Article

Enhancing Tidal Wave Predictions at the Estuary of the Nakdong-River using Fixed-lag Smoother

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Abstract: Predicting the tidal wave is essential not only to better understand hydrological cycle at the boundary between land and ocean but also to improve energy production in the coastal area. As affected by various factors such as astronomical, meteorological, and hydrological effects, predicting the tidal wave at the estuary remains uncertain. In this study, we present a novel method to improve short-term tidal wave predictions using a fixed-lag smoother based on sequential data assimilation (DA). The proposed method is implemented for the tidal wave predictions at the estuary of Nakdong River. As a result, the prediction accuracy was improved by 63.9% with DA and the calibration using the regression. Although the accuracy of DA diminished for increasing forecast lead times, the 1-hr lead forecast by DA had still 44.4% improvement over the open loop without DA. Plus, the optimal conditions for the fixed-lag smoother were analyzed in terms of the order of a smoothing function and the length of assimilation window and forecast lead time. It was suggested that the optimal DA configuration could be obtained with the 8th order polynomial as a smoothing function assimilated under 6-hr or longer past and future DA windows.

Keywords: tidal wave predictions; estuary; fixed-lag smoother; data assimilation; sensitivity analysis

1. Introduction

An accurate prediction of the tidal wave is of importance not only to better understand hydrological cycle at the boundary between land and ocean but also to improve energy production in the coastal area. One of conventional techniques for tidal level prediction is a harmonic analysis method based on the least square estimation. The harmonic analysis based on the least square regression uses a superposition of sinusoidal functions describing the amplitude, phase, and frequency of the oscillatory components such as tides and sound. After Darwin's pioneering work [1] dating back to the late 19th century, the harmonic analysis was further improved by several researches [2,3]. In addition, various approaches including artificial neural networks, backpropagation neural networks, wavelet transformation, inaction methods, and hybrid models have been proposed to improve tidal wave prediction [4–13]. However, one of limitations of conventional tide level prediction is that it does not consider hydrological and meteorological factors. Especially, at an estuary, the river discharge and storm surge can significantly affect the water level which may exacerbate the accuracy of the tidal wave prediction. One of techniques to fill a gap in model prediction is data assimilation (DA) which updates model states or parameters using newly obtained observation [14]. There have been also advances in tidal wave prediction using DA approaches such as Kalman filtering [15–19]. Nevertheless, it is expensive to implement DA methods for specific model and application since the modeling procedure needs to be reorganized or reformulated to adjust model states or parameters when new observations are available [20]. Post-processing can be another option to reduce the model biases [21]. While DA dynamically changes model states or parameters, post-processing adjusts the simulation results using an additional statistical treatment with no changes in a prediction model. Yet, post-processing has been rarely implemented in the tidal wave prediction.

In this study, we present a novel method to improve the short-term tidal wave prediction using the fixed-lag smoother. While originated from DA, we use the proposed smoother as a post-processor producing the optimal solution under the assimilation window with no changes in model states. Before applying the smoother, the tidal wave simulation is adjusted using the regression equation to consider the impact of the river discharge and atmospheric pressure. The proposed method is implemented for the tidal wave predictions at the estuary of the Nakdong River. Also, the optimal conditions for the fixed-lag smoother are analyzed in terms of the order of function and the length of assimilation window and forecast lead time.

2. Study area and Data

The Nakdong River (basin area: 23,860 km^2, length: 525.15 km) is one of the longest rivers on the Korean Peninsula flowing through the entire Yeongnam region, meeting the South Sea at the estuary [22]. The estuary of the Nakdong River is characterized by a complex branch channel network where the movement of water is driven mainly by tidal water and freshwater discharge. In 1987, the estuary gate was constructed with the aim of reducing the saltwater damage to the irrigation area in the Gimhae Plain and supplying water to the industrial areas [23]. However, as ecological diversity and water quality have decreased since the construction of the estuary bank, a partial re-opening of the Nakdong Estuary Bank gate was planned and tested in recent years [24]. The movement of fresh and salt water is affected by both river discharge and tidal sea level in the estuary. Therefore, an accurate tide prediction is an important factor for the concentration and location of salt water to be under control while operating the estuary gate.

Figure 1. Location of the tide observation station at the estuary of Nakdong River in Busan, South Korea.

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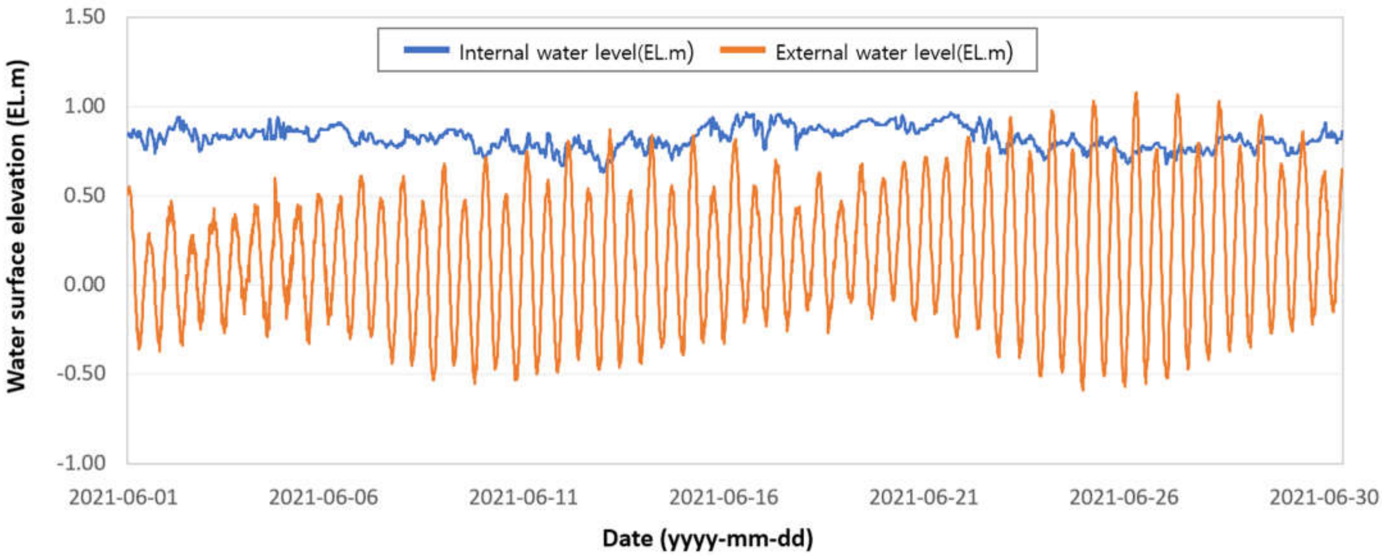

Figure 2. Example time series of observed water level in and out of the estuary gate.

As shown in Figure 1, the tidal observation station is located outside the Nakdong River estuary gate measuring the water level with the frequency of 10 minutes. Figure 2 illustrates the time series of the observed sea level (external water level) and river water level (internal water level) inside the estuary gate from June 1, 2021 to June 30, 2021 among all observations (June 1, 2021 to August 24, 2021) for visibility of the figure. Except for the complete opening of the estuary bank floodgate due to flooding, the river water level varies between EL.0.51 m and EL.1.01 m with the average of EL.0.76 m.

3. Methodology

In this section, we describe the tidal wave simulation method, an adjustment technique using regression, DA-based smoothing method, and the configuration of the DA experiments.

3.1. Tidal wave simulation using TASK2K and regression without DA

The tidal analysis software kit 2000 (TASK2K) based on harmonic decomposition has been applied for the operational prediction of the tidal level at the estuary of the Nakdong River. However, it was found that the difference between the observed and predicted water surface elevations became significantly larger up to about 13 cm when opening the Nakdong River estuary gate during the pilot operation in 2021. The large simulation error was considered to be affected by temporal changes of the discharge when opening the gate. Therefore, the additional adjustment was implemented considering the effects of discharge and atmospheric pressure via a regression relationship curve (Figure 3).

Figure 3 shows the relationship curve between the water surface elevation and the inland discharge from the Nakdong River. The black dotted lines represent the relationship and the time series of the observed water level (Tidal_Obs). The red lines represent those of TASK2K (Tidal_Cal_Orig) while the blue lines meaning those with additional calibration using regression (Tidal_Cal_Re). The simulation results via each method are analyzed in Section 4.

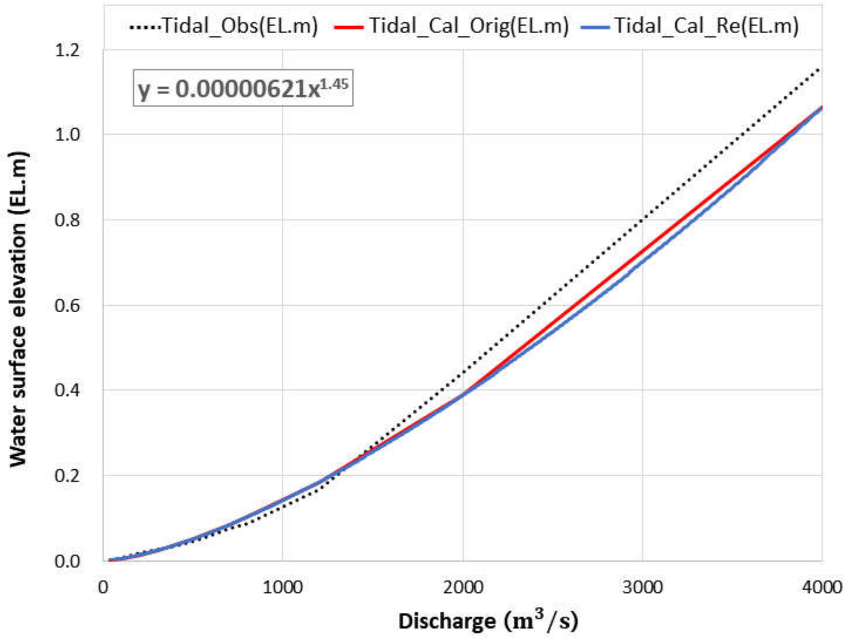

Figure 3. A relationship the relationship between river discharge and the water level.

3.2. Fixed-lag smoother

The water level at the estuary is heavily controlled by local physiography in addition to the tidal effect. Even after adjusted by the regression model, the tidal prediction may not fully capture the effect of hydro-meteorological conditions. In this work, we investigate how assimilating water level observations at the estuary may further improve the tidal wave predictions. For the DA technique, we use the fixed-lag smoother method to combine the base simulations by TASK2K with the observations under an assimilation window. The fixed-lag smoother minimizes the objective function J as follows:

$$\mathrm{J} = \frac{1}{n}\sum_{i}^{n}(O_i - P_i)^2 + \frac{1}{m}\sum_{j=n+1}^{n+m}\left(S_j - P_i\right)^2 \quad (1)$$

where O_i and P_i denote the observed and predicted water levels at timestep i, n and m denote the size of the assimilation window in the past and future times, respectively, and S_j denotes the original simulation by TASK2K at timestep j. P, the predicted water levels, is newly formulated by both past observations and future original simulations in every assimilation window.

Figure 4 illustrates how the fixed-lag smoother works for the improved tidal wave prediction. The assimilation window consists of two windows: past and future. A DA simulation line represented by a polynomial function is the optimal solution within the assimilation window. Unlike the conventional DA methods including filtering and smoother, the fixed-lag smoother for tidal wave prediction does not require updating state variables but provides the improved predictions optimized within the assimilation window. The simplified implementation procedure of the smoother is due to the fact that the errors of tidal wave prediction are mainly associated with the magnitude and the timing errors are relatively small.

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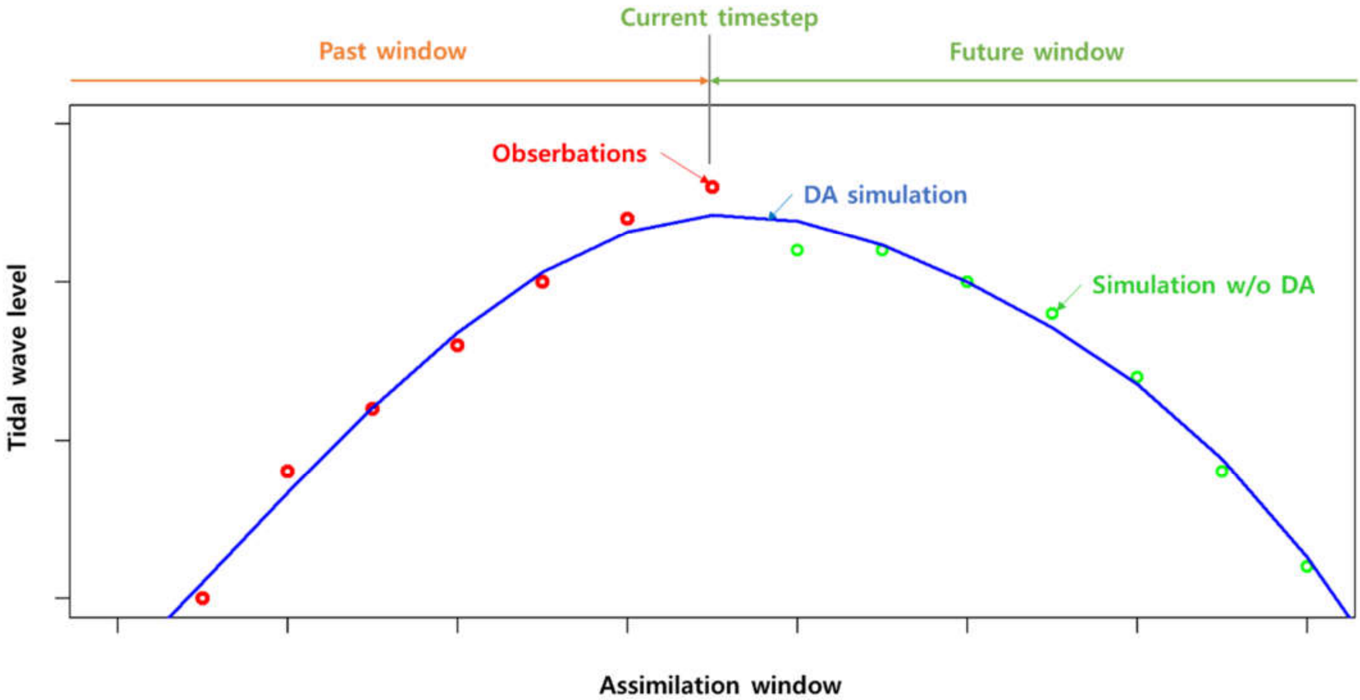

Figure 4. A schematic diagram of DA method via fixed-lag smoother.

3.3. Configuration of DA experiments

The fixed-lag smoother for tidal wave prediction provides an optimal solution within an assimilation window for a smoothing function and a prediction lead time. Since the tidal wave prediction is highly-nonlinear, configuration of DA may affect the performance of prediction. In this study, the effects of DA conditions were analyzed through multiple experiments considering polynomial order, DA window period, and lead time. In this sub-section, the configuration of DA experiments is presented.

The DA simulation experiments were implemented using the 10-minute interval observation data from June 1, 2021, to August 24, 2021. In the experiments, 1 step means 10 minutes. For example, a 24-step ahead lead time means a 4-hour ahead lead time. Table 1 shows the simulated conditions of each DA experiment. The fixed condition was expressed as Ｘ, and the variable condition was expressed as Ｏ. The polynomial order is a non-linear function that integrates observational and predictive data, which can affect the accuracy of tide prediction depending on the shape and order of the function. In the DA experiment 1, the effects of varying polynomial orders were assessed while the other factors such as sizes of DA window for past and future time steps are fixed. Also, the accuracy of DA may vary depending on the sizes of the past and future assimilation window, which would be assessed in the DA experiments 2. The effects of the assimilation window were assessed by two sub-experiments separating the past observation part and the future prediction part. In the DA experiment 2-1, the future prediction part was fixed and the effects of the varying size of past observation part were assessed. On the contrary, in the DA experiment 2-2, the past observation part was fixed while the future prediction part was changing. For the DA experiments 2, the polynomial order and lead time were fixed under the same conditions. In the DA experiment 3, it was assessed how long the effects of DA last with varying prediction lead times while the other conditions were fixed.

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Table 1. Summarized configuration of DA experiments.

Experiments	Polynomial order	Size of DA window for past time steps	Size of DA window for future time steps	Prediction lead time
DA experiment 1	O	X	X	X
DA experiment 2-1	X	O	X	X
DA experiment 2-2	X	X	O	X
DA experiment 3	X	X	X	O

4. Results

In this section, the results of tidal wave predictions and DA experiments were analyzed.

4.1. Comparison of tidal wave predictions

Figure 5 illustrates the timeseries of the observed and predicted surface water elevation. While Figure 5 (a) shows the timeseries of the full simulation period, Figures 5 (b), (c), and (d) present the enlarged views in the different time slots. It is found that the the original prediction (Tidal_Cal_Orig) is improved by the calibration using the regression (Tidal_Cal_Re) and further improved by the fixed-lag smoother (DA). The 1-hr lead forecast by DA (DA_1hr) is moving between two simulations: the regression (Tidal_Cal_Re) and the fixed-lag smoother (DA). As shown in Figures 5 (c) and (d), the improvement by DA becomes more obvious when the discrepancy between the observed and predicted water levels is large, usually at the higher and lower cycles of the tidal wave. In Figure 5, the results of the fixed-lag smoother (DA) and the 1-hr lead forecast by DA (DA_1hr) are obtained using the following configuration: 36 steps (6 hours) of the past and future DA window with a smoothing function of an 8^{th} polynomial line. The sensitivity of the DA configuration is analyzed in Sub-section 4.2 through multiple DA experiments.

Table 2 compares the performance of the predicted water surface elevation shown in Figure 5 assessed by the root mean square error (RMSE). The values of the RMSE are as follows: 0.108 m, 0.071 m, 0.039 m, 0.060 m in Tidal_Cal_Orig, Tidal_Cal_Re, DA, and DA_1hr, respectively. Compared to the open-loop original prediction (Tidal_Cal_Orig) as a base line, the results of the regression (Tidal_Cal_Re) and the fixed-lag smoother (DA) correspond to 34.3% and 63.9% improvement. The 1-hr lead forecast by DA (DA_1hr) shows 44.4% improvement over the open loop, which means there are benefits from DA even in the forecast while the accuracy of DA diminishes as the forecast lead times increase. To assess the performance at the extreme, the RMSE values are estimated for the observation ranges above 90% and below 10% quantiles. In the open loop (Tidal_Cal_Orig), the error for the extreme ranges is 0.012, 19.4% larger than the error for the whole ranges. However, the predictions are successfully enhanced by the calibration using the regression and DA. The percentile improvement over the open loop is 40.3%, 68.2%, and 49.6% by the calibrated (Tidal_Cal_Re), the DA analysis (DA), and the 1-hr lead DA forecast (DA_1hr), respectively. It is noteworthy that about 10% additional improvement is achieved in the 1-hr lead forecast when DA is applied for the calibration result by the regression.

Figure 6 compares the observed and simulated surface water elevation. While the open loop by TASK2K (Tidal_Cal_Orig) is most dispersed from a 1:1 line (Figure 6 (a), the fixed-lag smoother (DA) shows the most improved result demonstrating that the use of the fixed-lag smoother can further improve the prediction by the regression.

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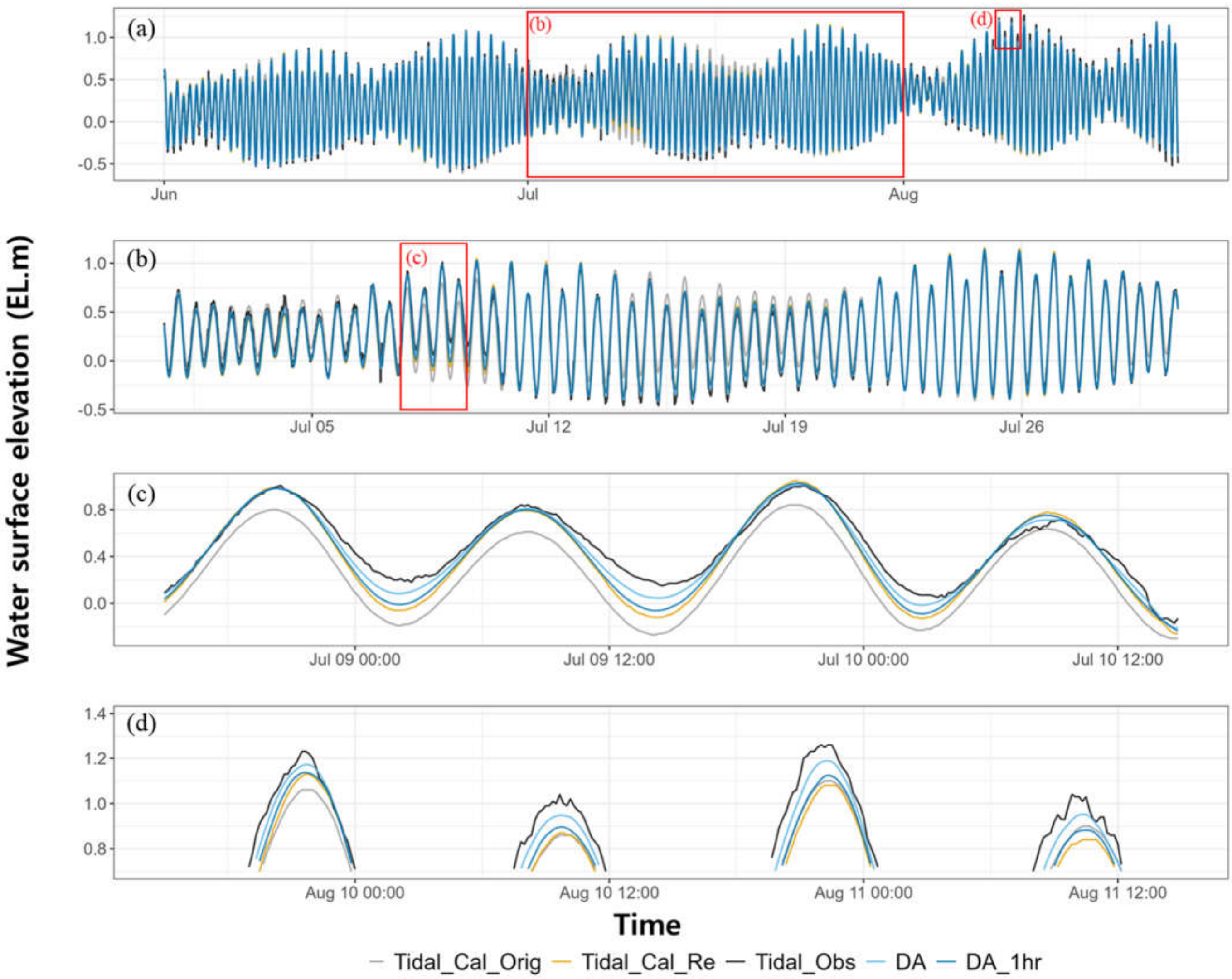

Figure 5. Predicted water surface elevation via multiple methods compared to observations.

Table 2. Comparison of the performance of the predicted water surface elevation.

Evaluation index	Evaluation range	RMSE of predicted water surface elevation (m)			
		Open loop (Tidal_Cal_Orig)	**Calibrated (Tidal_Cal_Re)**	**DA analysis (DA)**	**1-hr lead forecast by DA (DA_1hr)**
RMSE (m) (percentile improvement over open loop)	All data	0.108 (-)	0.071 (34.3% improved)	0.039 (63.9% improved)	0.060 (44.4% improved)
	Data above 90% and below 10% quantiles	0.129 (-)	0.077 (40.3% improved)	0.041 (68.2% improved)	0.065 (49.6% improved)

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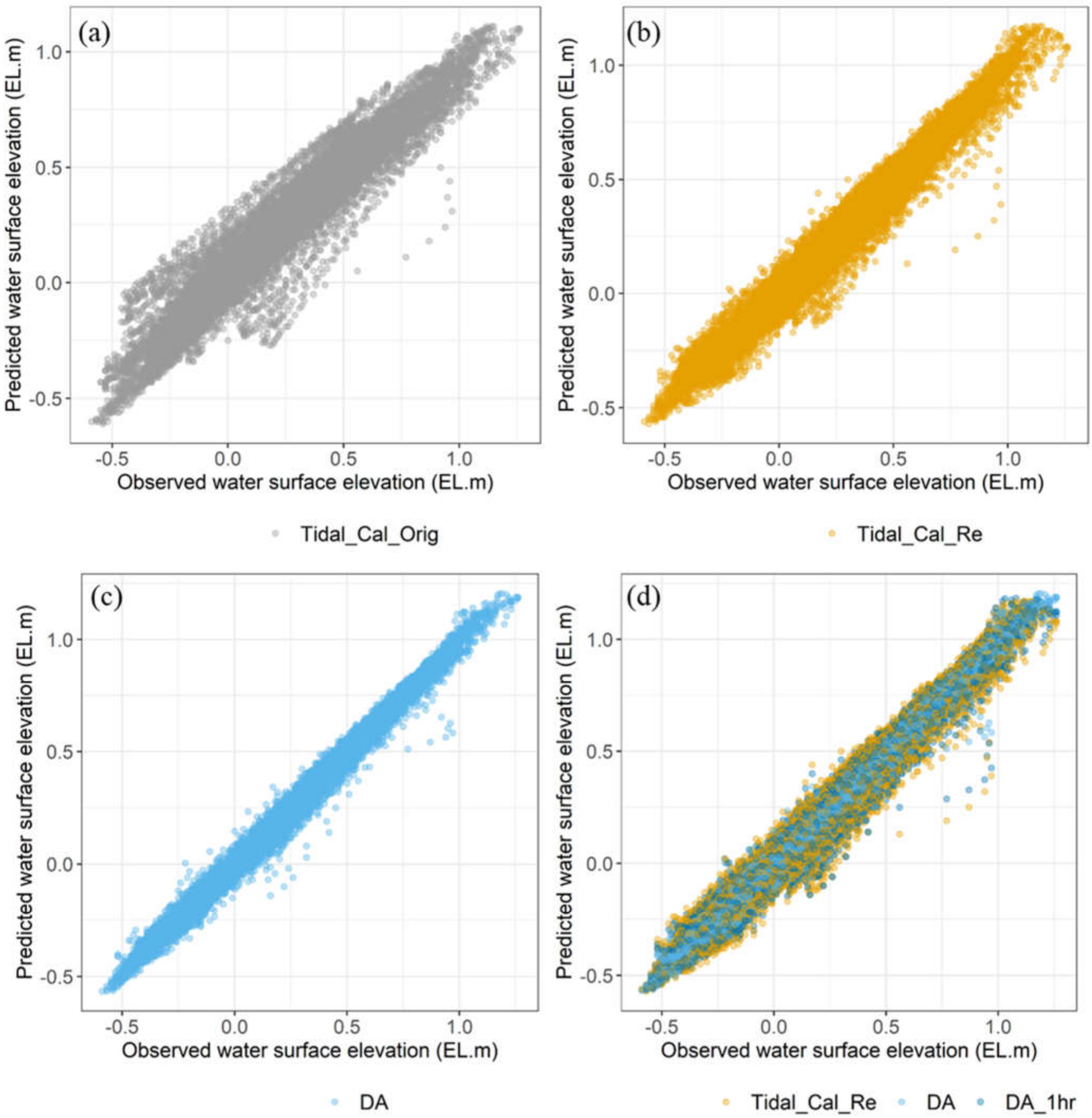


Figure 6. Comparison of the observed and predicted water surface elevation by: (a) the TASK2K simulation (Tidal_Cal_Orig), (b) the adjustment by the regression equation (Tidal_Cal_Re), (c) DA, and (d) the 1-hr lead forecast by DA (DA_1hr) with the regression and DA methods.

4.2. DA experiments for improved tidal wave preditions

Since the tidal wave generally has a time-series pattern, it is important to select the time-lag value appropriately to find the data portion that is highly correlated with the observations used in the model [25]. In this section, we investigate prediction accuracy based on the selection of appropriate polynomial order, lead time length and time-lag values when using the fixed-lag smoother techniques.

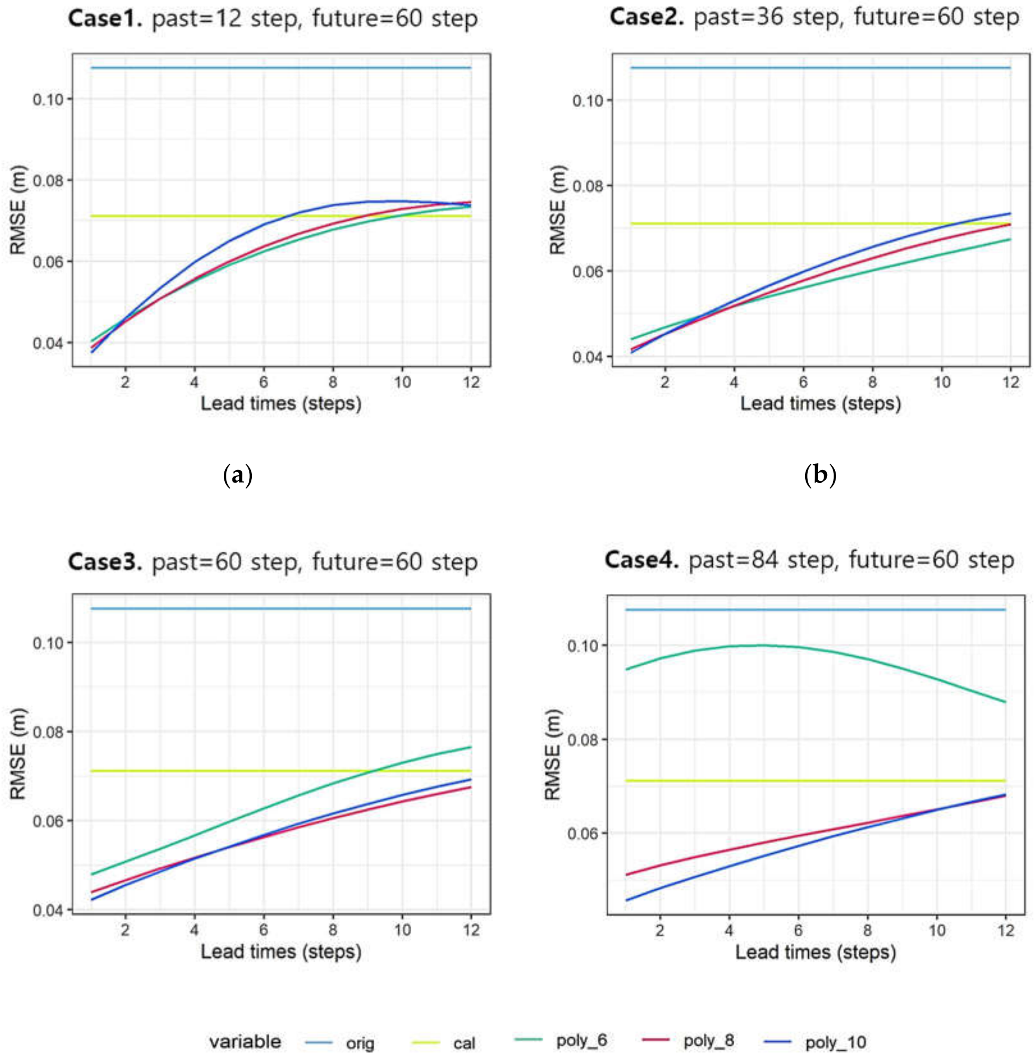
4.2.1. DA experiment 1: impact of the order of a polynomial

The effect of the polynomial order of the fixed-lag smoother function on the tidal wave forecast accuracy was analyzed. Since there are numerous combinations of DA configuration, some representative cases were shown to evaluate the performance of the DA with with the different polynomial orders of the smoother function over varying forecast lead times. In Figure 7, the size of the DA window for the "future" time steps is fixed as 60 steps (10 hours), while the sizes of the DA window for the past time steps change from 12 to 84 steps (2 hours, 6 hours, 10 hours, 14 hours). On the contrary, in Figure 8, the size of the DA window for the "past" time steps is fixed while the sizes of the DA window for the future time steps change.

When the size of the DA window for the past time steps is relatively small (e.g. Cases 1 and 2), the highest predictive performance is obtained with the 6th polynomial order. For Case 1, at the forecast lead time of 6 steps (1 hour), the values of the RMSE are 0.062 m, 0.064 m, and 0.069 m by the 6th, 8th, 10th polynomial orders; for Case 2, 0.056 m, 0.058 m, and 0.06 m by the 6th, 8th, 10th polynomial orders, respectively. However, the performance increases as the size of the past DA window becomes larger and the order of the polynomial is higher than 6th (e.g. Cases 3 and 4). It is also found that the accuracy with the 6th order polynomial deteriorates quickly when the size of the past DA window is larger than 10 hours. For Case 3, at the forecast lead time of 6 steps (1 hour), the values of the RMSE are 0.063 m, 0.056 m, and 0.057 m by the 6th, 8th, 10th polynomial orders; for Case 4, 0.1 m, 0.059 m, and 0.057 m by the 6th, 8th, 10th polynomial orders, respectively. For varying past DA windows, the smoothing function with the 8th order shows stable and improved results.

In Figure 8, the predictive performance of each polynomial order according to the future DA windows were analyzed. Similar to the sensitivity of the "past" DA window (Figure 7), as the size of the "future" DA window increases, the performance of the polynomial with the 6th order deteriorates but more quickly. Although the performance by the 8th and 10th order polynomials is similar, the 8th order polynomial is slightly better in the performance when the forecast lead time is longer than 1 hour.

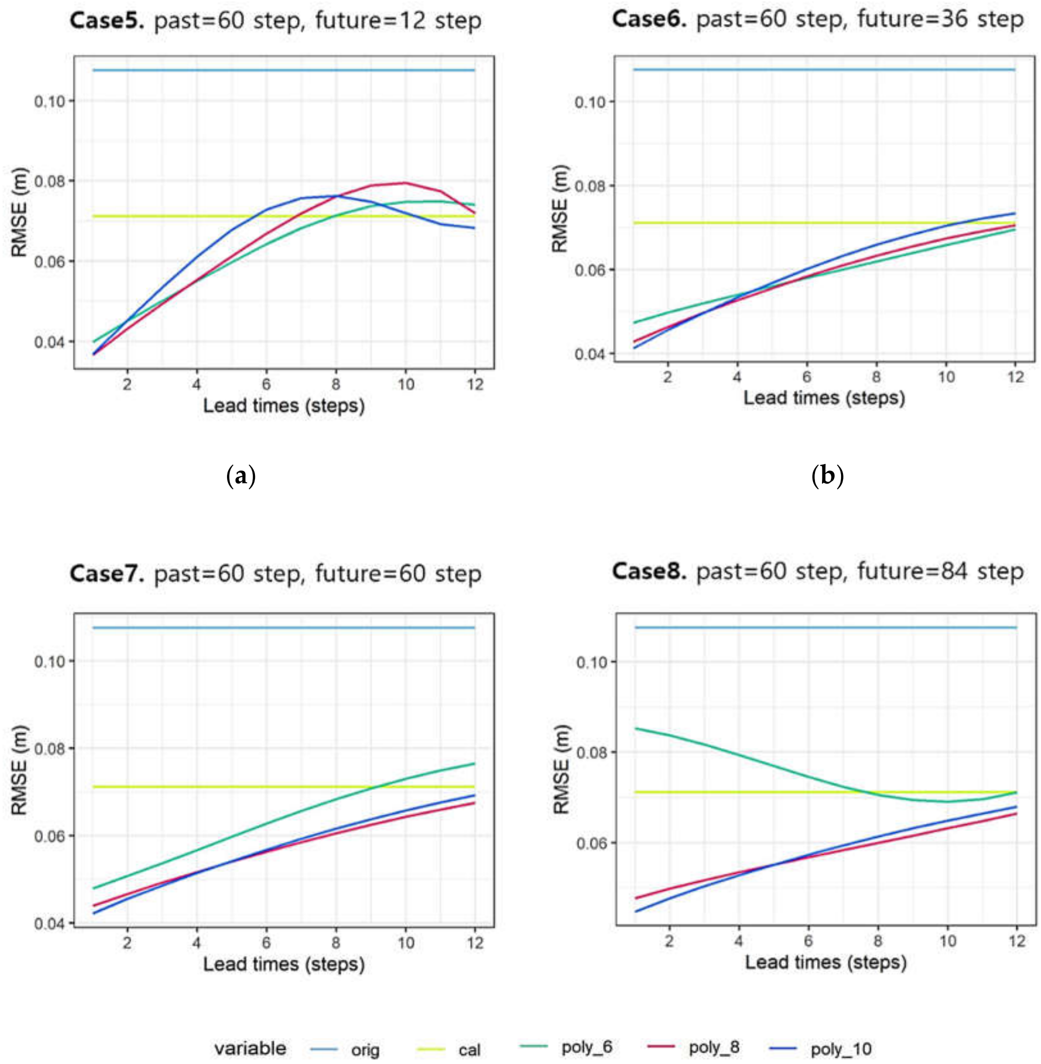
For Case 5, at the forecast lead time of 6 steps (1 hour), the values of the RMSE are 0.064 m, 0.067 m, and 0.073 m by the 6th, 8th, 10th polynomial orders; for Case 6, 0.058 m, 0.058 m, and 0.06 m by the same polynomial orders, respectively. In Cases 7 and 8, the performance was the best with the 8th polynomial at lead time 6 steps (1 hour). For Case 7, at the forecast lead time of 6 steps (1 hour), the values of the RMSE are 0.063 m, 0.056 m, and 0.057 m by the 6th, 8th, 10th polynomial orders; for Case 8, 0.075 m, 0.057 m, and 0.057 m by the same polynomial orders, respectively.



(a) (b)

(c) (d)

Figure 7. Tidal prediction performance by varying polynomial orders according to various past DA window periods: (a) 12 past steps, (b) 36 past steps, (c) 60 past steps, and (d) 84 past steps with the same 60 future steps for all cases.



(a) (b)

(c) (d)

Figure 8. Tidal prediction performance by varying polynomial orders according to various future DA window periods: (a) 12 future steps, (b) 36 future steps, (c) 60 future steps, and (d) 84 future steps with the same 60 past steps for all cases.

4.2.2. DA experiment 2: impact of DA windows

The prediction accuracy of DA depends on the window size and future prediction size of past observations used for DA. Therefore, to analyze the effect of DA window size on short-term tidal wave prediction accuracy, we performed two simulations separating the past observation interval and the future prediction interval. At first, the size of the past DA window changed with the fixed future DA window (Table 2 and Figure 9). Secondly, the size of the future DA window changed with the fixed past DA window (Table 3 and Figure 10). The same values for the polynomial order (8th) and the forecast lead time conditions were applied to DA experiment 2.

It was confirmed that most of the prediction accuracy decreased as the lead time increased. Both simulations found that the prediction accuracy by window size decreases with increasing lead time, and the RMSE (m) value is approximated to cal (0.071 m). Taken together, the accuracy of short-term tidal wave prediction using the fixed-lag smoother needs to be carefully selected as it shows a sensitive response according to the lead time length.

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Table 2. The conditions for DA experiment 2-1: varying past DA window.

Case	DA window past step	DA window future step	Lead time step	Polynomial order
DA_Past_2hr	12	72	12	8
DA_Past_6hr	36	72	12	8
DA_Past_10hr	60	72	12	8
DA_Past_14hr	84	72	12	8

* Table 2. is a showing the conditions of DA Window past, DA Window future, Lead time step, and polynomial order when the future observation period is fixed, and the past prediction period is adjusted.

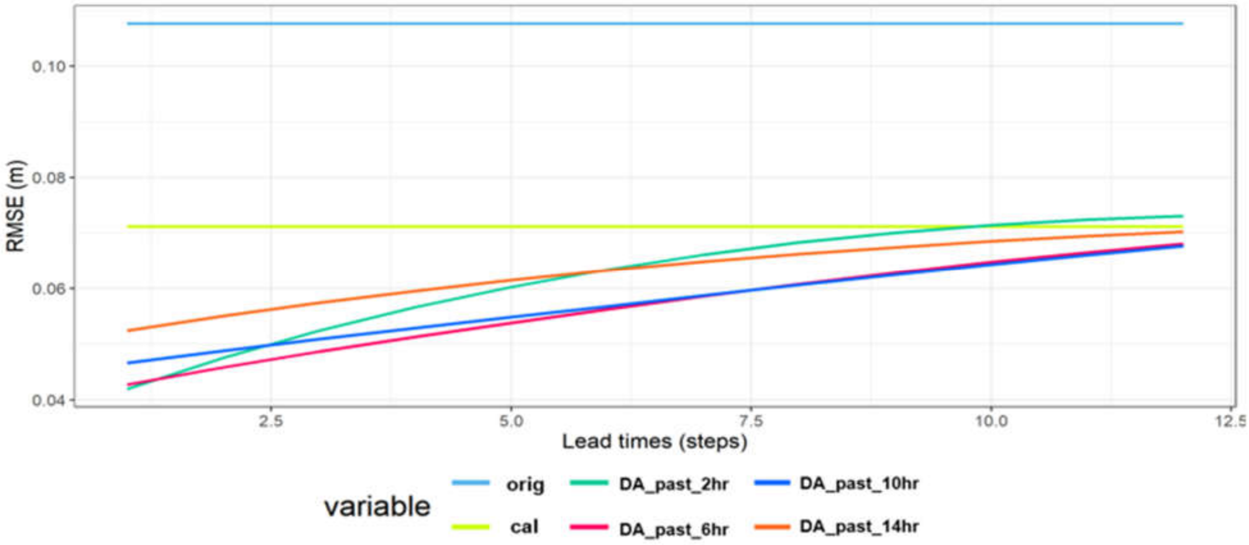

Figure 9. Performance of tidal wave prediction with varying past DA window size.

In Figure 9 shows the performance of tidal wave prediction with varying past DA window size. DA_past_2hr to DA_past_14hr means that the length of the past observation period is set at 24 steps intervals from 12 steps to 84 steps (2 hours, 6 hours, 10 hours, and 14 hours). The RMSE values of Tidal_Cal_Orig and Tidal_Cal_Re show prediction errors of 0.108 m and 0.071 m. The best performance at 6-step lead (1 hour) forecast is obtained using DA_past_6hr (0.056 m) followed by DA_past_10hr (0.057 m), DA_past_14hr (0.063 m), and DA_past_2hr (0.063 m). As a result of simulation of DA window past size control at 8^{th} polynomial, it was found that the optimal past DA window size for 1-hour lead time was 6 hours.

Table 3. The conditions for DA experiment 2-1: varying past DA window.

Case	DA window past step	DA window future step	Lead time step	Polynomial order
DA_future_2hr	60	12	12	8
DA_future_6hr	60	36	12	8
DA_future_10hr	60	60	12	8
DA_future_14hr	60	84	12	8

*Table 3. is a showing the conditions of DA Window past, DA Window future, Lead time step, and polynomial order when the past observation period is fixed, and the future prediction period is adjusted.

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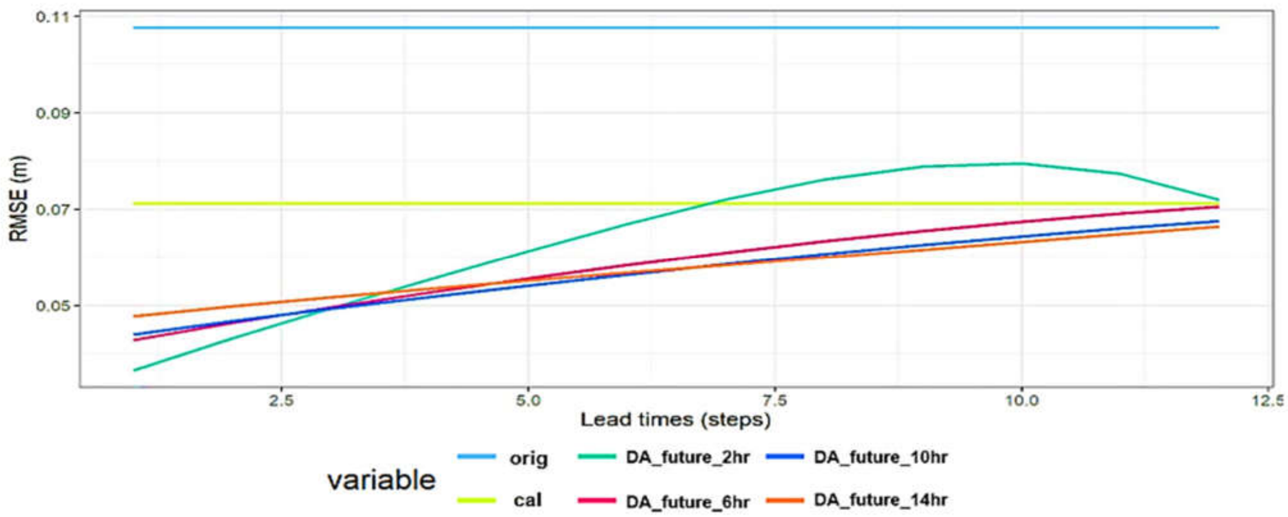


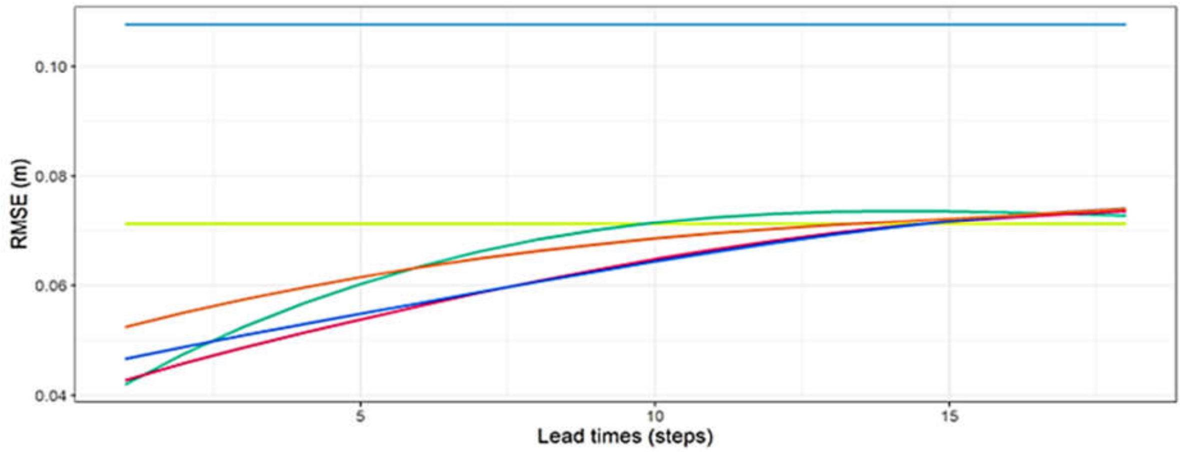
Figure 10. Performance of tidal wave prediction with varying DA window future size.

Figure 10 is the result of simulating the future prediction period and evaluating using the RMSE evaluation index for each lead time step. The past observation period was fixed at 60 steps, the 8th polynomial, and 12 steps (2 hours) of lead time. In Figure 10, DA_future_2hr to DA_future_14hr means that the length of the future predict part is set at 24 steps intervals from 12 steps to 84 steps (2 hours, 6 hours, 10 hours, and 14 hours). Ranking by the RMSE (m) values in lead time 1hr of the graph shows that 1. DA_future_10hr (0.056 m), 2. DA_future_14hr (0.057 m), 3. DA_future_6hr (0.058 m), 4. DA_future_2hr (0.067 m). As a result of simulation of DA window past size control at 8th polynomial, it was found that the optimal DA window past size for 1 hour of lead time was 10 hours.

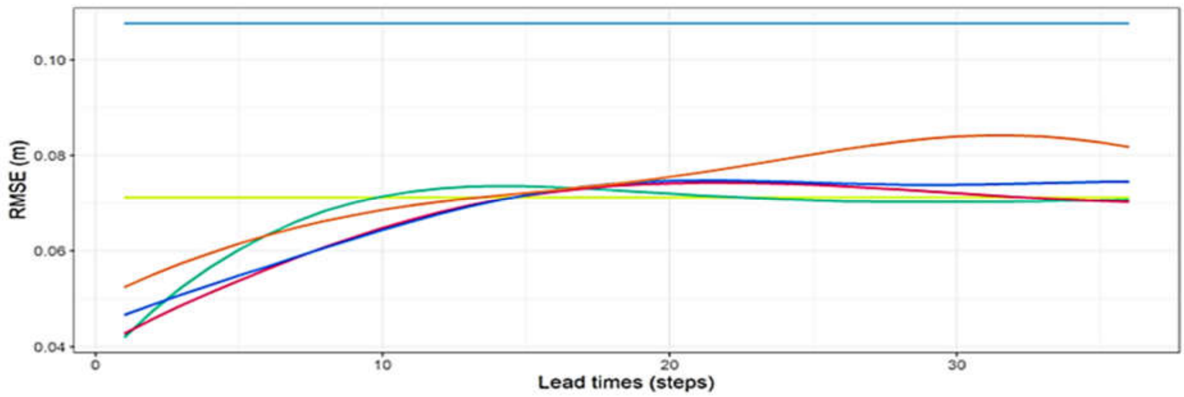
4.2.3. DA experiment 3: impact of lead time length

To analyze the effect of the lead time length on the fixed-lag smoother accuracy, the lead time length was adjusted and simulated. Figure 11 shows the result of RMSE evaluation of prediction performance corresponding to the length of the lead time interval. As shown in (a), (b), and (c) in Figure 11, The lead time stages were divided into 3 hours, 6 hours, and 12 hours. In this simulation, the length of the past observation period was adjusted to 2 hours, 6 hours, 10 hours, and 14 hours. The length of the future prediction interval was fixed at 12 hours and an 8th polynomial was applied.

The results of the estimation accuracy evaluation by lead time length were observed as follows. As shown in the previous results, the longer the lead time in common, the significantly reduced prediction accuracy. Furthermore, with the increase in lead time, it was approximated to the RMSE (0.071 m) of Tidal_Cal_Re. Interestingly, when the lead time was 12 hours, the value of the RMSE evaluation index in which the DA window exceeded 14 hours showed great variability, such as a waveform. Overall, these results suggest that setting the appropriate lead time and DA window length in the data has a significant impact on prediction accuracy. As a result of the three test simulations by lead time length, it seems appropriate to specify the lead time length within 2 hours in terms of prediction accuracy. Further, it is proposed that the length of DA window past is reduced as the prediction period becomes longer and the degree of polynomial is increased.



(a)



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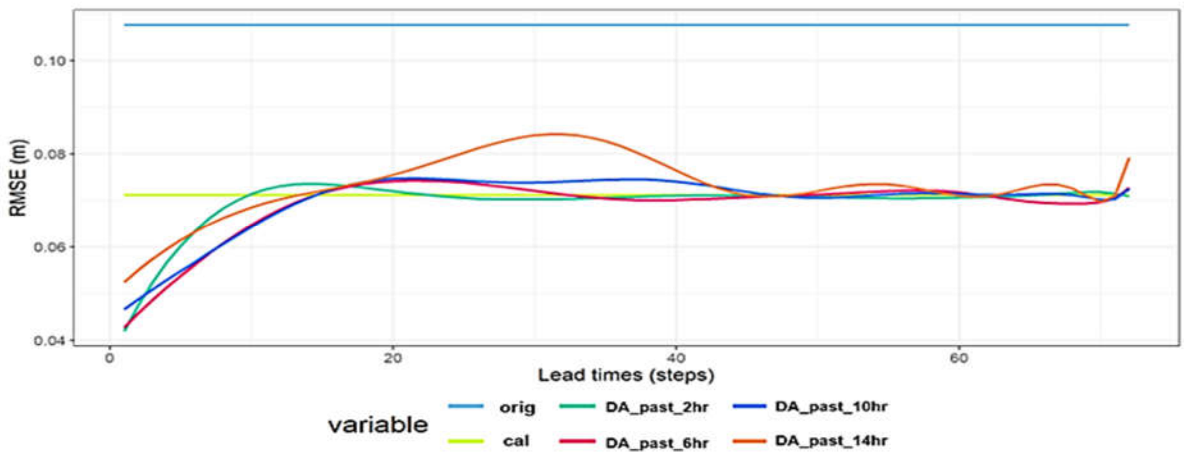

(c)

Figure 11. Performance of tidal wave prediction with varying DA lead times: (a) Up to 3 hours lead time; (b) Up to 6 hours lead time; (c) Up to 12 hours lead time.

5. Conclusions

Predicting the tidal wave is essential not only to better understand hydrological cycle at the boundary between land and ocean but also to improve energy production in the coastal area. In this study, the fixed-lag smoother method was proposed, implemented, and evaluated for improving the short-term tidal wave prediction. The proposed fixed-lag smoother, based on sequential DA, can be used as a post-processor with no changes of model structure. This study analyzed the optimal conditions for DA in terms of polynomial order, DA window length, and the lead time length. As a result, the prediction accuracy was improved by 63.9% with DA and the calibration using the regression. Although the accuracy of DA diminished for increasing forecast lead times, the 1-hr lead forecast by DA (DA_1hr) had still 44.4% improvement over the open loop without data assimilation. For the extreme ranges of the tidal wave, the benefit from data assimilation was more obvious from the fact that 40.3%, 68.2%, and 49.6% relative improvement was obtained by the the regression (Tidal_Cal_Re), the DA analysis (DA), and the 1-hr lead DA forecast (DA_1hr). About 10% additional improvement could be expected in the 1-hr lead forecast on top of the calibration using the regression.

In addition, the optimal conditions for the fixed-lag smoother were analyzed in terms of the order of a smoothing function and the length of assimilation window and forecast lead time. It was suggested that the optimal DA configuration could be obtained with the 8^{th} order polynomial. Also, it was stable to set the DA window length to 6 hours or more. The attention should be paid to appropriate lead time decisions when aiming to improve short-term wave prediction accuracy.

The empirical results of this study provide a new understanding of polynomial order determination and optimal lead time proposal, and optimal observation interval length determination for DA window past and future. In this study, the fixed-lag smoother was applied to the three-month observation data period from June to August 2021, but it is necessary to verify the additional tidal observation data measurement period. In addition, since DA techniques can be linked to data-based models as well as physical models, the application of links with various models that can optimally improve the accuracy of tide prediction should be considered. This is advantageous in determining the optimal DA window length by applying the optimal parameter extraction technique and will be analyzed through further studies.

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