

## Article

# A Self-Adaptive Multiple Exposure Image Fusion Method for High Reflective Surface Measurement

Xiaobo Chen <sup>1,\*</sup>, Hui Du <sup>2,\*</sup>, Jinkai Zhang <sup>3</sup>, Xiao Yang <sup>4</sup>, and Juntong Xi <sup>1</sup>

<sup>1</sup> School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China; xiaoboc@sjtu.edu.cn(X.C.); jtxi@sjtu.edu.cn(J.X.).

<sup>2</sup> Percipio Technology Limited, 112 Liangxiu Rd, Shanghai 201203, China; sdduhui@hotmail.com (H.D.)

<sup>3</sup> School of Mechanical Engineering, Jinan University, Jinan, Shandong 250022, China; me\_zhangjk@ujn.edu.cn

<sup>4</sup> School of Artificial Intelligence, Optics and ElectroNics (iOPEN), Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China; xyang@nwpu.edu.cn

\* Correspondence: xiaoboc@sjtu.edu.cn (X.C.); sdduhui@hotmail.com (H.D.)

**Abstract:** Fringe projection profilometry(FPP) has been extensively applied in various fields for its superior fast speed, high accuracy and high data density. However, measuring some objects with high reflective surfaces or high dynamic range surfaces remains challenging for FPP. Some multiple exposure image fusion methods have been proposed and successfully improved the measurement performance for these kinds of objects. Normally, these methods have a relatively fixed sequence of exposure settings determined by practical experiences or trial and error experiments, which may decrease the efficiency of the entire measurement process and may have less robustness to various environmental lighting conditions and object reflective properties. In this paper, a novel self-adaptive multiple exposure image fusion method is proposed with two main aspects of improvement on adaptively optimizing the initial exposure and the exposure sequence. By introducing the theory of information entropy, combined with the analysis of the characterization of fringe image entropy, an adaptive initial exposure searching method is first proposed. Then, an exposure sequence generation method based on dichotomy is further described. On the base of these two improvements, a novel self-adaptive multiple exposure image fusion method for FPP as well as its detailed procedures are given. Experimental results validate the performance of the proposed self-adaptivity multiple exposure image fusion method by measuring the objects with different surface reflectivity and in different ambient lighting conditions.

**Keywords:** Multiple exposure image fusion; Fringe projection profilometry; High reflective surface measurement

## 1. Introduction

Fringe projection profilometry (FPP) has been widely applied in three-dimensional shape measurement in many fields such as manufacturing [1, 2], medicine [3], law enforcement [4] and entertainment [5], due to its advantages of noncontact operation, full-field measurement, high accuracy and high efficiency. However, the measurement of high reflective surfaces or high dynamic range surfaces using FPP has always been an open problem. Capturing surface details in dark areas requires higher camera exposure values, whereas recovering ground truth in bright regions needs lower exposure values. For conventional cameras, this may lead to some low contrast fringe regions or saturated regions existing in the captured images. These regions have a low signal-noise ratio, which may further result in measurement data missing or accuracy decreasing. Therefore, finding effective measurement methods for high reflective or high dynamic surfaces has always been an important research focus in the area of FPP.

Various methods have been proposed to solve this problem. One category of the methods, named multiple exposure fusion methods, improves measurement performance

by capturing fringe images with different camera exposures. Zhang and Yau put forward a high dynamic range scanning method taking advantage of pixel-by-pixel phase retrieval merit for the phase-shifting algorithm [6]. This technique captures a sequence of fringe images with different exposure times. The brightest but not saturated pixels are selected to construct the fused fringe images which are used to compute the final phase map. Ekstrand and Zhang proposed a method that could evaluate the effect of exposure according to the feedback from the measured object reflectivity [7]. Defocusing binary projection is introduced to enlarge the camera exposure time selection range. Long et.al. presented a method that uses the magnitude of a non-principal frequency component to identify saturated pixels [8]. The magnitude function of the frequency component is presented by Fourier analysis. The magnitude of the non-principal frequency component is deduced and utilized as the saturation criteria.

Another category of the methods is to measure the shiny surfaces by adjusting the projected fringes accordingly. Waddington and Kofman presented a camera-independent method of avoiding image saturation according to modifying projected fringes after complex calibration procedures [9]. Babaie et al. captured the fringe images by recursively controlling the intensity of the projection pattern pixel-wisely [10]. The reflected images captured by the camera are used as references. Lin et al. proposed an adaptive method by adjusting the pixel-wise intensity of the projected fringe patterns based on the saturated pixels in the captured images [11].

The third category of the methods is to extend the dynamic range by hardware, which can be categorized into two types-chip level extension and affiliated hardware extension. In terms of the chip level extension, Orly [12], Cheng [13] and Bub [14] proposed different methods to extend the dynamic response range of the pixels. When it comes to the affiliated hardware method, digital-micromirror-device(DMD) chips [15-18] and spatial light modulators [19, 20] are usually implemented to modulate the intensity and direction of spatial light.

Among all these categories, the multiple exposure fusion methods become one of the most important research directions for their superior flexibility, high accuracy and hardware-independent characteristics. There are two key factors in the sequence of exposure settings for the multiple exposure fusion methods: initial exposure and exposure sequence. The initial exposure is considered as the highest value in the exposure sequence, and it is critical for the quality of the fusion images and the overall processing efficiency. A higher initial exposure means more unnecessary series of fringe images have to be captured, which contributes nothing to the quality of the fusion images, but makes the overall process longer. A lower initial exposure may lead to poor quality of the obtained fusion images. For the exposure sequence, the large sequence with small exposure steps improves the quality of the fusion images, but it requires more operating time. A short sequence with large exposure steps may result in lower quality fusion images. There is no such criterions to balance the fusion image quality and the processing efficiency. Thus, the initial exposure and the exposure sequence are very important to the performance of the multiple exposure fusion method.

Current multiple exposure fusion methods always have a relatively fixed sequence of exposure settings normally determined from practical experiences or multiple trial and error experiments. However, when measuring new objects with different surface reflectivity or in different ambient lighting conditions, the initial exposure and the exposure sequence must be adjusted accordingly to ensure the multiple exposure fusion performance. In such cases, obviously, the trial and error experiments are time-consuming to adjust the initial exposure and the exposure sequence, also the initial exposure and the exposure sequence cannot be precisely optimized based on manual adjustments based on practical experiences. This proposes a critical demand for self-adaptive, fast, automatic methods to optimize the sequence of exposure settings according to the measured object surfaces adaptively, thereby improving the overall processing efficiency as well as the measurement quality.

In order to solve this problem, a novel self-adaptive multiple exposure image fusion method is proposed in this paper. By introducing the information entropy theory into the analysis of the fringe images, which builds the theoretical foundation for the selection of initial exposure, a self-adaptive initial exposure searching method is first presented. Also, an exposure sequence generation method based on dichotomy is proposed. On the base of the aforementioned two methods, a self-adaptive multiple exposure image fusion method for FPP as well as its detailed procedures are presented. The remainder of this paper is organized as follows. Section 2 presents the principle of the proposed method. Specifically, Section 2.1 presents the initial exposure searching method based on information entropy. Section 2.2 describes the exposure sequence generation method based on dichotomy. Section 2.3 proposes the self-adaptive multiple exposure image fusion method for high reflective surface measurement. Section 3 shows the experiments and results analysis. Section 4 concludes this paper.

## 2. The Proposed method

### 2.1. Initial exposure searching based on information entropy

For the multiple exposure image fusion methods, the selection of the initial exposure value is one of the key aspects that determine the measurement performance. High initial exposure may increase the pixel grayscale values in the low-reflectivity areas, but it will also increase the number of overexposure pixels. And the multiple exposure fusion methods generally adopt a high to low approach to vary the exposure values and analyze the fringe images. With a too high initial exposure, more exposure changes are required to obtain a fused image, reducing the entire measurement efficiency. And if the initial exposure is set too low, the quality of the final fused image will be affected, leading to data missing and accuracy decreasing.

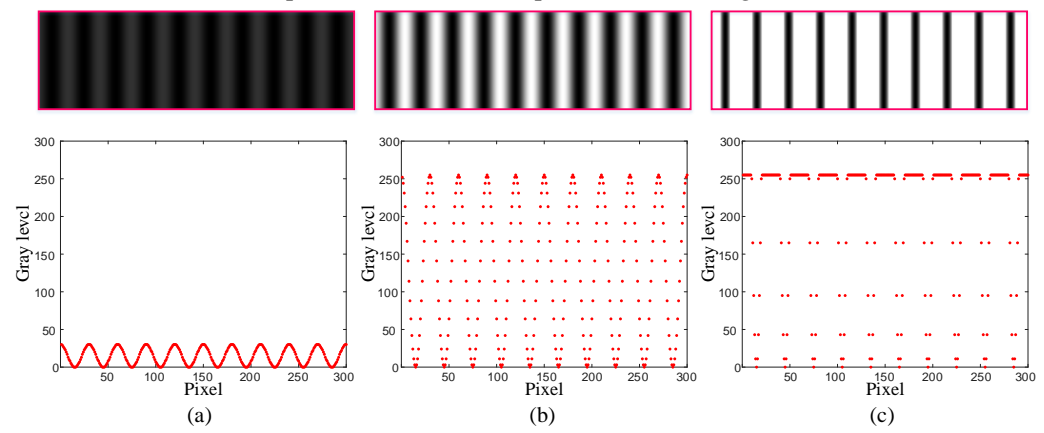
The present methods usually select an appropriate initial exposure through multiple experiments or practical experiences. However, this kind of method is not very flexible in case the illumination environment changes or some objects with different surface reflectivity are measured because the initial exposure should be reset correspondingly. This reduces the adaptability of the current multiple exposure methods. Therefore, there is an urgent need to develop a self-adaptive initial exposure searching method to enhance the adaptive capability of current multiple exposure image fusion methods.

#### 2.1.1. Selecting criteria for initial exposure

An appropriate initial exposure value should not cause too many saturated or low gray value pixels in the captured images. As shown in Fig.1, in order to observe the fringe image intensity distribution in different exposure conditions, three simulated images are obtained by adjusting the modulated gray value in low, normal and high exposure conditions. The upper row shows the simulated images, and the down row shows the intensity variation scatter diagram in one pixel row of the images. The scatters diagram in Fig.1 (a) shows that most of the pixels in the image are concentrated in a narrow gray value interval for the low exposure case. There is little intensity difference between pixels, which makes the simulated fringe image be very blurred. Fig.1(b) shows that the gray value of the fringe image pixels for the normal exposure condition are distributed in the entire interval between 0 and 255, and the stripes can be seen very clearly in the corresponding image. In Fig.1(c), it is illustrated that the intensity of most pixels is near 255 since the exposure value is too high. Therefore the fringe is no longer sinusoidal. An appropriate initial exposure value should maximize the variability between pixels in the acquired images and make the range of the pixels' gray value be as wide as possible. The opposite means that there are too many overflow pixels or too many pixels with low gray values in the obtained initial exposure image. The former will lead to more adjustment times of

exposure value and longer measurement time, and the latter will easily lead to a quality reduction of the fusion images, both of which should be avoided.

Based on the above analysis, the selection criterion of initial exposure value for the multiple exposure image fusion method can be concluded. That is, the selected initial exposure value should make the maximum variability of the pixel grayscale values in the acquired sequence of images. The larger the variability of the pixel gray values in the image, the more obvious the sinusoidal fringe feature is. In FPP, fringe stripes are the carrier for encoding and decoding phase information, which itself is the information for 3D reconstruction. In other words, maximizing the variance of pixel gray values in an image means maximizing the amount of information in the image. Thus, the criterion for selecting the initial exposure value in the multiple exposure image fusion method can be further clarified as the initial exposure should be chosen in such a way as to maximize the amount of information in the acquired set of initial exposure value images.



**Figure 1.** Simulated fringe images in different exposure conditions: (a) Low exposure fringes, (b) Normal exposure fringes, (c) High exposure fringes

### 2.1.2. Information entropy metric for initial exposure selecting criteria

As aforementioned, the criterion for selecting the initial exposure value in the multiple exposure image fusion method should be such that the amount of information in the corresponding image sequence at this exposure is maximized. This implies the need to determine a quantitative index for measuring the amount of information in the images.

In physics, the concept of entropy is used to represent the uniformity of energy distribution in space. The more uniform the energy is distributed, the higher the entropy of the system is. Shannon, the founder of information theory, introduced this concept into the field of informatics and proposed to use information entropy to measure the uncertainty of an information source [21]. Suppose that the probability of occurrence of each event in a probabilistic event set is  $p_1, p_2, \dots, p_n$  respectively, and suppose that there exists such a metric  $H$  that can describe the degree of uncertainty of the information, which can be given as follows [21], where  $n$  is the number of the event in the probabilistic event set.

$$H = - \sum_{i=1}^n p_i \log_2 \left( \frac{1}{p_i} \right) \quad (1)$$

This formula is used to express the metric of information uncertainty in information theory, which is termed information entropy. The magnitude of information entropy can be expressed to express the information amount in the information source. Specifically, in the field of image processing, the pixel intensity is a probabilistic event in Shannon's definition of information entropy. The intensity of pixels in an image at different image coordinates is denoted by  $X_i$ , where  $i = 1, 2, \dots, k$ ,  $k$  is the number of pixel grayscale levels. Let  $E$  be the image information entropy, which can be expressed as follows.

$$E = \sum_{i=1}^k p_i \log_2 \left( \frac{1}{p_i} \right) = p_1 \log_2 \left( \frac{1}{p_1} \right) + p_2 \log_2 \left( \frac{1}{p_2} \right) + \dots + p_k \log_2 \left( \frac{1}{p_k} \right) \quad (2)$$

Where  $p_i$  is the grayscale probability. For an 8-bit depth image, its maximal grayscale is 256, and its maximum image entropy equals 8, only when the gray levels are evenly distributed in the image, which is not a common case in real applications. For any given scene, the image information entropy will vary with the variation of exposure. The amount of information will reach the maximum when the image entropy becomes maximal.

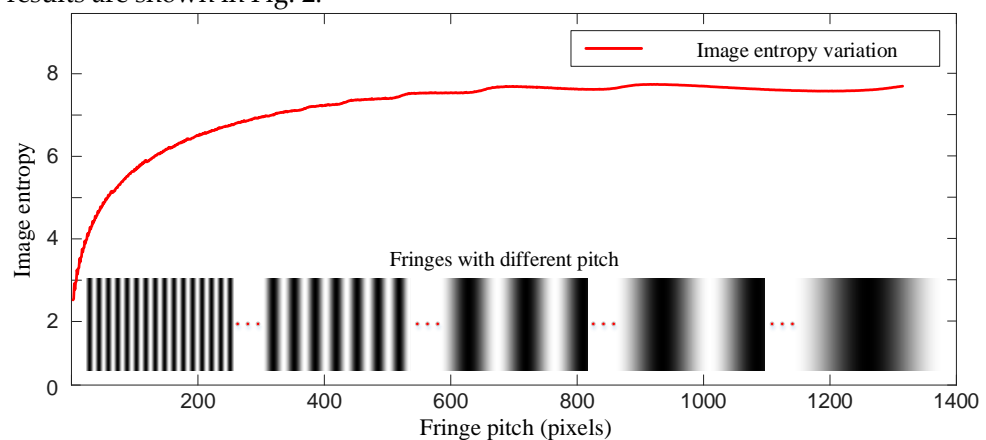
Based on the above analysis, the image information entropy can be used as the metric of the amount of information in an image. The greater the image entropy, the more information amount contained in the image. Thus, the selection criteria of initial exposure in the multiple exposure image fusion method can be expressed as determining a suitable exposure such that the image information entropy in the acquired image sequence at this exposure value reaches its maximum.

### 2.1.3. Variation characteristics for fringe image information entropy

As one of the most common encoding and decoding methods for FPP, three-frequency three-step phase shifting is adopted in this study. And when determining the initial exposure value, in order to improve the algorithm efficiency, the fringe image at a certain frequency should be used as the reference image for the information entropy calculation. This requires the study of the information entropy characteristics of the fringe images at different frequencies, the fringe images with different phase shift steps at the same frequency, and the three-frequency three-step phase shift fringe images captured when the exposure value changes.

#### 1) Information entropy for multi-frequency fringe images

In the multi-frequency phase-shifting method, the frequency of fringe images is reflected in the stripe pitch used to produce the fringe image. To study the fringe image information entropy variation for different frequencies, a range of fringe images with varied stripe pitch are artificially produced. According to the Nyquist sampling theorem and further taking into account the effect of noise [22], the minimum stripe pitch is set to 10. And the maximum stripe pitch is set to 1280 since it's the pixel column size for common DMD chips. Total 1271 fringe images with the strip pitch  $\lambda$  ranging from 10 to 1280 are used for further analysis. The image entropy of each fringe image is calculated and the results are shown in Fig. 2.



**Figure 2.** Fringe image information entropy as the fringe pitch changes.

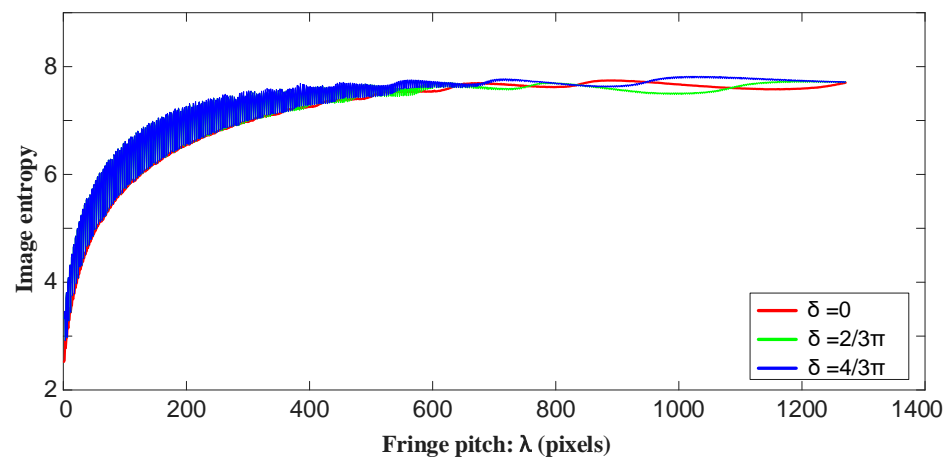
As can be seen from the figure, the information entropy of the striped image rises and fluctuates with the increase of the stripe pitch, showing an increasing trend. However, the curve plot has many fluctuation intervals, in which the image information entropy

fluctuates to some degree. As the fringe pitch increases, the period of fluctuation interval also increases gradually. In terms of variation rate, it can be seen from the figure that in the interval  $\lambda \in [10, 100]$ , the fringe image entropy increases rapidly with the increase of fringe pitch. When it comes to the interval  $\lambda \in [600, 1280]$ , the increasing speed of image information entropy reduces gradually. After that, the image entropy stabilizes between 7 and 8 and fluctuates in a small range.

## 2) Information entropy for three-step phase-shift fringe images

To study the information entropy variation for the phase shifting fringe images concerning the change of fringe pitch, three groups of fringe images with three-step phase shifts ( $\delta_1=0, \delta_2=2\pi/3, \delta_3=4\pi/3$ ) are produced. For each group of fringe images, its incremental step is set to 1, and a total of 3813 fringe images are produced. The information entropy for each phase shift concerning the variation of fringe pitch is presented in Fig.3.

As shown in Fig.3, the information entropy of  $\delta_1=2\pi/3$  and  $\delta_2=4\pi/3$  is generally similar to that of  $\delta_0=0$ , which shows the change characteristic of the composite function of logarithm and sine functions. There is also a certain degree of fluctuation, its period becomes larger as the pitch increases. In terms of the fluctuation amplitude, in the range of  $\lambda \in [0, 600]$ , the curve of  $\delta_0=0$  gives the smallest amplitude. The fluctuation of the other two curves is larger, and the fluctuation average is above that of  $\delta_0=0$ . This means that the phase-shifting images are more sensitive to fringe pitch change. In most cases, the entropy of the fringe image with phase shift is larger than that of the fringe image without phase shift. For the range of  $\lambda \in [600, 1280]$ , the fringe image entropy changes alternately among the three phase shifts.



**Figure 3.** Variation curve of information entropy for three-step phase-shifting fringes.

## 3) Information entropy for phase-shifting fringe images at different exposures

To further clarify the variation characteristics of information entropy of three-frequency three-step phase-shifting fringe images when exposure changes, the simulation images and real images are used to study the variation characteristics, respectively. The simulated images are used to study the variation characteristics without considering the surface optical properties of the measured object. Then the real images are utilized to verify the applicability of the conclusions drawn from this ideal case.

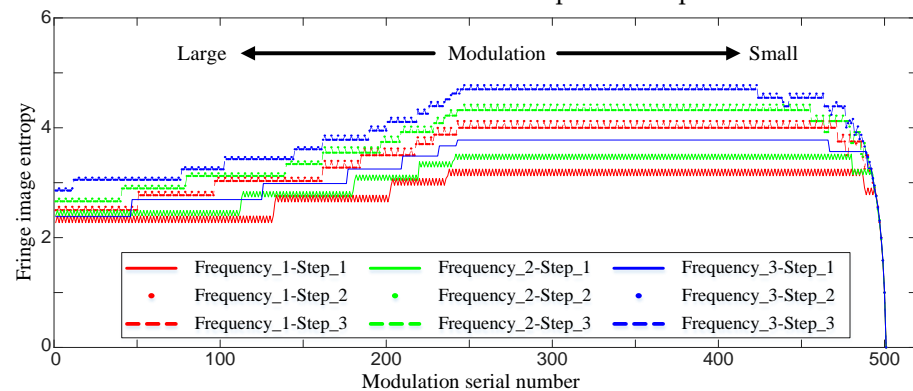
The encoding scheme of the sinusoidal fringes can be expressed as

$$I = I' + I'' \cos(\varphi(x) + \delta) \quad (3)$$

Where  $I$  is the fringe image intensity,  $I'$  is the average intensity,  $I''$  is the intensity modulation,  $\varphi$  is the principle phase,  $x$  is the pixel position,  $\delta$  is the phase shift. The intensity variation of the fringe images with the change of exposure can be simulated by modifying the intensity modulation. And in the multiple exposure image fusion method, the exposure values are usually adjusted in the order from the highest to the lowest values

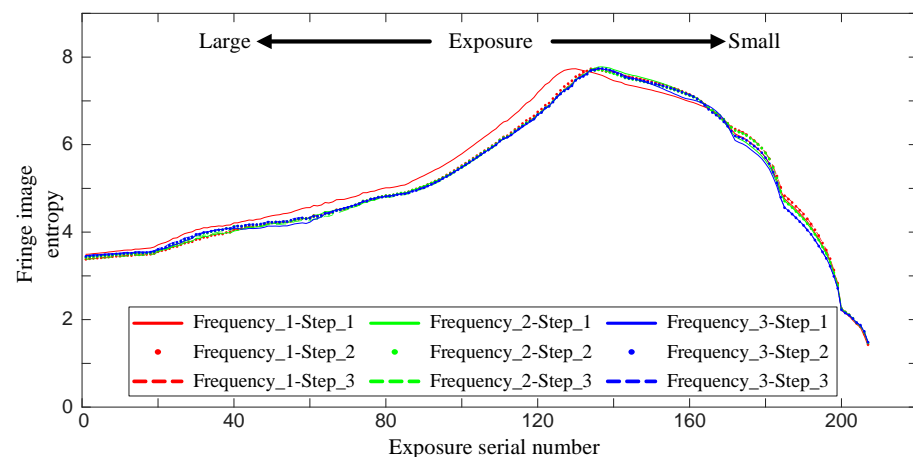
to obtain high quality fused fringe images. Therefore, the light intensity modulation values are also adjusted in the order from high to low, and in the simulation, the fringe images are obtained at 500 light intensity modulation values, and the information entropy of a total number of 4500 fringe images at different frequencies and phase shifts are calculated. The results are shown in Fig.4.

As can be seen from the figure, the information entropy of three-frequency three-step fringe images tends to increase and then decrease as the modulation value decreases. Specifically, the information entropy of the fringe images first increases in a step-like manner. After reaching the maximum, it stabilizes in a small range of fluctuation as the modulation value decreases at the beginning and descends rapidly when the modulation value is small to a certain degree. In terms of the speed of reaching the maximum, the larger the fringe pitch is, the later the maximum is reached and the smaller the corresponding modulation is. For the same fringe pitch, the image entropy of the two steps with phase shifts reaches the maximum later than that of the step without phase shifts.



**Figure 4.** Entropy variation diagram of three-frequency three-step phase-shifting simulated fringe images with the change of modulation.

To verify the applicability of the above conclusions in a real scene, a white plane is measured using the multiple exposure image fusion method. The exposure is adjusted from high value to low value, and three-frequency three-step phase-shifting fringe images at 207 different exposures are captured. The information entropy of the captured fringe images is calculated and the results are shown in Fig.5. As can be seen, with the decrease of the exposure, the fringe image entropy also shows a trend of first increasing and then decreasing. And the larger the fringe pitch, the slower the stripe image entropy reaches its maximum, which is the same as the results obtained using the simulated fringes.



**Figure 5.** Entropy variation of three-frequency three-step phase-shifting real captured fringe images with the change of exposure.

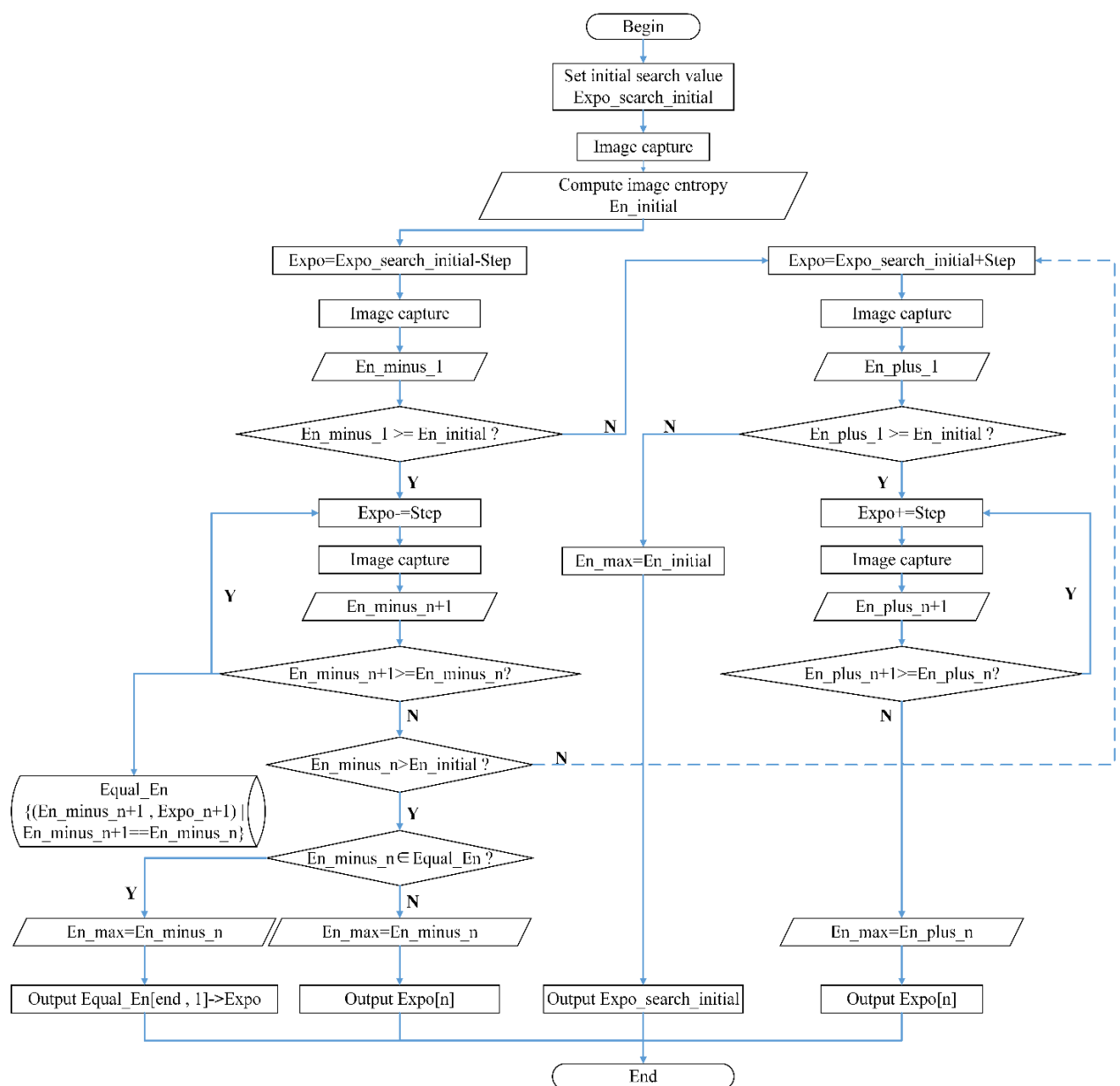
To enhance the measurement speed of the multiple exposure image fusion method for FPP, one phase-shifting image under a certain frequency should be selected and analyzed as the reference fringe image for the initial exposure calculation. And when the information entropy of this reference fringe image reaches the maximum, it should be ensured that the image entropy of all other fringe images is at or adjacent to the maximum. Additionally, in order to get the highest possible quality fused fringe images, an as large as possible exposure value is preferred. Based on the above criteria, combined with the analysis results for the variation characteristics of the fringe images entropy, the third phase-shifting fringe image with the largest fringe pitch is selected as the reference image for initial exposure calculation in this study.

#### 2.1.4 The initial exposure searching algorithm based on information entropy

Based on the above discussion, the variation characteristics of the fringe image information entropy concerning the change of pitch and exposure are clarified and the reference image used to search the initial exposure is determined. Thus, an adaptive initial exposure searching algorithm based on information entropy for multiple exposure image fusion is proposed in this paper, as shown in Fig.6. The flow of the algorithm as well as the procedures can be detailly presented as follows.

- 1) An initial value for exposure searching is set manually or automatically according to the adjustment range of the camera. This initial value is denoted as  $Expo\_search\_initial$ .
- 2) The reference fringe image of the third phase shift with the largest fringe pitch at the initial exposure is acquired. The image information entropy  $En\_initial$  is calculated.
- 3) Searching along the direction the exposure reduces. Taking  $Step$  as the exposure searching step, and let  $Expo = Expo\_search\_initial - Step$ . Capture the images under this exposure and calculate the entropy  $En\_minor\_1$  for the reference fringe image.
- 4) Determine the searching direction according to the relationship between  $En\_minor\_1$  and  $En\_initial$ . If  $En\_minor\_1 < En\_initial$ , according to the variation characteristics of the fringe image entropy with the change of exposure, it will never reach the maximum in this direction. Therefore the searching in this direction stops and the procedure goes to 8). If  $En\_minor\_1 \geq En\_initial$ , this means that the entropy may reach the maximum in this direction. The procedure continues searching in this direction and goes to 5).
- 5) Continue to search in the direction of exposure reduction. Reduce the exposure by an interval of  $Step$  and calculate the fringe image entropy. Then compare the two values acquired before and after the adjustment of exposure. If  $En\_minus\_n + 1 \geq En\_minus\_n$ , the exposure is further reduced by  $Step$  and the loop continues. If the two values are equal, the image entropy and corresponding exposure will be saved to the data set  $Equal\_En$  in a form of key-value pair. If  $En\_minus\_n + 1 < En\_minus\_n$ , the procedure goes to 6).
- 6) If  $En\_minus\_n > Equal\_En$ , the procedure goes to 7). Otherwise, the procedure stops searching in the current direction and switches to the opposite direction the exposure increases. Taking  $Expo\_search\_initial$  as the initial exposure searching value, the procedure goes to 8).
- 7) If  $En\_minus\_n$  belongs to the data set  $Equal\_En$ , set  $En\_max$  to  $Equal\_minus\_n$ . The exposure value of the first entropy-exposure pair in the last equivalent entropy-exposure pair of  $Equal\_En$  is selected as the desired exposure for the maximum entropy. The procedure then ends here. Otherwise, set  $En\_max$  equal to  $Equal\_minus\_n$ , and output the corresponding exposure value, then the procedure ends.

- 8) Search in the direction the exposure value increases. Let  $Expo = Expo\_search\_initial + Step$ , and capture corresponding fringe images and calculate the image entropy. If  $En\_plus\_1 \geq En\_initial$ , then continue to search in this direction and the procedure goes to 9). Otherwise, Let  $En\_max = En\_initial$ , and output the corresponding exposure  $Expo\_search\_initial$ . This means that the initial exposure set at the beginning of the searching algorithm makes the image entropy reaches the maximum. The procedure then ends here.
- 9) Continue the searching loop in the rising direction of the exposure. Increase the exposure and calculate the image entropy. If  $En\_plus\_n+1 \geq En\_plus\_n$ , increase the exposure and the loop continues. Otherwise,  $En\_max$  is set to be equal to  $En\_plus\_n$ . The corresponding exposure  $Expo[n]$  will be outputted and the procedure ends here.



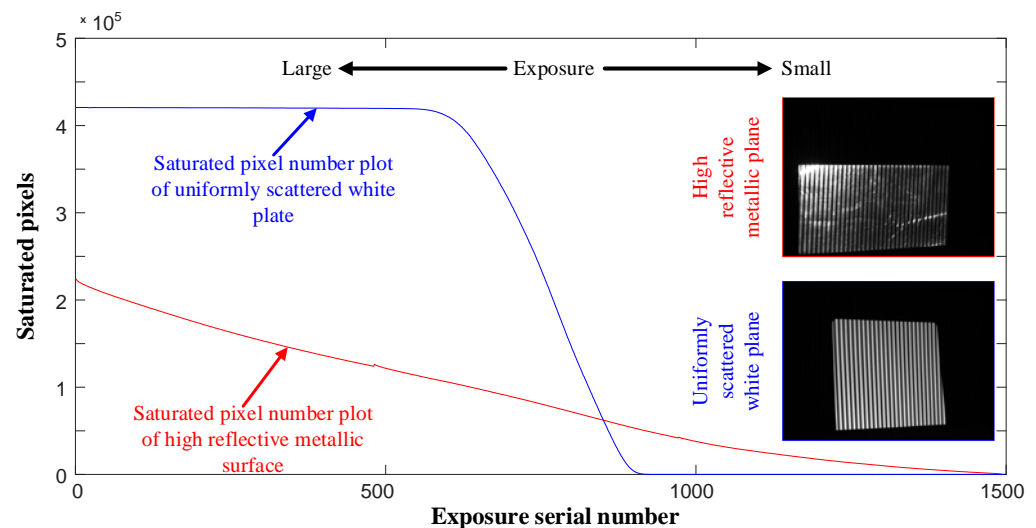
**Figure 6.** Flow chart of the initial exposure searching algorithm based on entropy.

## 2.2 Exposure sequence generation based on dichotomy

For the multiple exposure fusion methods for the high reflective surface measurement, the selection of a proper exposure sequence is another key factor that affects the measurement effect. In theory, the higher number of exposure value adjustments and the wider the exposure coverage range is, the higher the quality of the fusion fringe image will be. However, from the perspective of improving the measurement efficiency, it is not possible to increase the number of exposure value adjustments indefinitely. When the exposure adjustment step is too small, the change in the number of saturated pixels will become insignificant. A balance between measurement efficiency and fusion image quality is generally achieved through experiments, which seriously reduces the algorithm efficiency. To improve the automation of the multiple exposure method, an exposure sequence generation method based on dichotomy is proposed in this study.

In order to study the variation rule of the saturated pixel number concerning the exposure values for the highly reflective metal surfaces, a reflective metal surface was selected for measurements at a range of exposure values. The exposure value was adjusted from high to low in steps of 10 to obtain a raster stripe image at 1481 exposures, and the saturated pixels in the images were counted. As a comparison, the same exposure values parameters were used to measure a white uniformly scattering plane, and the number of saturated pixels was counted. The results are shown in Fig. 7.

Compared to the saturated pixels number variation curve of the metallic high-reflective surface, the curve of the uniform scattering plane has an obvious slope. The reason for this difference is that the uniformly scattering surface has a similar reflectance at all points, and as the exposure value decreases, the grayscale values of the image pixels also tend to change uniformly. The number of saturated pixels declines rapidly when the exposure value decreases to a certain level. When measuring the metallic high-reflective surface, for the existence of a specular reflection lobe and specular reflection crest in the direction of the camera, there always exists saturated pixels in the images. This leads to a slight variation in the saturated pixel number curve, which suits most of the machining metallic surfaces. This makes it suitable for constructing the exposure sequence based on the dichotomy method.



**Figure 7.** Variation diagram of the saturated pixel number with the change of exposure time.

Let the upper limit and lower limit of the exposure be denoted as  $Expo\_begin$  and  $Expo\_end$ , respectively. And suppose that the number of the exposure values in the exposure sequence is more than three. Then the exposure sequence generation procedure based on dichotomy is proposed as follows.

- 1) Set the desired total number of exposure  $N$  and determine  $Expo\_begin$  and  $Expo\_end$  ;
- 2) Build the initial exposure sequence  $\{Expo\_begin, Expo\_end\}$  . The exposure number in the current exposure sequence is denoted as  $Num\_count$  ;
- 3) Update the exposure sequence. Compute the average of the upper and lower limit of exposure and insert it into the sequence. In this way, the updated exposure sequence  $S1$  can be generated.

$$S1 = \{Expo\_begin, Expo\_1\_1, Expo\_end\} \quad (4)$$

$$Expo\_1\_1 = \text{floor}[(Expo\_begin + Expo\_end)/2] \quad (5)$$

- 4) Continue to update the exposure sequence.

$$Expo\_2\_1 = \text{floor}[(Expo\_begin + Expo\_1\_1)/2] \quad (6)$$

$$Expo\_2\_2 = \text{floor}[(Expo\_1\_1 + Expo\_end)/2] \quad (7)$$

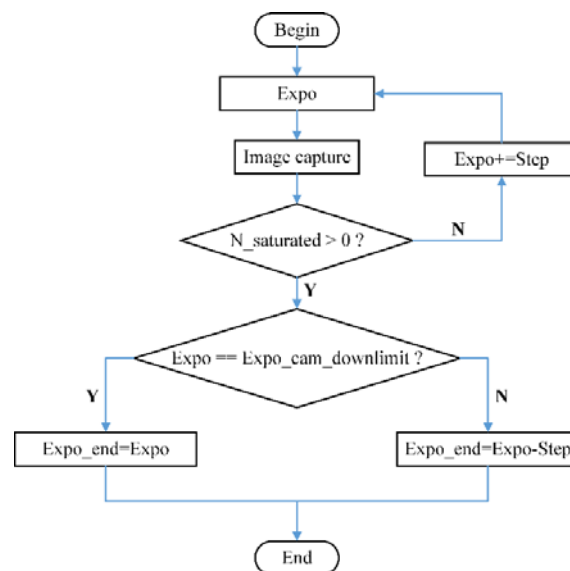
Then the updated exposure sequence can be expressed as

$$S2 = \{Expo\_begin, Expo\_2\_1, Expo\_1\_1, Expo\_2\_2, Expo\_end\} \quad (8)$$

- 5) If  $Num\_count > N$  , then the procedure ends and outputs the exposure sequence. Otherwise, continue the dichotomy method until the aforementioned inequation works. The final generated exposure sequence can be denoted as  $S\_final$  .

### 2.3 Self-adaptive multiple exposure image fusion algorithm for FPP

Based on the previous initial exposure searching procedures based on information entropy and the exposure sequence generation procedures based on dichotomy, a self-adaptive multiple exposure image fusion algorithm for FPP is finally proposed in this section, which is detailed described as follows.



**Figure 8.** Procedures for searching the ending exposure.

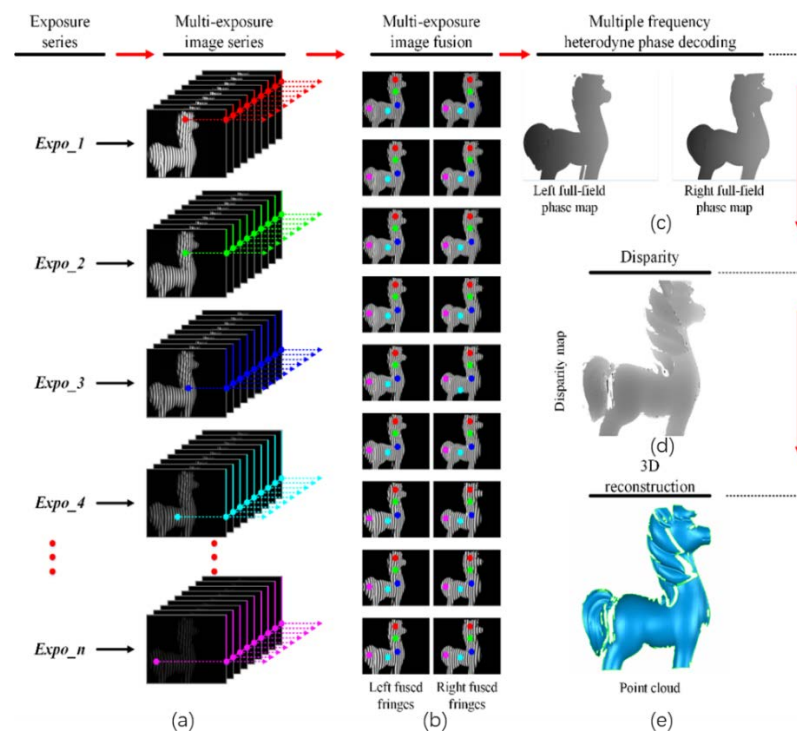
- 1) Determine the beginning and ending exposure values. Determine the initial exposure  $Expo\_initial$  using the method proposed in section 2.1. Then the beginning exposure can be given by:

$$Expo\_begin = k\_initial * Expo\_initial \quad (9)$$

Where  $k_{initial}$  is a weighting factor which is normally greater than one. By using this item, the intensity level of the pixels which are outside the saturated area of the image captured under the beginning exposure could be enhanced.

The procedure to determine the ending exposure  $Expo\_end$  is given by analyzing the number of saturated pixels, as shown in Fig.8. Let the current exposure be denoted by  $Expo$ . The searching procedure begins with capturing the images at the lower exposure limit of the camera  $Expo\_cam\_lowerlimit$ , and calculating the number of saturated pixels  $N_{saturated}$ . If  $N_{saturated}$  is greater than zero, and if there exists  $Expo = Expo\_cam\_lowerlimit$ , then the ending exposure  $Expo\_end$  is set to be  $Expo$ . Otherwise,  $Expo\_end = Expo - M$ , where  $M$  is the exposure adjustment step. If the number of saturated pixels is still equal to zero, then  $Expo = Expo + M$ . Then the aforementioned operations will be repeated until the ending exposure is determined. The procedure ends then.

- 2) Generate the exposure sequence automatically. After giving the exposure times, the exposure sequence will be generated by the method proposed in section 2.2.
- 3) Project the fringes and capture the images according to the generated exposure sequence in Step(2). Suppose  $\{Expo\_1, Expo\_2, Expo\_3, \dots, Expo\_n\}$  are the  $n$  exposure values from the exposure sequence, as shown in Fig. 9(a), then  $n$  groups of images, each of which has nine phase shift fringe images, are captured at each of the exposure values.



**Figure 9.** Step (3)-(6) of the high reflective surface measurement method based on self-adaptive multiple exposure image fusion.

- 4) Multiple exposure image fusion. Image fusion starts from the image group of three-frequency three-step fringe images captured at the largest exposure value  $Expo\_1$  by the pixel-by-pixel analysis. If a certain position pixel at this exposure value is not over-exposed in all the nine fringe images of this group, this pixel with its intensities in the nine phase shift images will be selected to form the final fused images. Otherwise, if this pixel is overexposed in at least one image of the nine fringe images in this group, then the image group at a lower exposure value is selected to repeat the above

judgment procedures. As shown in Fig. 9(b), a set of fused three-frequency three-step phase-shifting fringe images for both left and right cameras can finally be obtained in this way.

- 5) Phase decoding. Unwrap the fused three-frequency three-step phase-shifting fringe images and get the full-field phase map, as shown in Fig. 9(c). The details of the phase decoding is presented in reference [23].
- 6) Point cloud reconstruction [23]. Compute the disparity map according to the left and right full-field phase map and the point cloud is finally constructed, as shown in Fig. 9(d) and Fig. 9(e).

Thus, a complete set of the self-adaptive multiple exposure fusion methods for FPP for measuring high-reflective metal surfaces has been established. The self-adaptivity of this method is assured by the initial exposure searching algorithm based on image information entropy and the exposure sequence generation algorithm based on dichotomy. This makes the method more flexible in the measurement of surfaces with different reflectivity and suits better in different lighting environments.

The developed algorithm needs to work in conjunction with the FPP systems. It needs the fringe images captured by the FPP systems as the inputs for analysis, and generates the exposure values as the output to instruct the FPP systems to capture the images. This process is repeated during the initial exposure searching and exposure sequence generation. The algorithm can be developed using any programming language depending on how easily it can be integrated into a specific FPP system and make the entire process automatic.

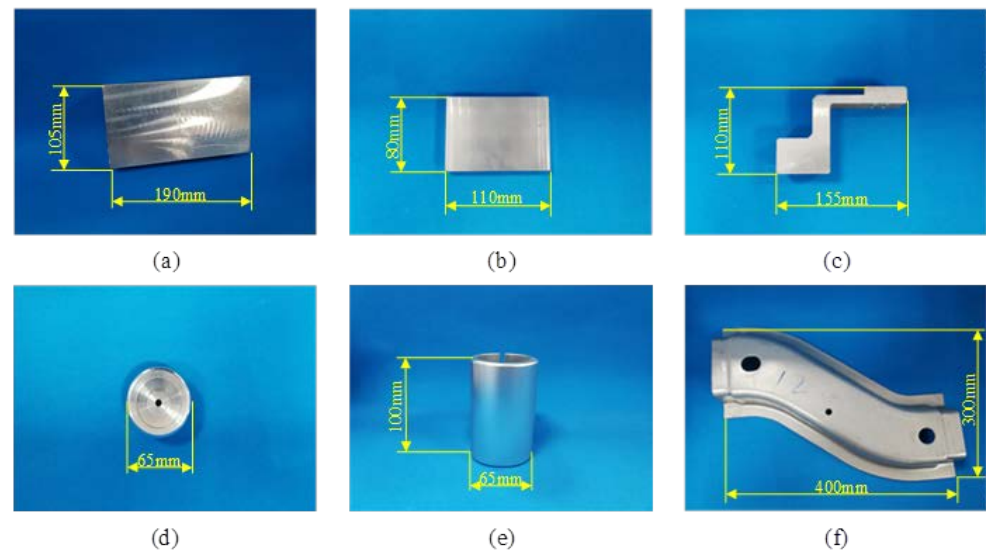
The total cost time of the entire process to determine the optimized initial exposure and exposure sequence mainly depend on the FPP hardware system because the process includes the interacting with the FPP hardware to project patterns and capture images. The computational time on analyzing the fringe images cost very little. For our system, the entire process is very fast and can output the results within seconds. Once the optimized initial exposure and exposure sequence are obtained, the FPP system can start its measurement according to the exposure values.

### 3.3. Experiments and discussions

To verify the performance of the proposed methods, two experiments are designed and implemented. The first experiment is designed for the verification of the initial exposure searching algorithm and the second is for the verification of the proposed self-adaptive multiple exposure image fusion method for FPP by evaluating the measurement performance. The FPP system has two Basler aca1300-30gm cameras with  $1280 \times 960$  resolution.

#### 3.1 Adaptivity verification of the proposed initial exposure searching algorithm

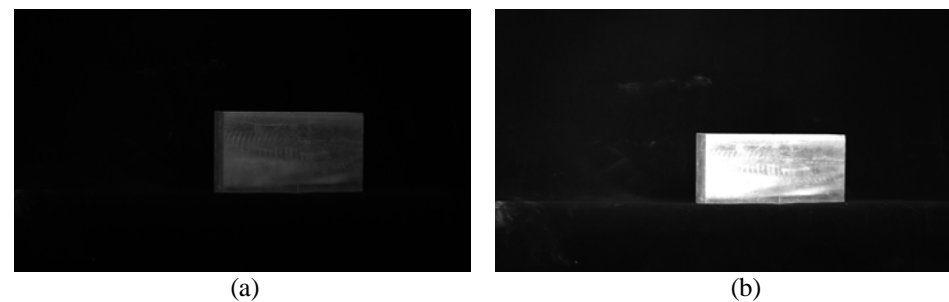
To verify the adaptivity and accuracy of the proposed initial exposure searching algorithm based on information entropy, six metal workpieces with different sizes, shapes and surface reflectivity are used as the experimental objects as shown in Fig. 10. The surfaces of workpieces No.1-No.4 are the original machined surfaces, the workpiece No.5 is a cylinder after oxidation treatment, and the workpiece No.6 is an automobile body structural part treated by the oxidation process. Workpiece No.1 to No.4 are utilized to verify algorithm performance in different lighting conditions. The suitability of the algorithm for surfaces with different reflectivity is verified by using workpiece No.1 to workpiece No.6. Taking these workpieces as the test objects, the algorithm adaptivity in different lighting conditions and with different surface reflectivity is verified as follows.



**Figure 10.** Metal test workpieces: (a)-(f) correspond to workpiece No. 1-6, respectively, with workpieces No. 1-4 having the original machining surface and No.5-6 having the oxidation treatment surface.

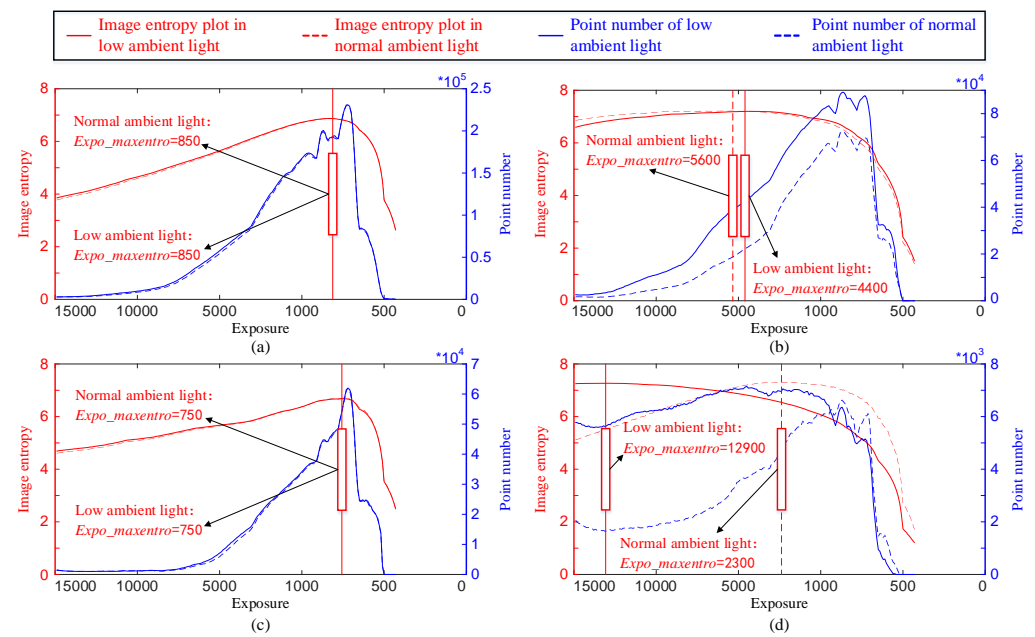
### 3.1.1 Algorithm adaptivity verification in different ambient lighting conditions

To verify the algorithm adaptivity in different ambient lighting conditions, workpieces No.1 to No.4 are selected as the experimental objects. The maximum information entropy of the fringe images is searched in few ambient lighting and normal ambient lighting, respectively. The ambient light scene is shown in Fig.11.



**Figure 11.** Experiment scenes with different ambient lighting conditions: (a) low lighting condition, (b) Normal lighting condition

The fringe image entropy of each workpiece was obtained at a total of 229 exposures to verify the accuracy of the maximum entropy searching algorithm. This exposure sequence decreases from maximum 15000 to minimum 20, with a 100-step decrease when the exposure was greater than 1000 and a 10-step decrease when the exposure was less than 1000. Additionally, to observe the relationship between the maximum of the point cloud number and the initial exposure value obtained by the proposed algorithm, the point numbers of the point clouds obtained at these 229 exposures are also counted. The searching algorithm was executed with 5000 as the initial exposure value and 50 as the searching step. The results are shown in Fig. 12. The reason why total 229 exposures is used because a complete distribution of image information entropy with respect to the exposure value needs to be obtained. It is considered as a standard to evaluate the result accuracy of the maximum entropy searching algorithm, and thus to further evaluate the proposed initial exposure searching algorithm. In actual measurement, it doesn't need as many exposures to optimize the initial exposure. Normally, twenty exposures are sufficient to accurately calculate the initial exposures in the experiments.



**Figure 12.** Evaluation of the initial exposure searching algorithm in different lighting conditions: (a)-(d) correspond to workpiece No. 1-4, respectively.

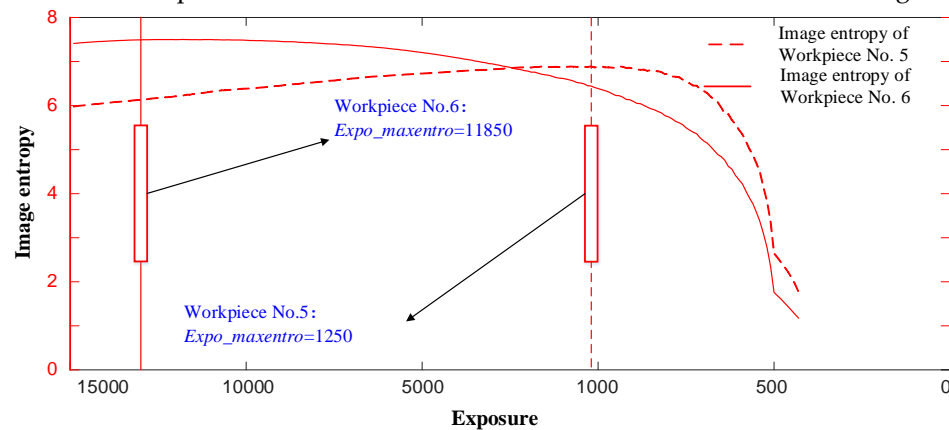
As can be seen from the figure, the maximum value of the fringe image entropy is accurately obtained for each of the workpieces and the corresponding initial exposure value is given. For workpieces No.1 and No.3, the fringe image entropy variation curves under normal ambient lighting are very close to that under low ambient lighting conditions, and thus the initial exposure values obtained by the searching algorithm in different lighting conditions are equal. The image entropy change curves of workpiece No. 2 show a certain degree of difference between the different lighting conditions, thus the final obtained initial exposure values are also slightly different. And as for workpiece 4, its fringe image entropy change curve obtained in normal ambient lighting is different from that in low ambient illumination. Compared with the image entropy curve in low ambient illumination condition, the curves in normal ambient illumination arrives at their peak later. And the curves in normal ambient illumination descend faster than that in low ambient illumination.

As for the variation characteristics of the number of the point cloud, it also rises firstly and decreases after arriving at the peak. And the curve of the point cloud number arrives at the peaks later than the curve of the image entropy. It means that after the fringe image entropy reaches the maximum value, the number of the point cloud will continue to maintain the trend of increasing and reach the maximum after the exposure value drops to a certain value. This illustrates that the three-frequency three-step phase shift method used in this study is more adaptable to the lighting conditions and can guarantee the decoding quality in a weaker lighting condition. Also, it means that the exposure value corresponding to the maximum point number of the point cloud, denoted by  $Expo\_max\_pointnum$ , would be less than the initial exposure obtained from the proposed algorithm. This ensures that the exposure  $Expo\_max\_pointnum$  is always within the upper limit and lower limit of the exposure sequence, which makes the fusion algorithm make full use of the fringe images captured at this exposure value. Additionally, it ensures that more pixels with high intensity could be selected for the fused image, which further improves the quality of the full-field phase map and point cloud.

### 3.1.2 Algorithm verification for surfaces with different reflectivity

In 3.1.1, the performance of the proposed algorithm is verified in different ambient lighting conditions using four workpieces with the original machining surfaces, which are

high-reflective. To verify the algorithm's adaptivity for the surfaces with different reflectivity, experiments are performed with workpiece No. 5 and No. 6 with oxidized surfaces under normal lighting conditions. And their fringe image entropy curves are produced with the same parameters as those used in 3.1.1. The results are shown in Fig.13.



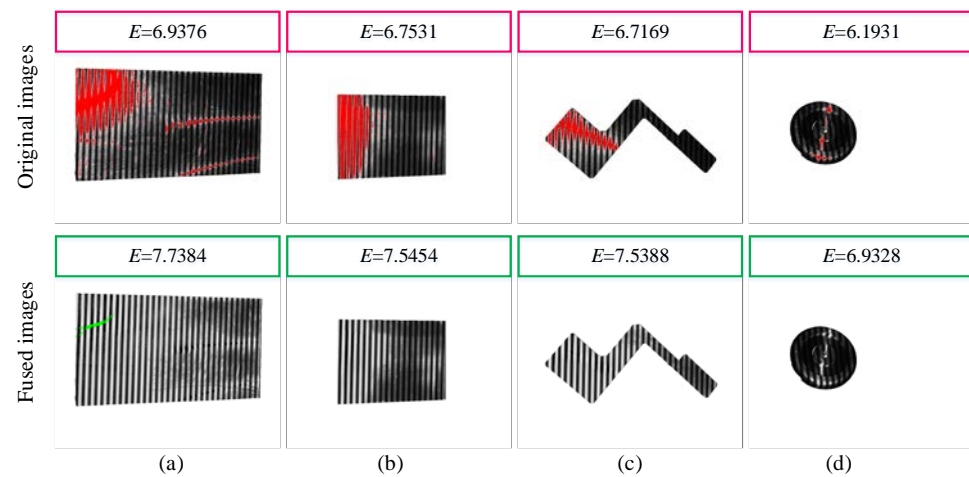
**Figure 13.** Evaluation of the initial exposure searching algorithm for different surface reflectivity.

As can be seen from the figure, the initial exposure value search algorithm finds the maximum entropy of the streak image and obtains the corresponding exposure value for both workpieces 5 and 6 with surface oxidation. This, combined with the previous results for the machined original surfaces of workpiece 1-4, demonstrates that the proposed algorithm can successfully find the maximum entropy of the fringe images for the measured objects with different surface reflectance and can obtain the corresponding exposure value as the initial exposure value. Thus, the proposed initial exposure searching algorithm based on fringe image entropy is adaptive for both different ambient lighting conditions and different surface reflectivity, which ensures the adaptiveness, robustness and accuracy of the obtained results for the initial exposure value.

### 3.2 Measurement verification of the adaptive multiple exposure fringe fusion method

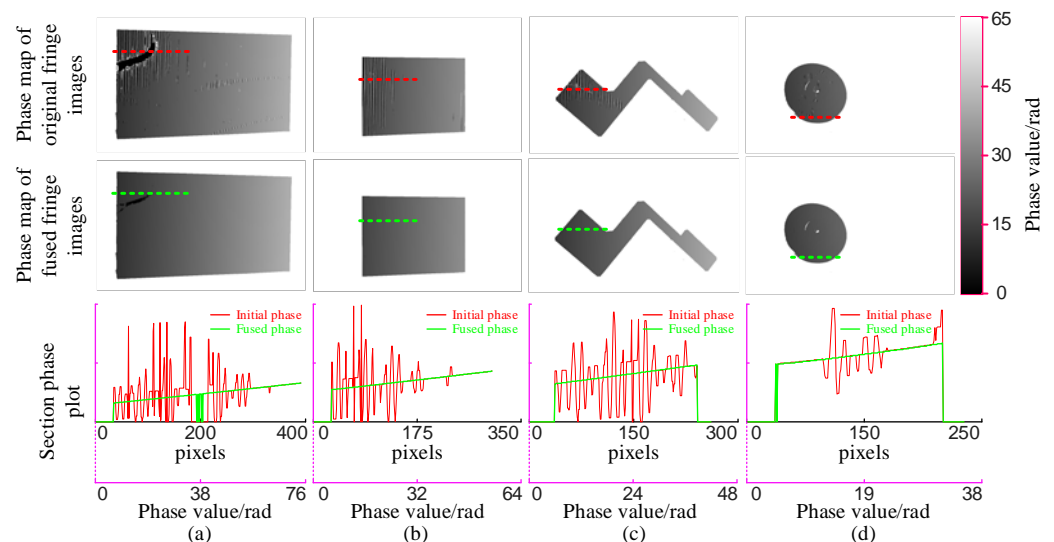
To verify the measurement performance of the proposed adaptive multiple exposure fringe fusion method for FPP, the workpieces No.1-No.4 are measured under normal ambient lighting condition. To demonstrate the algorithm's performance sufficiently, the original fringe images with maximal entropy obtained in 3.1 are selected to be compared with the fused fringe images. The saturation threshold is set to 250 and the overexposure pixels in the original images are represented in red. And the area which fails in getting rational complemented pixels in the fused image is marked with green color. The image entropy before and after the fusion is calculated and the results are shown in Fig.14.

As can be seen from the figure, there are overexposure areas, presented in red, in the original fringe images for all the four workpieces. This may greatly reduce the sinusoidal intensity distribution in these areas, which leads to significant phase errors during image decoding. In contrast, almost all of these areas are well restored in the fused fringe images except for a small part of workpiece 1, presented as the green area, which cannot be restored because of the extremely high surface reflection. Also, there are a large number of areas where the fringe contrast is very low due to the darkness of the image, as shown in the right part of the images of the four workpieces. Also, the fringe contrast in these areas was significantly improved in the fused images. To evaluate this effect quantitatively, the image entropy  $E$  is calculated before and after the image fusion. From the results, it can be seen that the entropy of each fused image is significantly improved compared to the original image, with the improvement being 11.5%, 11.7%, 12.2%, and 11.9%, respectively, which means that the fused images can provide more fringe information for the later phase decoding, thus improve the quality of the obtained phase map.



**Figure 14.** Comparison of the original fringe images with maximal entropy with the fused fringe images: (a)-(d) correspond to workpiece No. 1-4, respectively.

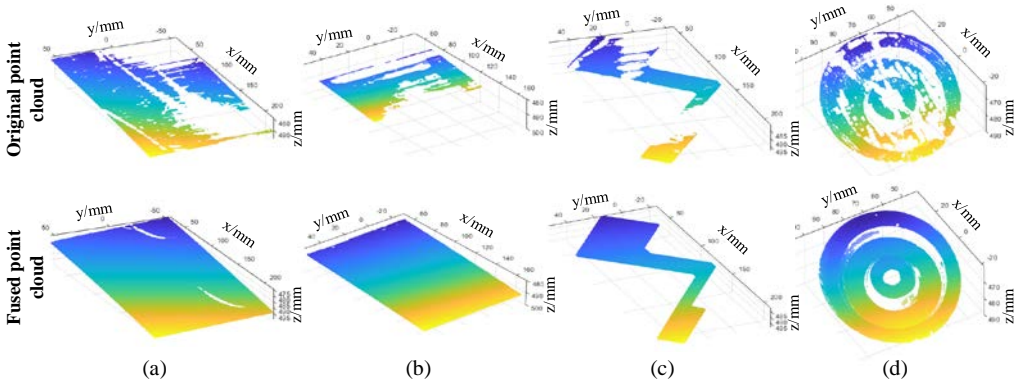
The phase unwrapping was performed using these fringe images before and after the fringe fusion. The cross sections of the unwrapped phase maps at the same positions are obtained for evaluation, as shown in Fig.15. The quality of the full-field phase maps obtained using fused stripe images are significantly improved compared to the unwrapped results obtained using the original fringe images. Taking workpiece No.1 as an example, in the upper left region of the phase map unwrapped from the original fringe images, there exists distinct visual fold area. And in this area, there is a large region where it fails in phase unwrapping because of the strong surface reflection. This folded and black region in the phase map is corresponding to the red connected region of workpiece 1 in Fig.14. And it can be seen that there is a large error in this area due to the presence of high reflectivity. On the contrary, the fold areas in original fringe phase map are effectively eliminated in the fused phase map. As can be seen, all the regions of workpiece 1 have good unwrapping results, except for the extremely high reflective region marked in green in Fig.14. And the quality of phase maps are also greatly improved for workpiece 2 to 4.



**Figure 15.** Unwrapped phase maps obtained from the initial fringe images versus those from the fused fringe images: (a)-(d) correspond to workpiece No. 1-4, respectively.

To compare the phase unwrapping error of the original fringe images and the fused fringe images more intuitively, the cross-section data is plotted in Fig.15. The selected section for the phase map before and after the fusion algorithm is marked with the red dotted

line and green dotted line respectively. From the phase variation curve, there exists a distinct phase disturbance in the cross-section phase curve of the original phase map. This means that there is an obvious phase error in this area, which will lead to an error in 3D reconstruction. Comparatively, the section phase curve of the fused phase map is smooth and the quality of phase unwrapping has been greatly improved.



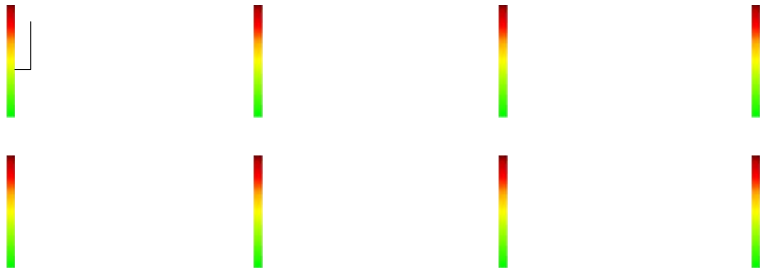
**Figure 16.** Point clouds reconstructed based on the original fringe images and the fused fringe images: (a)-(d) correspond to workpiece No. 1-4, respectively.

As shown in Fig. 16, the point cloud is reconstructed based on the original fringe images and the fused fringe images. As can be seen, lots of points are missing in the reconstructed point cloud reconstructed from the original fringe images. This stems from the phase error for the existence of a high-reflective area, which results in the wrong matching and matching failure in the stereo match step, and further leads to the missing of point cloud data. In contrast, the completeness of the point cloud reconstructed from the fused fringe images is improved dramatically. The point number of the original point cloud and the fused point cloud are given in Table 1.

**Table 1.** Point number reconstructed from original fringe images and fused fringe images.

	Workpiece No.1	Workpiece No.2	Workpiece No.3	Workpiece No.4
Initial point cloud	192535	23534	50454	4782
Fused point cloud	418859	129824	98169	33894

To analyze the 3D reconstruction results more accurately, the aforementioned point clouds are fitted to obtain the fitting surface as the ground truth. Then the error of the point cloud before and after the fusion algorithm is calculated and the color range maps are constructed as shown in Fig. 17. Simultaneously, to evaluate the error quantitatively, the mean error and root mean square error (RMSE) is calculated, as listed in Table 2.



**Figure 17.** Error of the point clouds reconstructed from the original fringe images and fused fringe images: (a)-(d) correspond to workpiece No. 1-4, respectively.

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**Table 2.** Error of the point clouds reconstructed from the original and fused images.

	Error item	Workpiece No.1	Workpiece No.2	Workpiece No.3	Workpiece No.4
Initial point cloud	Mean Error/mm	0.0639	0.0478	0.0537	0.0496
	RMSE/mm	0.1125	0.0942	0.0937	0.0786
Fused point cloud	Mean error/mm	0.0332	0.0308	0.0281	0.0313
	RMSE	0.0358	0.0362	0.0297	0.0344

Compared to the point clouds of the original fringe images, the mean error and RMSE of the fused point clouds are greatly reduced. For example, the mean error of the four workpieces is about 0.05mm to 0.06mm. Whereas the mean error of fused point clouds is reduced to around 0.03mm. The RMSE of the original point cloud is around 0.09mm, whereas the RMSE of the fused point cloud is around 0.03mm. This means that the accuracy of the fused point cloud is also improved compared with the original point cloud. It's mainly because the fused fringe images present better quality especially in the reflective areas, thus the point noise can be greatly suppressed.

4. Conclusions

In this paper, a self-adaptive multiple exposure image fusion method is proposed to solve the challenges of measuring high reflective surfaces or high dynamic range surfaces for FPP. An adaptive initial exposure searching method is first proposed by introducing the theory of information entropy to the fringe image combined with the analysis of the characterization of fringe image entropy. To generate the proper exposure sequence automatically, an exposure sequence generation algorithm is proposed, which needs only a few parameters presetting. Based on these two algorithms, a novel self-adaptive multiple exposure image fusion method for FPP as well as its detailed procedures are given. To verify the performance of the proposed method, experiments are designed by measuring metal workpieces with different shapes and surface reflectivity. Compared with the current multiple exposure image fusion methods, the experimental results verify the self-adaptivity, efficiency and robustness of the method in the measurement for different reflectivity and different ambient lighting conditions.

In the cases when measuring different objects with different surface reflectivity or in different ambient light conditions, the initial exposure and the exposure sequence must be adjusted accordingly. Most existing multiple exposure image fusion methods typically have a relatively fixed sequence of exposure settings that are determined by practical experiences or trial and error experiments. The trial and error experiments to adjust the sequence of exposure settings are time-consuming, also the initial exposure and the exposure sequence cannot be precisely optimized based on practical experiences. Compared with the existing methods, the core novelty and main contribution of the method proposed in this manuscript is that it can optimize the initial exposure and the exposure sequence self-adaptively. This advantage makes the proposed method superior to others in terms of self-adaptivity for varying surface reflectivity and ambient lighting conditions. It also has the ability to optimize the exposure settings quickly and automatically rather than manually adjusting based on practical experiences or performing some trial and error experiments.

Although the proposed method works well, there still exists some issues to be further studied, for example, the affection of the geometry complexity on measurements has not been studied detailly. For the future works, the possibility to apply the proposed solution in some structured light measurement systems projecting other types of patterns need to be studied. The influences of object geometry complexity and the light intensity on the

proposed method need to be studied. And the criteria to use when deciding whether to modify the exposure setting or continue using the old one when the relative angle and position between the measured object and the light source change. Also, deep learning could be tried in the recovery of the saturation areas in the captured images. Methods based on hardware could be studied. For example, taking the super high dynamic range characteristic of the chips into consideration, the event-based camera has great potential to solve the problem of high-reflective surface measurement.

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**Data Availability Statement:** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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**Conflicts of Interest:** The authors declare no conflict of interest.

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