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Article

Predictive Optimal Control of Hybrid Line Haul Trucks

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Abstract: Fuel consumption, subsequent emissions and safe operation of class 8 vehicles are of prime importance in recent days. It is imperative that the vehicle operates in its true optimal operating region given a variety of constraints such as road grade, load, gear shifts, Battery State of charge (for hybrid vehicles), etc. In this paper a research study is conducted to evaluate the fuel economy and subsequent emission benefits when applying predictive control to a mild hybrid line haul truck. The problem is solved using a combination of dynamic programming with back tracking and model predictive control. The specific fuel saving features that are studied in this work are dynamic cruise control, gear shifts, vehicle coasting and torque management. These features are evaluated predictively as compared to a reactive behavior. The predictive behavior of these features are a function of road grade. The result and analysis shows significant improvement in fuel savings along with NOx benefits. Out of the control features dynamic cruise (predictive) control and dynamic coasting showed the most benefits while predictive gear shifts and torque management (by power splitting between battery and engine) for this architecture did not show fuel benefits but provided other benefits in terms of powertrain efficiency.

Keywords: dynamic program; fuel economy; global optimization; predictive control

1. Introduction

In recent days fuel consumption and emissions are two major challenges to the ever growing trucking segment. Either due to more stringent legislative norms across the globe or due to the global need for more energy efficient operations, the prime focus driving the trucking industry in recent years is alternate energy, cooperative platooning and better emissions management. There is a huge potential to reduce energy consumption, and thereby emissions as the byproduct of energy producing devices. There are a number of ways these kind of problem can be set up but the real challenge is how to robustly frame the problem in terms of the right objectives and addressing all the required constraints which will take into consideration the real world operating scenarios. A number of robust multi-objective non-linear optimal control strategy is analyzed and based on the current requirement and considering all trade offs, dynamic programming is selected to carry out the theoretical behavior of the controller. The following literature highlights the current state of technology. A simple optimization method for BEV is designed by Scalaretta, et. al [1]. Some other control levers are also discussed as direct, indirect methods using spectral collocation, shooting methods etc, [2], [3], [4]. The paper [5] assesses the impact of an eco-driving training program on fuel savings and reduction of CO2 emissions in a well-designed field trial. This methodology includes different types of road sections under various traffic conditions and a systematic method to evaluate the overall and specific impacts of eco-driving. The paper [6] presents a simulation study of various Battery Electric Vehicle (BEV) types to compare their performance when driving on real-road drive cycles to highly optimized eco-driving cycles. The results of the simulation confirmed that eco-driving has a high potential to reduce energy consumption for all types of BEVs. This study also compares the impact of eco-driving on conventional vehicles to comparable BEVs. The authors in [7] implemented strategies to minimize fuel consumption by limiting instantaneous vehicle specific power while maintaining average speed and conserving total distance. The paper [8] explains how a truck driver controls his vehicle with the motive of maintaining a desired velocity while keeping the fuel consumption as low as possible. This is achieved by estimating oncoming operation points of the powertrain and optimal choice of inputs. This information is used as an input in an algorithm for the implementation of a predictive gearshift program and predictive cruise controller. In the paper [9], a novel predictive technology is used to incorporate the cruise set speed along with a gear shift point. The numerical based algorithm used a combination of nonlinear dynamics constraint and objective cost. The mixed integer problem due to the gear choice is solved partially by the outer convexification process. Benefits are shown on real world and artificial routes. The paper [10] explores how information about future road slopes can be used in a heavy truck with an aim of reducing fuel consumption without increasing total travel time. The longitudinal behavior of the vehicle is controlled by determining accelerator and brake levels and also which gear to engage. In the paper [11], a novel predictive control scheme is used for energy management in hybrid trucks driving autonomously on the highway. This scheme uses information from GPS together with speed limits along the planned route to schedule charging and discharging of the battery, the vehicle speed, the gear and decision of when to turn off the engine and drive electrically. The paper [12] presents an optimal strategy for heavy-duty trucks that minimizes fuel consumption in urban ares. This strategy uses an online convex model predictive control strategy that balances a trade-off between reducing braking effort and tracking optimal velocity. The paper [13] introduces a model predictive control algorithm which attempts to reduce the cost of operation of heavy trucks with cruise control application based on road topology information obtained through GPS positioning and 3D maps. The paper [13] proposes implementation of predictive optimal algorithms operating the truck at economically favourable operation points by considering the costs of operation and dynamics of the vehicle. This approach considers GPS positioning and 3D maps for slope, curve and speed limit information of future road segments. The paper [14] proposes a model predictive control method to control the clutch engagement process effectively shorten the torque interruption, thus enhancing the gear downshift quality. The paper [15] explains a way of exploiting vehicular on-board prediction for a limited time horizon and minimizing the auxiliary energy consumption of the electric cooling system through real-time optimization. The paper [16] provides a comparison of three strategies using model predictive control in with the objective of minimizing fuel consumption for a heavy-duty truck. The three strategies are, a time-based formulation that penalizes braking effort in place of fuel consumption, a simplified approach to the first strategy, and a distance-based convex formulation that maintains a tradeoff between energy expenditure and tracking of the coarsely optimized velocity. In the operation of long-haul trucks, fuel costs have a large impact on total cost of ownership. The paper [17] attempts to solve the problem of obtaining a trade-off between minimizing the fuel consumption and simultaneously maximizing the vehicle speed thus eventually decreasing time-related fixed costs. The paper [18] explores learning-based predictive cruise control and the impact of this technology on increasing fuel efficiency for commercial trucks by implementing predictive cruise control model which uses future road conditions and solves for cost-effective course of action. The paper [19] provides a comparison of three strategies using model predictive control in with the objective of minimizing fuel consumption for a heavy-duty truck. Two of these three strategies can then be adapted to accommodate the presence of traffic and optimally navigate signalized intersections using infrastructure-to-vehicular communication. The paper [20] illustrates how optimizing the power split among different energy sources in electric trucks and following distance should be performed to ensure safety, drag reduction and energy consumption. The paper [21] investigates the fuel saving potential of predictive optimal control methods for the engine cooling system in

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conventional trucks. The advantages of this approach are the recovery of brake energy and the balance of energy sources in order to minimize total energy. The paper [22] attempts to reduce ECMS's calculation load by proposing an adaptive simplified ECMS based strategy for a parallel plug-in hybrid electric vehicle. The paper [23] proposes a novel real-time energy management strategy for parallel hybrid electric vehicles. This approach uses adaptive ECMS which sets the time-varying equivalent factor. Hybrid electric vehicles have been known to be a feasible option to reduce fuel consumption and emissions. The paper [24] proposes an adaptive energy management system consisting of off-line and online parts to improve the energy efficiency of a parallel hybrid electric bus. The offline part focuses on the recognizing the precision of driver's driving style based on the hybrid algorithm. The online part incorporates driver's driving style into equivalent consumption minimization strategy.
While a lot of focus is made individually in solving a fuel efficient problem with different constraints but none presented a holistic global optimal problem for a class 8 mild hybrid vehicle. In this work the attempt was to find the global predictive fuel efficient and emissions efficient behavior in terms of predictive control of cruise speed, gear shift, engine ON/OFF coasting and intelligent SOC management for a mild hybrid driven class 8 truck.

2. 1D Longitudinal Vehicle Dynamics

A simple 1 dimensional longitudinal forward torque model is used in this work. A class 8 heavy duty truck with a 48V mild hybrid system configured to run in a parallel power assist mode is used in this paper. The electric machine is connected to the driveshaft via a single gear/clutch assembly at the transmission output shaft. One key objective of using this unique configuration is that line haul applications are the major consumer of diesel fuel and produce the most emissions. It is also very important to explore the emissions reduction while studying key control features since the emissions standards are growing more stringent. Subsections below will discuss briefly about the sizing for different components in such a configuration.

2.1. Internal Combustion Engine

The engine is of a **15L diesel** family which has a power rating of **298-373 kW** and a torque rating of **1966-2508 N.m**. The fuel map is made up manually to mimic an engine efficiency **47%**, as shown in Figure 1. It is a **6 cylinder inline** configured system [25].

2.2. Electrification System

The electrification system in this configuration consists of a motor generator and an energy storage device. Since the chosen configuration is a **mild 48V** hybrid system the Motor of choice is a Borgwarner P2 Off Axis motor which supports a torque range up to **80 N.m**. Figure 1 shows the torque and power characteristics of the chosen motor as a function if its speed in RPM. It is worth noting that beyond **4000 RPM** the torque starts decreasing and power is flattened. The continuous power of the machine used is 15kW with peak torque raging between 50-80Nm.
There are several choices for a 48V energy storage system. In this work a simple configuration from A123 Systems is selected [27]. The battery is moderately sized with **8Ah** capacity and a nominal operating temperature of **25C**. At this settings it can provide continuous power of **15kW**. A simple thermal model for the battery is designed to model the heat loss by the battery. An active cooling system is also in place to increase the rate of heat loss by the battery. Since the battery is small and limited by power, proper heat management of the battery is necessary to utilize its full range of power capability. It is also worth mentioning that the battery is considered to always provide continuous power.

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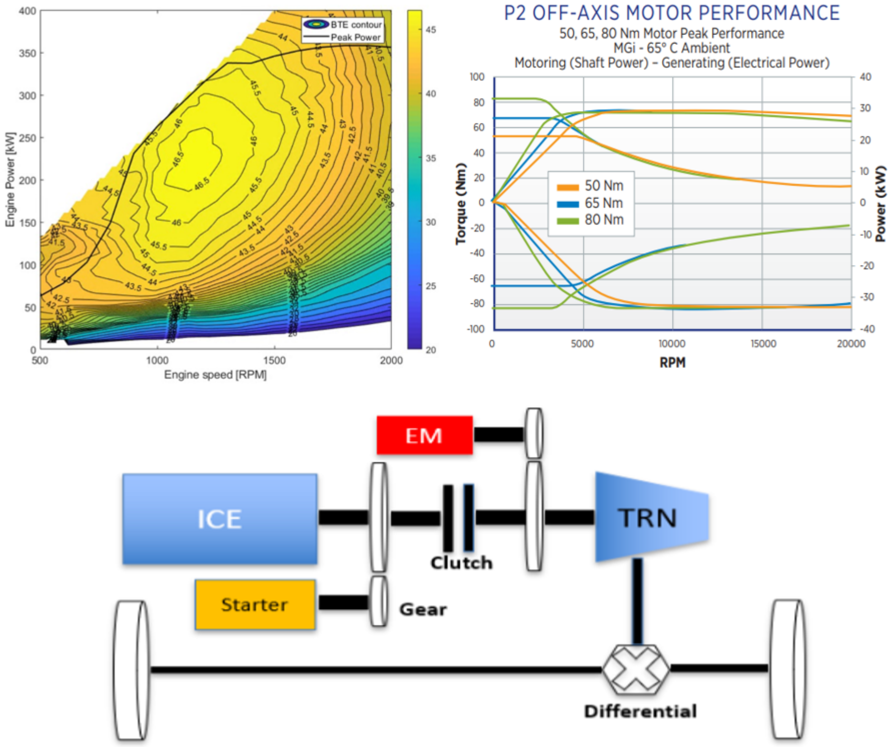

Figure 1. Powertrain Design[25][26]

The SOC is estimated using coulomb counting method [28][29][30] which is very efficient and simple way to calculate SOC.

$$SOC(s) = SOC(s-1) + \frac{1}{v(s)}\frac{I_c(s)}{Q_n}\Delta s \tag{1}$$

It is worth to note here that the SOC state is divided by the vehicle speed. This is done to reformulate all vehicle dynamics in distance domain. This change from time domain is necessary to solve the problem for an independent time solution. This fact will be discussed further in the problem formulation section.

2.3. Transmission System

The transmission system is a 12 speed overdrive system. There are 12 forward ratios and 2 reverse ratios. Only the top 4 gear ratios are used in this work since the velocity profile used is taken from highway drive. The top 4 gear ratios used are [0.776, 1, 1.3, 1.7]**EATON**©[31]. It can support a maximum Gross Vehicle Weight (GVW) of **49895 Kg** and supports a maximum torque of **2508 N.m**. The shift points for the transmission is made up using vehicle speed reference. The way it is derived as a function of vehicle speed and operator throttle so that at cruising speed the transmission stays at top gear. It is also done in a way to keep the engine speed within the best operable BTE region.

2.4. Drive line & Chassis

The chassis is from a typical line-haul application. A Gross Vehicle Weight (GVW) of **65000 lbs** is used in this study which fits nicely into the component requirements as well as a standard load carrying measure. The number of wheels are **18**.
A rear axle ratio of **2.64** is used which gives a lot of low end torque propagation at startup and also does not let the engine operating point go, too high at top gear. The optimization result is strongly coupled to these chosen components. Specifically the chassis components are key players in deciding the vehicle dynamics and optimal fuel numbers since they

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impact the vehicle speed directly. Table 1 shows the base vehicle parameters which are used in the simulation.

Parameter	Symbol	Value
Vehicle Mass	m	32.5 tonne
Effective mass in cruise gear	m_e	32.52 tonne
Wheel Radius	R_w	0.5m
Aerodynamic drag coefficient	$c_d A_f$	5
Rolling resistance coefficient	c_r	0.005
Air Density	ρ_a	$1.184 kg/m^3$
Gravitional acceleration	g	$9.81 m/s^2$
Engine Maximum Power	$P_{E_{max}}$	$325 kW$

Table 1. Vehicle Parameters

2.5. Force Balance

The different forces at the wheel is summed up and then divided by the equivalent vehicle mass to get the acceleration. Finally the acceleration is integrated to get the velocity of the vehicle which is used to feed back to the upstream controllers for a full closed loop dynamics.

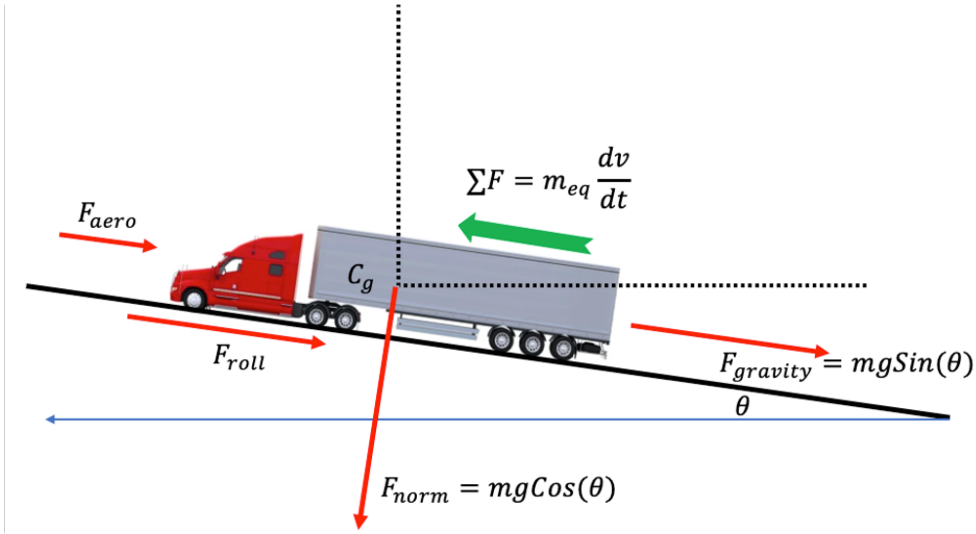

Figure 2. 1-D Longitudinal Forces on a Vehicle

The gravitational force as a function of the road grade is given by equation 2.

$$F_{drag} = m * g * sin(\theta) \tag{2}$$

where, θ is the road grade in *radians*

The aerodynamic drag is a direct function of vehicle speed and is given by equation 3

$$F_{aero} = \frac{1}{2}\rho * A_f * C_d * v^2 \tag{3}$$

where, A_f is the vehicle frontal area, C_d is the Drag Coefficient & ρ is the air density.

The road normal force is a function of road grade and is given by equation 4

$$F_{norm} = m * g * cos(\theta) \tag{4}$$

where, θ is the road grade in radians

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Hence, using the force balance principle and rearranging, the vehicle speed is given by equation 5

$$\nu = \int \frac{1}{m}[F_{tractive} - F_{drag} - F_{aero} - F_{norm}]dt \tag{5}$$

The optimal problem is solved in distance domain since the time in this solution is not fixed. Depending on the speed modulation the time for the entire route will change and hence the problem is changed from a fixed time problem to a fixed distance problem. Hence we convert equation 5 as

$$\nu = \sqrt{(2 * \int \frac{1}{m * \nu(s)}[F_{tractive} - F_{drag} - F_{aero} - F_{norm}]ds)} \tag{6}$$

where, the initial condition of the integration is Equation 6

$$v_0 = \frac{1}{2}v_0^2 \tag{7}$$

It is worth noting here that equation 6 makes vehicle speed a state of the system dynamics. The assumptions made throughout this section while designing the system dynamics are

- Rotational Compliance & Coupling Dynamics between components are not considered for the purpose of this research.
- Losses are considered constant instead of a function of any dependent variables.
- Map based logic is used in every calculation possible to eliminate the need of complex analytical design.

Since the research is based on energy level analysis the above considerations are justified. Hence the 5 continuous states are **Vehicle Speed**, **Vehicle Position**, **Engine Fuel Quantity**, **Battery SOC** & **Battery Temperature**. There is also another state which is the gear number but this is a discrete integer type state hence making the problem suitable for a mixed integer type non-linear problem. The control inputs are **Engine Throttle, Clutch Command, Brake Command,** & **Gear Shift Request**.
Power split between the Internal Combustion Engine and Electrical Energy Storage is decided by a simple splitting logic where the battery does what ever it can and the rest is provided by the engine. Similarly for regeneration the battery absorbs energy to its SOC based limits and the rest is consumed by the engine as motoring torque.

3. Problem Approach

A multi-objective minimization problem is solved in this work for a mixed integer type non-linear dynamical system. The objective is to achieve a fuel efficient solution based on "a-priori" knowledge of the road elevation for the entire route. Since better fuel efficient operation also indicates a better engine operating point in the Brake Thermal Efficiency (BTE) contours, we also anticipate to improve the emission. The reduced order vehicle dynamical model as described in the above section is used to solve the problem using 4 individual control levers. The controls are cruise set speed, clutch disengagement with engine either ON or OFF, dynamic gear shift and dynamically torque split (splitting power between engine and battery). A weighted sum of total fuel consumed and total trip time is used as the cost function. Rate of change of battery temperature is also added as a objective in the cost function to make sure the battery is operated in its most optimal operating zone. Equation 8, shows the cost function.

$$\min_{\forall u^* \in \mathcal{U}} \sum [\frac{\alpha}{\omega_{fc}}(\frac{\dot{m}_f(u)}{\mathcal{V}_s(u)}) + \frac{1-\alpha}{\omega_{tt}}(\frac{1}{\mathcal{V}_s(u)}) + \frac{\beta}{\omega_{bt}}(\frac{\dot{T}_{batt}(u)}{\mathcal{V}_s(u)})]\Delta x \tag{8}$$

where, $\dot{m}_f$ is the fuel rate, V_s is the vehicle speed, α is the tuning coefficients for fuel consumed and trip time, ω_{fc} & ω_{tt} are normalizing weights to transform the units in the

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same domain and Δx is the integration step in distance domain. $\dot{T}_{batt}$ is the rate of change of battery internal temperature and β is the independent tuning weight.
The dynamics in time domain is converted to distance domain by dividing the differential equations by Vehicle Speed $(v(s))$. Inclusion of time in the cost function is a measure of drivability. It is not acceptable to achieve a fuel efficient solution if the time constraints are not met. In other words the vehicle cannot take more time to cover the route, to save fuel and emissions.

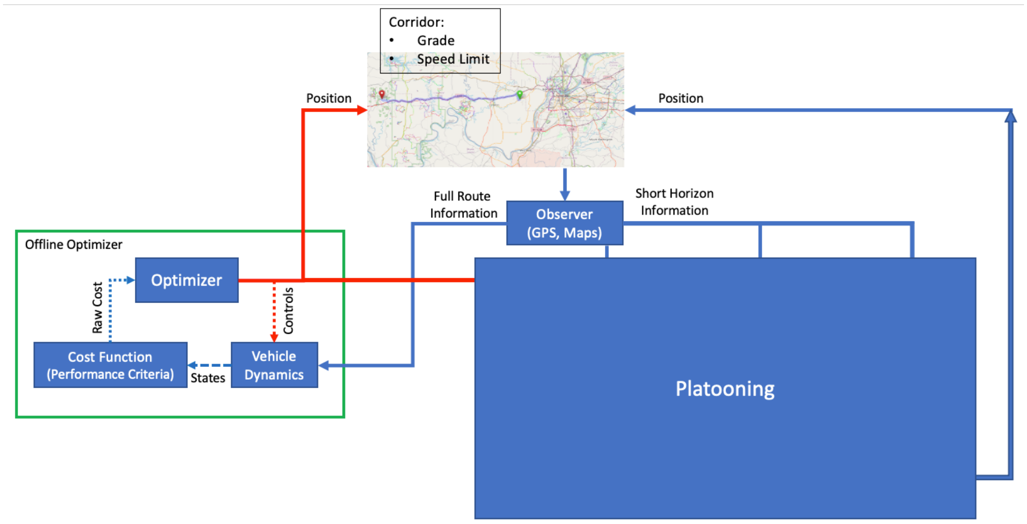

Figure 3. Overview of the problem architecture.

Figure 3, shows the high level architecture of the problem. The platooning problem is solved by the authors in another paper in a two step problem approach. In this work only the offline optimizer part of the problem is solved as detailed in the below sections. The look ahead road grade is fetched from the corridor information module, where it is assumed that the full route information is available. Now that we know the details of the problem and how we are going to approach those, we will lay down the individual problem in some more details. There are 4 control factors in this work which are implemented in a cascaded approach by introducing one control parameter at a time and then finally solving the problem with all the control parameters. The problem has 4 states $x(.)$ =[Vehicle Speed, Transmission Gear Number, Clutch State and Battery SOC], 4 controls $u(s)$ = [Throttle, Clutch Command, Gear Shift Command, Power Split Ratio]. Engine Speed is another derived state which is not explicitly needed by dynamic programming. Position in the route is another exogenous state which is used in the optimal model. Constraints that are modelled in this work are both soft and hard. Vehicle speed is limited between an absolute maximum and minimum threshold as a hard constraint. A soft root mean square type, second order norm constraint is also used which is based on the difference between baseline speed profile and the optimal speed profile. Additional constraints for coast problem is the duration and frequency of coast events. Since the predictive behavior can increase or decrease the vehicle speed from the cruise set speed, it is required to appropriately set the constraints on vehicle speed. Similarly the engine off coast can also increase speed beyond reasonable limits if not monitored correctly. Hence, there are vehicle and engine speed limits set up accordingly while solving the problem.

3.1. Dynamic Program Formulation

The problem is solved using dynamic programming and back tracking the cost, which is a potential solver for accurately solving global optimal problems with non-linear system dynamics. Dynamic programming based control problem is well established by Guzella, et.al [32],[33],[34]. Since dynamic programming has knowledge of the complete route and

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solves the problem in a backward fashion it is guaranteed to provide a global optimal solution.
Figure 4, shows the ***Cost-To-Go*** calculation and the selection of control variables. The iteration is done starting from the end as per dynamic programming principle [35][36][37]. Starting from the end of the route the cost is calculated at each position step which in our work is set up to be 20m which is selected based on a resolution study for the optimization. It is also worth noting that **20m** corresponds to **0.7s** at **65mph** isochronous speed. This gives a good resolution for capturing any vehicle dynamics in terms of finding the optimal solution. The corresponding control levers are chosen at each position step for which the calculated cost is minimum.
The output of this solver is the optimal value for throttle, clutch, power split and gear shift. This throttle control is used as input to the closed loop system to generate the optimal speed profile using a model predictive controller. At each step the minimum cost is obtained and added to the cost-to-go value for the forward closed loop control.

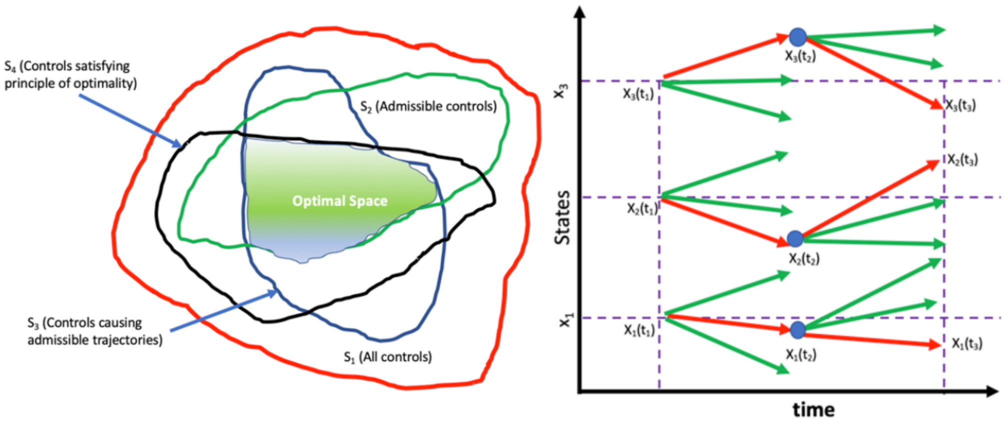

Figure 4. Illustration of Dynamic Programming Solver.

4. Detailed Problem and Subsequent Simulation Results

In this section the individual control levers are formulated one at a time and then with each subsequent problem one additional control is added. This setup help understand the problem better and the contribution and interaction of each added control factor. The route profile chosen for this work is a 86mile long section of I64 which has a good combination of road grade distribution.

4.1. Dynamic Speed Management

The first control lever used is **Cruise Set Speed**. The idea here is to dynamically modulate the cruise set speed around 65mph isochronous speed as a function of future road grade knowledge. This is similar to adaptive cruise control but is based on road grade information. Equation 9, shows the cost function as defined earlier,

$$\min_{\forall u^* \in \mathcal{U}} \sum [\frac{\alpha}{\omega_{fc}}(\frac{\dot{m}_f(u)}{\mathcal{V}_s(u)}) + \frac{1-\alpha}{\omega_{tt}}(\frac{1}{\mathcal{V}_s(u)}) + \frac{\beta}{\omega_{bt}}(\frac{\dot{T}_{batt}(u)}{\mathcal{V}_s(u)})]\Delta x \tag{9}$$

subject to,

$$\begin{aligned} \dot{x}(s) &= f(x(s), u(s), w(s)), \\ y(s) &= g(x(s), u(s), w(s)), \end{aligned} \tag{10}$$

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and, non-linear constraints,

$$\begin{aligned} v_{min} \leq & v(s) \leq v_{max}, \\ g_{min} \leq & g(s) \leq g_{max}, \\ soc_{min} \leq & SOC(s) \leq soc_{max}, \\ \omega_{eng,min} \leq & \omega_{eng}(s) \leq \omega_{eng,max}, \\ \tau_{eng,min}(\omega_{eng}) \leq & \tau_{eng}(s) \leq \tau_{eng,max}(\omega_{eng}), \end{aligned} \tag{11}$$

There are three states here $x(.)$ =[Vehicle Speed, Transmission Gear Number, Battery SOC], 1 control $u(s) = Throttle$ and the primary output $y(s)$ =[Optimal Vehicle Speed Target Trajectory. The search space is discretized between a minimum and maximum set of points for all these states. Engine Speed is also a state but it is a dependant state of the vehicle speed and hence it is not needed by the solver for the control problem. The engine speed is given by Equation 12,

$$\omega_{eng}(t) = \frac{v(t) * w_{rad}}{\nu * RAR} \tag{12}$$

where, ω_{eng} is engine speed in rad/s, ν is "gear ratio", RAR is Rear Axle Ratio and w_{rad} is wheel radius. Total time is also included in the objective cost to make sure that total time remains within baseline limits. So, if the truck without predictive control takes **"X" seconds** to cover the route, the optimal control should also be close to that **"X" seconds**.
The output of this solver is the optimal throttle value. This throttle control is used as input to the closed loop system to generate the optimal speed profile based on traditional model predictive control. The vehicle will no longer target a constant **65mph** cruise set speed in this case as the optimal throttle will let the vehicle dynamically increase speed and slow down in the route based on look ahead grade information. Since dynamic programming with back tracking is computationally heavy, parallel computation using multi core system is used where ever possible. One such situation is when the stage cost is calculated for discretized points. During this step the full set of points is divided into smaller sets and the problem is solved for those smaller sets in different cores of the system CPU.
Table 2 shows the key metrics for the Cruise Speed modulation problem. It shows an absolute fuel economy of 3.02% with a change of 0.07% in trip time. There is a reduction of 1% of aerodynamic work and 2.56% reduction in total cycle work. The brake thermal efficiency improved by 0.18%. Negative work reduction is mostly due to engine braking reduction. The % improvement numbers are against the baseline simulation where the vehicle cruise set speed is **65mph**.

Metrics	Units	VS	Δ
Fuel Consumed	Kg	26.4984	-0.8
Fuel Economy	mpg	9.86	3.02
Trip Time	s	4602.8	0.07
Aerodynamic Work	kWh	89.26	-1.01%
Cycle Work	kW	142.34	-2.56%
BTE	%	44.95	0.18%
Negative Work	kWh	-24.1	-18.66
EONOx	Kg	0.4104	-6.41

Table 2. Optimal Metrics for the Dynamic Speed Management Problem

Key observations from this problem is that the predictive cruise control modulates speed around uphills and downhills. Specifically it increases speed before entering a hill and decreases the speed before entering a down hill. In energy domain it is similar to gaining energy when it is easy to do in the flat section and then utilize the kinetic energy gained, to cover the uphill section to overcome the grade drag. Similarly during the downhill it is efficient to slow down a bit before entering the downhill to save energy (fuel) since it is expected to increase speed during the downhill and will have to brake thereby wasting energy which is gained at the expense of fuel. The main objective here is to reduce the negative work in the form of reduced engine braking. Speed modulation during the flat sections are not very common. During the flat section the truck follows the usual route speed limit which is 65mph or 29m/s in this work. The emissions are also improved as a passive component due to the engine operating point change. Now that the engine operates at a more better BTE zone consuming less fuel, we observed a better NOx numbers. The Normalized NOx reduction from baseline simulation is around 5% in this optimal problem formulation.

4.2. Dynamic Speed & Coast (Engine Idle + Engine Off) Management

The vehicle dynamics and the cost function for this problem is similar to what is used for speed management. An additional state and control parameter is added to the speed problem. To manage coast we need to know the current clutch state and then command the clutch to engage or disengage. One additional constraint here is the duration and frequency of coast events. Even though speed constraints will take care of how long coast events can be but there is a need for a constraint on how frequent the coast events could be. Hence a penalty on the frequency of clutch state change is added. This will prevent frequent coast events and thereby reduce oscillations in operation. Table 3 shows the metrics for the problem where only coasting is used as a control lever. 1.3% compensated fuel savings is achieved by using the coast problem in the engine off mode and around 0.9% by keeping the engine on while coasting. Most of the benefit is achieved by the reduction of cycle work, aerodynamic work and a reduction in negative work. There is a also reduction of Engine Out NOx in the order of 1.8% for both the engine on and engine off coasting scenarios. The difference in NOx reduction is not significant since the engine is tuned for ultra clean performance.

Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	27.03	-0.26	26.92	-0.37
Fuel Economy	mpg	9.66	0.98%	9.7	1.39%
Trip Time	s	4604.1	0.09%	4602.6	0.06%
Aerodynamic Work	kWh	89.7	-0.52%	89.1	-1.19%
Cycle Work	kW	144.97	-0.76%	144.76	-0.91%
BTE	%	44.87	0.09	44.99	0.21
Negative Work	kWh	-27.21	-8.17%	-26.1	-11.91%
EONOx	Kg	0.4304	-1.82%	0.4297	-1.98%

Table 3. Comparison of key metrics for the Coast Management problem only with Engine Idle and Engine Off Condition. The Δ% is the comparison with the baseline simulation

Table 4 shows the key energy domain metrics for the speed and coast problem together for the entire I64 portion of the route. This in an important metrics to look at since the negative work done is the loss in energy which is gained at the expense of either fuel or electric energy. Since dynamic programming did not show the reason why the fuel benefits are occurring it is important to compare the reduction in negative work which clearly indicates where the fuel economy is coming from along with the improvement in engine BTE. Analyzing the distributed speed and coast problems alone, it is evident that when speed and coast problems are solved together the fuel saving benefits are additive. The speed and coast problem solved together achieved 3.6% for the Engine idle coast case and 4.4% for the Engine off coast scenario. It is also worth noting that the engine off case benefits are also additive.

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Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	26.3345	-0.96	26.14	-1.15
Fuel Economy	mpg	9.92	3.64%	9.99	4.41%
Trip Time	s	4604.2	0.1%	4603.7	0.08%
Aerodynamic Work	kWh	87.52	-2.94%	87.53	-2.93%
Cycle Work	kW	141.92	-2.85%	140.25	-3.99%
BTE	%	45.09	0.31	44.89	0.11
Negative Work	kWh	-21.76	-26.56%	-22.12	-25.35%
EONOx	Kg	0.4021	-8.28%	0.4002	-8.71

Table 4. Comparison of key metrics for the Vehicle Speed and Coast Management problem with Engine Idle and Engine Off Condition. The Δ% is the comparison with the baseline simulation

Figure 5, shows the time domain evolution of various signal for the optimal problem. The plot shows a section of the I64 route. Subplot 2 shows the gear number for the two scenarios and it shows near similar behavior which indicates similar engine operating conditions in the torque curve. Subplot 3 shows the difference in the vehicle speed for the two different problem along with the reference speed target generated by the optimal solver. Subplot 6 shows the grade power for the two optimal problems (Engine Off and Engine Idle). The plots are identical as expected since the grade is same for the two problem. Subplot 7 in Figure 5, shows that the coast zones for engine off problem are not exactly similar to the coast zones for the engine idle problem. This indicates that predictive coast with engine off and engine idle are two separate problem in terms of optimality. The black vertical boxes highlight are difference in behavior of between the two problem.

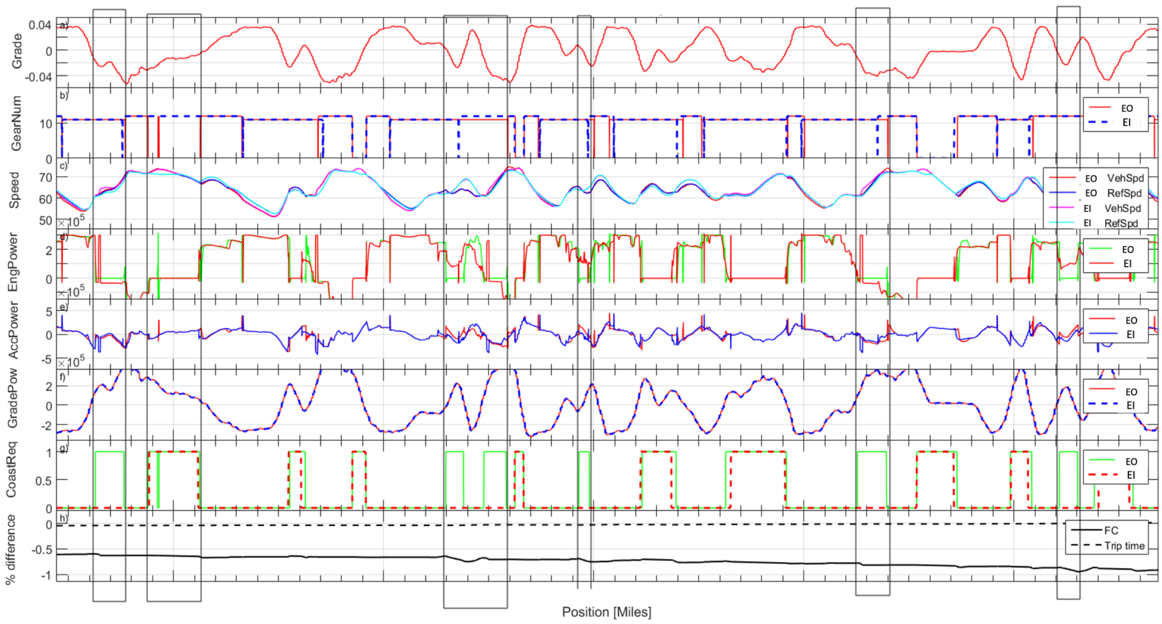

Figure 5. Performance Results for Optimal Solution compared to Baseline rule based control. The plot is zoomed version of stitched version of different sections in the route.

4.3. Dynamic Speed, Coast (Engine Idle + Engine Off) & Gear Management

In this third problem we have included predictive gear control as a third lever along with speed and coast controls. The objective function remains same with the addition of an extra control input which is the gear shift command. Gear shift command can take 3 possible states (up shift, hold gear & down shift). The objective here is to find if shifting the gear with the knowledge of road grade in the route will help achieve any fuel benefits

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and/or drivability improvements. Analytically it is not expected to gain fuel benefits unless the fuel maps are tuned to include high Brake Thermal Efficiency (BTE) zones for lower gears. It is also worth pointing out here that the engine efficiency maps used for this work has its peak BTE zone around a range of engine speed which corresponds to top gear ratio of the transmission system. This means that if we get down to a lower gear the system will compromise on fuel savings. Hence to achieve the minimum cost of fuel savings the system will not down shift. This is also seen with the optimal solution. This problem did not provide any fuel benefits but neither did penalize fuel savings. It was a hard problem to tune for achieving at least the same fuel economy as with the previous problem and with this tuning it is observed that the system down shifts a little early while in positive grade, and stays at a lower gear a little more after coming out of positive grade. The interaction between gear shift for this problem and clutch disengagement for the coasting problem is handled through the addition of appropriate penalties. On the performance side it is observed that with dynamic gear shifts the truck was able to maintain a higher speed in the uphill sections. This is illustrated in Figure 6. The red plot is the vehicle speed for the optimal solution and it shows clear reduction in lug-back in the up hill section.

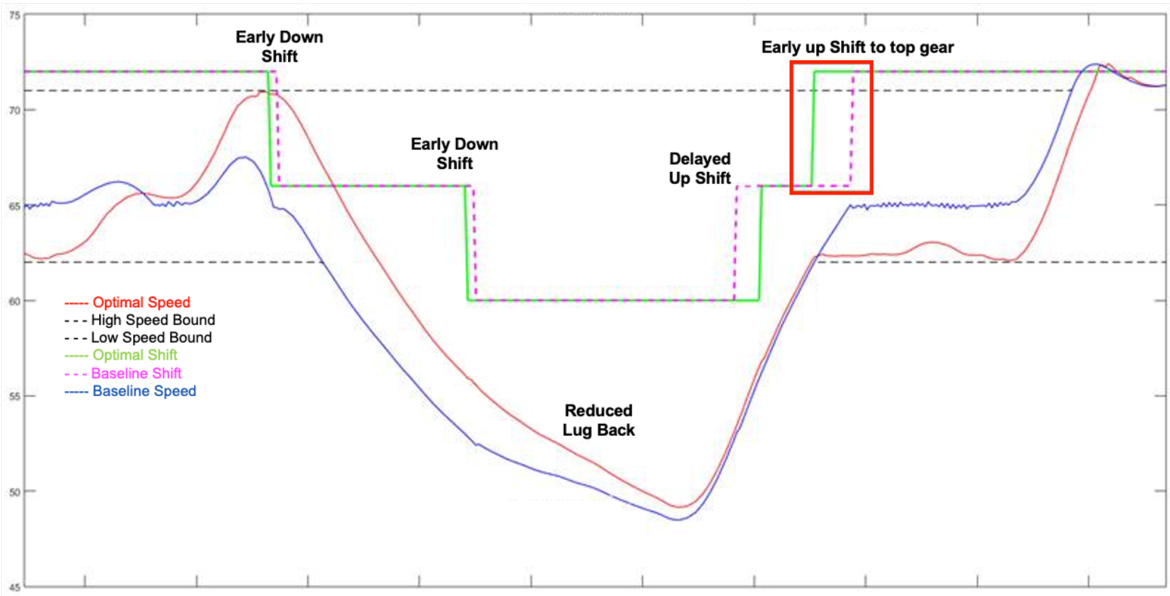

Figure 6. Predictive Optimality in Gear Management. The problem shows the predictive gear shift behavior in one of the up-hill section.

On an average a $2mph$ less speed reduction is achieved in all the uphills.

Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	26.34	-0.95	26.1	-1.19
Fuel Economy	mpg	9.92	3.62%	10.01	4.57%
Trip Time	s	4597.3	-0.05%	4605.3	0.12%
Aerodynamic Work	kWh	90.29	0.13%	87.54	-2.92%
Cycle Work	kW	142.021	-2.78%	140.218	-4.02%
BTE	%	45.11	0.33	44.94	0.17
Negative Work	kWh	-22.3	-24.74%	-23.93	-19.24%
EONOx	Kg	0.41.	-6.48%	0.4082	-6.89

Table 5. Comparison of key metrics for the Vehicle Speed, Coast & Gear Management problem with Engine Idle and Engine Off Condition. The Δ% is the comparison with the baseline simulation

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Table 5 shows the metrics for this problem with engine idle coasting as well as engine off coasting. As discussed earlier we notice insignificant improvement in compensated fuel economy. There is less engine out NOx reduction as compared to the Coast only problem. This is due to the increase in lower gear operation. The impact of NOx improvement is not at all substantial to justify that addition of predictive knowledge for gear management can improve NOx production in the system. In fact for some tuning cases it increased the NOx production a bit due to the gear operation at a lower gear. This is analytically justified as well since a lower gear operation means better performance rather than a better BTE zone operation. The good observation is that even with the fuel efficient tuning for the optimal parameters it did not penalized NOx production drastically.

4.4. Dynamic Speed, Coast (Engine Idle + Engine Off), Gear & Torque (Power Split) Management

Lastly the problem is solved for dynamically varying torque demand between the engine and battery system. This is done base don the predictive knowledge of the road grade and the battery system temperature. Analytically it is not expected to provide significant fuel savings since the electrification system is quite limited in power.
In this problem there are no additional states involved but there is an extra control input for the dynamic solver. This new control input is power split ratio. The way this ratio is defined in the problem is by discretizing the entire hybrid power range including the charge and discharge limits. Hence the electric power range of -20kW to +20kW, is discretized with equal grid size. The resolution of the grid size matters since it impact the results based on how dynamic and responsive the particular control input is.

Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	26.34	-0.95	25.995	-1.30
Fuel Economy	mpg	9.92	3.63%	10.05	5.00%
Trip Time	s	4597.2	-0.06%	4601.5	0.04%
Aerodynamic Work	kWh	90.298	0.14%	89.21	-1.07%
Cycle Work	kW	141.89	-2.87%	141.67	-3.02%
BTE	%	45.07	0.29	45.59	0.82
Negative Work	kWh	-22.78	-23.12%	-23.97	-19.1%
EONOx	Kg	0.4	-8.76%	0.4023	-8.23

Table 6. Comparison of key metrics for the Vehicle Speed and Coast Management problem with Engine Idle and Engine Off Condition. The Δ% is the comparison with the baseline simulation

Table 6 shows the key metrics for this problem. As discussed earlier there is no substantial increase in fuel benefits. Figure 7 shows the total time spent in each gear for the individual problems. The plot enumerations are Bsln: No Optimal Behavior, VS: Dynamic Speed, VSC: Dynamic Speed & Coast, VSG: Dynamic Speed & Gear, VSCG: Dynamic Speed, Coast & Gear, VSCGP: Dynamic Speed, Coast, Gear & Power Split. This metrics provides an understanding of which gear is predominantly being exercised by each problem. Since downshifting to a lower gear will take the operation outside of the maximum BTE zone it is not expected that the gear problem will try to shift down for a better fuel efficient solution. Hence for this kind of BTE map a more fuel efficient solution is practically not possible. Gear problem can expected to provide a better drivability by helping to reduce lug backs in heavy hill. It is seen in Figure 7 that all the problem types are trying to increase top gear operation since fuel saving will be more due to the BTE contour positioning. It is also interesting to observe that the problems with gear management is reducing the time in (top-1) gear. The coast management problem alone is the only problem which is not able to increase top gear operation much as compared to the other problems. This is due to the fact that with coast management problem since the vehicle is not predictively modulating speed and gear the speed drops are more which causes the gear to shift down more.

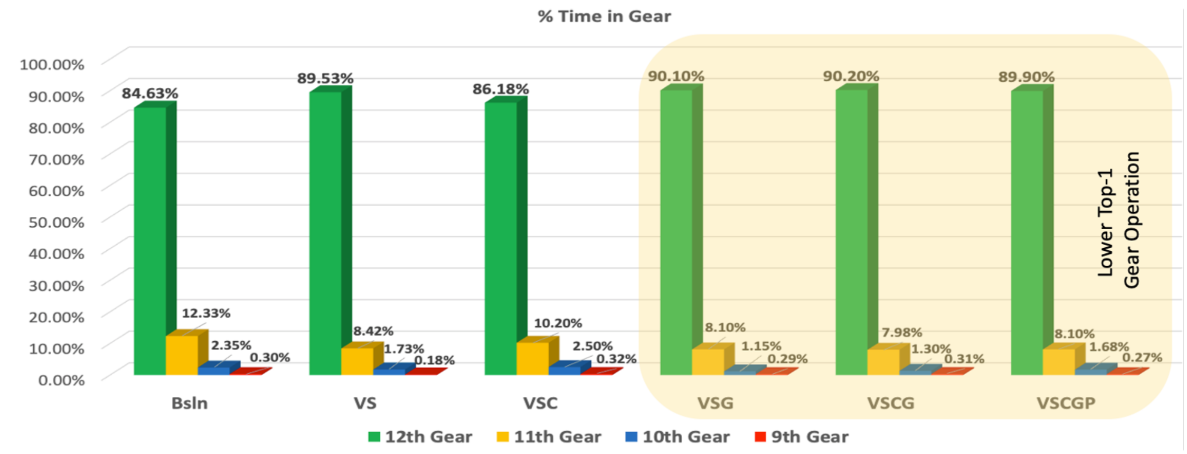

Figure 7. % Time in top 4 Gear for each DOE. The comparison has to be between the top 2 gears. Predictive gear tries to operate more at a lower gear while Fuel Economy tends to operate at a higher gear.

Figure. 8 shows the key metrics for the complete problem with Engine idle coasting condition. The plot shows the absolute fuel economy for each problem when compared to baseline case. The orange plot is the % change in trip time. The green plot is a measure of relative fuel economy which is the difference between the absolute fuel economy and the % change in trip time. This is done to make sure that negative trip time is compensated accordingly. The plots also shows the reduction in cycle work and the improvement in Brake Thermal Efficiency in each case. Figure. 9 shows the reduction in aerodynamic drag and the reduction in Engine Out NOx numbers. The complete problem achieved a 3.7% fuel economy and a NOx reduction of 8.3%. The corresponding BTE improvement in this case is much lower and is close to 0.3%.

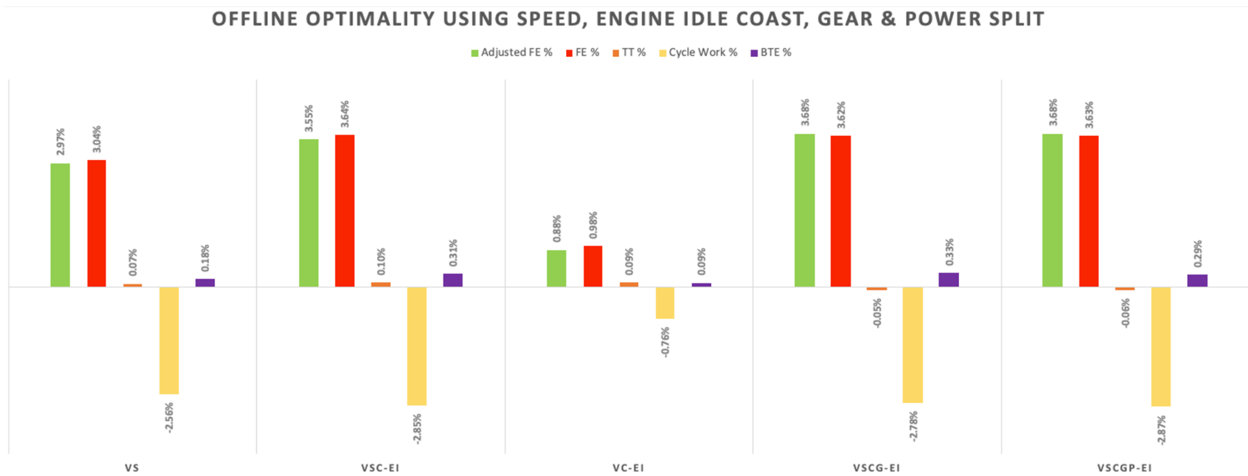

Figure 8. Key metrics showing the comparison of benefits along with Cycle work and BTE for the complete set of problem including Speed, Coast, Gear and Power Split Management. This scenario is with with Engine Idle Coast.

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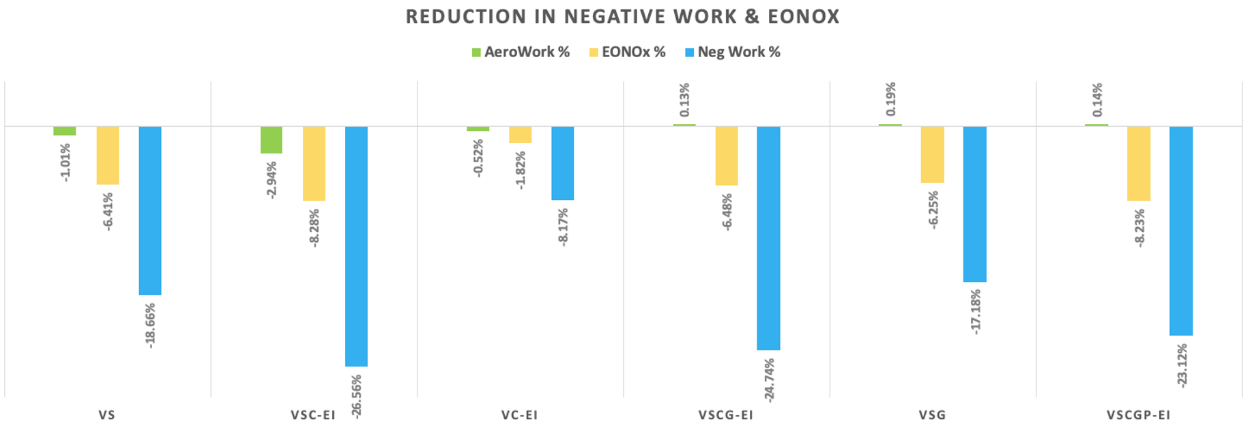

Figure 9. Reduction in Aerodynamic Work along with associated EONOx Reduction. The last bar plot shows the reduction in Negative Work which includes Engine braking, Motoring Losses and Service Braking.

Figure 10 and Figure 11 captures the detailed metrics for all the problems stacked up for the engine off coasting case. Overall an impressive 5% fuel economy is achieved with the predictive features working together with engine off coasting condition. This benefit is mostly contributed by 0.8% improvement in BTE, 3% reduction in cycle work and 19% reduction in negative work. There is also an associated NOx reduction with each control levers. NOx reduction was 8.75%. This is due to the fact that engine BTE has improved.

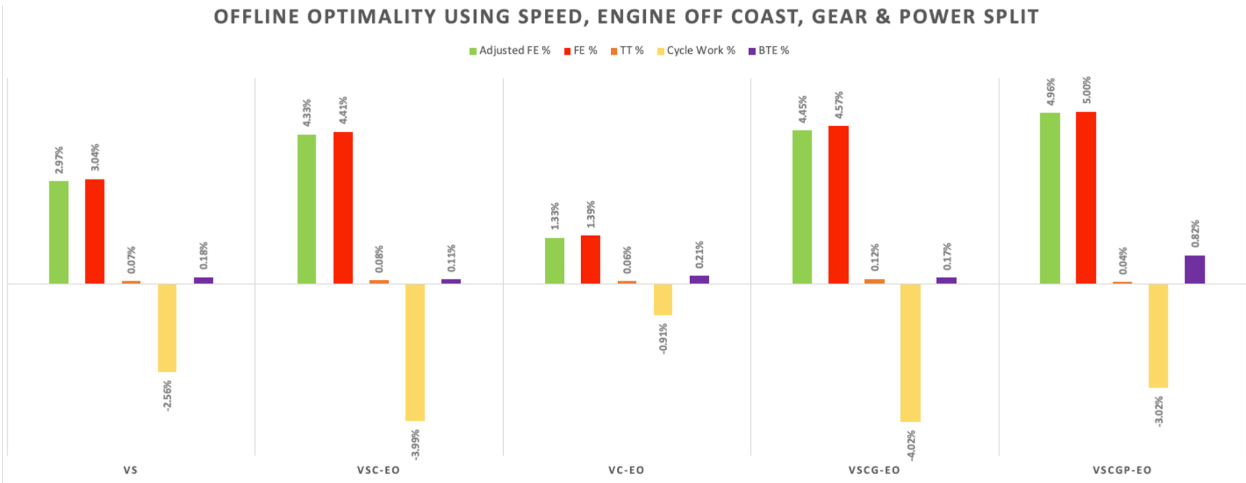

Figure 10. Key metrics showing the comparison of benefits along with Cycle work and BTE for the complete set of problem including Speed, Coast, Gear and Power Split Management. This scenario is with with Engine Off Coast.

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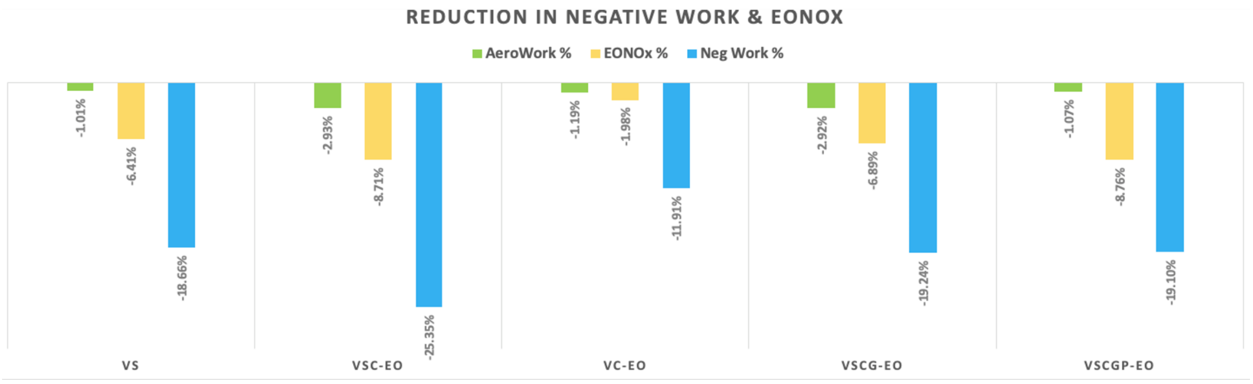

Figure 11. Reduction in Aerodynamic Work along with associated EONOx Reduction. The last bar plot shows the reduction in Negative Work which includes Engine braking, Motoring Losses and Service Braking.

Figure 12 shows the coast metrics for various problems. The bars show the percentage of time in coast by each problem and the line plot shows the number of coast events in each problem. Engine off and engine idle coast metrics show similar behavior in terms of time in coast and number of coast events. Interestingly the coast alone problems has some good amount of coasting events but could not provide a lot of benefit simply because of the fact that the net fuel economy is not related to coast events alone but is a combined factor of multiple scenarios including cycle work reduction, negative work reduction, BTE improvements and aerodynamic drag reduction. Further a couple of very large coast events were also observed which may not be feasible in real environment due to physical engine operation restrictions. Nevertheless, the metric gives an overview of the coast event distribution across various problem set.

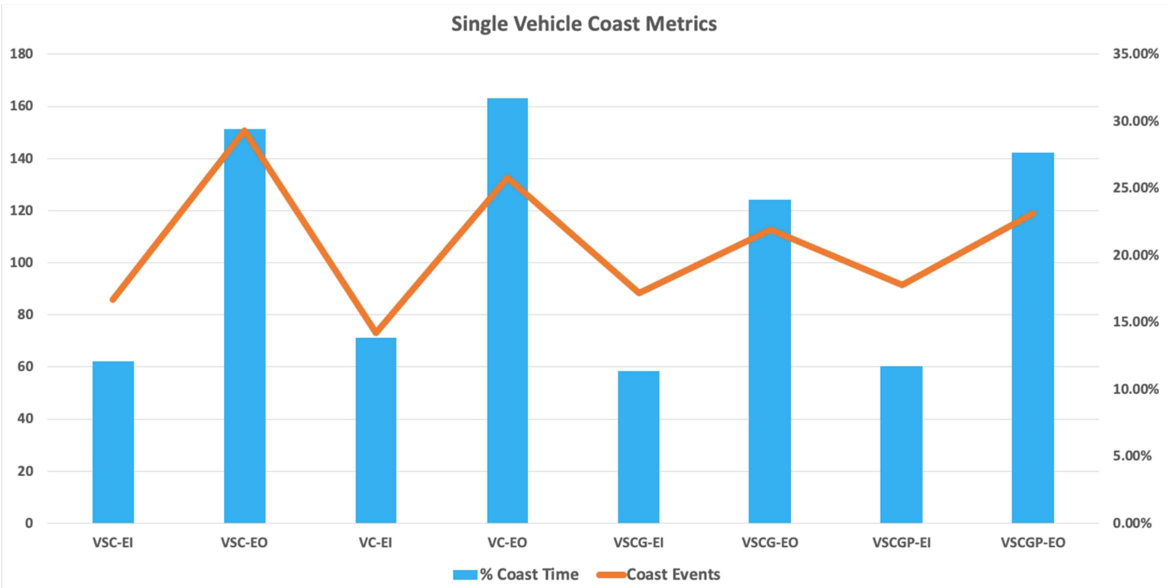

Figure 12. Coast Metrics for all combination of problems with Coast formulation. The bars show the % time in Coast for each problem and the plot shows the number of coast events.

Figure 13 shows the % change in aerodynamic work as a function of % improvement in fuel economy. There is no concise co-relation between the the two in terms of the different problems. This is because the trip time is balanced with baseline trip time. Hence the overall increase/decrease in speed tends to balance each other. The dynamic speed+coast problem show typically more aerodynamic work reduction. This is due to the fact that they

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are only slowing down the vehicles whenever possible by going to coasting along with speed modulation. 421 422

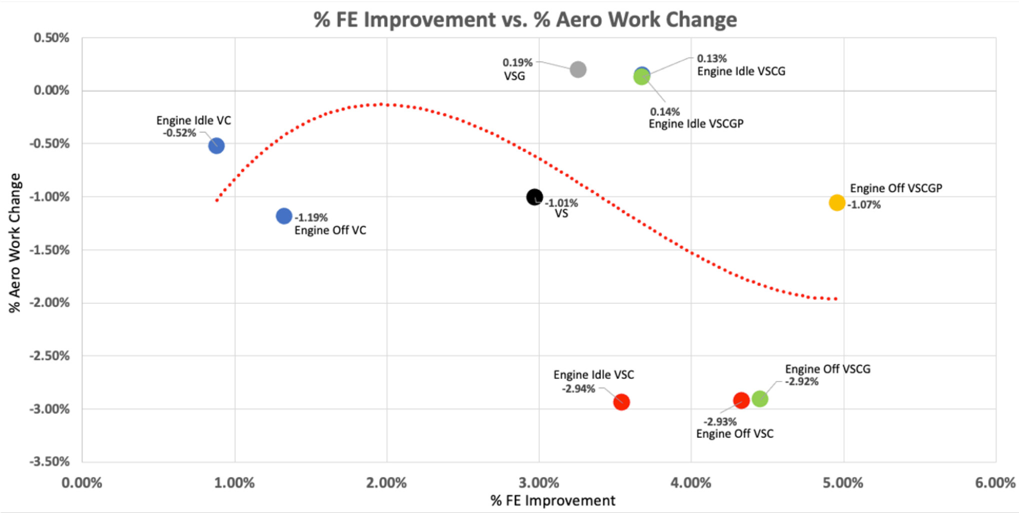

Figure 13. % Change in aerodynamic work as a function of % fuel economy improvement for various optimal problem setups.

Figure 14 shows similar trends as compared to the cycle work reduction. It shows the metric is correlated to speed modulation. Since negative work is due to the speed band operating at regions beyond the engine braking limits, with the coast alone problem vehicle speed is not intentionally modulated to a higher or lower value at the expense of the fuel hence the reduction is less as compared to the baseline results. In this case the speed modulation typically follows the baseline numbers. The other problems have a lesser spread with the engine idle problem as compared to the engine off problem. It is noted that there is a linear trend in fuel economy and negative work reduction for all problems except the problems with the addition of the gear modulation. 423 424 425 426 427 428 429 430 431

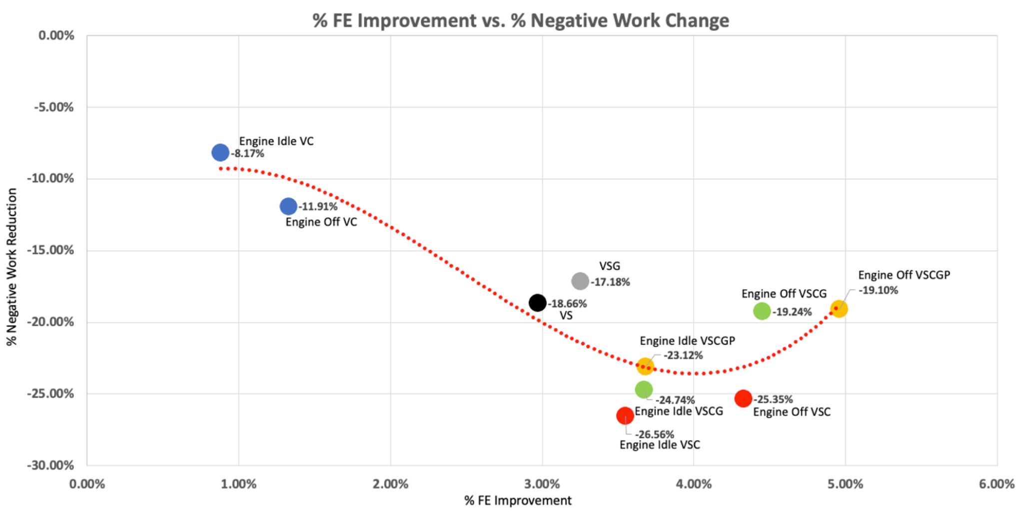

Figure 14. % Reduction in negative work as a of function of % fuel economy improvement for various optimal problem setups.

Figure 16 shows the reduction in total cycle work of the engine as a result of the predictive knowledge of the road grade. The bubbles shows the reduction in Engine Out NOx as a function of the reduction in % Cycle Work by the engine. Though it can also be seen that the reduction is more in case of Engine off case which is due to the fact that 432 433 434 435

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the engine idle work is taken away in this case. In case of the Engine idle scenario the reduction for all the problems are around -2.75% while the problem with engine idle coast only is around -0.76% while with the Engine off scenario the problems with vehicle speed along with coast, gear and power split provides added reduction as compared to vehicle speed alone problem only. This clearly demonstrate the fact that the problem with engine idle and engine off case are completely different in behavior and cannot be determined by interpreting zero fuel consumption by engine idle problem during the idle sections. This is an important observation. Similar trends are also observed with the negative work reduction for both the engine off and engine idle coast cases. Negative work in this case is comprised of engine braking work and motoring work.

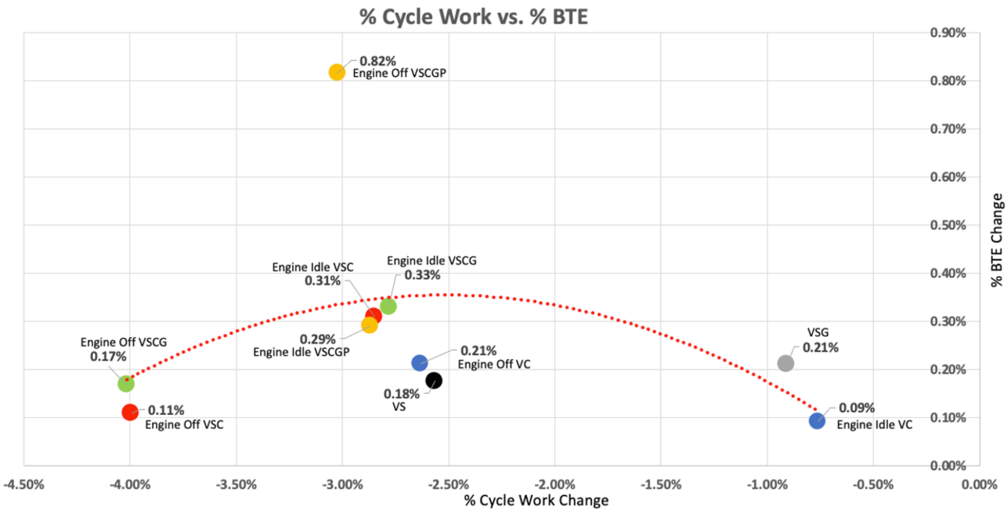

Figure 15. % Change in Brake Thermal Efficiency as a function of % change in Cycle work for different combination of optimal problems.

Similarly, Figure 15 shows the variation of Cycle work reduction to Brake Thermal Efficiency (BTE) improvement. There is no strong co-relation between the problems and the general behavior.

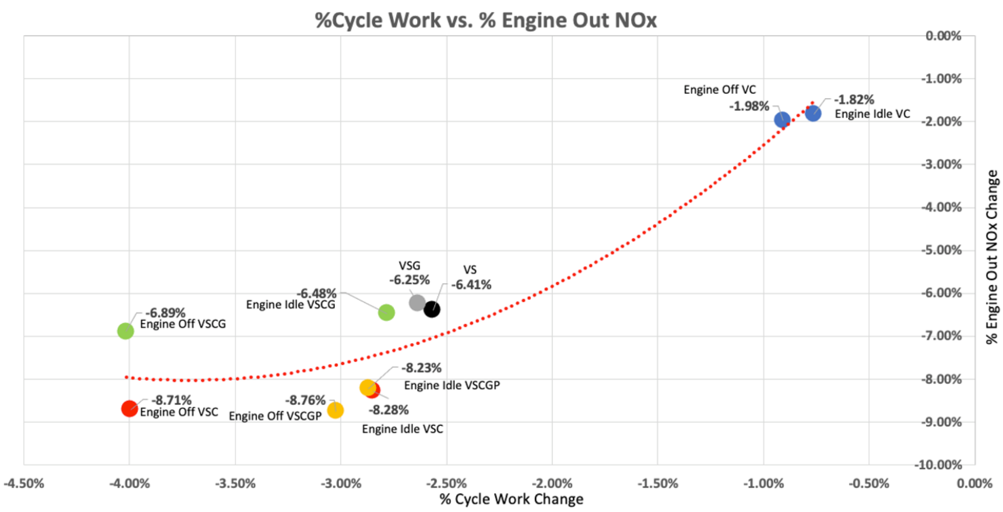

Figure 16. % Reduction in cycle work as a function of % improvement in fuel economy for various optimal problem setups.

Another quick analysis done in this work is to run the same problem on a shorter section of the route. This was done to understand the look ahead distance required for optimal behavior. The complete route is divided into two sections of 40 mile each, one for the first half and the next for the second half. Table 7 shows the % Fuel Economy numbers for the two sections of the route.

Table 7. Predictive fuel economy numbers for different section of the route

Route Section	FE	Trip Time	Full Route FE	Full Route TT	Coast Events
1^{st} 40 miles	2.41	-0.05	5.00	0.04	Decreased
2^{nd} 40 miles	2.39	0.02			Increased
Hilly 10 mile	0.053	-0.86			None
Flat 10 mile	1.03	0.27			Regular

The results from Table 7 shows that the over all behavior and fuel economy numbers stays near similar if we shorten the route to half. Since the route is not exactly symmetrical the numbers are not equally divided. The coast events also reduced a little for the first half of the route and increased marginally for the second section. This is solely due to the fact that the grade profile is not similar. It is also noted that the optimal control shows similar physical behavior during the very short hilly section where there was no coast events observed and the vehicle speed modulation was also not effective. The predictive gear played a role by reducing the lug back. It is noted that the Fuel Economy is not at all achieved in this section. While in the flat section there is usual behavior of coast events and the problem was able to achieve around 1% benefit. There is also more slow down of the vehicle due to the fact that there was coast events which slowed the speed down. Overall if these results are compared with the full route solution it is not observed that the benefits are hugely sacrificed. Specifically for the 40 miles route it is noted that the benefits are almost equally divided between the two segments and adds up to get close to the full route benefits.

5. Conclusion & Further Work

This research indicates that predictively applying control action with a-priori knowledge of road grade can provide increased fuel economy without negatively impacting vehicle performance. Dynamic cruise and coast control provides most benefits while predictively controlling gear and torque (power split) does not provide any significant fuel benefit but improved drivability and powertrain efficiency. The major outcome of the work are:

- Predictive road grade knowledge can help design control algorithms that will enable fuel savings depending on road grade profile,
- Vehicle cruise speed can be increased within acceptable bounds (calibrated for drivability) before entering an uphill,
- Vehicle cruise speed can be reduced within calibratible bounds before entering a downhill,
- Down shift gear to a lower value predictively before hitting speed lug back in up-hill,
- Up Shift gear predictively while still on uphill and before completely coming out of the hill,
- Engine can be disengaged and turned off in mild down grade,
- Engine can be disengaged for short duration during flat section of route with predictive speed modulation (increase speed then disengage),

This analysis is also precursor to predictive platooning systems. The usage of this formulation in platooning system is discussed in another paper by the same authors.

Conflicts of Interest: The authors declare no conflict of interest

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