical definition (1) of α to the 5th significant digit. Therefore, using Equations (10) and (11), we can express the negative reciprocal of the 2nd fine structure constant α_2^{-1} that emerged in Equation (7) as

$$\alpha_2^{-1} = -\pi - \alpha_1^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.178. \tag{12}$$

But how can this negative value be interpreted physically?

If $\alpha^{-1} = (q_P/e)^2$ (1) is valid also for the negative α_2^{-1} (9) or (12) then it requires an introduction of the imaginary Planck charge iq_{Pi}, so that its square would yield

$$i^2 q_{Pi}^2 = e^2\alpha_2^{-1}. \tag{13}$$

Furthermore, almost all physical constants of $(4\pi\epsilon_0\hbar c)/e^2$ in the *physical* definition of the fine structure constant (1) are positive[1], whereas the charge e is squared. Only the velocity can be negative, as it is a *directional* quantity. Therefore, if

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} = \frac{4\pi\epsilon_0\hbar c}{e^2}, \tag{14}$$

then

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} = \frac{4\pi\epsilon_0\hbar c_n}{e^2}, \tag{15}$$

where c_n is the negative speed of light in vacuum that, using Equations (10) with (1) and (15), amounts

$$\begin{gathered}\frac{4\pi\epsilon_0\hbar c}{e^2} + \frac{4\pi\epsilon_0\hbar c_n}{e^2} = -\pi \\ c_n = -\frac{e^2}{4\epsilon_0\hbar} - c \approx -3.066653 \times 10^8 \text{ [m/s]},\end{gathered} \tag{16}$$

which is greater that the speed of light in vacuum c in modulus, whereas their average

$$\frac{c + c_n}{2} \approx -3.436417 \times 10^6 \text{ [m/s]} \tag{17}$$

is in the range of the Fermi velocity.

Therefore, using c_n (16) (or the value of the elementary charge e in (13)), the modulus of the imaginary Planck charge (13) amounts

$$|q_{Pi}| = \sqrt{4\pi\epsilon_0\hbar\,|c_n|} \approx 1.8969 \times 10^{-18} \text{ [C]} > q_P. \tag{18}$$

Furthermore, the negative speed of light in vacuum c_n (16) introduces all the remaining base Planck units defined by square roots containing c raised to an odd (1, 3, 5) power, that redefined with $c_n < 0$ become imaginary

$$|\ell_{Pi}| = \sqrt{\frac{\hbar G}{|c_n|^3}} \approx 1.5622 \times 10^{-35} \text{ [m]} < \ell_P, \tag{19}$$

$$|m_{Pi}| = \sqrt{\frac{\hbar\,|c_n|}{G}} \approx 2.2012 \times 10^{-8} \text{ [kg]} > m_P, \tag{20}$$

$$|t_{Pi}| = \sqrt{\frac{\hbar G}{|c_n|^5}} \approx 5.0942 \times 10^{-44} \text{ [s]} < t_P, \tag{21}$$

$$|T_{Pi}| = \sqrt{\frac{\hbar\,|c_n|^5}{G k_B^2}} \approx 1.4994 \times 10^{32} \text{ [K]} > T_P. \tag{22}$$

With algebraic definitions of α (11) and α_2 (12) transmittance T (2), reflectance R (3) and absorptance A (4) of monolayer graphene for normal EMR incidence can be expressed just by π.

For $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (11) they become

$$T(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)^2}{(8\pi^2 + 2\pi + 3)^2} \approx 97.746\%, \tag{23}$$

$$A(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)}{(8\pi^2 + 2\pi + 3)^2} \approx 2.241\%, \tag{24}$$

while for $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi$ (12) they become

$$T(\alpha_2) = \frac{4\left(4\pi^2 + \pi + 2\right)^2}{(8\pi^2 + 2\pi + 3)^2} \approx 102.279\%, \tag{25}$$

$$A(\alpha_2) = \frac{4\left(4\pi^2 + \pi + 2\right)}{(8\pi^2 + 2\pi + 3)^2} \approx -2.292\%, \tag{26}$$

with

$$R(\alpha) = R(\alpha_2) = \frac{1}{(8\pi^2 + 2\pi + 3)^2} \approx 0.013\%. \tag{27}$$

[1] vacuum permittivity ϵ_0 is a measure of how *dense* is an electric field; objects that do not change their measure with respect to orientation (as compared to volumes, for example) are densities. Thus, ϵ_0 cannot be negative. The Planck constant h is the uncertainty principle parameter. Thus, it cannot be negative; negative probabilities do not seem to withstand Occam's razor.

Obviously $T(\alpha) + A(\alpha) + R(\alpha) = T(\alpha_2) + A(\alpha_2) + R(\alpha_2) = 1$ as required by the law of conservation of energy (6), whereas each conservation law is associated with a certain symmetry, as asserted by Noether's theorem. Nonetheless, physical interpretation of $T(\alpha_2) > 1$ and $A(\alpha_2) < 0$ requires further research. We note in passing that $A(\alpha) > 0$ implies a *sink*, whereas $A(\alpha_2) < 0$ implies a *source*, whereas the opposite holds true for the transmittance T.

Perhaps, the negative absorptance and transmittance exceeding 100% for α_2 (9), (12) could be explained in terms of graphene spontaneous emission but this issue requires further research. Particularly in the context of emergent dimensionality [11–13].

III. DISCUSSION

We have shown that the reflectance of graphene under the normal incidence of electromagnetic radiation (EMR), expressed as the quadratic equation with respect to the fine structure constant α must introduce the *2^nd^* negative fine structure constant α_2.

It is shown that the sum of the reciprocal of this 2^{nd} fine structure constant α_2 with the reciprocal of the *physical* fine structure constant α (1) is independent of the reflectance value R and remarkably equals simply $-\pi$.

Particular algebraic definition of the *physical* fine structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (11), containing the free π term, when introduced to this sum, yields $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi < 0$.

Assuming universal validity of the physical definition of the fine structure constant α (1), the 2^{nd} fine structure constant α_2 (12) defines the negative speed of light c_n (16) and introduces the imaginary set of base Planck units (19)-(22). The average of this speed and the speed of light is in the range of the Fermi velocity (10^6 m/s).

This paper is a cleanup of the research presented in [14] and [15].

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Appendix: Other Quadratic Equations

The quadratic equation for the sum of transmittance (2) and absorptance (4), putting $C_{TA} \doteq T + A$, is

$$\frac{1}{4} C_{TA}\pi^2\alpha^2 + (C_{TA} - 1)\pi\alpha + (C_{TA} - 1) = 0, \tag{A.1}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} + \sqrt{1 - C_{TA}}\right)} \approx 137.036, \tag{A.2}$$

and

$$\alpha_2^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} - \sqrt{1 - C_{TA}}\right)} \approx -140.178, \tag{A.3}$$

whereas their sum $\alpha^{-1} + \alpha_2^{-1} = -\pi$ is also independent of T and A.

Other quadratic equations do not feature this property. For example, the sum of $T + R$ (5) expressed as the quadratic equation and putting $C_{TR} \doteq T + R$, is

$$\frac{1}{4}(C_{TR} - 1)\pi^2\alpha^2 + C_{TR}\pi\alpha + (C_{TR} - 1) = 0, \tag{A.4}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} + 2\sqrt{2C_{TR} - 1}} \approx 137.036, \tag{A.5}$$

and

$$\alpha_{TR}^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} - 2\sqrt{2C_{TR} - 1}} \approx 0.0180, \tag{A.6}$$

whereas their sum

$$\alpha_{TR_1}^{-1} + \alpha_{TR_2}^{-1} = \frac{-\pi C_{TR}}{C_{TR} - 1} \approx 137.054 \tag{A.7}$$

is dependent on T and R.

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