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Article

Nonlinear Pauli Equation

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Abstract: In the framework of the self-consistent Maxwell-Pauli theory, the non-linear Pauli equation is obtained. Stationary and nonstationary solutions of the nonlinear Pauli equation for the hydrogen atom are studied. We show that spontaneous emission and the related rearrangement of the internal structure of an atom, which is traditionally called a spontaneous transition, have a simple and natural description in the framework of classical field theory without any quantization and additional hypotheses. The behavior of the intrinsic magnetic moment (spin) of an electron wave in an external magnetic field is considered. We show that, according to the self-consistent Maxwell-Pauli theory, in a weak magnetic field, the intrinsic magnetic moment of an electron wave is always oriented parallel to the magnetic field strength vector, while in a strong magnetic field, depending on the initial orientation of the intrinsic magnetic moment, two orientations are realized: either parallel or antiparallel to the magnetic field strength vector.

Keywords: classical field theory; unified Maxwell-Pauli theory; non-linear Pauli equation; non-linear Schrödinger equation; spin behavior in a magnetic field

1. Introduction

Quantum mechanics in its modern form arose as a result of attempts to explain the spectra of spontaneous emission of atoms. Unsuccessful attempts to explain these spectra within the framework of classical mechanics and classical electrodynamics have formed a firm conviction that classical physics is not able to explain the laws of the microcosm and this requires not only new mathematical methods, but also new physical concepts, such as quantization of matter, including electromagnetic radiation, as well as new philosophical ideas, such as, for example, wave-particle duality, which has no analogues in classical physics.

At the same time, along with the refined quantum approach, semiclassical theories have been actively developed and continue to develop, which consider the electromagnetic field (including electromagnetic radiation) as a classical field, fully described by Maxwell equations, while the electron is described by the wave equations of quantum mechanics: Schrödinger, Klein-Gordon, Pauli or Dirac equations, depending on the detail of the theory. Note that in most semiclassical theories, quantization of non-relativistic matter is not explicitly used, but solutions of the wave equation or its reduced forms - matrix equations are considered, i.e., in fact, non-relativistic matter is described as some continuous in space and time ψ-field. At the same time, reasoning about the “quantization” of non-relativistic matter can arise only at the stage of interpreting solutions from the standpoint of corpuscular representations, although in most cases in semiclassical theories quantization is not considered at all, even in the form of interpretation.

Semiclassical theories [1-25] have made significant progress in describing the basic quantum effects that make up the experimental foundation of modern quantum mechanics, such as the Compton effect [1-4,18], photoelectric effect [5,21], thermal radiation [22,25], spontaneous emission and spontaneous transitions [6-12, 15-17, 19, 24], light-matter interactions [7, 20], induced emission [6, 7, 20, 22], Lamb shift [6, 7, 9], the Lamb-Retherford experiments [23], etc. It can be argued that at present there is not a single basic quantum effect that has not been described within the semiclassical theory. Thus, in fact, the myth is dispelled that the so-called quantum processes cannot be described within the framework of the concepts of classical physics, and this certainly requires quantization.

At the same time, most semiclassical theories are characterized by incoherence: by refusing to quantize the electromagnetic field, they retain (sometimes implicitly) quantum representations with respect to nonrelativistic matter (electrons).

It was shown in [17-23] that it is possible to construct a completely classical theory that successively and consistently describes and explains all the basic quantum effects within the framework of classical field theory without any quantization or other additional hypotheses. This theory is based on the original idea of Schrödinger [26, 27] that the wave functions ψ (scalar, spinor or bispinor) described by the wave equations allows constructing real parameters that can be interpreted as the electric charge density and electric current density, which are continuously distributed in space. Taking this into account, at least from a formal mathematical point of view, we can talk about some real electrically charged material (for example, electronic) field ψ, continuously distributed in space, which is described by the wave equation, just as the classical electromagnetic field is described by Maxwell equations [18,19]. In this case, the atom can be considered (at least from a formal mathematical point of view) as an open cavity resonator that holds the wave field ψ due to the electrostatic field of the nucleus [19]. According to classical electrodynamics, charges and currents continuously distributed in space create an electromagnetic field, which, in turn, must act on them, changing the field ψ, and, hence, its charge density and current density. Thus, the potentials of the electromagnetic field included in the wave equations of quantum mechanics are a superposition of the potentials of the external electromagnetic field (created by external sources) and the potentials of its own electromagnetic field, created by an electrically charged material field ψ.

As shown in [19], such a field ψ must be described by the equation

$$i\hbar\frac{\partial\psi}{\partial t}=\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}_\Sigma\right)^2-e(\varphi_\Sigma-\varphi_0)\right]\psi \qquad (1)$$

where

$$\varphi_0(\mathbf{r})=-e\int\frac{|\psi(\mathbf{r}',t)|^2}{R}dV' \qquad (2)$$

is the scalar potential of its own electrostatic field created by an electrically charged wave ψ;

$$\varphi_\Sigma=\frac{e}{r}-e\int\frac{|\psi_{t-R/c}|^2}{R}dV' \qquad (3)$$

$$\mathbf{A}_\Sigma=\frac{1}{c}\int\frac{\mathbf{j}_{t-R/c}}{R}dV' \qquad (4)$$

are the scalar and vector potentials of the total (external, i.e., atomic nucleus and intrinsic, i.e., electron wave) electromagnetic field, obtained by the formal solution of Maxwell equations, taking into account the distributed charges and currents of the electron wave; $\mathbf{j}$ is the electric current density of the electron wave, which is defined by the well-known expression of quantum mechanics [28]; $R=|\mathbf{r}-\mathbf{r}'|$.

In the case when spontaneous emission can be described in the dipole approximation, equation (1), taking into account (2)–(4), is reduced to the form [19]

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{1}{2m}\Delta\psi-\frac{e^2}{r}\psi+\frac{2e}{3c^3}\mathbf{r}\dddot{\mathbf{d}}\,\psi \qquad (5)$$

Where

$$\mathbf{d}=-e\int\mathbf{r}|\psi|^2dV \qquad (6)$$

is the dipole moment of the electron wave in the hydrogen atom.

Equation (5), (6) is a nonlinear Schrödinger equation, and allows correctly describing the spontaneous emission spectrum of an atom, as well as all the basic quantum effects [18-23], which are traditionally described in the framework of QED using the second quantization apparatus [29]. For example, the rearrangement of the structure of the hydrogen atom during spontaneous emission, which in QED is called "spontaneous transition" and is traditionally explained by a zitterbewegung caused by fluctuations of the QED-vacuum, from the point of view of equation (5) turns out to be a completely trivial effect that has a simple explanation within the framework of classical electrodynamics [19]. Similarly, it was shown in [22] that all known laws of thermal radiation, from which, in fact, quantum physics began, follow directly from equation (5) without using any additional hypotheses, including the energy quantization hypothesis.

A feature of equation (1) is that it excludes its own electrostatic field (2) from the total electromagnetic field, but takes into account the non-stationary (radiative) component of its own electromagnetic field.

Despite the success of equation (1) (or, what is the same, equation (5)) in explaining a number of basic "quantum" regularities [18-23], from the point of view of classical electrodynamics, it remains inexplicable why the electrically charged electron field ψ does not "feel" its own electrostatic field, but at the same time "feels" the electrostatic field of other charges and the non-stationary (radiation) component of its own electromagnetic field, which occurs during spontaneous emission.

To eliminate this paradox, a self-consistent Maxwell-Pauli theory was proposed in [30], which retains all the useful properties of equations (1) and (5).

The theory [30] is described by the equations

$$i\hbar\frac{\partial\Psi}{\partial t}=\left[\frac{1}{2m_e}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}_\Sigma\right)^2-e\varphi_\Sigma-e\boldsymbol{\sigma}\mathbf{G}_\Sigma+\frac{e\hbar}{2m_ec}\boldsymbol{\sigma}\mathbf{H}_\Sigma\right]\Psi \tag{7}$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{G}_\Sigma}{\partial t^2}-\Delta\mathbf{G}_\Sigma+\varkappa^2\mathbf{G}_\Sigma=4\pi e(\Psi^*\boldsymbol{\sigma}\Psi) \tag{8}$$

$$\mathbf{H}_\Sigma=\mathbf{H}+\mathbf{H}_e,\mathbf{E}_\Sigma=\mathbf{E}+\mathbf{E}_e,\mathbf{G}_\Sigma=\mathbf{G}+\mathbf{G}_e \tag{9}$$

$$\varphi_\Sigma=\varphi+\varphi_e,\ \ \mathbf{A}_\Sigma=\mathbf{A}+\mathbf{A}_e \tag{10}$$

$$\mathrm{rot}\mathbf{H}_\Sigma=\frac{1}{c}\frac{\partial\mathbf{E}_\Sigma}{\partial t}+\frac{4\pi}{c}\mathbf{j} \tag{11}$$

$$\mathrm{div}\mathbf{E}_\Sigma=4\pi\rho \tag{12}$$

where

$$\mathbf{H}_\Sigma=\mathrm{rot}\mathbf{A}_\Sigma,\ \ \mathbf{E}_\Sigma=-\frac{1}{c}\frac{\partial\mathbf{A}_\Sigma}{\partial t}-\nabla\varphi_\Sigma \tag{13}$$

$$\rho=-e\Psi^*\Psi \tag{14}$$

$$\mathbf{j}=\frac{e\hbar}{2m_ei}\ [(\nabla\Psi^*)\Psi-\Psi^*\nabla\Psi]-\frac{e^2}{m_ec}\mathbf{A}_\Sigma\Psi^*\Psi-\frac{e\hbar}{2m_e}\mathrm{rot}(\Psi^*\boldsymbol{\sigma}\Psi) \tag{15}$$

index Σ refers to total fields; the index e refers to the its own fields created by the electron wave described by the spinor $\Psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$; parameters without index refer to external fields created by external (with respect to the field Ψ) charges, currents and spins; $\varkappa$ is a constant with the dimension reciprocal of the length and satisfying the condition [30]

$$\varkappa a_B\ll 1 \tag{16}$$

where $a_B=\frac{\hbar^2}{m_ee^2}$ is the Bohr radius. The constant $\varkappa$, which satisfies condition (10), makes the field $\mathbf{G}$ short-range, exponentially decaying at distances of the order of a_B from the source. The constant $\varkappa$ is introduced into equation (8) in order to explain why the field $\mathbf{G}$ has not been observed experimentally until now, and also why the electron field of an atom does not "feel" its own electrostatic field, but at the same time "feels" the electrostatic fields created by other atoms and ions.

Further, we use synonyms: Pauli field, electron field, electron wave, denoting the same physical object: a classical electrically charged field described by equation (7). In what follows, for brevity, the components of the spinor Ψ will be called polarizations of

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the electron wave. Thus, in the Pauli approximation, the electron wave has two polarizations.

The system of equations (7)-(15) is closed and self-consistent. It differs from a simple formal union of the Maxwell and Pauli equations in that the Pauli equation (7) includes an additional term $-e\boldsymbol{\sigma}\mathbf{G}_\Sigma$ containing a new real vector field $\mathbf{G}_\Sigma$ that satisfies equation (8). In this paper, we will show that this fundamentally changes the solutions of the combined system of Maxwell-Pauli equations and allows correctly describing the experimentally observed effects.

It is easy to see that the system of equations (7)-(15) is gauge invariant:

$$\mathbf{A}_\Sigma \to \mathbf{A}_\Sigma + \nabla f,\ \ \varphi_\Sigma \to \varphi_\Sigma - \frac{1}{c}\frac{\partial f}{\partial t}\ ,\ \ \Psi \to \Psi \exp\left(-\frac{ie}{\hbar c} f\right) \tag{17}$$

where f is an arbitrary function; in this case, the strength of the electric and magnetic fields, the field $\mathbf{G}_\Sigma$, the density of the electric charge, the current density and other physical characteristics of the electron field do not change.

The Maxwell-Pauli theory (7)-(15) is not relativistically invariant. The relativistically invariant Maxwell-Dirac theory, from which Eqs. (7)–(15) follow, was constructed in [31].

Note that in the theory [30] described by equations (7)–(15), there is one indefinite parameter $\varkappa$ that satisfies condition (16). This parameter is external to this theory. It can be found, for example, by comparing the theoretical results with the corresponding experimental data.

In this paper, for definiteness, we consider the case

$$\varkappa = 0 \tag{18}$$

which corresponds to the long-range field $\mathbf{G}$. The case $\varkappa \neq 0$ will be investigated in the next papers of this series.

It follows from equations (11) and (12) that ρ and $\mathbf{j}$ defined by relations (14) and (15) are indeed the electric charge density and electric current density of the Pauli field.

Equation (7), taking into account (14) and (15), implies the continuity equation

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\mathbf{j} = 0 \tag{19}$$

expressing the charge conservation law, which is consistent with Maxwell equations (11) and (12).

The electric charge of the Pauli field contained in some region of space Ω is equal to $q = \int \rho dV$, where the integral is taken over the region Ω. In particular, the electric charge of the electron field of an electrically neutral atom whose nucleus has the charge Ze is equal to $\int \rho dV = -Ze$, where $Z = 1,2,\ldots$, and the integral is taken over the entire space. Taking into account (14), we come to the conclusion that the wave function in an electrically neutral atom satisfies the normalization condition

$$\int \Psi^*\Psi dV = Z \tag{20}$$

In particular, for the hydrogen atom $Z = 1$.

As shown in [30], the classical electron field Ψ has an angular momentum and a magnetic moment, which have two components:

$$\mathbf{L} = \mathbf{L}_{or} + \mathbf{S} \tag{21}$$
$$\mathbf{M} = \mathbf{M}_{or} + \boldsymbol{\mu} \tag{22}$$

where $\mathbf{L}_{or}$ and $\mathbf{M}_{or}$ are the convective (orbital) components of the angular momentum and the magnetic moment of the electron field associated with currents, while $\mathbf{S}$ and $\boldsymbol{\mu}$ are the intrinsic angular momentum (spin) and the associated intrinsic magnetic moment of the electron field. The intrinsic angular momentum (spin) of the electron field is continuously distributed in space with a density

$$\mathbf{s} = \frac{\hbar}{2}\Psi^*\boldsymbol{\sigma}\Psi \tag{23}$$

while the intrinsic magnetic moment of the electron field is continuously distributed in space with a density

$$\mathfrak{m} = -\frac{e\hbar}{2m_e c}\Psi^*\boldsymbol{\sigma}\Psi \tag{24}$$

From (23) and (24) follows the relation

$$\mathfrak{m} = -\frac{e}{m_e c}\mathbf{s} \tag{25}$$

showing that the internal (spin) gyromagnetic ratio of the electron field at all points in space is the same and equal to

$$\gamma_e = -\frac{e}{m_e c} \tag{26}$$

In theory [30, 31], the internal angular momentum (spin) and the associated internal magnetic moment are properties of the classical electron field itself and cannot be reduced to motions and, moreover, to rotations of any particles.

The convective (orbital) components of the angular momentum and magnetic moment of the electron field are related by the relationship [30]

$$\mathbf{M}_{or} = -\frac{e}{2m_e c}\mathbf{L}_{or} \tag{27}$$

coinciding with the corresponding relation for classical charged matter (for example, classical particles) [32]. Hence it follows that the convective (orbital) gyromagnetic ratio for the electron field is determined by the classical formula

$$\gamma_{or} = -\frac{e}{2m_e c} \tag{28}$$

Thus, the spin (intrinsic) gyromagnetic ratio of the electron field (26) is twice the convective (orbital) gyromagnetic ratio (28). In the classical Maxwell-Pauli field theory [30], this fact is a natural property of the electron field, which is an electrically charged magnetic matter continuously distributed in space, and does not lead to paradoxes.

Further, we will show that the self-consistent system of equations (7)-(15) allows describing all basic quantum effects within the framework of classical field theory.

2. Nonlinear Pauli equation

2.1. Formal solution of the Maxwell equations

According to (9) and (10), the electromagnetic field and the **G**-field can be decomposed into an external field (created by external sources) and an intrinsic field created by the electrically charged electron wave itself.

The solution of Eq. (8) for case (18) for a localized electron field has the form [32]

$$\mathbf{G}_e = e\int\frac{(\Psi^*\boldsymbol{\sigma}\Psi)_{t-R/c}}{R}dV' \tag{29}$$

Using the formal solutions (3) and (4) of Maxwell equations, we write the relations for the intrinsic electromagnetic field of the electron wave

$$\mathbf{A}_e = \frac{1}{c}\int\frac{\mathbf{j}_{t-R/c}}{R}dV' \tag{30}$$

$$\varphi_e = -e\int \frac{(\Psi^*\Psi)_{t-R/c}}{R}dV' \qquad (31)$$

Taking into account (29) and (31), one obtains

$$(e\varphi_e + e\boldsymbol{\sigma}\mathbf{G}_e)\Psi = \begin{pmatrix} -2e^2\psi_1\int\frac{|\psi_2|^2_{t-R/c}}{R}dV' + 2e^2\psi_2\int\frac{(\psi_1\psi_2^*)_{t-R/c}}{R}dV' \\ -2e^2\psi_2\int\frac{|\psi_1|^2_{t-R/c}}{R}dV' + 2e^2\psi_1\int\frac{(\psi_1^*\psi_2)_{t-R/c}}{R}dV' \end{pmatrix} \qquad (32)$$

Let us expand the integrand in (30) into a series in powers of the parameter R/c:

$$\mathbf{A}_e = \mathbf{A}_0 - \frac{1}{c^2}\frac{\partial}{\partial t}\int \mathbf{j}dV + \frac{1}{2c^3}\frac{\partial^2}{\partial t^2}\int R\mathbf{j}dV' - \frac{1}{6c^4}\frac{\partial^3}{\partial t^3}\int R^2\mathbf{j}dV' + \cdots \qquad (33)$$

where

$$\mathbf{A}_0 = \frac{1}{c}\int\frac{\mathbf{j}}{R}dV' \qquad (34)$$

is the quasi-static field; the remaining terms in expansion (33) describe non-stationary (radiative) effects.

From the continuity equation (19), taking into account (6), it follows that for an electron field localized in space

$$\int \mathbf{j}dV = \dot{\mathbf{d}} \qquad (35)$$

Taking into account (35), we write relation (33) in the form

$$\mathbf{A}_e = \mathbf{A}_0 - \frac{1}{c^2}\ddot{\mathbf{d}} + \frac{1}{2c^3}\frac{\partial^2}{\partial t^2}\int R\mathbf{j}dV' - \frac{1}{6c^4}\frac{\partial^3}{\partial t^3}\int R^2\mathbf{j}dV' + \cdots \qquad (36)$$

Let us perform the gauge transformation (17) with the function

$$f = \frac{1}{c^2}\mathbf{r}\ddot{\mathbf{d}} \qquad (37)$$

Then, taking into account relation (36), the scalar and vector potentials of the electromagnetic field take the form

$$\varphi_\Sigma = \varphi + \varphi_e - \frac{1}{c^3}\mathbf{r}\dddot{\mathbf{d}} \qquad (38)$$

$$\mathbf{A}_\Sigma = \mathbf{A} + \mathbf{A}_0 + \frac{1}{2c^3}\frac{\partial^2}{\partial t^2}\int R\mathbf{j}dV' - \frac{1}{6c^4}\frac{\partial^3}{\partial t^3}\int R^2\mathbf{j}dV' + \cdots \qquad (39)$$

Let us calculate $\mathbf{A}_0$. To do this, we represent the current density (15) taking into account (39) in the form

$$\mathbf{j} = \mathbf{j}_c - \frac{e^2}{m_e c}\mathbf{A}\Psi^*\Psi - \frac{e^2}{m_e c}\mathbf{A}_e\Psi^*\Psi + c\mathrm{rot}\boldsymbol{\mathfrak{m}} \qquad (40)$$

where we introduced the vector of convective electric current [30]

$$\mathbf{j}_c = \frac{e\hbar}{2m_e i}\left[(\nabla\Psi^*)\Psi - \Psi^*\nabla\Psi\right] \qquad (41)$$

If the external magnetic field varies only slightly at distances of the order of the size of an atom, we can write

$$\mathbf{A} = \frac{1}{2}\mathbf{H}\times\mathbf{r} \qquad (42)$$

Substituting (40) and (42) into (34), one obtains

$$\mathbf{A}_0 = \frac{1}{c}\int \frac{\mathbf{j}_c}{R} dV' - \frac{e^2}{2m_e c^2}\mathbf{H} \times \int \mathbf{r}\frac{\Psi^*\Psi}{R} dV' - \frac{e^2}{m_e c^2}\int \mathbf{A}_e \frac{\Psi^*\Psi}{R} dV' + \int \frac{\mathrm{rot}\boldsymbol{\mathfrak{m}}}{R} dV' \quad (43)$$

For a localized electron field, we take into account that $\int \frac{\mathrm{rot}'\boldsymbol{\mathfrak{m}}}{R} dV' = \mathrm{rot}\int \frac{\boldsymbol{\mathfrak{m}}}{R} dV'$, where the prime refers to the integration variables.

Then relation (43) takes the form

$$\mathbf{A}_0 = \frac{1}{c}\int \frac{\mathbf{j}_c}{R} dV' + \frac{e^2}{2m_e c^2}\mathbf{H} \times \int \frac{\mathbf{R}}{R}\Psi^*\Psi dV' - \frac{e^2}{2m_e c^2}\mathbf{H} \times \mathbf{r}\int \frac{\Psi^*\Psi}{R} dV' - \frac{e^2}{m_e c^2}\int \mathbf{A}_e \frac{\Psi^*\Psi}{R} dV' - \frac{\mu_B}{e}\mathrm{rot}\mathbf{G}_0 \quad (44)$$

where $\mu_B = \frac{e\hbar}{2m_e c}$ is the Bohr magneton;

$$\mathbf{G}_0 = e\int \frac{\Psi^*\boldsymbol{\sigma}\Psi}{R} dV' \quad (45)$$

A simple estimate shows that the term on the right-hand side of (43), which contains $\mathbf{A}_e$ under the integral, has the order $\alpha^2\mathbf{A}_0$, and can be discarded.

As a result, taking into account (2), one obtains

$$\mathbf{A}_0 = \frac{1}{c}\int \frac{\mathbf{j}_c}{R} dV' + \frac{e}{2m_e c^2}\mathbf{H} \times \left(e\nabla \int R\Psi^*\Psi dV' + \mathbf{r}\varphi_0\right) - \frac{\mu_B}{e}\mathrm{rot}\mathbf{G}_0 \quad (46)$$

Using (36), we calculate the intrinsic magnetic field of the electron wave $\mathbf{H}_e = \mathrm{rot}\mathbf{A}_e$.

Taking into account that $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and using (35), one writes

$$\int \mathbf{j} \times \mathbf{R} dV' = \int \mathbf{j} dV' \times \mathbf{r} - \int \mathbf{j} \times \mathbf{r}' dV' = \dot{\mathbf{d}} \times \mathbf{r} + 2c\mathbf{M}$$

where

$$\mathbf{M} = \frac{1}{2c}\int \mathbf{r} \times \mathbf{j} dV \quad (47)$$

is the magnetic moment of the electron field (22).

As a result, one obtains

$$\mathbf{H}_e = \mathbf{H}_0 - \frac{1}{2c^3}\frac{\partial^2}{\partial t^2}\int \frac{\mathbf{j} \times \mathbf{R}}{R} dV' + \frac{1}{3c^4}\frac{d^4\mathbf{d}}{dt^4} \times \mathbf{r} + \frac{2}{3c^3}\dddot{\mathbf{M}} \quad (48)$$

where

$$\mathbf{H}_0 = \frac{1}{c}\int \frac{\mathbf{j} \times \mathbf{R}}{R^3} dV' \quad (49)$$

The terms on the right-hand side of equation (48), starting from the second, are the radiative components of the intrinsic magnetic field.

Equation (7) with parameters (9), (10), (32), (38), (39), (46) and (48) is a closed non-linear Pauli equation.

Consider the nonlinear Pauli equation as applied to atoms.

2.2. Nonlinear Pauli equation for atoms

Taking into account (38), (39), and (42), we rewrite the Pauli equation (7) for the atom with the atomic number Z in the form

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[\frac{1}{2m_e}\left(\frac{\hbar}{i}\nabla + \frac{e}{2c}\mathbf{H} \times \mathbf{r} + \frac{e}{c}\mathbf{A}_e\right)^2 + \frac{e}{c^3}\mathbf{r}\dddot{\mathbf{d}} - \frac{Ze^2}{r} - e\varphi - e\boldsymbol{\sigma}\mathbf{G} - (e\varphi_e + e\boldsymbol{\sigma}\mathbf{G}_e) + \frac{e\hbar}{2m_e c}\boldsymbol{\sigma}\mathbf{H} + \frac{e\hbar}{2m_e c}\boldsymbol{\sigma}\mathbf{H}_e\right]\Psi \quad (50)$$

where $(e\varphi_e + e\boldsymbol{\sigma}\mathbf{G}_e)\Psi$ is determined by relation (32), while the vectors $\mathbf{A}_e$ and $\mathbf{H}_e$ are determined by relations (39) and (48). The scalar potential φ describes the electromagnetic field external to the atom, while the scalar potential of the electrostatic field of the atomic nucleus $\frac{Ze}{r}$ is separated into a separate term. In particular, if the electric field changes only slightly at distances of the order of the Bohr radius, one can write

$$\varphi = -\mathbf{rE} \qquad (51)$$

As is known [29], the Pauli equation is the first approximation in the expansion of the Dirac equation in terms of a small parameter, the fine structure constant α. For this reason, all terms in equation (50) having order α^2 and higher must be discarded. They can be taken into account only within the framework of the Dirac equation or in approximations of the Dirac equation following the Pauli equation [31].

Let us estimate the order of terms in equation (50).

The main terms of Eq. (50) (which determine the solution already in the Schrödinger approximation) have orders of magnitude

$$\hbar\nabla \sim \frac{\hbar}{a_B} = \frac{m_e e^2}{\hbar} \qquad (52)$$

$$\frac{e^2}{r} \sim \frac{e^2}{a_B} = \frac{m_e e^4}{\hbar^2} \qquad (53)$$

Obviously, the term (32) (if it is not equal to zero) has the order (53).

The rest of the terms are of order

$$\frac{e}{c}\mathbf{A}_e \sim \frac{e}{c}\mathbf{A}_0 \sim \frac{e}{c^2}\int \frac{\mathbf{j}_c}{R} dV' \sim \frac{e\mu_B}{c^2}\mathrm{rot}\mathbf{G}_0 \sim \frac{e^2\hbar}{m_e c^2 a_B^2} = \alpha^2 \frac{m_e e^2}{\hbar} \sim \alpha^2 \hbar\nabla \qquad (54)$$

$$\frac{e\hbar}{2m_e c}\mathbf{H}_e = \frac{e\hbar}{2m_e c}\frac{1}{c}\int \frac{\mathbf{R}\times\mathbf{j}}{R^3} dV' \sim \frac{e\hbar}{2m_e c}\int \frac{\mathfrak{m}}{R^3} dV' \sim \frac{e\hbar}{2m_e c}\int \frac{\mathbf{R}(\mathfrak{m}\mathbf{R})}{R^5} dV' \sim \frac{e\hbar}{2m_e c}\mathfrak{m} \sim \frac{e^2\hbar^2}{m_e^2 c^2 a_B^3} = \alpha^2 \frac{m_e e^4}{\hbar^2} \sim \alpha^2 \frac{e^2}{r} \qquad (55)$$

Thus, the terms in the Pauli equation (50) associated with the intrinsic magnetic field of the electron wave can be discarded as small in the approximation under consideration.

As a result, in the considered dipole approximation, equation (50) takes the form

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m_e}\Delta + \frac{e\hbar}{i2m_e c}\mathbf{H}(\mathbf{r}\times\nabla) + \frac{e^2}{8m_e c^2}|\mathbf{H}\times\mathbf{r}|^2 - \frac{Ze^2}{r} + \frac{e}{c^3}\mathbf{r}\dddot{\mathbf{d}} - (e\varphi_e + e\boldsymbol{\sigma}\mathbf{G}_e) + e\mathbf{rE} - e\boldsymbol{\sigma}\mathbf{G} + \frac{e\hbar}{2m_e c}\boldsymbol{\sigma}\mathbf{H}\right]\Psi \qquad (56)$$

where $(e\varphi_e + e\boldsymbol{\sigma}\mathbf{G}_e)$ is determined by relation (32).

In addition, taking into account (54), in the approximation under consideration

$$\mathbf{j} = \frac{e\hbar}{2m_e i}\left[(\nabla\Psi^*)\Psi - \Psi^*\nabla\Psi\right] - \frac{e^2}{2m_e c}\mathbf{H}\times\mathbf{r}\Psi^*\Psi - \frac{e\hbar}{2m_e}\mathrm{rot}(\Psi^*\boldsymbol{\sigma}\Psi) \qquad (57)$$

In equations (56) and (57), the terms containing the external electromagnetic field are retained, even if the corresponding terms are small compared to (52) and (53). This allows taking into account the influence of an external electromagnetic field on the processes occurring in the atom.

Equation (56) is a direct analog of equation (5) obtained in [19], however, in contrast to equation (5), in which the intrinsic electrostatic field (2) of an electron wave is artificially excluded from consideration, in equation (56) it is naturally compensated by the field $\mathbf{G}_e$.

3. Hydrogen atom

3.1. Stationary states of the hydrogen atom

Consider a hydrogen atom ($Z = 1$) in free space, i.e. when

$$\mathbf{A} = 0;\ \varphi = 0; \mathbf{G} = 0 \qquad (58)$$

We will consider the stationary states of the hydrogen atom. In this case, all derivatives with respect to time on right-hand side of equation (56) are equal to zero. As a result, the nonlinear Pauli equation (56) takes the form

$$i\hbar\frac{\partial\Psi}{\partial t}=\left[-\frac{\hbar^2}{2m_e}\Delta-\frac{e^2}{r}-(e\varphi_0+e\boldsymbol{\sigma}\mathbf{G}_0)\right]\Psi \qquad (59)$$

Consider the solution of Eq. (59) which has the form

$$\Psi=\binom{a_1}{a_2}\psi \qquad (60)$$

where a_1 and a_2 are arbitrary constants; $\psi(\mathbf{r},t)$ is some function.

Taking into account the normalization condition (20), we conclude that the constants a_1 and a_2 characterize the distribution of the electric charge of the electron wave between its polarizations ψ_1 and ψ_2. In particular, for the hydrogen atom, the function ψ is normalized to unity, i.e., $\int|\psi|^2dV=1$, and the constants a_1 and a_2 satisfy the condition

$$|a_1|^2+|a_2|^2=1 \qquad (61)$$

which has a simple physical meaning [19-23]: the total electric charge of an electron wave in a hydrogen atom, regardless of how it is distributed over polarizations, is equal to $-e$.

For solution (60), expression (32) is identically equal to zero, and equation (59) turns into the usual linear Schrödinger equation for the hydrogen atom with respect to the function ψ.

Its general solution is

$$\psi(\mathbf{r},t)=\sum_n c_n\psi_n(\mathbf{r})\exp(-i\omega_n^{(0)}t) \qquad (62)$$

where c_n are the constants characterizing the excitation amplitude of the eigenmode n; $\psi_n(\mathbf{r})$ and $\omega_n^{(0)}$ are eigenfunctions and eigenvalues of the stationary linear Schrödinger equation

$$\hbar\omega_n^{(0)}\psi_n=-\left(\frac{\hbar^2}{2m_e}\Delta+\frac{e^2}{r}\right)\psi_n \qquad (63)$$

The functions $\psi_n(\mathbf{r})$ form an orthonormal basis:

$$\int\psi_n^*\psi_k dV=\delta_{nk} \qquad (64)$$

Thus, the hydrogen atom has a discrete set of eigenmodes that have eigenfrequencies $\omega_n^{(0)}$ and are described in space by eigenfunctions $\psi_n(\mathbf{r})$. This means that, at least formally, the hydrogen atom can be considered as a classical open cavity resonator which holds the wave field ψ and having a discrete set of eigenmodes and their corresponding eigenfrequencies [19].

Taking into account the normalization condition for the wave function ψ, which reflects the fact that the total charge of the electron wave in the hydrogen atom is $-e$, one obtains

$$\sum_n|c_n|^2=1 \qquad (65)$$

That is, the parameters c_n describe the distribution of the electric charge of the electron wave over the eigenmodes of the hydrogen atom. In particular, the value $-e|c_n|^2$ is equal to the electric charge of the electron wave contained in the mode n [19].

If only one eigenmode n is excited in the hydrogen atom, i.e.

$$\psi(\mathbf{r},t)=\psi_n(\mathbf{r})\exp(-i\omega_n^{(0)}t) \qquad (66)$$

we will say that it is in a pure state [19]. Thus, for a pure state of the atom, $c_n = 1$ while $c_k = 0$ for all $k \neq n$ in solution (62). On the contrary, if several eigenmodes in the same polarization are simultaneously excited in the hydrogen atom, i.e. two or more parameters c_n are nonzero in solution (62), then we will say that the atom is in a mixed state [19].

According to [30], the energy of the Maxwell-Pauli field in the hydrogen atom in the absence of an external magnetic field (i.e., at $\mathbf{A}_\Sigma = 0$) in the approximation under consideration for case (18) is equal to

$$\mathcal{E} = \int \left(\frac{1}{8\pi} \mathbf{E}_\Sigma^2 + \frac{\hbar^2}{2m_e} \nabla\Psi^* \nabla\Psi - e\Psi^* \boldsymbol{\sigma} \Psi \mathbf{G}_0 + \frac{1}{8\pi} \left(\frac{1}{c^2} \frac{\partial \mathbf{G}_0}{\partial t} \frac{\partial \mathbf{G}_0}{\partial t} + \frac{\partial \mathbf{G}_0}{\partial x_k} \frac{\partial \mathbf{G}_0}{\partial x_k} \right) \right) dV \quad (67)$$

Let us take into account that in this case $\mathbf{E}_\Sigma = -\nabla\varphi_\Sigma$, where, according to equation (12), the scalar potential φ_Σ satisfies the equation

$$\Delta\varphi_\Sigma = 4\pi e \Psi^* \Psi - 4\pi e \delta(\mathbf{r}) \quad (68)$$

Then, integrating in (67) by parts, one obtains

$$\mathcal{E} = \int \left(\frac{1}{8\pi} \nabla(\varphi_\Sigma \nabla\varphi_\Sigma) - \frac{1}{8\pi} \varphi_\Sigma \Delta\varphi_\Sigma + \frac{\hbar^2}{2m_e} \nabla(\Psi^* \nabla\Psi) - \frac{\hbar^2}{2m_e} \Psi^* \Delta\Psi - e\Psi^* \boldsymbol{\sigma} \Psi \mathbf{G}_0 + \frac{1}{8\pi c^2} \frac{\partial \mathbf{G}_0}{\partial t} \frac{\partial \mathbf{G}_0}{\partial t} + \frac{1}{8\pi} \frac{\partial}{\partial x_k} \left(\mathbf{G}_0 \frac{\partial \mathbf{G}_0}{\partial x_k} \right) - \frac{1}{8\pi} \mathbf{G}_0 \Delta \mathbf{G}_0 \right) dV = \int \left(-\frac{1}{8\pi} \varphi_\Sigma \Delta\varphi_\Sigma - \frac{\hbar^2}{2m_e} \Psi^* \Delta\Psi - e\Psi^* \boldsymbol{\sigma} \Psi \mathbf{G}_0 + \frac{1}{8\pi c^2} \frac{\partial \mathbf{G}_0}{\partial t} \frac{\partial \mathbf{G}_0}{\partial t} - \frac{1}{8\pi} \mathbf{G}_0 \Delta \mathbf{G}_0 \right) dV \quad (69)$$

Taking into account equations (8), (59) and (68), as well as the solution of equation (68) $\varphi_\Sigma = \frac{e}{r} + \varphi_0$, one obtains

$$\mathcal{E} = \int \left(i\hbar \Psi^* \frac{\partial \Psi}{\partial t} + \frac{1}{2} \frac{e^2}{r} \delta(\mathbf{r}) + \frac{1}{2} \frac{e^2}{r} \Psi^* \Psi + \frac{1}{2} e \varphi_0 \delta(\mathbf{r}) + \frac{1}{2} \Psi^* (e\varphi_0 + e\boldsymbol{\sigma} \mathbf{G}_0) \Psi + + \frac{1}{8\pi c^2} \left(\frac{\partial \mathbf{G}_0}{\partial t} \frac{\partial \mathbf{G}_0}{\partial t} - \mathbf{G}_0 \frac{\partial^2 \mathbf{G}_0}{\partial t^2} \right) \right) dV \quad (70)$$

When calculating (70), we take into account that for solution (60) relation (32) is identically equal to zero, and for the stationary Maxwell-Pauli field $\frac{\partial \mathbf{G}_0}{\partial t} = 0$ and

$$\varphi_0(\mathbf{r} = 0) = -e \int \frac{1}{r} \Psi^* \Psi dV = \int \varphi_0 \delta(\mathbf{r}) dV$$

Then one obtains

$$\mathcal{E} = i\hbar \int \Psi^* \frac{\partial \Psi}{\partial t} dV + \int \frac{1}{2} \frac{e^2}{r} \delta(\mathbf{r}) dV \quad (71)$$

The second term in (71) describes the energy of the electrostatic field of the hydrogen atom nucleus and diverges. This is due to the fact that the nucleus of the hydrogen atom, in contrast to the Pauli field, is considered as a point electric charge. If we take into acount that the nucleus (proton) has a finite (though small) size and the electric charge of the proton is distributed continuously in the region of small size, then the second integral in (71) will have a finite value. If we assume that the distribution of the electric charge in the atomic nucleus, and, hence, the self-electrostatic energy of the nucleus do not depend on the state of the Pauli field, then the second integral has a constant value, which can be ignored in processes in which the structure of the atomic nucleus does not change (which takes place in ordinary atomic processes and in chemical reactions). In this case, the energy of the Maxwell-Pauli field for the hydrogen atom is

$$\mathcal{E} = i\hbar \int \Psi^* \frac{\partial \Psi}{\partial t} dV \tag{72}$$

Relation (72) exactly coincides with the well-known expression for energy in quantum mechanics [28]. At the same time, in quantum mechanics, expression (72) for the energy of a "quantum object" is actually postulated: it is not strictly derived, but justified based on the correspondence principle, which cannot be considered as a physical law. In the theory under consideration [30], the Maxwell-Pauli field energy (72) was derived strictly within the framework of the classical field theory without any additional assumptions and hypotheses.

Note that relation (72) determines the energy of not only the Pauli field ("electron"), but the total energy of the Maxwell-Pauli field, which includes the energy of the electromagnetic field, the energy of the electron field (Pauli field), the energy of the **G**-field and the energy interaction of the electron field with the electromagnetic field and the **G**-field. Remarkable is the fact that the total energy of the Maxwell-Pauli field for the hydrogen atom is expressed in its entirety only through the spinor Ψ.

Using (72), we can calculate the energy of the Maxwell-Pauli field for a hydrogen atom in a pure state (66). Taking into account (60), (61), (64) and (66), one obtains

$$\mathcal{E}_n = \hbar\omega_n^{(0)} \tag{73}$$

If the hydrogen atom is in a mixed state (62), then the energy (72) of the Maxwell-Pauli field

$$\mathcal{E} = \sum_n \hbar\omega_n^{(0)} |c_n|^2 \tag{74}$$

Taking into account that the parameters c_n can vary continuously, we conclude that the energy of an atom also takes continuous values, depending on the redistribution of the electron wave between the excited eigenmodes (see Section 3.3).

3.2. Angular momentum and magnetic moment of the Pauli field in the hydrogen atom

Let us consider the intrinsic angular momentum (spin) $\mathbf{S}$ of the electron field in the hydrogen atom described by solution (60).

In this case, direct calculation gives

$$\Psi^* \boldsymbol{\sigma} \Psi = \mathbf{v}|\psi|^2 \tag{75}$$

where

$$\mathbf{v} = (a_1^* a_2 + a_1 a_2^*, i(a_1 a_2^* - a_1^* a_2), |a_1|^2 - |a_2|^2) \tag{76}$$

moreover, according to (61), $|\mathbf{v}| = 1$.

Taking into account relations (23), (75) and the normalization condition $\int |\psi|^2 dV = 1$, we write the intrinsic angular momentum (spin) of the electron field in the form

$$\mathbf{S} = \frac{\hbar}{2}\mathbf{v} \tag{77}$$

Thus, in a natural way, within the framework of classical field theory, we obtained a well-known result: the intrinsic angular momentum (spin) of the electron field in the hydrogen atom has a constant value equal to $\hbar/2$. In this case, the unit vector $\mathbf{v}$ indicates the direction of the intrinsic angular momentum of the electron field.

Consider a unit spinor

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{78}$$

satisfying condition (61), which can be rewritten as

$$a^*a = 1 \quad (79)$$

As is known [33, 34], a unit spinor can always be represented as

$$a = \begin{pmatrix} \cos(\theta/2)\exp(i\chi/2) \\ i\sin(\theta/2)\exp(-i\chi/2) \end{pmatrix} \quad (80)$$

where θ and χ are some parameters.

Using (76) and (80), one obtains

$$\mathbf{v} = a^*\boldsymbol{\sigma}a = (\sin\theta\sin\chi, \sin\theta\cos\chi, \cos\theta) \quad (81)$$

Thus, the parameters θ and χ in the spinor (80) are the angles of the unit vector $\mathbf{v}$ in the spherical coordinate system.

Taking into account (25), one obtains the vector of the intrinsic magnetic moment of the Pauli field in the hydrogen atom

$$\boldsymbol{\mu} = -\mu_B\mathbf{v} \quad (82)$$

Thus, the intrinsic magnetic moment of the Pauli field in the hydrogen atom has a constant modulus equal to the Bohr magneton μ_B and is directed antiparallel to the unit vector $\mathbf{v}$.

In general, the parameters a_1 and a_2 can change over time, satisfying the condition (61). This is equivalent to the rotation of the unit vector $\mathbf{v}$ in space. This case takes place for an atom in an external magnetic field and will be considered in Section 4.

If the parameters a_1 and a_2 do not depend on time, then the direction of the vector $\mathbf{S}$ remains unchanged.

In this case, by rotating the coordinate system, spinor (60) can always be reduced to one of the following special cases.

(i) The z axis of the coordinate system is chosen parallel to the vector $\mathbf{S}$. Then $a = \begin{pmatrix}1\\0\end{pmatrix}$.

(ii) The z axis of the coordinate system is chosen antiparallel to the vector $\mathbf{S}$. Then $a = \begin{pmatrix}0\\1\end{pmatrix}$.

(iii) The x axis of the coordinate system is chosen parallel to the vector $\mathbf{S}$. Then $a = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$.

(iv) The x axis of the coordinate system is chosen antiparallel to the vector $\mathbf{S}$. Then $a = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$.

(v) The y axis of the coordinate system is chosen parallel to the vector $\mathbf{S}$. Then $a = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$.

(vi) The y axis of the coordinate system is chosen antiparallel to the vector $\mathbf{S}$. Then $a = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$.

Thus, in the absence of an external magnetic field, for a single-electron Pauli field (i.e., an electron field whose charge is $-e$) described by spinor (60), one can always introduce such a direction of the z axis that the intrinsic angular momentum of the electron field is equal to $\pm\hbar/2$.

This result is in complete agreement with the concepts of classical field theory and classical vector algebra.

3.3. Spontaneous emission and spontaneous transitions

The traditional explanation of spontaneous emission, for example, of a hydrogen atom is based on the notion that an electron (as a particle) can only exist in an atom at discrete energy levels that are stationary. Being on one of the stationary energy levels, the electron does not emit electromagnetic waves, however, it can make spontaneous abrupt transitions from one energy level to another, while emitting an energy quantum equal to the energy difference of those levels between which the transition takes place. The reason for the spontaneous transition, and therefore spontaneous emission, is considered to be a phenomenon called zitterbewegung: the interaction of an electron with fluctuations of hypothetic QED-vacuum, which cause the electron to leave a stationary energy level and move to a new level.

In the theory [18-23], spontaneous emission and the related rearrangement of the atom structure, which is traditionally called spontaneous transition, are a natural consequence of classical electrodynamics: spontaneous emission is caused by oscillations of the electric dipole moment of the electric charge of an electron wave distributed in space, and the rearrangement of the atom structure is a consequence of interaction of an electron wave with its own radiation electromagnetic field [19-23].

Let us consider how spontaneous emission and spontaneous transitions are described within the framework of the classical Maxwell-Pauli field theory [30] under condition (18).

In the general case, any state of an electron wave (Pauli field) in a hydrogen atom can be expanded according to (62) in terms of eigenfunctions of equation (63).

Let us show that the mixed states of the hydrogen atom (i.e., solutions (62) with several non-zero parameters c_n) are nonstationary; only pure states of the hydrogen atom (66) are stationary. Moreover, we will show that only the pure state with the lowest eigenfrequency (ground mode) is stable, while the rest of the pure states are unstable.

Consider the spontaneous emission of a hydrogen atom. According to classical electrodynamics [32], the intensity of electric dipole emission

$$I_{\mathbf{EH}} = \frac{2}{3c^3}\ddot{\mathbf{d}}^2 \qquad (83)$$

where $\mathbf{d}$ is the electric dipole moment of an atom (6).

Using solution (62), one obtains

$$\mathbf{d}(t) = \sum_k \sum_n c_k^* c_n \mathbf{d}_{nk} \exp(-i\omega_{nk} t) \qquad (84)$$
$$\ddot{\mathbf{d}}(t) = -\sum_k \sum_n \omega_{nk}^2 c_k^* c_n \mathbf{d}_{nk} \exp(-i\omega_{nk} t) \qquad (85)$$

where

$$\omega_{nk} = \omega_n^{(0)} - \omega_k^{(0)} \qquad (86)$$
$$\mathbf{d}_{nk} = \mathbf{d}_{kn}^* = -e \int \mathbf{r}\psi_k^*(\mathbf{r})\psi_n(\mathbf{r}) dV \qquad (87)$$

Substituting (85) into (83) and averaging over time, one obtains

$$I_{\mathbf{EH}} = \frac{4}{3c^3}\sum_k \sum_{n>k} \omega_{nk}^4 |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 \qquad (88)$$

According to classical electrodynamics [32], the vector $\ddot{\mathbf{d}}$ determines the electric and magnetic fields of the dipole radiation. Then it follows from relations (85) and (88) that only discrete frequencies ω_{nk} are present in the spectrum of spontaneous (dipole) emission of the hydrogen atom, while the intensity of spontaneous emission of the atom at the frequency ω_{nk}

$$(I_{\mathbf{EH}})_{nk} = \frac{4}{3c^3}\omega_{nk}^4 |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 \qquad (89)$$

Relations (88) and (89) coincide with the corresponding relations obtained in quantum electrodynamics [29] for the intensity of spontaneous emission of a hydrogen atom, however, when deriving relation (88), the hypotheses about jump transitions of an

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electron from one energy level to another and about the quantum character of emission have not been used. Thus, in order to explain the discrete spectrum of spontaneous emission of a hydrogen atom and obtain the correct expression for the intensity of spontaneous emission on different lines of the spectrum, the hypothesis of the quantum nature of spontaneous emission is redundant.

Let us calculate the energy carried away by the **G**-field during dipole emission of an atom. As shown in [30], the component of the energy flux density associated with the **G**-field is equal to

$$\mathbf{J}_{\mathbf{G}} = -\frac{1}{4\pi}\frac{\partial G_{ek}}{\partial t}\nabla G_{ek} \qquad (90)$$

where $k = 1,2,3$ is the vector index.

At large distances r from the emitting atom, we write

$$R = |\mathbf{r} - \mathbf{r}'| = r - \mathbf{r}'\mathbf{n} \qquad (91)$$

where $\mathbf{n} = \frac{\mathbf{r}}{r}$ is the unit vector in the direction of $\mathbf{r}$; $r \gg r'$.

Using (91), we write relation (29) in the considered approximation in the form

$$\mathbf{G}_e = \frac{e}{r}\int\left((\Psi^*\boldsymbol{\sigma}\Psi)_{t-r/c} + \frac{(\mathbf{r}'\mathbf{n})}{c}\frac{\partial}{\partial t}(\Psi^*\boldsymbol{\sigma}\Psi)_{t-r/c} + \frac{1}{2}\left(\frac{\mathbf{r}'\mathbf{n}}{c}\right)^2\frac{\partial^2}{\partial t^2}(\Psi^*\boldsymbol{\sigma}\Psi)_{t-r/c} + \cdots\right)dV' \qquad (92)$$

Substituting (75) into (92), and taking into account the normalization condition $\int|\psi|^2 dV = 1$, one obtains

$$\mathbf{G}_e = \frac{e\boldsymbol{\nu}}{r} - \frac{1}{rc}\dot{\boldsymbol{\nu}}(\mathbf{nd}) - \frac{1}{rc}\boldsymbol{\nu}(\mathbf{n}\dot{\mathbf{d}}) \qquad (93)$$

where the vectors $\boldsymbol{\nu}$ and $\mathbf{d}$ are taken at the moment $t' = t - r/c$. The remaining discarded terms are small in the considered dipole approximation.

We will consider distances r from the atom that satisfy the conditions $|\dot{\boldsymbol{\nu}}|r/c \gg 1$ and $|\dot{\mathbf{d}}|r/c \gg |\mathbf{d}|$. Then

$$\frac{\partial \mathbf{G}_e}{\partial t} = \frac{e\dot{\boldsymbol{\nu}}}{r} - \frac{1}{rc}\ddot{\boldsymbol{\nu}}(\mathbf{nd}) - \frac{2}{rc}\dot{\boldsymbol{\nu}}(\mathbf{n}\dot{\mathbf{d}}) - \frac{1}{rc}\boldsymbol{\nu}(\mathbf{n}\ddot{\mathbf{d}}) \qquad (94)$$

$$\nabla G_{ek} = -\frac{e\dot{\nu}_k}{rc}\mathbf{n} + \frac{1}{rc^2}\ddot{\nu}_k\mathbf{n}(\mathbf{nd}) + \frac{2}{rc^2}\dot{\nu}_k\mathbf{n}(\mathbf{n}\dot{\mathbf{d}}) + \frac{1}{rc^2}\nu_k\mathbf{n}(\mathbf{n}\ddot{\mathbf{d}}) \qquad (95)$$

Substituting (94) and (95) into (90), one obtains

$$\mathbf{J}_{\mathbf{G}} = \frac{1}{4\pi r^2 c}\mathbf{n}\left(e\dot{\boldsymbol{\nu}} - \frac{1}{c}\ddot{\boldsymbol{\nu}}(\mathbf{nd}) - \frac{2}{c}\dot{\boldsymbol{\nu}}(\mathbf{n}\dot{\mathbf{d}}) - \frac{1}{c}\boldsymbol{\nu}(\mathbf{n}\ddot{\mathbf{d}})\right)^2 \qquad (96)$$

The energy carried away by the **G**-field through a spherical surface of radius $r \to \infty$ centered on the nucleus of an atom

$$I_{\mathbf{G}} = r^2\oint(\mathbf{n}\mathbf{J}_{\mathbf{G}})d\Omega \qquad (97)$$

where Ω is the solid angle.

Taking into account (96), we obtain

$$I_{\mathbf{G}} = \frac{1}{4\pi c}\oint\left(e\dot{\boldsymbol{\nu}} - \frac{1}{c}\ddot{\boldsymbol{\nu}}(\mathbf{nd}) - \frac{2}{c}\dot{\boldsymbol{\nu}}(\mathbf{n}\dot{\mathbf{d}}) - \frac{1}{c}\boldsymbol{\nu}(\mathbf{n}\ddot{\mathbf{d}})\right)^2 d\Omega \qquad (98)$$

Let us first consider the case when the direction of the spin does not change, i.e. $\dot{\boldsymbol{\nu}} = 0$.

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For $\dot{\mathbf{v}} = 0$ and $|\mathbf{v}| = 1$, relation (98) takes the form

$$I_{\mathbf{G}} = \frac{1}{4\pi c^3} \oint \left(\mathbf{n}\ddot{\mathbf{d}}\right)^2 d\Omega \qquad (99)$$

A change in the direction of the dipole moment vector can occur in an external electromagnetic field. If there is no external electromagnetic field, then the direction of the vector $\mathbf{d}$ does not change. In this case $(\mathbf{nd}) = |\mathbf{d}| \cos\theta$ and $\left(\mathbf{n}\ddot{\mathbf{d}}\right) = \left|\ddot{\mathbf{d}}\right| \cos\theta$. Substituting $d\Omega = 2\pi \sin\theta\, d\theta$ into (99), one obtains

$$I_{\mathbf{G}} = \frac{1}{3c^3} \ddot{\mathbf{d}}^2 \qquad (100)$$

As follows from (100), the intensity of the electric dipole emission of the **G**-field is commensurate with the intensity of electromagnetic radiation (83). This means that if the **G**-field really exists, it can be fixed experimentally, moreover, it can be used to transfer energy and information, like electromagnetic radiation, which opens up new technological possibilities. At the same time, we do not have any experimental data on the existence of the **G**-field so far. Assuming that the **G**-field still exists (otherwise we cannot explain why the electron wave does not feel its own electrostatic field), this may be due to several reasons [30]. First, condition (18) may not be satisfied, and then we are dealing with a short-range **G**-field, which manifests itself only within the atom. This case will be considered in the next papers of this series. Another possible reason may be the peculiarity of the interaction of the **G**-field with matter (electron field). As shown in [30], the **G**-field interacts very weakly with matter and practically does not interact with electric charges and currents. For this reason, it cannot be registered by conventional physical devices, the principle of operation of which is based on the force interaction of fields with electric charges and currents. At the same time, according to the theory under consideration, the **G**-field interacts with the intrinsic (spin) magnetic moment of the electron wave, but does not interact with the convective magnetic moments [30]. This fact can be used as a basis for creating devices for recording and emitting a wave **G**-field.

As follows from the results of [30], the energy loss during dipole radiation of an atom is associated with only two components of the energy flux density: electromagnetic radiation and **G**-field radiation. The remaining terms in the energy flux density of the Maxwell-Pauli field [30] contain the field Ψ, which decreases at infinity faster than r^{-2} during dipole radiation, and therefore do not contribute to the energy loss.

Thus, the total intensity of radiation of an atom (the energy lost by an atom per unit time) during dipole radiation $I = I_{\mathbf{EH}} + I_{\mathbf{G}}$. Taking into account (83) and (100), one obtains

$$I = \frac{1}{c^3} \ddot{\mathbf{d}}^2 \qquad (101)$$

It follows from (101) that, under condition (18), the total intensity of energy emission by an atom in the dipole approximation consists of 2/3 of electromagnetic emission and 1/3 of **G**-field emission.

It follows from relations (85) and (93) that the spectrum of spontaneous (dipole) emission of a hydrogen atom carried by the **G**-field also contains only discrete frequencies ω_{nk}. Using (88) and (89) it is easy to calculate the intensity components $(I_{\mathbf{G}})_{nk}$ and I_{nk} at frequencies ω_{nk}. In particular,

$$I_{nk} = \frac{2}{c^3} \omega_{nk}^4 |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 \qquad (102)$$

Then

$$I = \sum_k \sum_{n>k} I_{nk} \qquad (103)$$

The results obtained allow drawing some conclusions, which at first glance may seem trivial, however, it should be borne in mind that they were obtained in the framework of classical field theory without any quantization.

As follows from (85), (88) and (93), spontaneous emission of the hydrogen atom (electromagnetic radiation and radiation associated with the **G**-field) is observed only at discrete frequencies (86) and only in those cases when the conditions$|c_k|^2 \neq 0, |c_n|^2 \neq 0$ and $|\mathbf{d}_{nk}|^2 \neq 0$. The first two conditions mean that a hydrogen atom can spontaneously radiate only if at least two eigenmodes are simultaneously excited in it, i.e. when the atom is in a mixed state. The condition $|\mathbf{d}_{nk}|^2 \neq 0$ means that the spontaneous emission spectrum does not contain those frequencies (86) for which $|\mathbf{d}_{nk}|^2 = 0$, even if $|c_k|^2 \neq 0$ and $|c_n|^2 \neq 0$, that is even if the atom is in a mixed (excited) state, in which the n and k modes are simultaneously excited. In the traditional interpretation of quantum mechanics, in which spontaneous emission is interpreted as jump-like transitions of electrons (charged particles) between discrete energy levels, the condition $|\mathbf{d}_{nk}|^2 = 0$ defines the so-called “forbidden transitions”. The well-established term “forbidden transitions” is based on a certain subjective interpretation of experimental data, which goes back to Bohr's naive theory. In an objective analysis of experimental data, it is more correct to state the observed effect or phenomenon without using any interpretation. For this reason, it is more correct to speak not about “forbidden transitions”, but about “forbidden frequencies” ω_{nk}, at which spontaneous emission is not observed (i.e., the intensity of spontaneous emission at these frequencies is equal to zero). Thus, the frequencies ω_{nk}, for which the condition $|\mathbf{d}_{nk}|^2 = 0$ is satisfied, are forbidden and are absent in the spectrum of spontaneous emission. We see that the “forbidden” frequencies in the spectrum of spontaneous emission have a simple and clear explanation in the framework of classical electrodynamics without any quantization.

As follows from (83) and (100), spontaneous emission occurs not in the form of discrete energy quanta, but continuously in the form of classical electromagnetic waves and classical **G**-field waves: during dipole emission, an atom continuously loses energy at a rate (101), which has two components: electromagnetic (83) and associated with the **G**-field (100). The loss of energy should lead to a rearrangement of the structure of the atom (Pauli fields), which in the traditional interpretation of quantum mechanics is considered as jump-like quantum transitions of an electron from one energy level to another.

Taking into account that in the classical field theory under consideration, emission occurs continuously in the form of classical waves (electromagnetic waves and **G**-field waves), we conclude that the rearrangement of the atom structure (more precisely, the rearrangement of its Maxwell-Pauli field) occurs continuously.

Thus, if an atom is in a mixed state, it loses energy, and this is accompanied by a rearrangement of its internal structure, which, as follows from (74), consists in a change in the parameters c_n characterizing the excitation amplitude of the atom’s eigenmodes. This process continues until the atom passes into the pure state (66), in which only one eigenmode is excited. In this pure state, the atom can be indefinitely long, because its electric dipole moment remains constant and, consequently, there is no energy loss due to dipole emission. We note that the spontaneous dipole emission will also cease when the atom is in a mixed state with two excited eigenmodes n and k, for which $|\mathbf{d}_{nk}|^2 = 0$. We do not consider this case here.

Let us assume that the atom is in the pure state (66) corresponding to the eigenmode n. The energy $\mathcal{E}_n$ of the Maxwell-Pauli field of an atom in this mode is equal to (73). Suppose, due to some external influence, the eigenmode k was also excited, for which $\mathcal{E}_k < \mathcal{E}_n$, i.e. $\omega_k^{(0)} < \omega_n^{(0)}$. As a result, the atom ended up in a mixed state with two excited modes n and k. According to (83) and (100), the atom will spontaneously emit electromagnetic waves and **G**-field waves, losing energy at the rate (102), even if the k mode is weakly excited, i.e. even if $|c_k|^2 \ll |c_n|^2 \approx 1$. In this case, the structure of the atom (the Maxwell-Pauli field) will be rearranged, which will consist in the fact that the amplitude $|c_k|^2$ will increase, while the amplitude $|c_n|^2$ will decrease, at the same time, according to (65) $|c_k|^2 + |c_n|^2 = 1$. As shown above, the parameter $|c_k|^2$ is equal to the fraction of the electric charge of the electron wave contained in the k mode. Thus, the rearrangement of the structure of the hydrogen atom during spontaneous emission is reduced to the flow of the electric charge of the electron wave from the n mode to the k mode, and continues until the entire electric charge flows into the k mode, i.e. until it becomes $|c_n|^2 = 0$. Thus, we see that the pure states of the atom (66), although they are stationary, turn out to be unstable: even a weak excitation of an eigenmode with a lower eigenfrequency is sufficient to initiate spontaneous emission and spontaneous overflow of an electron wave into an eigenmode with a lower eigenfrequency. An exception to this rule is the eigenmode with the lowest eigenfrequency (lowest energy), i.e. ground mode: any mixed state in which the ground mode is excited will eventually, due to spontaneous emission, go into the ground state in which only the ground mode is excited. If the atom is in the ground state and some other eigenmodes are excited, then over time, due to spontaneous emission, the atom will return to the ground state. Thus, the ground state of the atom is absolutely stable.

Let us consider how the rearrangement of the structure of an atom (electron field) in the process of spontaneous emission is described from the point of view of the nonlinear Pauli equation.

In the absence of an external electromagnetic field and an external **G**-field, equation (56) has the form

$$i\hbar\frac{\partial\Psi}{\partial t}=\left[-\frac{\hbar^2}{2m_e}\Delta-\frac{e^2}{r}+\frac{e}{c^3}\mathbf{r}\dddot{\mathbf{d}}-(e\varphi_e+e\boldsymbol{\sigma}\mathbf{G}_e)\right]\Psi \qquad (104)$$

Consider solutions (60) of equation (104) at constant values of the parameters a_1 and a_2. In this case, the unit vector $\boldsymbol{\nu}$ has a constant direction, expression (32) is identically equal to zero, and equation (104) turns into a nonlinear Schrödinger equation for the hydrogen atom:

$$i\hbar\frac{\partial\psi}{\partial t}=\left(-\frac{\hbar^2}{2m_e}\Delta-\frac{e^2}{r}+\frac{e}{c^3}\mathbf{r}\dddot{\mathbf{d}}\right)\psi \qquad (105)$$

Note that here the term containing the electric dipole moment has the factor $\frac{e}{c^3}$ instead of $\frac{2e}{3c^3}$ as in equation (5). This is due to the presence of a **G**-field satisfying condition (18), which was not taken into account when deriving Eq. (5).

We seek the solution of equation (105) in the form of expansion (62) in terms of eigenfunctions of the linear Schrödinger equation (63), but with time-dependent coefficients c_n:

$$\psi(\mathbf{r},t)=\sum_n c_n(t)\psi_n(\mathbf{r})\exp\left(-i\omega_n^{(0)}t\right) \qquad (106)$$

Substituting (106) into (105), taking into account (63), one obtains

$$i\hbar\sum_n \dot{c}_n(t)\psi_n(\mathbf{r})\exp\left(-i\omega_n^{(0)}t\right)=\frac{e}{c^3}\mathbf{r}\dddot{\mathbf{d}}\sum_n c_n(t)\psi_n(\mathbf{r})\exp\left(-i\omega_n^{(0)}t\right) \qquad (107)$$

We multiply equation (107) by $\psi_k^*(\mathbf{r})$ and integrate over the entire space, taking into account (64) and (87). As a result, one obtains

$$i\hbar\dot{c}_k(t)=-\frac{1}{c^3}\sum_n c_n(t)\left(\dddot{\mathbf{d}}\mathbf{d}_{nk}\right)\exp(-i\omega_{nk}t) \qquad (108)$$

Taking into account (84), one obtains

$$\dddot{\mathbf{d}}(t)=i\sum_k\sum_n \omega_{nk}^3 c_k^* c_n \mathbf{d}_{nk}\exp(-i\omega_{nk}t) \qquad (109)$$

We assume that the characteristic time of changing the parameters $c_n(t)$ is significantly less than the period $T_{nk}=\frac{2\pi}{\omega_{nk}}$, i.e. that the condition

$$T_{nk}|\dot{c}_k(t)| \ll c_k(t) \tag{110}$$

In this case, we can average equation (108) over fast oscillations with frequencies ω_{nk}, assuming that the parameters $c_n(t)$ are constant. Taking into account (109), one obtains

$$\dot{c}_k = c_k \sum_n \gamma_{nk} |c_n|^2 \tag{111}$$

where

$$\gamma_{nk} = \frac{\omega_{nk}^3}{\hbar c^3} |\mathbf{d}_{nk}|^2 \tag{112}$$

is the damping rate of the spontaneous emission, associated with the excited eigenmodes n and k. Obviously, $\gamma_{nk} > 0$, if $\omega_{nk} > 0$ while $\gamma_{nk} < 0$, if $\omega_{nk} < 0$.

Equation (111) can be rewritten as

$$\frac{d|c_k|^2}{dt} = |c_k|^2 \sum_n 2\gamma_{nk} |c_n|^2 \tag{113}$$

Taking into account that $\gamma_{nn} = 0$, the formal solution of equation (111) has the form

$$c_k(t) = c_k(0) \exp\left(\sum_n \gamma_{nk} \int_0^t |c_n|^2 dt\right) \tag{114}$$

Equation (113) implies that if all excited eigenmodes (i.e. eigenmodes with $|c_n|^2 \neq 0$) have eigenfrequencies $\omega_n^{(0)} > \omega_k^{(0)}$ (i.e. $\gamma_{nk} > 0$ for all n), then the right side of equation (113) is non-negative and the amplitude $|c_k|^2$ of the eigenmode k increases monotonically with time. Taking into account (65), this is possible only by reducing the amplitudes of all modes $n \neq k$. The amplitude $|c_k|^2$ will increase until $|c_k|^2 = 1$ and, accordingly, $|c_n|^2 = 0$ for all $n \neq k$ are reached, i.e. until the entire electric charge of the electron wave flows into the lower mode k. After that, spontaneous emission and rearrangement of the structure of the hydrogen atom will stop. This implies that all pure states of the hydrogen atom, with the exception of the ground state (with the lowest eigenfrequency), are unstable.

Simple estimates [19-23] show that $\frac{\gamma_{nk}}{\omega_{nk}} \sim \alpha^3 \ll 1$, where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant. It is this condition that allows spliting the process of solving the Schrödinger equation into two stages: first, find the eigenfrequencies and eigenmodes of the atom, based on the assumption that the atom is in a stationary state (even if it is in a mixed state), and then calculate the change in the structure associated with the spontaneous emission. Note that usually in quantum mechanics, when solving the Schrödinger (Klein-Gordon, Pauli, Dirac) equations, it is limited only to the first stage.

We multiply equation (113) by $\hbar\omega_k^{(0)}$ and sum over all k. As a result, taking into account (74), one obtains

$$\frac{d\mathcal{E}}{dt} = \sum_k \sum_n \omega_k^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 \tag{115}$$

Taking into account that

$$\sum_k \sum_n \omega_k^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 = -\sum_n \sum_k \omega_n^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 = -\sum_k \sum_n \omega_n^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2$$

One can write

$$\sum_k \sum_n \omega_k^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 = \frac{1}{2} \sum_k \sum_n \omega_k^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 + \frac{1}{2} \sum_k \sum_n \omega_k^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 = \frac{1}{2} \sum_k \sum_n \omega_k^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 - \frac{1}{2} \sum_k \sum_n \omega_n^{(0)} \frac{2\omega_{nk}^3}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 - \frac{1}{2} \sum_k \sum_n \frac{2\omega_{nk}^4}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2 = - \sum_k \sum_{n>k} \frac{2\omega_{nk}^4}{c^3} |c_k|^2 |c_n|^2 |\mathbf{d}_{nk}|^2$$

Then, taking into account (102) and (103), one writes equation (115) as

$$\frac{d\mathcal{E}}{dt} = -I \qquad (116)$$

Equation (116) expresses the energy conservation law for the hydrogen atom: the energy of the Maxwell-Pauli field in the hydrogen atom decreases due to spontaneous emission at a rate I. Thus, the results obtained are in full accordance with the energy conservation law.

Equations (111) and (113) give a complete quantitative description of spontaneous emission and the related rearrangement of the structure of the hydrogen atom (what is commonly called a spontaneous transition) within the framework of classical field theory. In this case, the damping rate of the spontaneous emission (112), obtained in the framework of classical field theory, is one and a half times greater than the damping rate of the spontaneous emission, which is derived in quantum electrodynamics [29] based on quantum concepts. This is due to the fact that in quantum electrodynamics the **G**-field satisfying condition (18) is not taken into account.

Thus, we come to the conclusion that spontaneous emission occurs in full accordance with classical field theory (in particular, with classical electrodynamics), and in order to explain its cause, there is no need to invoke additional hypotheses, such as zitterbewegung and QED-vacuum.

Moreover, to explain the spectrum of spontaneous emission and its quantitative characteristics, there is no need to use the hypothesis of quantization of fields (electron and electromagnetic), but only the concepts and ideas of classical field theory are sufficient.

4. Hydrogen atom in an external magnetic field

The solution of the nonlinear Pauli equation (56) for a hydrogen atom in an external magnetic field will be considered in more detail in the next papers of this series.

In this paper, we consider the behavior of the intrinsic angular momentum vector of a hydrogen atom in an external magnetic field.

As shown in [30], the change in the intrinsic angular momentum $\mathbf{S}$ of the electron field of an atom is described by the equation

$$\frac{d\mathbf{S}}{dt} = -\mu_B \int \Psi^* \boldsymbol{\sigma} \Psi \times \mathbf{H}_\Sigma dV + e \int \Psi^* \boldsymbol{\sigma} \Psi \times \mathbf{G}_\Sigma dV \qquad (117)$$

The first term on the right-hand side of equation (117) describes the usual classical torque acting on the magnetic moment in an external magnetic field. The second term is related to the **G**-field and has no analogues in classical electrodynamics.

Consider the case $\mathbf{G} = 0$, i.e. when there is no external **G**-field. Then, in the considered approximation, equation (117) takes the form

$$\frac{d\mathbf{S}}{dt} = \mu_B \mathbf{H} \times \int \Psi^* \boldsymbol{\sigma} \Psi dV + e \int \Psi^* \boldsymbol{\sigma} \Psi \times \mathbf{G}_e dV \qquad (118)$$

Let us expand the integrand in (29) into a power series with respect to R/c. As a result, one obtains

$$\mathbf{G}_e = \mathbf{G}_0 - \frac{e}{c} \frac{\partial}{\partial t} \int \Psi^* \boldsymbol{\sigma} \Psi dV' + \frac{e}{2c^2} \frac{\partial^2}{\partial t^2} \int R \Psi^* \boldsymbol{\sigma} \Psi dV' - \frac{e}{6c^3} \frac{\partial^3}{\partial t^3} \int R^2 \Psi^* \boldsymbol{\sigma} \Psi dV' + \cdots \qquad (119)$$

We consider the solution of the nonlinear Pauli equation in the form (60), considering the parameters a_1 and a_2 as functions of time. In this case, relations (75) and (77) hold, where the unit vector $\mathbf{v}$ depends on time, i.e., rotates in space.

In this case, equation (118) and relation (119) take the form

$$\dot{\mathbf{v}} = -\gamma_e \mathbf{H} \times \mathbf{v} + 2\frac{e}{\hbar}\mathbf{v} \times \int |\psi|^2 \mathbf{G}_e dV \qquad (120)$$

$$\mathbf{G}_e = \mathbf{G}_0 - \frac{e}{c}\dot{\mathbf{v}} + \frac{e}{2c^2}\frac{\partial^2}{\partial t^2}\left(\mathbf{v}\int R|\psi|^2 dV'\right) - \frac{e}{6c^3}\frac{\partial^3}{\partial t^3}\left(\mathbf{v}\int R^2|\psi|^2 dV'\right) + \cdots \qquad (121)$$

where, taking into account (2),

$$\mathbf{G}_0 = e\mathbf{v}\int \frac{|\psi(\mathbf{r}',t)|^2}{R} dV' = -\mathbf{v}\varphi_0(\mathbf{r}) \qquad (122)$$

Substituting (121) into equation (120), one obtains

$$\dot{\mathbf{v}} = -\gamma_e \mathbf{H} \times \mathbf{v} - 2\alpha\mathbf{v} \times \dot{\mathbf{v}} + \frac{\alpha}{c}\mathbf{v} \times \int |\psi|^2 \frac{\partial^2}{\partial t^2}\left(\mathbf{v}\int R|\psi|^2 dV'\right)dV - \frac{\alpha}{3c^2}\mathbf{v} \times$$
$$\int |\psi|^2 \frac{\partial^3}{\partial t^3}\left(\mathbf{v}\int R^2|\psi|^2 dV'\right)dV + \cdots \qquad (123)$$

where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant.

Restricting ourselves to the first two terms on the right-hand side of equation (123), one obtains

$$\dot{\mathbf{v}} = -\gamma_e \mathbf{H} \times \mathbf{v} - 2\alpha\mathbf{v} \times \dot{\mathbf{v}} \qquad (124)$$

The first term in equation (124) describes the classical Larmor precession of spin in an external magnetic field with a frequency

$$\mathbf{\Omega}_v = \frac{e}{m_e c}\mathbf{H} \qquad (125)$$

Equation (124) has the form of the Landau–Lifshitz–Gilbert equation [35, 36] with a damping parameter 2α. Note that, in contrast to the Landau–Lifshitz–Gilbert equation [35, 36], which is phenomenological, equation (124) is not phenomenological, but naturally follows from the theory [30].

Let us multiply equation (124) on the left vectorially by $\mathbf{v}$. As a result of simple transformations, one obtains

$$\mathbf{v} \times \dot{\mathbf{v}} = \gamma_e \mathbf{v} \times \mathbf{v} \times \mathbf{H} + 2\alpha\dot{\mathbf{v}} \qquad (126)$$

Substituting (126) into equation (124), one obtains

$$\dot{\mathbf{v}} = -\frac{\gamma_e}{1+4\alpha^2}\mathbf{H} \times \mathbf{v} - \frac{2\alpha\gamma_e}{1+4\alpha^2}\mathbf{v} \times \mathbf{v} \times \mathbf{H} \qquad (127)$$

Equation (127) has the form of the Landau–Lifshitz equation [35].

Assuming the vector $\mathbf{H}$ to be constant, we consider the scalar product of equation (127) and $\mathbf{H}$. As a result, one obtains

$$\frac{d(\mathbf{vH})}{dt} = \frac{2\alpha}{1+4\alpha^2}\gamma_e\left(\mathbf{H}^2 - (\mathbf{vH})^2\right) \qquad (128)$$

Taking into account that $(\mathbf{vH}) = H\cos\theta$, where θ is the angle between the vectors $\mathbf{v}$ and $\mathbf{H}$, we reduce equation (128) to the form

$$\frac{d\theta}{dt} = -\frac{2\alpha}{1+4\alpha^2}\gamma_e H \sin\theta \qquad (129)$$

Equation (129) has a solution

$$\cos\theta = \frac{(1+\cos\theta_0)\exp\left(\frac{4\alpha}{1+4\alpha^2}\gamma_e Ht\right)-(1-\cos\theta_0)}{(1+\cos\theta_0)\exp\left(\frac{4\alpha}{1+4\alpha^2}\gamma_e Ht\right)+(1-\cos\theta_0)} \quad (130)$$

where θ_0 is the initial value of the angle θ.

Taking into account that $\gamma_e < 0$, it follows from equation (129) that the angle θ monotonically increases and tends to the limit value $\theta_\infty = \pi$. The characteristic time of rotation of the vector $\mathbf{v}$ in an external magnetic field is of the order of

$$t_\nu = \frac{m_e c}{2\alpha e H} \quad (131)$$

Let us compare this time with the period of the Larmor precession of the vector $\mathbf{v}$ around the magnetic field vector $\mathbf{H}$.

Using (125), one obtains

$$|\boldsymbol{\Omega}_\nu| t_\nu = (2\alpha)^{-1} \gg 1 \quad (132)$$

Thus, the period of the Larmor precession of the vector $\mathbf{v}$ is much shorter than the characteristic time of the change in the angle θ in an external magnetic field. From this point of view, we can say that the Larmor precession of the vector $\mathbf{v}$ occurs quasi-stationary.

Thus, according to equation (127), the vector $\mathbf{v}$ (vector of intrinsic angular momentum $\mathbf{S}$) of a hydrogen atom in an external magnetic field performs a Larmor precession around the vector $\mathbf{H}$ with an angular frequency (125) and, at the same time, slowly turns, tending to take a direction antiparallel to the vector $\mathbf{H}$. Accordingly, the vector of intrinsic magnetic moment $\boldsymbol{\mu}$ (82) performs a Larmor precession and tends to take a direction parallel to the vector $\mathbf{H}$, i.e. $(\boldsymbol{\mu}\mathbf{H}) \to \mu_B H$ at $t \to \infty$. Such a behavior of the magnetic moment has a simple explanation from the point of view of classical physics. As is known, any system tends to the state with the lowest potential energy, and the potential energy of a magnetic dipole in a magnetic field is $U = -(\boldsymbol{\mu}\mathbf{H})$. At constant $|\boldsymbol{\mu}| = \mu_B$ and $\mathbf{H}$, such a state is the state in which the magnetic moment is parallel to the vector $\mathbf{H}$.

Due to the turn of the intrinsic magnetic moment of the atom in an external magnetic field, the potential energy of the atom changes by $\Delta U = -\Delta(\boldsymbol{\mu}\mathbf{H}) = -\mu_B H + (\boldsymbol{\mu}\mathbf{H})_0$, where $(\boldsymbol{\mu}\mathbf{H})_0 = \mu_B H\cos\theta_0$ is the initial value of the potential energy of the magnetic moment. Thus,

$$\Delta U = -\mu_B H(1+\cos\theta_0) \quad (133)$$

Let us calculate the energy carried away by the field $\mathbf{G}$ during dipole radiation of an atom located in an external magnetic field. In the dipole approximation, the radiation intensity of the field $\mathbf{G}$ is determined by relation (98). In the Section 3.3, we considered the case when the direction of the spin does not change ($\dot{\mathbf{v}} = 0$), but the dipole moment of the atom changes ($\dot{\mathbf{d}} \neq 0$). Let us now consider the case when $\dot{\mathbf{v}} \neq 0$ while the atom does not have an electric dipole moment: $\mathbf{d} = 0$ (for example, the atom is in a pure *s*-state).

In this case, relation (98) takes the form

$$I_{\mathbf{G}} = \frac{e^2|\dot{\mathbf{v}}|^2}{c} \quad (134)$$

If the atom is in an external magnetic field $\mathbf{H}$, then using (127) one obtains

$$|\dot{\mathbf{v}}|^2 = \frac{\gamma_e^2}{(1+4\alpha^2)} H^2 \sin^2\theta \quad (135)$$

where θ is the angle between the vectors $\mathbf{v}$ and $\mathbf{H}$.
Thus,

$$I_{\mathbf{G}} = \frac{e^2}{(1+4\alpha^2)c}\gamma_e^2 H^2 \sin^2\theta \qquad (136)$$

Because under the action of the field $\mathbf{G}$, the intrinsic magnetic moment of the atom tends to take a direction parallel to the vector $\mathbf{H}$, the radiation intensity (136) rapidly decreases. As a result, the total energy carried away by the field $\mathbf{G}$ is equal to $\Delta\mathcal{E}_{\mathbf{G}} = \int_0^\infty I_{\mathbf{G}} dt$. Using (129), one obtains

$$\Delta\mathcal{E}_{\mathbf{G}} = -\frac{1+4\alpha^2}{2\alpha\gamma_e H}\int_{\theta_0}^{\pi} I_{\mathbf{G}}\frac{d\theta}{\sin\theta} \qquad (137)$$

Substituting (136) into (137), one obtains

$$\Delta\mathcal{E}_{\mathbf{G}} = \mu_B H(1+\cos\theta_0) \qquad (138)$$

Comparing (138) with (133), we see that the difference in potential energy resulting from the turn of the intrinsic magnetic moment in an external magnetic field is completely carried away by the wave field $\mathbf{G}$.

If the angles of the magnetic moments of atoms are distributed randomly and uniformly in the range $[0,\pi]$, then, averaging (138) over all angles θ_0, we obtain that atoms, being in an external magnetic field, on average lose energy carried away by the field $\mathbf{G}$, equal to

$$\overline{\Delta\mathcal{E}_{\mathbf{G}}} = \mu_B H \qquad (139)$$

Let us take into account the third term in expansion (121):

$$\mathbf{G}_e = \mathbf{G}_0 - \frac{e}{c}\dot{\mathbf{v}} + \frac{e}{2c^2}\frac{\partial^2}{\partial t^2}\left(\mathbf{v}\int R|\psi|^2 dV'\right) \qquad (140)$$

In particular, if the atom is in a pure state, then $|\psi|^2$ does not depend on time, and relation (140) takes the form

$$\mathbf{G}_e = \mathbf{G}_0 - \frac{e}{c}\dot{\mathbf{v}} + \frac{e}{2c^2}\ddot{\mathbf{v}}\int R|\psi|^2 dV' \qquad (141)$$

In this case, equation (123) takes the form

$$\dot{\mathbf{v}} = \gamma_e\mathbf{v}\times\mathbf{H} - 2\alpha\mathbf{v}\times\dot{\mathbf{v}} + \frac{\alpha b}{c}\mathbf{v}\times\ddot{\mathbf{v}} \qquad (142)$$

where

$$b = \int\int R|\psi(\mathbf{r})|^2|\psi(\mathbf{r}')|^2 dV'\,dV \qquad (143)$$

By analogy with equation (127), equation (142) can be reduced to the form

$$\dot{\mathbf{v}} = \frac{\gamma_e}{1+4\alpha^2}\mathbf{v}\times\mathbf{H} + \frac{\alpha b}{(1+4\alpha^2)c}\mathbf{v}\times\ddot{\mathbf{v}} - \frac{2\alpha}{1+4\alpha^2}\gamma_e\mathbf{v}\times\mathbf{v}\times\mathbf{H} - \frac{2\alpha^2 b}{(1+4\alpha^2)c}\mathbf{v}\times\mathbf{v}\times\ddot{\mathbf{v}} \quad (144)$$

To calculate the derivative $\ddot{\mathbf{v}}$, we take into account that the second and subsequent terms on the right-hand side of (144) are small compared to the first one. As a result, one can write approximately

$$\dot{\mathbf{v}} \approx \frac{\gamma_e}{1+4\alpha^2}\mathbf{v}\times\mathbf{H} \qquad (145)$$

$$\ddot{\mathbf{v}} \approx \frac{\gamma_e}{1+4\alpha^2}\dot{\mathbf{v}} \times \mathbf{H} \approx -\left(\frac{\gamma_e}{1+4\alpha^2}\right)^{\mathbf{2}} \mathbf{H} \times \mathbf{v} \times \mathbf{H} = -\left(\frac{\gamma_e}{1+4\alpha^2}\right)^{\mathbf{2}} \left(\mathbf{v}\mathbf{H}^2 - \mathbf{H}(\mathbf{v}\mathbf{H})\right) \quad (146)$$

Substituting (146) into (144), one obtains

$$\dot{\mathbf{v}} = \frac{\gamma_e}{1+4\alpha^2}\left(1 + \frac{\alpha b\gamma_e}{(1+4\alpha^2)^2 c}(\mathbf{v}\mathbf{H})\right)\mathbf{v} \times \mathbf{H} - \frac{2\alpha}{1+4\alpha^2}\gamma_e\left[1 + \frac{\alpha b\gamma_e}{(1+4\alpha^2)^2 c}(\mathbf{v}\mathbf{H})\right]\mathbf{v} \times \mathbf{v} \times \mathbf{H} \quad (147)$$

Assuming the vector $\mathbf{H}$ to be constant, we multiply equation (147) by it. As a result, one obtains

$$\mathbf{H}\dot{\mathbf{v}} = -\frac{2\alpha}{1+4\alpha^2}\gamma_e\left[1 + \frac{\alpha b\gamma_e}{(1+4\alpha^2)^2 c}(\mathbf{v}\mathbf{H})\right]\left[(\mathbf{v}\mathbf{H})^2 - \mathbf{H}^2\right] \quad (148)$$

Taking into account that $(\mathbf{v}\mathbf{H}) = H\cos\theta$, one obtains

$$\frac{d\theta}{dt} = -\frac{2\alpha}{1+4\alpha^2}\gamma_e \mathrm{H}\left(1 + \frac{\alpha b\gamma_e}{(1+4\alpha^2)^2 c}H\cos\theta\right)\sin\theta \quad (149)$$

Equation (149) differs from equation (129) by an additional factor $\left(1 + \frac{\alpha b\gamma_e}{(1+4\alpha^2)^2 c}H\cos\theta\right)$ on the right-hand side.

When

$$H > -\frac{\left(1+4\alpha^2\right)^2 c}{\alpha b\gamma_e} = (1+4\alpha^2)^2\frac{m_e c^2}{\alpha b e} \quad (150)$$

the right-hand side of equation (149) changes sign as the angle θ changes from zero to π. In this case, when

$$\cos\theta < -\frac{\left(1+4\alpha^2\right)^2 c}{\alpha b\gamma_e H} \quad (151)$$

the right-hand side of equation (149) is positive and the angle θ increases until it reaches the value $\theta = \pi$. Thus, the magnetic moment of the electron wave turns in the magnetic field until it becomes parallel to the vector $\mathbf{H}$. On the contrary, when

$$\cos\theta > -\frac{\left(1+4\alpha^2\right)^2 c}{\alpha b\gamma_e H} \quad (152)$$

the right-hand side of equation (149) is negative, and the angle θ decreases until it reaches the value $\theta = 0$. Thus, the magnetic moment of the electron wave turns in the magnetic field until it becomes antiparallel to the vector $\mathbf{H}$.

As a result, in a field that satisfies condition (150), we will observe only two possible orientations of the magnetic moment: parallel and antiparallel to the vector $\mathbf{H}$. If the field $\mathbf{H}$ is inhomogeneous (as in the Stern and Gerlach experiments), this will lead to the separation of the atomic beam in space into two parts, one of which corresponds to the parallel and the other to the antiparallel orientation of the intrinsic magnetic moment of the electron wave of the atom with respect to the vector $\mathbf{H}$.

Thus, the fulfillment of condition (150) actually means an explanation of the results of the Stern and Gerlach experiments.

If condition (150) is not satisfied, then the magnetic moment of the atom is always oriented parallel to the field $\mathbf{H}$. In this case, the results obtained at the beginning of this section remain valid and the atomic beam does not split into two parts in a nonuniform external magnetic field.

Let us estimate the right-hand side of inequality (150) for the hydrogen atom in the ground state.

As shown in the Appendix, in this case

$$b = \frac{35}{16} a_B \tag{153}$$

Substituting (153) into the right-hand side of (150), one obtains

$$(1 + 4\alpha^2)^2 \frac{m_e c^2}{\alpha b e} = 2 \times 10^{13} \text{gauss} \tag{154}$$

which is a billion times higher than the value of the magnetic field strength in the Stern and Gerlach experiments. Thus, condition (150) is not satisfied in the Stern and Gerlach experiments.

At the same time, equation (149), and hence also equation (142), with a different value of the coefficient b, shows a possible mechanism leading to a two-valued result in the Stern and Gerlach experiments. This explanation of the Stern and Gerlach experiments is understandable and seems very tempting.

There is another way to explain the results of the Stern and Gerlach experiments within the framework of this theory. If, in addition to the external magnetic field $\mathbf{H}$, there is an external field $\mathbf{G}$, then in all equations (142)-(152) the vector $\mathbf{H}$ should be replaced by the vector $\mathbf{H} + \frac{2e}{\hbar\gamma_e}\mathbf{G}$. In particular, condition (150) must be satisfied not by the modulus of the magnetic field strength $\mathbf{H}$, but by the modulus $\left|\mathbf{H} + \frac{2e}{\hbar\gamma_e}\mathbf{G}\right|$. In this case, the Stern and Gerlach experiments can be explained if we assume that the external field $\mathbf{G}$ in these experiments was sufficiently strong and, at the same time, homogeneous. In this case, the field $\mathbf{G}$ affects the orientation of the intrinsic magnetic moment of the atom, but does not affect the spatial separation of the atomic beam, because it is affected by the gradient of the vector $\mathbf{H} + \frac{2e}{\hbar\gamma_e}\mathbf{G}$ [30].

5. Concluding remarks

In this work, we did not consider such "quantum" phenomena as light-atom interaction, the Compton effect, the photoelectric effect, thermal radiation, and the Lamb-Riserford experiment. They were considered in detail in [18–23] on the basis of the nonlinear Schrödinger equation (5), which, as we have shown, is a direct consequence of the theory [30].

Thus, we have shown that the united Maxwell-Pauli theory [30], considered as a classical field theory, naturally describes and explains all the basic "quantum" phenomena (spontaneous "transitions" and the associated discrete spectrum of spontaneous emission, light-atom interactions, Compton effect, photoelectric effect, thermal radiation, Lamb-Riserford experiment, spin and spin magnetic moment of an atom, etc.) without any quantization and without using additional hypotheses. In particular, to explain and describe the so-called spontaneous "transitions" and spontaneous emission, we did not need such a hypothetical object as QED-vacuum and the related zero-point fluctuations and zitterbewegung: this theory does fine without them.

Note that the theory under consideration is completely deterministic: in it, as Einstein stated, "God does not play dice." This is another fundamental difference between the theory under consideration and the probabilistic theory based on linear wave equations (Schrödinger, Pauli, Klein-Gordon and Dirac) and the Copenhagen interpretation.

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Appendix

Let us calculate the integral (143).

Let's move on to nondimensional coordinates, where the Bohr radius a_B is taken as the scale. In this case

$$b = b_0 a_B \tag{A1}$$

where

$$b_0 = \int\int |\mathbf{r} - \mathbf{r}'||\psi(\mathbf{r})|^2 |\psi(\mathbf{r}')|^2 dV'\, dV \tag{A2}$$

Is the nondimensional parameter, which is calculated using nondimensional coordinates. In particular, for the ground state of the hydrogen atom

$$|\psi(\mathbf{r})|^2 = \frac{1}{\pi}\exp(-2r) \tag{A3}$$

We introduce the function

$$F(\mathbf{r}) = \int |\mathbf{r} - \mathbf{r}'||\psi(\mathbf{r}')|^2 dV' \tag{A4}$$

Then we write (A2) in the form

$$b_0 = \int F(\mathbf{r})|\psi(\mathbf{r})|^2 dV \tag{A5}$$

Obviously,

$$\Delta F = 2\Phi(\mathbf{r}) \tag{A6}$$

where

$$\Phi(\mathbf{r}) = \int \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} dV' \tag{A7}$$

The function $\Phi(\mathbf{r})$ satisfies the equation

$$\Delta\Phi = -4\pi|\psi(\mathbf{r})|^2 \tag{A8}$$

Obviously, as $r \to \infty$, the functions $F(`\mathbf{r})$ and $\Phi(\mathbf{r})$ behave as

$$F(\mathbf{r}) \to r \tag{A9}$$

and

$$\Phi(\mathbf{r}) \to \frac{1}{r} \tag{A10}$$

In addition, these functions have bounded limits as $r \to 0$, which can be easily calculated using (A3):

$$F(0) = 4\int_0^\infty r^3 \exp(-2r)\, dr = \frac{3}{2} \tag{A11}$$

$$\Phi(0) = 4\int_0^\infty r \exp(-2r)\, dr = 1 \tag{A12}$$

Because $|\psi(\mathbf{r})|^2$ depends only on r, then the function $\Phi(\mathbf{r})$ also depends only on r. Therefore, equation (A8), taking into account (A3), takes the form

$$\frac{d^2 r\Phi}{dr^2} = -4r\exp(-2r) \tag{A13}$$

The solution of equation (A13) taking into account conditions (A10) and (A12) has the form

$$\Phi = -\frac{1}{r}(r+1)\exp(-2r) + \frac{1}{r} \tag{A14}$$

Then equation (A6) takes the form

$$\frac{d^2 rF}{dr^2} = -2(r+1)\exp(-2r) + 2 \tag{A15}$$

The solution of equation (A15) taking into account conditions (A9) and (A11) has the form

$$F = -\frac{1}{2r}(r+2)\exp(-2r) + r + \frac{1}{r} \tag{A16}$$

Substituting (A3) and (A16) into (A5), one obtains

$$b_0 = \frac{35}{16} \tag{A17}$$

or, taking into account (A1),

$$b = \frac{35}{16} a_B \tag{A18}$$

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