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Solar system-like systems with special reference to relativistic quantum mechanics and weak gravitational field

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Abstract

This paper attempts to describe the large-scale (solar system-like, astronomical-scale) systems of the known world using the physical models and mathematical tools of relativistic quantum mechanics. The value of H_x can be introduced and approximated as an analogue of the constant $\hbar$. Based on the quantum mechanical approach, the proper time scale of the solar system can be determined.

Subject headings: Solar system-like systems, Relativistic Quantum Mechanics, Weak Gravity, Proper Time Scale of the Solar System

Introduction

The unified theory of general relativity and quantum mechanics has been and is still being worked on by countless researchers. In this paper, we have chosen a different path, in which special relativity provides an appropriate basis. Between special relativity and quantum mechanics, between the two approaches, there is a proven link. Theory of relativity and quantum mechanics are successfully linked in the work of Dirac [9]. From the point of view of relativistic quantum mechanical description and physical world view, it is the weak gravitational field that can be successfully described. For the purpose of the present paper, it is important to point out that, starting from the mathematical-physical foundations originally formulated by Dirac and later developed further, the possibility of quantization for weak gravitational fields has been demonstrated. One important and fundamental reference can be mentioned, which is attributed to Bronstein. His original article [8] is one of the first to formulate a proven link between relativistic quantum mechanics and weak gravitational fields. All this has been successfully demonstrated by modern research, using only special relativity to derive the "gravitomagnetic potential" [51].

In most cases, the approaches formulated in the mathematical "language" of the general theory(s) of gravity do not seem to yield useful results. Let us try to speak a "different language": the original mathematical tools used by relativistic quantum mechanics. The present paper focuses on the now classical Dirac formulation and continues it from a new physical point of view. It is very important to state now that the mathematical description of relativistic quantum mechanics established by Dirac does not have a distinguished scale.

If we ask the question where a weak gravitational field exists, the answer is simple: the set of solar system-like systems, the stars, as we know it (in other words, the world as we know it, the Universe) is almost entirely characterised by a weak gravitational field. This justifies the question of whether it is possible to extend the physical-mathematical description of quantum mechanics to the Universe (the whole of it, as defined by the weak gravitational field).

Such an extension could be justified if we could apply to solar system-like systems a physical model where masses (stars, planets) could be considered as mass points and also have charge (and also be considered as indistinguishable mass points and have quantized orbits in solar system-like systems). The first two criteria are easily met, at least in thought. 'Considered

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as mass points' is trivial, 'having charge' appears in many works. For other important aspects, see later.

1. Relativistic quantum mechanics and gravitational potential

The simplest way to describe the physical model of an atom with a single electron is to use the mathematical tools of quantum mechanics. Let's start from this. Consider an *e*-charged particle of mass μ moving in a system with central mass *M*. Assume that the electromagnetic effects and the gravitational effects act independently of each other, simultaneously. The energy is the sum of the kinetic energy of the μ-mass particle and the potential energy (*V*)

$$H = \frac{1}{2}\mu\left(v_x^2 + v_y^2 + v_z^2\right) + V(x,y,z) \tag{1}$$

The energy operator can be written as:

$$\mathbf{H} = -\frac{1}{2\mu}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(x,y,z) \tag{2}$$

The Hamiltonian function corresponding to the above equation of motion is:

$$\mathbf{H} = \frac{1}{2\mu}\left(\underline{p} - \frac{e}{c}\underline{A}\right)^2 + U \tag{3}$$

Assume that the centre of mass μ moves in the weak gravitational field of the central mass *M*. Here $\underline{p}$ is the momentum vector (the vector potential $\underline{A}$ is zero in our case).

In the gravitational equations, we assume a (weak) static gravitational field (the tensor component g_{ik} is time independent, there is no energy flux, there is a simple linear relation between g_{44} and the potential energy). The description of the movement of particles in such weak gravitational fields yields the same results as in the U_2 gravitational potential of the classical Newtonian theory. Given the above, the following Hamiltonian function can be written for weak gravitational fields. Here we depart from the usual quantum mechanical approach in the first taste. The only deviation from the original approach is the equation $U = eU_1+U_2$:

$$\mathbf{H} = c\sqrt{\mu^2c^2 + \mathbf{p}^2} + eU_1 + U_2 \tag{4}$$

where $U_1 = U_1(r) = -\frac{e}{r}$ the Coulomb potential, $U_2 = -\frac{f_x M\mu}{r}$ the gravitational potential

In the above equation, $\mathbf{p}$ denotes a (differential)operator. In the following, the steps of the derivation of [5] are followed. To carry out the square-root, operators similar to Pauli's spin operators can be introduced in quantum mechanics. Following the linearization performed by Dirac:

$$\sqrt{\mathbf{p}_x^2 + \mathbf{p}_y^2 + \mathbf{p}_z^2} = \boldsymbol{\sigma}_x\mathbf{p}_x + \boldsymbol{\sigma}_y\mathbf{p}_y + \sigma_z\mathbf{p}_z \tag{5}$$

For the above reasons, it can finally be written that:

$$\mathbf{H} = eU_1 + U_2 + c\rho_1(\boldsymbol{\sigma}_x\mathbf{p}_x + \boldsymbol{\sigma}_y\mathbf{p}_y + \boldsymbol{\sigma}_z\mathbf{p}_z) + \rho_3\mu_0c^2 \tag{6}$$

The energy *E* operator:

$$\mathbf{E} = -\frac{\hbar}{i}\frac{\partial}{\partial t} \tag{7}$$

Using the well-known form of the energy operator) the equation-of-state:

$$\mathbf{H}\psi = E\psi$$

which can be written as follows:

$$\frac{\hbar}{i}\frac{\partial\Psi}{\partial t}+\left[\frac{\hbar c}{i}\rho_1\left(\boldsymbol{\sigma}_x\frac{\partial}{\partial x}+\boldsymbol{\sigma}_y\frac{\partial}{\partial y}+\boldsymbol{\sigma}_z\frac{\partial}{\partial z}\right)+\rho_3\mu_0c^2+U\right]\Psi=0 \quad (8)$$

Formally, the equation above differs from the usual Dirac equation in the U_2 term, where $U=eU_1+U_2$. (The consequences of this difference are explained in more detail below). The rest of the solution is partly based on [9].

2. Derivation of the radial part of the relativistic Dirac equation

The state function Ψ depends on the variables x,y,z,t and the variables s=±1/2 and r==±1/2, so the state-function can be written as:

$$\Psi=\Psi(x,y,z,t,r,s) \quad (9)$$

Expressed in terms of the eigenfunctions, we obtain the following (here we have assumed that spin and motion are independent, so the eigenfunctions can be separated into coordinate-dependent and spin-dependent parts, and the same is true for the eigenfunctions ξ, η acting on the r-degree of freedom):

$$\Psi=\Psi_1(x,y,z,t)\alpha\xi+\Psi_2(x,y,z,t)\beta\xi+\Psi_3(x,y,z,t)\alpha\eta+\Psi_4(x,y,z,t)\beta\eta \quad (10)$$

In fact, the four coupled partial differential equations in (10) can be transformed into two coupled partial differential equations, since Ψ_1 and Ψ_2 respectively Ψ_3 and Ψ_4 are interchangeable. The stationary solutions of the above equation are found in the following form:

$$\Psi=\Phi(x,y,z,r,s)e^{-\frac{i}{\hbar}Et} \quad (11)$$

The corresponding derivations can be found in many quantum mechanics literatures [1], [6], [7], [19], [23]. Taking the above into account, we have to solve the following eigenvalue equation:

$$\left(U(r)+c\alpha\mathbf{p}_r+c\frac{i}{r}\alpha\rho_3\mathbf{K}+\rho_3\mu_0c^2\right)\Psi=E\Psi \quad (12)$$

Ψ can be written as:

$$\Psi=\frac{1}{r}\begin{bmatrix}F(r)\\G(r)\end{bmatrix} \quad (13)$$

The corresponding mathematical procedures lead to a system of equations consisting of two ordinary differential equations, the solution of which can be basically carried out according to the quantum mechanics literature:

$$\frac{dG}{dr}-K\frac{G}{r}=\frac{1}{\hbar c}\left(\mu_0c^2-E+U(r)\right)F \quad (14)$$

$$\frac{dF}{dr}+K\frac{F}{r}=\frac{1}{\hbar c}\left(\mu_0c^2+E-U(r)\right)G \quad (15)$$

where

$$K=\mp\left(j+\frac{1}{2}\right) \quad (16)$$

The derivation presented so far is fully consistent with the details of the literature cited above. Details of the differences in U(r) are given in the next section. The solution of the radial equations is available in many books on quantum mechanics. The only important purpose of the detailed description of the solution presented here is to point out the point where the original quantum mechanical interpretation can be changed to a more general interpretation of "large-scale systems". We now extend the interpretation of U(r) to include the gravitational potential in the potential energy:

$$U(r) = -\frac{1}{r}(e^2 + f_x M\mu_0) \tag{17}$$

$$\frac{1}{\hbar c}U(r) = -\frac{1}{r}A \tag{18}$$

where, because of the correct dimensions:

$$A = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}Z + f_x\frac{M\mu_0}{\hbar c} \tag{19}$$

Here f_x is the gravitational constant. In the following, *A* in (19) will play a role in the interpretation of the solution. We assume Z=1 and substitute it back into (14) and (15):

$$\frac{dG}{dr} - K\frac{G}{r} = \left(\frac{\mu_0 c}{\hbar}(1 - \frac{E}{\mu_0 c^2}) - A\frac{1}{r}\right)F \tag{20}$$

$$\frac{dF}{dr} + K\frac{F}{r} = \left(\frac{\mu_0 c}{\hbar}(1 + \frac{E}{\mu_0 c^2}) + A\frac{1}{r}\right)G \tag{21}$$

The solution procedure follows the descriptions in the literature, i.e. the form of the asymptotic solution (given by a large r value) is:

$$F = rf$$
$$G = rg \tag{22}$$

$$\lambda = \frac{\mu_0 c}{\hbar}\sqrt{1-\varepsilon^2} \tag{23}$$

where:

$$E_0 = \mu_0 c^2 \tag{24}$$

$$\varepsilon = \frac{E}{E_0} \tag{25}$$

Let:

$$x = 2\lambda r \tag{26}$$

thus:

$$\frac{1}{r} = 2\lambda\frac{1}{x} \tag{27}$$

and

$$dx = 2\lambda dr \tag{28}$$

According to the above:

$$\frac{dG}{dr} = \frac{dG}{dx}\frac{dx}{dr} = 2\lambda\frac{dG}{dx} \tag{29}$$

and also:

$$\frac{dF}{dr} = \frac{dF}{dx}\frac{dx}{dr} = 2\lambda\frac{dF}{dx} \tag{30}$$

Substitute equations (29) and (30) into equations (20) and (21):

$$2\lambda\frac{dG}{dx} - K\frac{G}{x}2\lambda = \left(\frac{\mu_0 c}{\hbar}(1 - \frac{E}{\mu_0 c^2}) - A\cdot 2\lambda\frac{1}{x}\right)F \tag{31}$$

$$2\lambda\frac{dF}{dx}+K\frac{F}{x}2\lambda=\left(\frac{\mu_0 c}{\hbar}(1+\frac{E}{\mu_0 c^2})+A\cdot 2\lambda\frac{1}{x}\right)G \tag{32}$$

Continue with the simplifications, divide the equations by *2λ*:

$$\frac{dG}{dx}-K\frac{G}{x}=\left(\frac{1}{2\lambda}\frac{\mu_0 c}{\hbar}(1-\varepsilon)-A\frac{1}{x}\right)F \tag{33}$$

$$\frac{dF}{dx}+K\frac{F}{x}=\left(\frac{1}{2\lambda}\frac{\mu_0 c}{\hbar}(1+\varepsilon)+A\frac{1}{x}\right)G \tag{34}$$

Substituting (23) and with minor transformations (33) we can write:

$$\frac{1}{\sqrt{1-\varepsilon}}\left(\frac{dG}{dx}-K\frac{G}{x}\right)=\left(\frac{\hbar}{2\mu_0 c}\frac{\mu_0 c}{\hbar}\frac{\sqrt{1-\varepsilon}\sqrt{1-\varepsilon}}{\sqrt{1-\varepsilon}\sqrt{1+\varepsilon}}\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\right)F\frac{1}{\sqrt{1+\varepsilon}} \tag{35}$$

After simplifications:

$$\frac{1}{\sqrt{1-\varepsilon}}\left(\frac{dG}{dx}-K\frac{G}{x}\right)=\left(\frac{1}{2}-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\right)F\frac{1}{\sqrt{1+\varepsilon}} \tag{36}$$

Similarly, substituting (23) with minor modifications, (34) can be written:

$$\frac{1}{\sqrt{1+\varepsilon}}\left(\frac{dF}{dx}+K\frac{F}{x}\right)=\left(\frac{\hbar}{2\mu_0 c}\frac{\mu_0 c}{\hbar}\frac{\sqrt{1+\varepsilon}\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}\sqrt{1+\varepsilon}}\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}+A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\right)G\frac{1}{\sqrt{1-\varepsilon}} \tag{37}$$

After simplifications:

$$\frac{1}{\sqrt{1+\varepsilon}}\left(\frac{dF}{dx}+K\frac{F}{x}\right)=\left(\frac{1}{2}+A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\right)G\frac{1}{\sqrt{1-\varepsilon}} \tag{38}$$

We need to solve the coupled equations (36) and (38). According to mathematicians - and we believe them - the solution should be found in the following form:

$$G=\sqrt{1-\varepsilon}\cdot e^{-\lambda r}\left(\varphi_1-\varphi_2\right) \tag{39}$$

$$F=\sqrt{1+\varepsilon}\cdot e^{-\lambda r}\left(\varphi_1+\varphi_2\right) \tag{40}$$

Substituting from (26) to (28):

$$G=\sqrt{1-\varepsilon}\cdot e^{-\frac{1}{2}x}\left(\varphi_1-\varphi_2\right) \tag{41}$$

$$F=\sqrt{1+\varepsilon}\cdot e^{-\frac{1}{2}x}\left(\varphi_1+\varphi_2\right) \tag{42}$$

Form the derivatives of (41) and (42):

$$\frac{dG}{dx}=\sqrt{1-\varepsilon}\cdot e^{-\frac{1}{2}x}\left(-\frac{1}{2}\varphi_1+\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}-\frac{d\varphi_2}{dx}\right) \tag{43}$$

$$\frac{dF}{dx}=\sqrt{1+\varepsilon}\cdot e^{-\frac{1}{2}x}\left(-\frac{1}{2}\varphi_1-\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}+\frac{d\varphi_2}{dx}\right) \tag{44}$$

Substitute (41), (42), and (43) into (36):

$$\frac{1}{\sqrt{1-\varepsilon}}\left(\sqrt{1-\varepsilon}\cdot e^{-\frac{1}{2}x}\left(-\frac{1}{2}\varphi_1+\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}-\frac{d\varphi_2}{dx}\right)-\frac{K}{x}\sqrt{1-\varepsilon}\cdot e^{-\frac{1}{2}x}\left(\varphi_1-\varphi_2\right)\right)$$
$$=\left(\frac{1}{2}-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\right)\frac{1}{\sqrt{1+\varepsilon}}\sqrt{1+\varepsilon}\cdot e^{-\frac{1}{2}x}\left(\varphi_1+\varphi_2\right) \tag{45}$$

With further simplifications and dividing by $e^{-\frac{1}{2}x}$ we get:

$$-\frac{1}{2}\varphi_1+\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}-\frac{d\varphi_2}{dx}-\frac{K}{x}(\varphi_1-\varphi_2)=\left(\frac{1}{2}-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\right)(\varphi_1+\varphi_2) \quad (46)$$

$$-\frac{1}{2}\varphi_1+\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}-\frac{d\varphi_2}{dx}-\frac{K}{x}(\varphi_1-\varphi_2)=\frac{1}{2}\varphi_1-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\varphi_1+\frac{1}{2}\varphi_2-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\varphi_2 \quad (47)$$

Further simplified:

$$\frac{d\varphi_1}{dx}-\frac{d\varphi_2}{dx}=\varphi_1\left(1+\frac{K}{x}-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\right)+\varphi_2\left(-\frac{K}{x}-A\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\frac{1}{x}\right) \quad (48)$$

Now we do the same with (41), (42) and (44), substituting into (38):

$$\frac{1}{\sqrt{1+\varepsilon}}\left(\sqrt{1+\varepsilon}\cdot e^{-\frac{1}{2}x}\left(-\frac{1}{2}\varphi_1-\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}+\frac{d\varphi_2}{dx}\right)+\frac{K}{x}\sqrt{1+\varepsilon}\cdot e^{-\frac{1}{2}x}(\varphi_1+\varphi_2)\right)$$
$$=\left(\frac{1}{2}+A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\right)\frac{1}{\sqrt{1-\varepsilon}}\sqrt{1-\varepsilon}\cdot e^{-\frac{1}{2}x}(\varphi_1-\varphi_2) \quad (49)$$

Again, with further simplifications and dividing by $e^{-\frac{1}{2}x}$:

$$-\frac{1}{2}\varphi_1-\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}+\frac{d\varphi_2}{dx}+\frac{K}{x}(\varphi_1+\varphi_2)=\left(\frac{1}{2}+A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\right)(\varphi_1-\varphi_2) \quad (50)$$

$$-\frac{1}{2}\varphi_1-\frac{1}{2}\varphi_2+\frac{d\varphi_1}{dx}+\frac{d\varphi_2}{dx}+\frac{K}{x}(\varphi_1+\varphi_2)=$$
$$=\frac{1}{2}\varphi_1+A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\varphi_1-\frac{1}{2}\varphi_2-A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\varphi_2 \quad (51)$$

Further simplified:

$$\frac{d\varphi_1}{dx}+\frac{d\varphi_2}{dx}=\varphi_1\left(1-\frac{K}{x}+A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\right)+\varphi_2\left(-\frac{K}{x}-A\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}\frac{1}{x}\right) \quad (52)$$

Add equations (48) and (52):

$$2\frac{d\varphi_1}{dx}=\varphi_1\left(2-\frac{K}{x}+\frac{K}{x}+\left(\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}-\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\right)A\frac{1}{x}\right)+\varphi_2\left(-2\frac{K}{x}+\left(-\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}-\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\right)A\frac{1}{x}\right) \quad (53)$$

Simplified:

$$\frac{d\varphi_1}{dx}=\varphi_1\left(1-\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A\frac{1}{x}\right)+\varphi_2\left(-\frac{K}{x}-\frac{1}{\sqrt{1-\varepsilon^2}}A\frac{1}{x}\right) \quad (54)$$

Subtract equation (48) from (52):

$$2\frac{d\varphi_2}{dx}=\varphi_1\left(-2\frac{K}{x}+\left(\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}+\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\right)A\frac{1}{x}\right)+\varphi_2\left(\left(-\frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}}+\frac{\sqrt{1+\varepsilon}}{\sqrt{1-\varepsilon}}\right)A\frac{1}{x}\right) \quad (55)$$

$$\frac{d\varphi_2}{dx}=\varphi_1\left(-\frac{K}{x}+\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A\frac{1}{x}\right)+\varphi_2\left(\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A\frac{1}{x}\right) \quad (56)$$

We continue with equations (54) and (56). Find the functions φ_1 and φ_2 in power series from:

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$$\varphi_1 = x^s \sum_{p=0}^{\infty} a_p x^p \tag{57}$$

$$\varphi_2 = x^s \sum_{p=0}^{\infty} b_p x^p \tag{58}$$

Express (57) and form the derivative:

$$\varphi_1 = a_0 x^s + a_1 x^{s+1} + a_2 x^{s+2} \ldots.. \tag{59}$$

$$\frac{d\varphi_1}{dx} = s a_0 x^{s-1} + (s+1) a_1 x^s + (s+2) a_2 x^{s+1} \ldots.. \tag{60}$$

$$\frac{d\varphi_1}{dx} = \sum_{p=0}^{\infty} (s+p) a_p x^{s+p-1} \tag{61}$$

Substitute into (54):

$$\sum_{p=0}^{\infty} (s+p) a_p x^{s+p-1} = \left(1 - \frac{\varepsilon}{\sqrt{1-\varepsilon^2}} A \frac{1}{x}\right) \sum_{p=0}^{\infty} a_p x^{p+s} + \left(-K - \frac{1}{\sqrt{1-\varepsilon^2}} A\right) \sum_{p=0}^{\infty} b_p x^{p+s-1} \tag{62}$$

$$\sum_{p=0}^{\infty} (s+p) a_p x^{s+p-1} = \sum_{p=0}^{\infty} a_p x^{p+s} - \frac{\varepsilon}{\sqrt{1-\varepsilon^2}} A \sum_{p=0}^{\infty} a_p x^{p+s-1} - \left(K + \frac{1}{\sqrt{1-\varepsilon^2}} A\right) \sum_{p=0}^{\infty} b_p x^{p+s-1} \tag{63}$$

Now we can write the following for the coefficients x^{s+p-1}:

$$(s+p) a_p = a_{p-1} - \frac{\varepsilon}{\sqrt{1-\varepsilon^2}} A a_p - \left(K + \frac{1}{\sqrt{1-\varepsilon^2}} A\right) b_p \tag{64}$$

Expass (58) similarly to (59) we get:

$$\varphi_2 = b_0 x^s + b_1 x^{s+1} + b_2 x^{s+2} \ldots.. \tag{65}$$

$$\frac{d\varphi_2}{dx} = s b_0 x^{s-1} + (s+1) b_1 x^s + (s+2) b_2 x^{s+1} \ldots.. \tag{66}$$

$$\frac{d\varphi_2}{dx} = \sum_{p=0}^{\infty} (s+p) b_p x^{s+p-1} \tag{67}$$

Substituteg into (56):

$$\sum_{p=0}^{\infty} (s+p) b_p x^{s+p-1} = \left(-K + \frac{1}{\sqrt{1-\varepsilon^2}} A\right) \sum_{p=0}^{\infty} a_p x^{p+s-1} + \left(\frac{\varepsilon}{\sqrt{1-\varepsilon^2}} A\right) \sum_{p=0}^{\infty} b_p x^{p+s-1} \tag{68}$$

Now we can write the following for the coefficients x^{s+p-1}:

$$(s+p) b_p = \left(-K + \frac{1}{\sqrt{1-\varepsilon^2}} A\right) a_p + \frac{\varepsilon}{\sqrt{1-\varepsilon^2}} A b_p \tag{69}$$

Equations (64) and (69), for p=0:

$$a_0 s = a_{-1} - \frac{\varepsilon}{\sqrt{1-\varepsilon^2}} A a_0 - \left(K + \frac{1}{\sqrt{1-\varepsilon^2}} A\right) b_0 \tag{70}$$

$$b_0 s = \left(-K + \frac{1}{\sqrt{1-\varepsilon^2}} A\right) a_0 + \frac{\varepsilon}{\sqrt{1-\varepsilon^2}} A b_0 \tag{71}$$

After $a_{-1} = 0$ (if we can believe mathematicians - and of course we do):

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$$a_0\left(s+\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A\right)+b_0\left(K+\frac{1}{\sqrt{1-\varepsilon^2}}A\right)=0 \quad (72)$$

$$a_0\left(K-\frac{1}{\sqrt{1-\varepsilon^2}}A\right)+b_0\left(s-\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A\right)=0 \quad (73)$$

The system of equations (72) and (73) has a solution if the determinant of the coefficient is zero, i.e.:

$$\begin{vmatrix} s+\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A & K+\frac{1}{\sqrt{1-\varepsilon^2}}A \\ K-\frac{1}{\sqrt{1-\varepsilon^2}}A & s-\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A \end{vmatrix}=0 \quad (74)$$

The value of the determinant after the corresponding operation:

$$s^2-\frac{\varepsilon^2}{1-\varepsilon^2}A^2-\left(K^2-\frac{1}{1-\varepsilon^2}A^2\right)=0 \quad (75)$$

$$s^2-K^2-\left(\frac{\varepsilon^2}{1-\varepsilon^2}-\frac{1}{1-\varepsilon^2}\right)A^2=0 \quad (76)$$

$$s^2-K^2+A^2=0 \quad (77)$$

$$s=\pm\sqrt{K^2-A^2} \quad (78)$$

Only a positive sign is possible, i.e.:

$$s=\sqrt{K^2-A^2} \quad (79)$$

In fact, it is because of this brief correlation (79) that we have presented the whole derivation above in such detail. This result can be used as a basis for the derivation of the solar system constant H_x

3. Approximation of the values of H_x

3.1 The value of H_x based on relativistic quantum mechanics

So far, at least according to the mathematical formulae, there is no deviation from the usual quantum mechanical derivations. Let us take a closer look at equation (79) above. From the previous (16) and (19) it can be seen that

$$A=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}Z+f_x\frac{M\mu_0}{\hbar c} \qquad \textit{see previous} \quad (19)$$

$$K=\mp\left(j+\frac{1}{2}\right) \qquad \textit{see previous} \quad (16)$$

Moreover, as we have seen, the correlation under the root (79) must be positive. At the scale of the atomic dimensions, after the substitutions, the value of A is 1/137,037 (the usual notation in quantum mechanics is α, a dimensionless number). Since j=0 is the smallest possible value in (16), this gives K=1/2, and A≤1/2 follows. Let us examine the substitutions in solar system-like systems. We have data for one such system, with the following known values: f_x=6,672*10^{-11} [$m^3/kg/s^2$] the gravitational constant, M=M_{Sun}=1,9891*10^{30} [kg] the mass of the Sun, c=2,997925*10^8 [m/s] the speed of light. The value of μ_0 may be questionable – the „smaller mass", since we have a one-mass model (Z=1), can be interpreted as the total mass of all possible orbiting the Sun, simplified: the total mass of the planets. This value is:

$\mu_0 \cong 2{,}668992*10^{27}$ [kg]. We can assume that whatever the value of the charge (*e*), in our case its product is negligible. The original value of $\hbar$ out of the question here, condition (79) would not be fulfilled. Thus $\hbar$ is replaced by H_x to denote the factor for the solar system, i.e.:

$$H_x = f_x \frac{M\mu_0}{Ac} \tag{80}$$

Substituting the above, the *possible minimum value* of the constant for the solar system is:

$$H_{xmin1} \geq 0{,}2363*10^{40} \text{ [Js]} \tag{81}$$

3.2 The value of H_x based on the Schrödinger-equation

The value of H_x can be approximated using another quantum mechanical model. Using the mathematical model of Schrödinger's equation of quantum mechanics, the radial part of the wave function, here denoted H_x instead of $\hbar$, is:

$$\frac{d^2R}{dr^2} + \left[\frac{2\mu}{H_x}\left(E - V(r) - \frac{l(l+1)}{r^2}\right)\right]R = 0 \tag{82}$$

The total solution is the product of the radial part and the spherical part:

$$\varphi(r,\theta,\chi) = \frac{1}{r}R(r)Y_l^m(\theta,\chi) \tag{83}$$

The process of the solution is guided by the referenced literature:

$$\varphi(r,\theta,\chi) = C\frac{1}{r}\xi^{l+1}L_{n+1}^{(2l+1)}e^{-\frac{1}{2}\xi}Y_l^m(\theta,\chi) \tag{84}$$

where $L_{n+1}^{(2l+1)}$ denotes the generalized Laguerre polynomials and $\xi = \frac{2r}{nr_0}$

After appropriate derivations and normalization the probability density function is:

$$\rho = R_{n,l}^2 = N_{n,l}^2\left[\xi^{2l}L_{n+1}^{(2l+1)2}(\xi)e^{-\frac{2\xi}{2}}\right] \tag{85}$$

Deriving from the value of ξ we get:

$$r_0 = n\frac{H_x^2}{\mu}\frac{1}{\frac{e^2}{4\pi\varepsilon_0} + f_x M\mu} \tag{86}$$

Neglecting the e^2 term and for n=1 (i.e. considering a single point of mass and no significant electric potential), the most probable radial distance:

$$r_0 = \frac{H_x^2}{\mu}\frac{1}{f_x M\mu} \tag{87}$$

Expressing H_x:

$$H_x \geq \mu\sqrt{r_0 M f_x} \tag{88}$$

In our rough model, we have the central mass M (the mass of the Sun), the mass of the particle μ (the mass of a planet), and r_0, the distance of the mass μ from the central mass M, where: $f_x = 6{,}672*10^{-11}$ [$m^3/kg/s^2$] the gravitational constant, M = $1{,}9891*10^{30}$ [kg] the mass of the Sun. Here again, we are dealing with a single-mass model, we can use the Earth's mass and orbital radius, in which case: μ = $5{,}9742*10^{24}$ [kg], the mass of the Earth, r_0 = 1 AU = $1{,}496*10^{11}$ [m] the Sun-Earth mid-distance. Given the above data, the *minimum possible value* of H_x is:

$$H_{xmin2} \geq 2{,}661962*10^{40} \text{ [Js]} \tag{89}$$

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If we calculate with the total mass of the planets orbiting the Sun and the distance data of e.g. the Oort cloud, then it is possible to calculate: μ_0=2,668992*10^{27} [kg] and r_0 = 1,496*10^{11} [m] the Oort cloud outer boundary, by substitution:

$$H_{xmin3} \geq 3{,}7607*10^{45} \text{ [Js]} \tag{90}$$

The approximations in Sections 4.1 and 4.2 are rather crude approximations of a complex system, but they are adequate for determining the minimum magnitude of H_x.

3.3 The value of H_x based on the measured natural frequency of the Sun

Based on the original definitions of quantum mechanics, the following relations can be written for a point of mass:

The Planck relation

$$E = h*\nu \tag{91}$$

The Einstein relation:

$$E = \mu*c^2 \tag{92}$$

As it follows from these: *„…any particle of mass* μ *is, in a sense a very precise 'clock' which ticks away with a frequency* ν *that is proportional to this mass…"* [38]. Based on the above relationships, this frequency is:

$$\nu = \mu\frac{c^2}{h} \tag{93}$$

where:

$$h = \hbar*2*\pi \tag{94}$$

$$\hbar = \mu\frac{c^2}{2\pi\nu} \tag{95}$$

In our case, using the Sun's natural frequency, ν_{Sun}, we obtain the following equation:

$$H_x = M\frac{c^2}{2\pi\nu_{Sun}} \tag{96}$$

where c = 2,9978*10^8 [m/s] speed of light, M = 1,9891*10^{30} [kg] the mass of the Sun. We should know the natural frequency of the Sun. If we accept - as a known first approximation - the „five-minutes" frequency as a valid value, then ν_{Sun} ≅ 3,3 mHz = 0,0033 Hz = 0,0033 [1/s], with these data:

$$\mathbf{H_x \cong 8{,}62*10^{48}}\ [\boldsymbol{Js}] \qquad \text{where } H_x > H_{xmin1}, H_{xmin2}, H_{xmin3} \tag{97}$$

4. The quantum mechanical model of solar system-like systems

Almost every physical, quantum mechanical, astronomical work since about the beginning of the 20th century mentions how the atomic structures are surprisingly similar, in many ways, to the 'solar system-like' structures. The ratio of central mass to orbiting masses, the ratio of distances between the central large mass and the orbiting smaller masses, etc. Then almost all works state that the physical laws describing 'solar system' motions (such as the Newtonian model and general relativity) are of course quite different from the relations describing atomic structures (relativistic quantum mechanics).

Let us continue this analogy. What would be the consequence of stating that the solar system is like a set of electrons revolving around an atomic nucleus?

Many works on quantum mechanics (and astronomy, cosmology) deal with the idea that quantum mechanical relations, 'laws', cannot be extended to the 'macro-world'. A crucial question may be what the 'macro-world' is from a quantum mechanical point of view. In what follows, we will not 'extend' quantum mechanics - rather, we will take a close look at the system under consideration, set up the mathematical model, clarify the boundary conditions and apply all this to approximate reality as closely as possible. We do not 'extend' quantum mechanics: we simply look at exactly what systems its laws apply to, and under what conditions.

In order to treat solar-system-like systems with quantum mechanical tools, i.e. for the analogy to work correctly, it is necessary that each element of the system be charged, identical particles that are indistinguishable, have mass, spin, magnetic moment, and the system is quantized. If all this is fulfilled, it would follow that the equations of motion of the atomic system can be applied to the solar system. An important claim of the work presented here is that solar system-like systems (possibly with individual actors Sun, Mercury, Venus, Earth, etc..) have charge. Measurements and models support the claim that they have charge. Of this 'elementary charge', which is a property of the constituent parts of the solar system, it is only necessary to state here that it exists.

The question immediately arises: is it legitimate to talk about the charge of an entire planet as such? The planet is made up of the elements we know, which may have a charge. We call the total charge of all the parts that make up the planet, measured from the outside at a distance, the charge '*e*' of the centre of mass we are considering. The probability that the total charge of the parts that make up mass μ is exactly zero is very small. That is, the point of mass μ may have a charge, seen from a distance, the magnitude of which is provisionally called '*e*'.

The next question to be answered is whether we can consider solar-system-like systems as mass points. The answer is certainly yes, if we think on a very 'large scale' and try to observe them from a terribly distant point. Even detection would be difficult, and from very far away they could well be considered as mass points of negligible size. This model is already quite similar to the legitimate applicability of the quantum mechanics approach. The model we consider is the following:

If you look at stars and planets from far enough away, they can be considered a centre of mass. As a first approximation, consider a fictitious system, or a 'solar system-like' system with a single planet. Viewed from far enough away, very far away, very outside the system, we can consider the planet and the central larger mass as a centre of mass.

The reality that can be described by the physical-mathematical tools of quantum mechanics must have, in a very simplified and concise way, the following properties: the 'particles', the agents of the systems we are here and now considering (originally, starting from atomic scale) have: mass, magnetic moment, spin, each particle is indistinguishable, has charge, and the system is quantized. The most important properties are therefore given to justify the use of the conceptual framework and notation of relativistic quantum mechanics [35], [41]. If a physical system has these properties, then the entire toolbox, mathematical apparatus and models of quantum mechanics can be legitimately applied to it [12],[21], [34]. Once again, it is important to note that the original mathematical model is not scale-bound.

Solar system-like systems meet the requirements listed above. Each member of a solar system-like system has, without a doubt, mass, magnetic moment, spin. Any source in the astronomical literature can support this. Immediately, questions and perhaps doubts may arise with regard to some of the above points. Let us consider them in turn.

4.1 The requirement of Indistinguishability

It is very easy to imagine that, viewed from a distance and sampled at very large intervals, all the motions in our solar system are merged, and the position and velocity of each point of mass cannot be measured separately. In other words, the basic conditions of quantum mechanics are met. If we look at several such systems simultaneously, sampled at very large intervals (by

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very large intervals we mean e.g. thousands, hundreds of thousands, millions, etc. of earth-years), the motion of the systems cannot be identified individually, only their totality can be described by laws, and these, according to the conceptual framework of quantum mechanics, are: we must speak of 'identical' particles, of blurry, cloud-like formations in space (and time), i.e. of location probabilities.

Indistinguishability is not immediately apparent, for example when you are standing on Earth and see how different, say, Jupiter is. But quantum mechanics does not 'stand on the electron' either. It examines the atom-electron with an 'instrument' (gigantic in size compared to individual atoms) built from atoms-electrons. It determines probability distributions, it sees an electron cloud. If we move away from the solar system, not so far, just a few hundred thousand light years, and extend the sampling frequency of the 'measurement' a little (not every minute, not every year, but much less frequently, with much larger time steps, even at intervals of several hundred thousand years), we immediately see an untraceable, indistinguishable 'planetary cloud'. The time scale of sampling must be much larger than the orbital frequency of the 'planetary cloud'. As in the case of electrons. So the indistinguishability condition is already valid.

4.2 The requirement of Charge

'Charge' is one of the features of the elements of a solar system-like system that is not immediately perceptible, not immediately visible. All that can be said is - solar system-like systems have charge. This is supported by the measurements and facts presented in the cited literature.

So then: suppose the Earth is a 'big lumpy electron'. What is what on the scale - is easily explained and understood. Ignore the fact that the Earth has countless complex features. What is 'missing', as far as we know, is the charge, or rather the property called charge, which is intrinsic to the (small) electron. That is, the Earth as a whole must have a charge in order to be treated as an 'electron'. More specifically, the elements of solar system-like systems must have a property called 'charge'. If it exists, then from this point of view it is legitimate to treat, calculate and interpret the whole system using a quantum mechanical model. Completely independently of the model discussed here, the charge of the individual solar system elements is known in the literature. For the moment, we refer only to the fact that the desired charge exists.

Sufficient data are available on the Sun's charge. Studies of the solar corona have theoretically known for about a hundred years that the Sun (and stars in general) have a charge. In their articles [4], [10], [36] Neslusan, referring to previous literature, also gives the magnitude of the charge as a function of solar (stellar) mass $Q_r = 77.043 * M_r$, where Q_r is the global electrostatic charge of the star within a given r-radius sphere and M_r is the mass within a given r-radius sphere. The formula is set up for an ideal star at rest - it can be accepted as an approximation. In any case, there is no doubt that the Sun (and other stars) have charge. Nevertheless, according to the author, *„…In conclusion, it seems to be desirable to remember the global electrostatic charge as a significant physical property of every star in various stellar studies…”*. Several authors and articles seem to accept these ideas [40], [43]; [49]; [50].

There are many measurements of the electrical properties of the individual actors and masses (mass points, planets) in the solar system. The ionospheres of planets with atmospheres (e.g. Venus, Earth, etc.) have a property described as a 'global electric circuit' [2], [3]. There are many measurements of global electric circuits on Earth [42] , Mars [14]. The global electric circuit as a property can be extended to planets with dusty surfaces. The link with cosmic rays

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can be supported by measurements [44]. Of course, it may be a legitimate question whether this property can be considered as a 'charge' to the system 'from the outside, at a distance' using our modelling approach. The answer here is that yes, it is considered to be a charge. The cases outside the solar system will be dealt with below, but it should be noted that Aplin's book referred to has an entire chapter on the possible ionisation of exoplanets' possible atmospheres.

Our argument is strongly supported by the fact that the charge of the Sun is positive, while the charge of the Earth is negative. That is, they attract each other. The magnitude of this force, which may be very small, is not the primary issue for our model - the question is not how large the charge is, but whether it exists. The charge of the other planets can only be assumed to be negative for now. Convincing data on all this could be collected by measurements.

As possible evidence for stellar charges, the only property and data on planetary nebulae (PNe) that we would highlight here is the morphology of PNe. Several attempts have been made to classify the morphology of PNe. Among these classifications, the works of [46-47], [29-31], [45], [22], [26-28] are noteworthy. According to the simplified classifications above [32], the main morphologies are classified as spherical (26%), elliptical (61%), bipolar-quadrupolar (13%) PN systems, with frequency indicated. There are several physical-mathematical models in the literature to explain the diverse morphology of PN systems. Some of these models have been very successful, with many important results being provided by the ISW (Interacting Stellar Winds) models of [24-25] and their modified and improved versions. The already cited book [25], while offering a multifaceted answer to the morphological manifold, also poses a chapter-long question for us. The last of these questions reads, in summary: *"…The morphology problem: what is the origin of the diverse morphologies observed in PN? PN possess definite symmetries in their appearances, but they come in a variety of shapes…"*.

There is no definitive, fully developed, clear-cut explanation for why the PN manifold is so diverse and why it has such geometric shapes. Most strikingly, the early diversity of morphology in the PN formation process is already present during the pre-planetary life stage (PPN - Protoplanetary Nebulae). Since the PPN shows a similar morphological diversity as the mature PN, this suggests that a shape-determining mechanism must be at work early in the developmental stage.

The quantum mechanical model can provide a new perspective on morphological diversity. Quantum mechanics modelling used the conceptual framework, physical models and mathematics of relativistic quantum mechanics. (However, models and calculations based on non-relativistic quantum mechanics are sufficiently accurate for speeds much lower than the speed of light (c) and provide results that are hardly different from those of relativistic quantum mechanics - for example, for calculations of the radial probability density distributions of the hydrogen atom, the solutions based on the modified Dirac equation hardly differ from the results based on the Schrödinger equation. This implies that the nature of the radial probability density distributions in three-dimensional (3D) representations can be well approximated by non-relativistic models. We compared these non-relativistic three-dimensional probability density distributions, which are much easier to compute and represent, with the morphological diversity of planetary nebulae. Each quantum mechanical 3D state was matched to a morphological PN shape. The 3D probability density distributions show a surprisingly good similarity with the observed morphological shapes of PN. All morphological shapes can be explained by the quantum mechanical model [16].

The special property of charge is that it can be locally attractive anywhere, but globally repulsive everywhere. One of the main possible consequences is: *"…One way to make a fundamental model of the dark energy is to assume the dark energy is a new force which can*

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be described by the same laws used in particle physics...(....)...This type of force is modeled by what is called a scalar field..." [48]. Taylor's sentences above almost demand the existence of a scalar field. This scalar field in our case could be the charge of solar system-like systems and their constituent elements (stars, planets, etc.). If we can assume that solar system-like systems have a charge, then dark energy could possibly be explained by such scalar fields. Likewise, according to Penrose, who while apparently disagreeing, correctly describes *"...we also cannot rule out the possibility that...dark energy...is a scalar field..."* [39].

4.3 The requirement of Quantization

So far, it has not been discussed whether solar system-like systems can be considered as systems with quantized states. We can only know one such system with sufficient precision - our own star system. For many hundreds of years we have known that the orbits of planets are not arbitrary - they follow a certain regularity. Many theorists have tried to explain this phenomenon. The fundamental causes of the planetary distribution according to the "Titius-Bode law" are not fully understood. One of the best comprehensive accounts of the numerous possible theories has been collected by [37].

As shown in [18] and [11], the diversity of theories can be explained by "scale invariance". The contradictions and possibilities of other models are outlined in [20]. However, of the myriad of theoretical possibilities, none contradicts the assumption that quantum mechanics suggests that the radial probability density distribution can be considered as an explanation. In any case, it is striking that of the two inner planets (Mercury and Venus), Venus has the opposite spin, the only one in the solar system. According to quantum mechanics, this is the only possible state of Venus' spin. Moreover, no two planets have the same spin and magnetic dipole moment vector directions [15].

5. The scale of the known Universe and the scale of Time

5.1 Space has no immanent (intrinsic, inherent) scale

Number of Particles – Number of Stars in the Universe

„How many stars are there in the Universe? ...For the Universe, the galaxies are our small representative volumes, and there are something like 10^{11} to 10^{12} stars in our Galaxy, and there is perhaps something like 10^{11} or 10^{12} galaxies. With this simple calculation, you get something like 10^{22} to 10^{24} stars in the Universe. This is only a rough number, as obviously not all galaxies are the same ..." [13]

Imagine you have about 22 litres of gas (gas molecules) in front of you, say air, (O_2, CO_2, CO, N_2, Ar, Ne, with possible dust particles included, etc.). Let this mixture of gases be in the "normal state", which means that there are about 6×10^{23} (Avogadro's number) gas molecules in the 22.42 litre volume (Avogadro number). Round up to 10^{24} molecules. The emphasis is on the number of molecules. In the 'Universe' as we know it today, the number of stars is about 10^{24} [13], [33]. That is, the number of solar system-like systems is the number of gas molecules in about 22 litres of gas. Now imagine that each atom, in our gas mixture above, is a star system (i.e. the nuclei - stars and the electrons - planets). Perhaps this gives a better sense, almost at arm's length, of the whole world we call the 'Universe'. It's a simple picture, the meaning of 'infinity' has suddenly changed, it's all tangible - but it also gives us a powerful sense of what 'true infinity' is - because if you have all the 'something-systems' (again, about 10^{24} of them) within that arm's length, it opens up incredible vistas of how big the World can be.

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When we say 'Universe', 'World', we can talk about the 'Currently Known Universe', which at the moment (in the year 2022) can be described as a sphere of diameter about $D_{Universe}$ = $8{,}61*10^{26}$ meters - with the solar system at the center.... (Obviously, in reality we know that the solar system is not the center, but we are here, we measure everything from here, we relate everything to this, this is where our scale comes from...).

Is there a system, a proportion, a perspective in which the Earth can be considered as small as an electron in our everyday world? If we dare to think that there is a scale where 'the Earth is as small as an electron', then, thinking along the lines of the number of pieces of solar system-like systems, scales, analogy, and applying the resulting scale and scales, the Universe as we know it today can be imagined as a small sphere of about 0.35 meters in diameter (see equation 99) containing about 10^{24} pieces of mass. The question is not whether this is really 'not very big', but where the 'infinity of the universe' is in comparison [17]. The volume of a normal (ideal) gas is V_{gas}=0,022 m3. By analogy, we can compare the size of this gas, its volume, with the size of the known Universe:

$$V_{gas} = \frac{4\pi}{3}\left(\frac{D_{gas}}{2}\right)^3 \qquad (98)$$

thus:

$$D_{gas} = 2\sqrt[3]{\frac{3V_{gas}}{4\pi}} = 2\sqrt[3]{\frac{3*0,022}{4\pi}} \cong 0{,}3476 \text{ m} \qquad (99)$$

The scale ratio $S_1=D_{Universe} / D_{gas} = 8{,}61*10^{26} / 0{,}3476 = 2{,}476*10^{27}$. By itself, this ratio has no particular significance. The numerical value may be uncertain, the 'normal state gas' analogy is rather crude, the number of gas molecules is quite 'generous' compared to the number of star systems, and the size of the Universe may have varied significantly over time. For the purposes of the present study, it could be pointed out that: space has no immanent (intrinsic) scale. The same applies to time.

5.2 Time has no immanent (intrinsic, inherent) scale

In order to draw conclusions about the Universe's proper time scale from the currently known measurement data, it is sufficient to use some well-known data from physics and astronomy, in addition to the above analogy. In the systems under study, there is a definite physical constant, observable in both systems, on which we can rely - the speed of light, *c*.

According to our Earth time calculation, our time concept and our time measurements, the unit of Earth time is 1 second (1 s). Hereafter, for ease of identification and reference, we will refer to this as the Earth secundum, i.e. : 1 s = 1 $s_{(Earth)}$

How long it takes for light to travel through the sphere of the Universe at a speed of *c* without interference:

$$T_{Universe} = \frac{D_{Universe}}{c} = \frac{8{,}61*10^{26}}{2{,}9978*10^{8}} = 2{,}872*10^{18}\ s_{(Earth)} \qquad (100)$$

How long does it take light to traverse a gas sphere of diameter 0,3476 m (see 99) at a speed *c* without interference:

$$T_{Normal\,gas} = \frac{D_{gas}}{c} = \frac{0{,}3476}{2{,}9978*10^{8}} = 0{,}116*10^{-8}\ s_{(Earth)} \qquad (101)$$

An analogy has been drawn between atomic-scale (atomic extent) systems and solar system-like systems. The claim (and proof? or rather the chance to support the claim?) relied on the fact that both systems can be discussed and described by the mathematical apparatus of

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relativistic quantum mechanics, and that their physical reality can be interpreted on the basis of this description. The comparison (analogy) was based on the comparison of the fundamental properties that are undoubtedly found in the two systems, such as the properties of size, spin, charge, magnetic moment, quantization. To decide whether this is a sustainable and legitimate analogy - or not - would be a matter for subsequent scientific criticism.

What has not been mentioned is Time. What has been mentioned is that time, like space, has no intrinsic, i.e. immanent scale. The only general possible measure of time, valid in all systems, is the speed of light *c*. Whatever the distance, whatever the size, if the light (a photon) travels this distance without disturbance, without collision, without deflection, we obtain the time characteristic of the given distance, of the given size. The scales of atomic size systems, the scales of Earth time measurement, solar system-like systems, the time scales of the Universe are different. This has to be taken into account when measuring in their proper time.

5.3 Characteristics of the Earth's second

It is well known that the 'Original Basis of Time Measurement', i.e. the origin of the second, comes from 1 Earth year, very simply put: 1 Earth year is about 365 Earth days, 24 Earth hours per day, 60 Earth minutes per hour, 60 Earth seconds per minute. That is:

1 EarthYear = 365*24*60*60 s = $3{,}1536 \cdot 10^7$ s = $3{,}1536 \cdot 10^7\ s_{(Earth)} = k \cdot 1\ s_{(Earth)}$

where k = $3{,}1536 \cdot 10^7$ - but to be more precise, the pre-1967 SI (Système International d'Unités) is the basis for the International System of Units:

> *„...the fraction 1/31,556,925.9747 of the tropical year for 1900 January 0 at 12 hours ephemeris time..."*

That is, what we call 'year' Earth-year is nothing more than our unit of time (1 second, 1 s) multiplied by the number k ≅ $3{,}1557 \cdot 10^7$

The numerical value of a second or a year can also be determined in other ways, for example by the basic property of a caesium atom according to the SI: *„...the duration of 9,192,631,770 [cycles] of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom..."*. The method of measurement and the precision with which we measure the earth second does not affect the point, the fact that *the origin of the second* is the 'earth year'.

How the Earth second is defined in the above terms, i.e. what method of measurement is chosen to measure it accurately and increasingly accurately, is not at all relevant to the present discussion. It makes absolutely no difference which characteristic of the caesium atom, for example, is currently the unit of the base value under the SI system, since the original value of the second is almost completely unchanged.

Let us try to use the common physical property - the speed of light (*c*) - to determine the natural unit of time for each system, their *proper inherent unit of time.*

Take an atomic system and measure the time it takes for light to 'pass through' this system at the speed of light. In this way, compare the 'seconds' of an atom, the Earth and the solar system. Let's carry out this *theoretical experiment* with the Neon atom. (A Neon atom with ten electrons is most analogous to a solar system with nine - or possibly ten - planets. This choice may be debatable - other, justifiable comparisons could be made.)

Let's 'measure' the following: a photon travels at speed *c* without disturbance for a distance of an atomic system, e.g. the diameter of a Neon atom, and measure how long it takes. The Van der Waals covalent size (radius) of a Neon atom is:

$$r_W = 58\ \text{pm} = 58 \cdot 10^{-12}\ \text{m}$$

The time it takes for light to travel this distance through the Neon atom:

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$$t_{w(Atom)} = \frac{2r_w}{c} = \frac{2*58*10^{-12}}{2,9978*10^8} = 3,87*10^{-19}\ s_{(Earth)} \tag{102}$$

From the above, we can say that the Earth second is based on the following: $1\ s_{(Earth)} = t_{w(Atom)} / 3,87*10^{-19}$, more precisely: $1\ s_{(Earth)} = 0,252*10^{19}\ s_{(Atom)}$

5.4 Characteristics of the Solar-System's second

Let us now consider the solar system in a similar (analogous) way to the above. Let us 'measure' the following: let a photon travel at *c*-speed without disturbance a corresponding distance the size of the solar system, and measure how long it takes. What would constitute the size, the radius of the Solar System? The distance of the Kuiper belt, the inner or outer radius of the Oort cloud, or the half distance of the star closest to the Sun? We could say any of the above is a possible choice - and that there are other reasonable ideas on size (1 AU = $1,496*10^{11}$ meter, Astronomical Unit – Sun-Earth distance):

Name	Specific Distance	Radius (m)
Neptune's distance	30,07 AU	$4,498*10^{12}$
Kuiper-belt	50 AU	$7,480*10^{12}$
Oort cloud inner extent	50 000 AU	$7,480*10^{15}$
Oort cloud outer boundary	100 000 AU	$1,496*10^{16}$
Proxima Centauri half distance	139 130 AU	$2,081*10^{16}$

The size of the solar system can be taken as two times the radius of the Oort cloud, following the previous points. The outer boundary of the Oort cloud is about: $r_{Oort} \cong 100\ 000$ AU = $10^5 * 1,496*10^{11}$ m. The diameter of the solar system: $D_{Solarsystem} = 2*r_{Oort} = 2*1,496*10^{16}$ m = $2,992*10^{16}$ m. As before, the time it takes for light to travel this distance:

$$t_{SolarSystem} = \frac{D_{SolarSystem}}{c} = \frac{2,992*10^{16}}{2,9978*10^8} = 0,998*10^8\ s_{(Earth)} \tag{103}$$

The value of one second in the solar system is $1\ s_{(SolarSystem)} \cong 10^8\ s_{(Earth)}$, according to the analogy above. That is, a single second in the solar system, measured in its proper natural time unit, is 10^8 Earth seconds, or about 100 000 000 Earth seconds, or:

$$1\ s_{(SolarSystem)} = 10^8/(3,1557*10^7) = 3,16887\ EarthYear \tag{104}$$

A single second in the solar system, measured in its proper natural unit of time, is equivalent to about 3,17 Earth years. It is safe to say that the basic time scale of the solar system is equivalent to the basic time scale of the Universe as we know it. If we can extend the above time step to the Universe as we know it, we obtain the following: a single second of the Universe in its proper time unit corresponds to 10^8 Earth seconds. The 'year' is an Earth-bound, Earth concept, as we have seen above, whatever number we multiply the second by. Let us apply this to the Universe. If the ratio *k* for seconds is true, then it is also true for 'year'.

1 UniverseYear ≅ 10^8 EarthYear

One 'year' of the Universe, measured in its proper natural unit of time, is equivalent to about 10^8 Earth years.

Summary

The key claim of the paper is:

> Solar system-like systems can be described by the physical models and mathematical tools of relativistic quantum mechanics. The basic relativistic equations derived by analogy are valid in weak gravitational fields (in the whole Universe), with the

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gravitational potential and the electric potential existing independently and simultaneously.

Some consequences of the analogy discussed here are as follows:

Based on the known natural frequency of the Sun, one can approximate the value of H_x for the solar system from the quantum mechanical model as an analogue of the constant $\hbar$.

According to the concept of time based on the analogy, one 'year' of the Universe in terms of proper time scale corresponds to 10^8 Earth years.

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