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# Can hidden electrodynamic field fluctuations preclude the derivation of Bell-type inequalities?

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## Abstract

We show that loophole-free Bell-type no-go theorems cannot be derived in theories involving local hidden fields. At the time of measurement, a contextuality loophole appears because each particle's electromagnetic field interacts with the field of its respective apparatus, preventing the expression of the probability density as a function independent of the orientation of the measuring devices. Then, we use the dynamical evolution of the probability distribution to show that the spin-correlation integral can neither be expressed in terms of initial Cauchy data restricted to the particles. A correlation loophole ensues, which prevents the usage of the non-contextual correlation integrals required to demonstrate the CHSH-Bell inequality. We obtain a new inequality not violated by quantum correlation functions of entangled spin pairs, and propose that Maxwell's electrodynamic field is the missing hidden variable triggering the coupled nonlinear oscillations of the particles, which bring about the synchronicities observed in the Einstein-Podolsky-Rosen-Bohm (EPRB) experiment.

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## I. INTRODUCTION

In the seminal work of Einstein, Podolsky and Rosen, the following dilemma is proposed [1]: Or complementary variables associated with non-commuting observables cannot be computed from quantum mechanics but have a simultaneous physical reality or, on the contrary, it is just that quantum mechanics does not fail in computing all aspects of reality, for the simple reason that some of these observables do not exist when certain experimental settings are considered. To this end, they adopt a sufficient empirical definition of reality, which states that a property is real if it can be measured without disturbing the system. Then, these authors show how quantum mechanics runs into a contradiction by using two entangled particles [1]. More specifically, they show that, if we assume that quantum mechanics is complete, we can overcome the lack of simultaneous reality imposed by the principle of uncertainty by making measurements only on one of the particles which, presumably, can not have any effect on the other particle (locality). Thus, beginning with the assumption of completeness, the assignation of realistic properties to non-commuting observables is achieved. It follows, consequently, that completeness and realism can be logically implied in entangled quantum mechanical systems. This contradicts the original dilemma, which ruled out any option apart from the mutual exclusion between completeness and realism, thus completeness must fail.

From an analytical point of view, there can be two solid attacks to their reasoning, concerning the two fundamental philosophical assumptions made in their work. Indeed, we can avoid their dilemma by renouncing either to their definition of realism or to locality (or both). Bohr criticized their definition of realism by appealing to the physical interaction between the measuring apparatus and the particles [2]. According to Bohr, two different experimental contexts were being used to measure non-commuting observables and, therefore, the two settings could not be invoked to attribute properties to the same particle. On the other front, Bell investigated the assumption of the separability between the two electrodynamic bodies, whose properties are to be measured [3]. Following an example advocated by Bohm and Ahronov [4], Bell studied the statistical correlation of different components of the spin, for a pair of entangled particles (see Fig. 1). He proved that statistical theories based on hidden classical magnitudes must obey certain inequalities, which are violated by quantum mechanical correlations of entangled pairs.

Many works have been published since the discovery of Bell's theorem [5], pointing out the many loopholes that experiments can suffer. Most of these loopholes have been experimentally closed [6], some of them in conjunction, and it is not expected that they can produce violations of the magnitude appearing in Bell tests [6]. However, here we adopt Bohr's *contextual* perspective and show that a loophole cannot be avoided from the very definition of measurement, when classical field theories are at stake. For this purpose, we reconsider Bell-type theorems using dynamical *hidden fields*. In these theories, the apparatus has an unavoidable effect on the particle when a measurement takes place, as a consequence of their electrodynamic interaction [7, 8]. A contextuality loophole is thus enforced, because we cannot impose *a priori* that the probability distribution of a hidden variable is independent of the experimental arrangement.

Furthermore, contrary to the assumption of probability distributions of stationary random hidden variables, the probability density of a hidden field defines a stochastic process [9] that evolves in time and differs for different experimental settings. Using this fact, we show that the correlation integral, when expressed in terms of the initial hidden data, is different from the integral used in the derivation of the CHSH-Bell inequality [10], unless a correlation loophole is closed. Finally, we extend our arguments to situations where *last-instant* and *free* choices can be made, regarding the orientation of the Stern-Gerlach apparatuses.

It is not the purpose of the present work to provide a quantitative mechanism that explains EPRB correlations, but just to show that Bell-type theorems cannot yet discard classical field theories as a foundation of quantum mechanics. Nevertheless, we suggest a qualitative explanation to the mysterious correlations between the measurements of the two electrodynamic particles by invoking the electromagnetic nature of entanglement and the concepts of self-oscillation and *synchronization* [7, 8, 11]. Some brief philosophical insights concerning realism are briefly pointed out in the discussion.

## II. HYPOTHESES IN BELL-TYPE THEOREMS

Bell's program is based on computing correlations of the spin of two entangled particles [3]. In particular, a singlet state of zero spin $S = S_1 + S_2 = 0$ is considered, which can be written as

$$|0,0\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle \right), \qquad (1)$$

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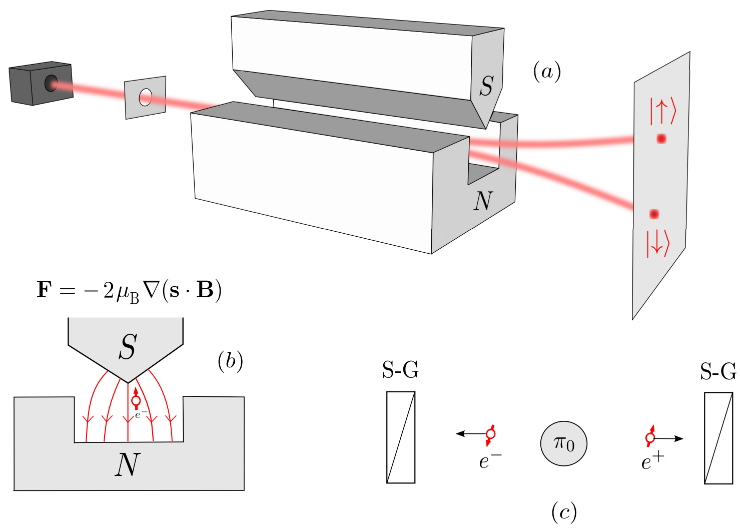


FIG. 1. **Stern-Gerlach apparatus**. (a) A beam of silver atoms (red) entering a S-G device, where some non-uniform magnetic field is present. The beam separates into two components, reflecting the different values of the spin that these atoms acquire during the process of measurement. (b) A section of the S-G device showing the magnetic fields and the evolution of the spin of an electron going through. (c) A disintegration of a $\pi_0$ meson into an electron-positron entangled pair. The two particles evolve in opposite senses towards two far away respective measuring apparatuses.

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are eingenstates of the operator $S_z$ representing the spin angular momentum component along the $z$-axis. As depicted in Fig. 1(c), from a physical point of view we can think of this entangled state as two electrodynamic particles, a positron and an electron, flying away from each other from the location where the disintegration of a neutral meson took place, for example. These two particles are heading towards two identical Stern-Gerlach (S-G) apparatuses.

In principle, each of these two measuring devices can be oriented along any possible direction described by the unit vector $a$ in the euclidean space, so that one measures the spin component $S_{1a} = S_1 \cdot a$, while the other can measure some other component of the spin $S_{2b} = S_2 \cdot b$, along the direction given by the unit vector $b$. Then, if we consider these two components of the spin, related to our two respective entangled particles, the correlation $C_{ab}$ of the product of spin projections can be computed as

$$C_{ab} = \langle 0,0| \, S_{1a} \otimes S_{2b} \, |0,0\rangle = -\frac{\hbar^2}{4} a \cdot b, \tag{2}$$

where Eq. (1) has been used, together with the expression $S_n = \hbar^2 n^i \sigma_i/2$, being $\sigma_i$ the

Pauli matrices and $n$ some unit vector in the three-dimensional euclidean space. Note that we are just computing a second-order moment using the random variables $S_{1a}$ and $S_{2b}$. For convenience, we shall neglect hereafter the $\hbar^2/4$ in the computation of correlations by considering units of $\hbar/2$, and simply assign values plus one or minus one to the spin.

Bell's point is that, for a deterministic hidden variable theory to exist, we must be able to compute the averages, the correlations and the moments of any order appearing in quantum mechanics, from the evolution of the system by using a probability distribution depending on the hidden variables $\lambda$. Any quantum operator must be defined as a function of these hidden variables in such a way that their knowledge would allow to precisely determine the value of the quantum operator and the average of any function of this and other operators, no matter how complicated. In particular, the correlation integral can be written as

$$C_{ab} = \int A_a(\lambda) B_b(\lambda) p(\lambda) d\lambda, \tag{3}$$

where $A_a(\lambda)$ denotes the result of measuring the spin of the first particle along the direction given by $a$. Given a value of $\lambda$, we must have a specific value of $A_a(\lambda)$, whether this value is one (spin up) or minus one (spin down). Correspondingly, we denote as $B_b(\lambda)$ the value of the spin of the second particle, when measured along the direction given by $b$. If we now consider four different orientations of the apparatuses $a, a', b, b'$, it can be shown after some algebraic manipulations [10] that

$$|C_{ab} + C_{ab'}| + |C_{a'b} - C_{a'b'}| \leq 2. \tag{4}$$

This inequality sets a restriction among the values that the correlations between two spin variables measured along different directions can have. By considering the quantum mechanical result $C_{ab} = -a \cdot b$, it is very easy to see that many values do not obey this inequality. A common counterexample is to let the four directions to be $\pi/4$ radians apart in a plane, in the order $b', a, b, a'$, so that the violation is maximized [6], yielding a value of $2\sqrt{2}$ in Eq. (4).

Four hypotheses can be clearly identified in Bell's work [3], regarding the expression of the correlation integral appearing in Eq. (3):

1. Hidden variables $\lambda$ can be anything, from simple constant vectors to dynamical vector fields evolving in spacetime $\lambda(x, t)$.

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2. There exists two functions $A_a(\lambda)$ and $B_b(\lambda)$ with outputs $\pm 1$, representing the two possible results of measuring the spin of any of the two particles, which depend on the hidden variables $\lambda$ related to them.

3. There exists a probability measure $p(\lambda)d\lambda$ that does not depend on the orientation of the apparatuses.

4. The principle of locality, which states that $A_a(\lambda)$ does not depend on $b$ and $B_b(\lambda)$ does not depend on $a$.

We now thoroughly discuss these four postulates in relation to the EPRB experiment. The first assumption is of great importance, since the main purpose of Bell's no-go theorem is to reject classical field theories as the foundation of quantum mechanics. It is evident that electrodynamic fields are specially concerned in this regard, since they do not explicitly appear in the Schödinger equation when studying the EPRB experiment, which involves two electrically charged bodies. Not to speak about other experiments involving correlated photon pairs, which are frequently represented without any allusion to the electromagnetic field.

The second assumption is undisputed as long as the correlations are computed at the time of measurement, since in a classical field theory the magnetic field inside the particle specifies its internal angular momentum. Together with the orientation of the measuring device, which also enforces a direction of the non-uniform magnetic field created by the SG apparatus, would suffice to determine the result of the measurement. However, as we shall see in Sec. IV, the expression of the functions $A_a(\lambda)$ and $B_b(\lambda)$ in terms of *initial* hidden field variables $\lambda$ related exclusively to the particles [10] is prevented by the contextuality of classical field theories.

Concerning the third assumption, it is evident that when Bell defines the probability measure $p(\lambda)$, he does not rigorously enter into the question about if these variables depend or not on any time-dependent parameter. If we assume that *any* initial $p(\lambda)$ does not evolve in time, then it is evident that we are also assuming that the hidden variables have constant values all over their trip from the place where the disintegration took place, until they arrive to the apparatus. This is frequently considered by supposing that measured observables correspond to properties possessed by the particles before and after measurement, which is tantamount to realism according to the definition provided by Einstein-Podolsky-Rosen [1].

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Quite the opposite, in field theories, which are contextual, this definition of reality becomes over-restrictive, because the apparatus has an active role in the production of the measured values of an observable.

It is crucial to recall that, if hidden variables are dynamical, the probability density must inherit such a dependence, as it occurs in the theory of stochastic processes [9]. For example, if the hidden variables depend on time $\lambda(t)$, we would have a probability density $p(\lambda, t)$, even if this time is made the same by resetting clocks at each row of the experiment. If hidden fields are at investigation, in principle we would have to consider a probability density $p(\lambda, x'_a, x'_b, t)$ defined over the entire space, where $x'_a$ and $x'_b$ would be the position at which the two point particles would be found at the time of measurement.

Alternatively, if particles are considered extended and made of fields, the probability density would have to be integrated throughout the region of space where these particles can be found at time $t$ of measurement. In this case, when we compute the correlation $C_{ab}$ using the distribution at the precise moment of arrival to the apparatus, we must face the question about if this probability is the same for all the experimental settings $a$ and $b$. In the next section we prove that a contextuality loophole produces a dependence of the probability density on the measurement setting at the time of measurement, even in Newtonian mechanical systems.

We might try to avoid the effect of the apparatus on the hidden variables by assuming that uncertainty emanates from a lack of control of the initial conditions. The knowledge of these hidden variables would entail a determination of the spin of the particles in the very moment in which they are created. This is an example that Bell has in mind, as he clearly manifests [3]. To this end, we must use some probability distribution describing the frequency with which these hidden variable values are uncontrollably picked as initial conditions. Then, the hidden variables would determine, at the moment of arrival to the apparatus, the values of the result of the experiment. In this case, the probability can be expressed as independent of the measurement settings only if hidden fields in the initial Cauchy hypersurface are uncorrelated. Quite the opposite, we suggest that correlated electrodynamic field fluctuations prevent the expression of Bell-type correlation integrals.

Here we assume the fourth hypothesis as evident, *i.e.*, that $A_a$ does not depend on the direction of the second apparatus (say $b$) at the time of measurement, and vice versa. Otherwise, the correlation could be perfectly explained by assuming that the well distanced apart experiments are communicating during the collapse of the simultaneous (in the laboratory frame) measurements of the two spins. This locality loophole was mostly closed in experiments with entangled photons [12, 13].

## III. BELL-TYPE CORRELATION INTEGRALS

Given the evidence appearing in Bell's works [14], it is obvious that he was perfectly aware wether the values of the hidden variables were given before the measurement process took place or not. Indeed, in such works Bell demonstrates a vast knowledge of Bohr's ideas, where he also states the misuse of the word "measurement", by recognizing that it is the interaction between the system and the measuring apparatus what determines the result of an experiment. In other words, hidden variables are contextual. Thus, in the first place, we show that a stationary probability density is unattainable, even in ordinary classical systems. This is important since many no-hidden variables theorems rely on this assumption [15], ascribing it to realism, as previously explained.

Evidently, if we make such an assumption, the main conclusion that can be drawn from the violation of such relations is that there exists no local hidden variables theory compatible with the predictions of quantum mechanics, as long as these hidden variables are assigned values prior to the measurement of the quantum physical magnitudes being studied. The assumption of no interference between the apparatus and the hidden variables is sometimes made because, otherwise, we would still have to explain why correlations are not destroyed by the collapse of the wave function. But it is really hard to ascertain if this assumption is correct in the absence of a clear physical picture of entanglement and of how such a collapse of the wave function takes place.

Consequently, in what follows we provide a possible picture of how a process of collapse can take place in a physical system. In this respect, we propose that the collapse of the wave function is a real dissipative process since, as it has been recently demonstrated, the wave function is a real force field related to internal and external electromagnetic forces, and not just a probabilistic entity [7, 16]. The present example is used just for illustration purposes, and it is not intended to replace the real case of fundamental particles under electrodynamic retarded potentials, even though it shares common features as a consequence of the dissipative character of the interaction between the apparatus and the physical system. In any case, we emphasize that with this example we are just trying to clear that an avoidance of the effect of the measuring apparatus on the hidden variables cannot be easily and generally done, without detailed experimental justification.

The experiment that we would like to propose is ostensibly simple. Consider that we let a coin fall freely along the vertical line with some initial angular velocity $\omega_0$, from some initial height $z_0$. The dynamics of the coin when it is falling is governed by Newton's second law and Euler's rotation equations, and can be described by using the variables $\theta(t)$ and $z(t)$ in the configuration space. We suppose that the value of the angle $\theta(t)$ is taken with respect to an axis of rotation that, for simplicity, we assume to be orthogonal with respect to the vertical axis. Then, with the purpose of measuring the angle (modulo $2\pi$) of the coin, we let our approximately rigid body evolve until it experiences an inelastic collision with a table underneath, which sets it to one of its two sides (the edge of the coin is thin enough). There can be only two possible outcomes of this measurement: heads ($\theta = 0$) or tails ($\theta = \pi$). We can safely affirm that, prior to the measurement, this system was in a combined state, oscillating between the two possible results of the experiment. Its interaction with the table, here playing the role of the apparatus, forces it to acquire one of the two possible outcomes that, following the simile, play the role of the eigenstates.

Of course, we could do things much better by using another apparatus with much smaller action than the table, so as not to disturb the coin. We must also recall that both, the final coordinates and the momenta of the two degrees of freedom $z$ and $\theta$ are known. Thus, no uncertainty principle is taking place here as in quantum mechanics. Moreover, we do not find here any interfering waves whose modes can be quantized, as it occurs with self-oscillating electrodynamic moving bodies [7, 8] and also in experiments with walking droplets [17, 18]. Nevertheless, this example allows to illustrate one first important point, which is that the outcome of the experiment was produced by the interaction with the table. Therefore, the result cannot be considered a property possessed by the system before the collision takes place. Interestingly, we note that this interaction is mediated by a dissipative phenomenon as a consequence of the electrodynamic nature of both constituents of the system: the coin and the table.

In the modern jargon of dynamical systems theory, we would say that the coin and the table comprise a nonlinear dynamical system that possesses two *attractors* in the phase space [19]: $(\theta, \omega, z, v_z) = (0, 0, 0, 0)$ and $(\theta, \omega, z, v_z) = (\pi, 0, 0, 0)$. It is also worth mentioning that, as a consequence of the dissipative dynamics, all the information concerning the angle and the angular momentum at the moment immediately previous to the contact between the two bodies is erased with the interaction. In other words, once an attractive limit set is attained, we have no empirical procedure to find out how this collapse towards the attractor occurred. This is just another manifestation of the irreversible nature of classical electrodynamics of moving sources, which is governed by the non-conservative functional differential equations arising from the field solutions of Jefimenko [8, 20].

Moreover, we can study how the initial conditions affect the fate of the result of the experiment, by computing the *basins of attraction*. Given a dynamical system with two or more attractors, a basin of attraction is a color plot defined over a region of the phase space, where every initial condition within a range of values is assigned a color, according to its asymptotic fate [21]. For example, in our case, we can use two colors, white and blue, if the experiment leads to heads or tails, respectively. It has been shown that, if we do not let the coin rebound on the table one or more times, the basins of attraction are manifestly separated by a smooth and a rather predictable boundary at our visual scale [19]. However, as we can see in the Fig. 30 in such reference, when we allow the coin to rebound twice on the table, the basins become increasingly random, and if we allow more rebounds, they completely mess up at the original scale [19]. Further inspection reveals that a fractal structure does not appear as we reduce the scale by zooming into the picture, as it commonly occurs with chaotic dynamical systems [21]. Therefore, from a classical point of view, the system is not unpredictable if sufficiently small scales are accessible to the experimenter [19]. On the contrary, for atomic systems, a sufficient reduction of the scale cannot be achieved in most cases. This occurs because fundamental particles are limiting structures and, as a consequence of this fact as well, it is impossible to reduce arbitrarily the action of the measuring apparatus on the system.

These last considerations lead us to the second important moral of the present analysis. The key point that we would like to stress is that, if our lack of knowledge of the final result of the coin orientation arises from our inability to control the initial conditions with which the coin is released in the beginning, a modification of the materials or the geometry of the table, here playing the role of the measuring apparatus, certainly can affect the nature of the potential results. These effects have two consequences. On the one hand, it modifies the function $B_b(\lambda)$, for example, to some other $B_c(\lambda)$, as Bell wisely devised when defining these variables.

On the other hand, since the dynamical evolution of the system is now guided by new differential equations, because of the different arrangement of the apparatus, the dynamically evolving solutions might not be the same. This introduces a dependence of the probability density on the measuring apparatus, which can be represented in the form $p_{ab}(\lambda, t)$, as it frequently occurs in *contextual* theories [22, 23]. In principle, $\lambda$ can also incorporate hidden variables of the apparatus. However, it must be highlighted that independence between the hidden variables related to the apparatus and those of its corresponding particle at the time of measurement is unattainable, contrary to expressions appearing in some contextual models [23].

If the reader hesitates to ascertain that this last argument is correct, perhaps he can consider the more familiar case of conservative physical systems. There, the evolution of a probability distribution of an ensemble of initial conditions $p(\lambda)$ is governed by Liouville's equation [24]. This hive of initial points distributed over the phase space, each representing a repetition of the apparently same experimental arrangement, evolve under the symplectic flow defined by Hamilton's equations. But then, we should not deny that, if the Hamiltonian changes throughout the evolution of the system, so it does the nature of the probability density $p(\lambda, t)$, which is advanced by the Liouvillian operator. In our more complicated case, which is not conservative neither Hamiltonian [7, 8], this change of the probability distribution is also reflected in the basins of attraction, which in turn affect the functions $A_a(\lambda)$ and $B_b(\lambda)$.

We now proceed to demonstrate Bell's correlation integral, by using the insights gained from the previous example. In the case of traditional mechanical systems (*e. g.*, Hamiltonian systems described by ordinary differential equations), where the feedback between the environmental electrodynamic fields and the dynamics of particles is neglected, particles can be represented by some finite set of generalized coordinates, that are affected by some potentials defined in the region where the apparatuses rest. Since the dynamics of the fields is being disregarded, we can neglect the explicit spatial dependence and write down the correlation as

$$C_{ab}(t) = \int A_a(\lambda) B_b(\lambda) p_{ab}(\lambda, t) d\lambda. \tag{5}$$

We can go back to a probability density in terms of the hidden initial conditions $\lambda_0$ and obtain a relation like the one used by Bell and his advocates. This can be done through a change of random variables in the form

$$p_{ab}(\lambda, t) = \int p(\lambda_0)\delta(\lambda_A - \Lambda^a_{At}(\lambda_0))\delta(\lambda_B - \Lambda^b_{Bt}(\lambda_0))d\lambda_0, \tag{6}$$

where the flow $\Lambda_t$ relates the hidden variables at the time of measurement and at the initial time $\lambda = \Lambda_t^{a,b}(\lambda_0)$, so that the vector $\lambda$ separates as a direct sum of two other vectors $\lambda_A$ and $\lambda_B$, which are uncorrelated. In other words, we assume that the flow can also be written as $\lambda_A = \Lambda^a_{At}(\lambda_0)$ and $\lambda_B = \Lambda^b_{Bt}(\lambda_0)$. Finally, we must consider that each observable depends on its respective and disconnected hidden variables as well (*i.e.*, $A_a(\lambda_A)$ and $B_b(\lambda_B)$ must hold), as suggested by the principle of locality. In this manner, we obtain the integral appearing in Eq. (3), by simply redefining the functions $A_a$ and $B_b$, and Bell's agenda, as appearing in the references [3, 10], can be perfectly carried out.

## IV. HIDDEN FIELD THEORIES

Now we assume that the hidden variables represent a vector field $\lambda(x, t)$, where $t$ is the present time in some inertial frame, while $x$ represents the position in the space. Just for the sake of clarity, here we accept that electrodynamic bodies are made of extended electromagnetic fields. This entails to think fundamental particles as some electromagnetic topological solitons [25]. Then, any other properties relying on their internal structure, as for example the spin, can be connected to the different physical properties of such gauge fields [26, 27]. For example, the spin could be related to the magnetic field configuration, while the charge would correspond to the topological charge of the vortex knot [28]. However, we recall that *no* particular assumptions are here made in relation to our argument concerning what particles really are. Our results are perfectly extensible to point particles with their properties (charge, mass and spin) all embedded in the point.

It is evident that the different orientations of the non-uniform magnetic field of a Stern-Gerlach apparatus will affect our dynamical hidden fields. Then, we have to rewrite the correlation function as

$$C_{ab}(t) = \iint A_a[\lambda_A(x'_a, t)]B_b[\lambda_B(x'_b, t)]p_{ab}[\lambda, t]\mathcal{D}\lambda_A\mathcal{D}\lambda_B, \tag{7}$$

where $t$ represents the time at which the particle's spin is first aligned with the S-G apparatus, while $x'_a$ and $x'_b$ are the spatial coordinates of the region where the two particles at

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such time instant are found. This integral is a functional integral over a set of hidden fields at the time of measurement, integrated over the region where most part of the particle's energy is stored. We also notice that the integral runs through all the possible values of the hidden fields $\lambda = (\lambda_A(x'_a, t), \lambda_B(x'_b, t))$ of the two particles, which have been gathered in one hidden variable for simplicity. Again, recall that the effect of the apparatus on the probability distribution appears explicitly in $p_{ab}[\lambda, t]$.

We now show that, when we consider fields that are defined all over the space, the derivation of the correlation integral is more complicated, yielding a different result. The hurdle arises because the hidden fields at the particle's position, when its magnetic moment has completely aligned with the external field of the Stern-Gerlach apparatus (*i.e.* when the collapse has completed), are determined from its causal past lying in some initial Cauchy hypersurface. As an example, the reader can consider the well-known theorem of Cauchy-Kovalévskaya [29].

In Fig. 2 we have represented the light cones of two entangled bodies that are measured at some instant of time to illustrate this effect. We see that when we attempt to express the correlation integral in terms of the initial hidden fields, which are defined on the aforementioned Cauchy surface, the probability density must inherit a dependence on the orientation of the apparatus. Indeed, the fields at the time of measurement are related to the initial fields through the Green's function $G(x', x, t)$ of the field theory and its time derivatives. For simplicity, we write this relation in the form

$$\lambda_a(x', t) = \int G(x', x, t)\lambda_0(x)dx. \tag{8}$$

Certainly, the propagator $G(x', x, t)$ will be zero between two regions that are not causally connected and, consequently, only the region inside the initial domain $\Omega_a$ affects the values of the hidden variables $\lambda_a(x', t)$, and similarly for $\lambda_b(x', t)$ and $\Omega_b$ (see again Fig. 2). If we now consider that the change of variables between the initial probability density and the probability density at the time of measurement is given by the mathematical relation $p_{ab}[\lambda, t] = \int \delta[\lambda_A(x'_a, t) - \lambda_a(x'_a, t)]\delta[\lambda_B(x'_b, t) - \lambda_b(x'_b, t)]p_{ab}[\lambda_0(x)]\mathcal{D}\lambda_0$, the correlation integral in terms of the initial fields can be written in the form

$$C_{ab} = \int \hat{A}_a[\lambda_0(x)]\hat{B}_b[\lambda_0(x)]p_{ab}[\lambda_0(x)]\mathcal{D}\lambda_0, \tag{9}$$

where this integral is again a functional integral over a set of uncontrollable initial hidden fields, defined on the initial Cauchy surface. We have introduced the functional $\hat{A}_a[\lambda_0(x)] = A_a[\int G(x', x, t)\lambda(x, 0)dx]$ and the same has been done for $B_b$.

Importantly, we highlight that, when defining these functionals, the spatiotemporal coordinates $t$ and $x'$ have been omitted. The temporal coordinate can be neglected by assuming that clocks are reset at the beginning of each experimental row and that measurements are always performed at the exact same instant of time. The spatial coordinate can be neglected if we assume that the measured particle is always at the same place and has the same orientation when the measurement is carried out. Obviously, these two restrictions are experimentally impossible to achieve [7, 8], and would hinder themselves the derivation of Bell's inequalities, as has been already proposed [30]. Nevertheless, here we take these impossible conditions for granted, to focus on a more severe loophole due to a correlation of hidden field fluctuations [31].

If we assume that the initial fields are only correlated in the domain $\Sigma_c = \Omega_a \cap \Omega_b$ depicted in Fig. 2, the CHSH-Bell inequality can be derived. We have to average the initial hidden fields over the regions $\Sigma_a = \Omega_a/\Sigma_c$ and $\Sigma_b = \Omega_b/\Sigma_c$. Indeed, under such hypothesis, the probability density $p_{ab}[\lambda_0(x)]$ can be expressed as a product of densities $p_a[\lambda_a(x)]p_b[\lambda_b(x)]p[\lambda_c(x)]$, where $\lambda_a(x) = \{\lambda_0(x) : x \in \Sigma_a\}$, $\lambda_b(x) = \{\lambda_0(x) : x \in \Sigma_b\}$ and $\lambda_c(x) = \{\lambda_0(x) : x \in \Sigma_c\}$. This is possible assuming that the fields are uncorrelated in spacelike separated sets. Similarly, we can express the functions $\hat{A}_a[\lambda_a(x), \lambda_c(x)]$ and the $\hat{B}_b[\lambda_b(x), \lambda_c(x)]$. This yields the functional integral

$$C_{ab} = \iiint \hat{A}_a[\lambda_a(x), \lambda_c(x)]\hat{B}_b[\lambda_b(x), \lambda_c(x)]p_a[\lambda_a(x)]p_b[\lambda_b(x)]p[\lambda_c(x)]\mathcal{D}\lambda_a\mathcal{D}\lambda_b\mathcal{D}\lambda_c, \quad (10)$$

which, after averaging out fluctuations $\bar{A}_a[\lambda_c(x)] = \int \hat{A}_a[\lambda_a(x), \lambda_c(x)]p_a[\lambda_a(x)]\mathcal{D}\lambda_a$, and the same for $\bar{B}_b[\lambda_c(x)]$, yields a Bell-type integral. As it has been pointed out in previous works [23, 32], these experiments are impossible to accomplish, since we would have to repeat each experimental row with exactly the same two entangled particles a great number of times, letting the electrodynamic fields in $\Sigma_a$ and $\Sigma_b$ change to average out their fluctuations. However, Eq. (10) allows to derive the CHSH-Bell inequality and must describe the same type of correlations.

The reader might wonder why there should be a dependence of $\hat{A}_a[\lambda_a(x), \lambda_c(x)]$ and $\hat{B}_b[\lambda_b(x), \lambda_c(x)]$ on the fields outside the particles $\lambda_a(x)$ and $\lambda_b(x)$ at all. The importance of the environmental fluctuating hidden fields in all the three sets $\Sigma_a$, $\Sigma_b$ and $\Sigma_c$, is explained as follows. On the one hand, it is crucial that these fluctuations are not very strong in the former regions. Otherwise, decoherence phenomena would appear. In both the former and the later regions, as it has been recently suggested, these field fluctuations, which travel at the speed of light and reach the particle all the way along their journey to the SG apparatus, can be indispensable to promote their coherent self-oscillatory motion [7, 8]. In turn, these oscillations can be crucial to explain the entanglement of the electrodynamic particles. We develop more thoroughly these ideas in Sec. VIII.

A correlation between $\Sigma_a$ (or $\Sigma_b$) and $\Sigma_c$ still allows to factorize Eq. 10 and to derive the CHSH-Bell inequality. Therefore, our main conclusion is that to prove this inequality for classical field theories, we must first show that hidden electrodynamic field fluctuations are uncorrelated for the spacelike separated sets in $\Sigma_a$ and $\Sigma_b$ in the Cauchy hypersurface. This assumption deserves more attention, since correlations of field fluctuations far from equilibrium can extend through very wide regions of spacetime in the Cauchy hypersurface [31]. The distance between $\Sigma_a$ and $\Sigma_b$ depends considerably on the nature of the experiment, being very small for experiments with photons receding from each other, and depending critically on the speed at which charges separate in experiments with entangled fermions.

As it has been already pointed out [31], this correlation loophole was overlooked by Shimony, Horne, and Clauser in a reply to a work by Bell [34], where they discuss another possible source of conspiracy [33]. The Eq. (9) prevents the derivation of the CHSH-Bell inequality, and unless the loophole is experimentally closed, we cannot safely affirm that classical field theories can be rejected as a foundation of quantum mechanics. First, it must be shown that the field fluctuations are uncorrelated or irrelevant to the production and maintenance of entangled pairs, which is at odds with recent findings explaining the origin of the wave-particle duality in terms of self-oscillations [7, 35]. Importantly, our derivation of Eq. (9) involves hidden fields that are local in the sense of classical field theories.

## V. LAST-INSTANT CHOICES

We might be tempted to avoid any possible effect on each particle by the environmental field fluctuations, which depend on its related apparatus, by making a last-instant choice, *i.e.*, by dynamically setting its orientation through some physical mechanism, right before the particle enters the S-G device [13]. The main purpose is to avoid a dependence of

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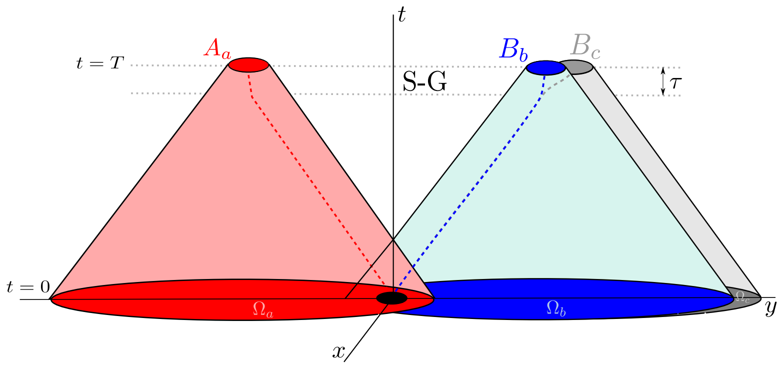


FIG. 2. **Light cones of the particles**. A (2+1)-dimensional spacetime diagram of an extended electrodynamic body (black disk) that splits into two bodies $A$ and $B$ (red and blue disks), which move far away from each other towards two respective measuring devices. The process of measurement is denoted as S-G. The dashed lines represent the worldlines of the particles, while their light cones at the instant of measurement are represented in red and blue, showing the sets of causally connected initial conditions $\Omega_a$ and $\Omega_b$ that affect each particle during its journey. When the orientation of the S-G apparatus at $B$ is switched from $b$ to $c$, the domain of initial conditions correspondingly switches from $\Omega_b$ to $\Omega_c$.

$p_{ab}[\lambda_a(x), \lambda_b(x), \lambda_c(x)]$ on $a$ and $b$, both in the hidden fields and the probability density. This would entail that the hidden fields appearing in $\hat{A}_a[\lambda_a(x), \lambda_c(x)]$ and $\hat{B}_a[\lambda_b(x), \lambda_c(x)]$ are also independent of $a$ and $b$.

Now, if the orientation of the apparatuses is set dynamically, the system of partial differential equations that describe the evolution of the fields are required to compute the evolution of such orientations, which can be expressed as functions of them in the form $a(\hat{x}_a, t) = \alpha(\lambda_A(\hat{x}_a, t))$ and $b(\hat{x}_b, t) = \beta(\lambda_B(\hat{x}_b, t))$. By composition, these functions become dynamical fields as well, where $\hat{x}_a$ and $\hat{x}_b$ represent the positions of the centre of the top part of each respective S-G (see Fig. 1), measured from the centre of mass of the apparatus, if desired.

In this case, the resulting change of variables relating the probability density at the time of measurement and the probability density defined on the initial Cauchy hypersurface is given by the relation $p_{ab}[\lambda, t] = \int \delta[\lambda_A(x'_a, t) - \lambda_a(x'_a, t)]\delta[\lambda_B(x'_b, t) - \lambda_b(x'_b, t)]\delta[a(\hat{x}_a, t) - \alpha(\lambda_A(\hat{x}_a, t))]\delta[b(\hat{x}_b, t) - \beta(\lambda_B(\hat{x}_b, t))]p_{ab}[\lambda_0(x)]\mathcal{D}\lambda_0$, where now we have gathered the four variables in the vector $\lambda = (\lambda_A(x'_a, t), \lambda_B(x'_b, t), \lambda_A(\hat{x}_a, t), \lambda_B(\hat{x}_b, t))$. However, we must not

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neglect the fact that, just as it occurred before, the initial probability density $p_{ab}[\lambda_0(x)]$ changes when a different orientation of the S-G apparatus results to measure the spin.

The reason of this dependence is that, in order to compute the correlations for a definite orientation of the apparatus, all those initial field configurations that lead to a different orientation are being disregarded. Consequently, a restriction of events in the sample space of initial fields is being imposed. Thus a dependence of the probability density $p_{ab}[\lambda_0(x)]$ on the orientations $a$ and $b$ cannot be circumvented. As the reader can verify himself, the replacement of $p_{ab}[\lambda, t]$ in a corresponding correlation integral similar to Eq. (7) leads to Eq. (9) anew.

## VI. RANDOM CHOICES

We can also assume that the orientations of the S-G apparatuses are set by an inherently random mechanism (freely, if desired) at some brief instant of time before the measurement takes place. The consideration of these stochastic hidden variable models is relevant because, despite the fact that one is inclined to think that randomness is just a byproduct of deterministic chaos [36], there exist complexity measures that distinguish between low-dimensional chaotic dynamical systems and computer-generated noise [37]. Nevertheless, and precisely because of the fact that rather simple low-dimensional chaotic dynamical systems are capable of generating white noise, we should not quickly discard that high-dimensional chaotic dynamical systems might be behind the supposed intrinsic randomness of computed-generated noise, neither of quantum noise [7, 8].

In case that stochastic fields are considered as fundamental [38], we can represent mathematically the selection of the orientation of the apparatus, and any other randomness in the experiment as well, by a *Langevin current* $j(x,t)$ acting on the dynamical fields, which ultimately decides what orientation of the S-G is used. This turns the system of equations describing the evolution of the fields $\lambda(x,t)$ into a system of stochastic partial differential equations. The relation between the hidden field configuration of the apparatus and the initial data $\lambda_0(x)$ is now given by the convolution

$$\lambda_a(\hat{x},t) = \int G(\hat{x},x,t)\lambda_0(x)dx + \int\int_0^t G(\hat{x},x,t')j_a(x,t')dxdt', \quad (11)$$

where in this example we have considered a situation in which the final orientation of the

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S-G is given by the vector $a$.

Nevertheless, the convolution (11) prevents the expression of the probability density as independent of the setting of the apparatus. As long as there is any deterministic component in the Green's function of the field's dynamical equations (no matter how small), in addition to the stochastic Langevin force, we cannot guarantee that the probability density depending on the initial hidden field configuration $\lambda_0(x)$ is the same for different ultimate orientations of the measuring apparatuses. This argument also concerns very complicated situations where human decision-making is used to generate random choices [39].

Importantly, we recall that a superdeterministic loophole is *not* here implied in any way [6], since we assume that the values of the magnetic field orientation of the S-G apparatus can be set stochastically and independently of each other. Simply put, the values of $a$ and $b$ are statistically independent and causally disconnected. This is irrelevant to our argument. Irrespective of how this choice is made, a particular setting of the apparatus affects the particle's hidden variables when its internal angular momentum is measured. Since different probability densities computed from the Cauchy data in its causal past correspond to different orientations of the SG devices, the correlation loophole induced by contextual hidden fields shall persist in stochastic hidden field variable models [22, 38].

## VII. A NEW INEQUALITY

On the basis of the expression represented in Eq. (5), the difference between correlations can be derived in a straightforward manner as

$$C_{ab}(t) - C_{ac}(t) = \int A_a(\lambda)B_b(\lambda)p_{ab}(\lambda,t)d\lambda - \int A_a(\lambda')B_c(\lambda')p_{ac}(\lambda',t)\; d\lambda'. \tag{12}$$

A change of variable, and the fact that probability densities are normalized, allow to write

$$C_{ab}(t) - C_{ac}(t) = \int \{A_a(\lambda)B_b(\lambda) - A_a(\lambda')B_c(\lambda')\} p_{abc}(\lambda,\lambda',t)d\lambda d\lambda', \tag{13}$$

where here we have introduced the joint probability density $p_{abc}(\lambda,\lambda',t) = p_{ab}(\lambda,t)p_{ac}(\lambda',t)$ of the the two hidden fields corresponding to each experiment. This probability density should not be confused with joint probability densities of incompatible experiments [5], since different hidden variables are used in repeated experiments even with one of the S-G apparatuses keeping its orientation. It manifests the mutual independence of different measurements, just as two consecutive coin tosses with the same coin are frequently considered independent processes. Nevertheless, we remind that this is only true assuming a certain inability to control the initial conditions [19], or unless the basins of attraction are very fractal due to an underlying chaotic dynamics.

If we again take the absolute value on both sides of the previous equations and we reason exactly as in Bell's work [3], we get the mathematical relation

$$|C_{ab}(t) - C_{ac}(t)| \leq \int |A_a(\lambda)A_b(\lambda)||1 - A_a(\lambda)B_b(\lambda)A_a(\lambda')B_c(\lambda')|p_{abc}(\lambda, \lambda', t)d\lambda d\lambda'. \quad (14)$$

Here we clearly see that conventional steps appearing in Bell-type theorems [3, 10] cannot be accomplished. Certainly, the first term still obeys the relation $|A_a(\lambda)A_b(\lambda)| = 1$, while the second is again always greater or equal than zero, what allows us to neglect the absolute value. We are thus lead to the relation

$$|C_{ab}(t) - C_{ac}(t)| \leq 1 - C_{ab}(t)C_{ac}(t). \quad (15)$$

Using again the relations for one-half spin particles, we get the inequation

$$|\cos\theta_{ab} - \cos\theta_{ac}| \leq 1 - \cos\theta_{ab}\cos\theta_{ac}. \quad (16)$$

The reader can verify and demonstrate that, no matter which angular variables $\theta_{ab}$ and $\theta_{ac}$ we consider in $[0, 2\pi]$, the previous inequality is always fulfilled. Therefore, there is no theoretical reason implying that a classical theory cannot violate Bell's inequalities, as long as their dynamical hidden fields are dependent on the measurement apparatus, as it occurs in classical electrodynamics.

## VIII. ENTANGLEMENT AS SYNCHRONIZATION

We now provide a qualitative explanation of why quantum collapse preserves the correlations of entangled pairs. Assuming that no communication between apparatuses is allowed (locality loophole), as recent experiments guarantee [40], the only possible solution that the authors can envisage is that such correlations are dynamically preserved during all the physical phenomenon. In other words, it must be the property of entanglement that guarantees the observed correlation at all times. From this point of view, the internal angular momentum of the two particles is evolving but both particles are electromagnetically *synchronized* to render the total conservation of the spin. This locking of the phases of their evolving

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internal angular momentum must not be a product of chance, but can only be maintained by means of some nonlinear interaction [41].

It has been recently demonstrated by deriving the quantum potential from classical electrodynamics that the wave-particle duality has its origin in the electrodynamic Liénard-Wiechert potentials [7, 8, 35]. These retarded potentials lead to state-dependent time-delayed differential equations, reflecting the memory effects arising from self-interactions of electrodynamic accelerated extended bodies. The feedback interaction of radiative and Coulombian fields can unleash a self-oscillatory dynamics [11], what manifests the excitable character of fundamental particles, which can operate far from equilibrium, specially in the presence of fluctuating external fields [8]. The resulting motion is a violent oscillation with frequency similar to the *zitterbewegung* oscillation appearing in Dirac's equation.

When several particles are considered in interaction, the delays of their self-interactions leading to such internal oscillation can couple to the delays affecting their mutual interactions. Since these delays depend on the kinematic variables of the bodies at different times, the dynamics of these fundamental particles becomes subsequently entangled. In other words, the internal oscillations of the particles can become synchronized or entrained, as it is this phenomenon technically named in the theory of nonlinear oscillations [11]. In the case of spin entangled pairs, a synchronization between the evolving internal magnetic moment of the electromagnetic particles is expected.

We can further assume that the process of collapse is of such nature that it destroys the entanglement following deterministic laws, even if there is a sensitivity to initial conditions. This means that the "channel" [16] or basin of attraction that determines which eigenmode of vibration (limit cycle) is selected, does not occur by chance, but it depends through a dynamical relation between both the system and the apparatus. The only condition that is mandatory is that the collapse takes place more or less simultaneously in the laboratory frame. In this last regard, the two particles must be kept entangled through their respective journeys, which is a rather difficult task to accomplish, given the tendency of these particles to decohere, by strongly coupling to their surrounding electromagnetic fluctuating environment when they approach other external bodies.

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## IX. DISCUSSION

We have shown that Bell-type inequations cannot be derived in classical theories involving local fields. This occurs because the hidden fields circumscribed to the particle, in terms of which the internal angular momentum is defined, are dynamically evolving. Their values are determined by the interaction of the particle's fields and the hidden fields of the contextual measuring devices [32, 42]. These hidden fields are then related to initial field data inside its causal past, which involve the apparatuses as well. This last fact implies a *correlation loophole* that is not usually considered when deriving Bell-type inequalities. It is reminiscent of arguments presented in previous works [31]. However, here we have not assumed that the fields need to be random, we have explicitly represented hidden fields all over the physical process and connected them to the contextual paradigm.

We also recall that the present arguments are extensible to other inequalities [10], which rely on correlation integrals as defined by Bell, and also to stochastic hidden variable models that are tantamount to them [43]. For the same reason, the present work evinces the non-Kolmogorovness of joint probability distributions [44] comprising more than one orientation of the same measuring apparatus, due to field correlations. Our work also rules out other formulations of no-hidden variables theorems, for example the Bell-Kochen-Specker theorems [15], since we have shown that stationary probability densities do not comply with classical electrodynamics. If the contextuality loophole is not circumvented, traditional experimental works carried out so far to test Bell's inequalities [12, 40], do not prove the impossibility of a foundation of quantum mechanics by means of classical field theories.

The point of view of the present work conforms very well with a description of reality in terms of two layers, as has been explained in previous essays [45]. The first layer is made of the fields, which are hidden, insofar as it is not possible to directly measure them. Following Bell, we could call this hidden fields the *beables* [34]. On the other hand, there is what we get to know by the affection of different parts of the field, one related to the system and another to the apparatus, which manifests as some perceivable change. Such interacting forces manifest as a dynamical effect of the fields (*e.g.* the appearance of a spot on a fluorescent screen), which is the only thing that is accessible to us directly, as opposed to the fields themselves. This would constitute the phenomenal part of reality, which is frequently called the *observables*.

But even if we decide not to do the measurement, the fields must be there as the precondition of any experimental knowledge, at least if we are willing to accept that this knowledge is nothing else than the result of a dynamical effect of the fields. Thus, whatever are the fundamental equations that constitute the dynamics of elementary fields, they must be used simultaneously for the representation of the system and the measuring apparatus, which have the same physical basis and, in this respect, form a unified inseparable physical reality.

The frequent avoidance of the representation of the apparatus and its fields in the quantum mechanical formalism, which reveals the non-completeness of quantum theory itself, is justified because of its tremendous size and complexity in relation to the system to be measured. This theoretical convenience, which lead Bohr to formulate the principle of correspondence, might be at the core of the measurement problem in quantum mechanics. Hopefully, our increasing ability to perform numerical simulations will help us to face these multi-scale problems, avoiding the appearance of the concept of measurement as a fundamental concept in physical theories [46].

## X. ACKNOWLEDGMENTS

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